Uncertainty Modelling for Instruments, Systems & Plants – An essential Guide to Optimising Performance

Abstract:

Knowledge of the uncertainty of Custody Transfer and Allocation measurement is typically calculated following the published Guidelines & Standards using Manufactures Data. But what if there are environmental issues regarding the Installation or Type of Equipment.

This Paper identifies and illustrates ways of using established uncertainty theory as a tool for modelling the design of measurement equipment, the combination of that equipment into flow measurement systems and/or the modelling of complex Plants/Allocation Systems. By using these tools measurement exposure can be expressed in terms of uncertainty, units measured or financial risk, thereby giving an insight into the advantages of one method of measurement over another. In a similar way Design Houses can determine the optimum solution for a measurement system and Operators the most cost effective solution for Plant/Allocation metering.

The Paper is subdivided into three main Sections:

1. *Instrument Uncertainties – Examining the physics of the meter and determining its sensitivity to various aspects of its installation.*

2. *Meter System Uncertainties – Examining the algorithms used to determine flow rate from instruments and using a combination of sensitivity factor and calibration tolerances.*

3. *Plant & Allocation Systems – Examining the connectivity between the individual metering system associated with a Plant/Allocation System and assessing the cumulative uncertainty effect on reported figures.*

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1.0 INTRODUCTION

1.1. GENERAL

In this day and age, engineers are under constant pressure to improve on what went before and nowhere is this more apparent in the Oil Industry than in the field of Fiscal & Allocation Hydrocarbon Measurement. Whether new installations or old, the emphasis is typically on minimising the "Uncertainty" of the measurement and therefore minimising the operators "Exposure" to unquantifiable losses (or gains).

Uncertainty is more formally defined as: "The interval within which the true value of the measured quantity can be expected to lie within a stated probability".

It is important to stress that Uncertainty is also dynamic; a measurement uncertainty will vary with changes to the active parameters. In flow metering that will mean that the measurement uncertainty varies with flow rate, pressure, temperature, density etc; similarly in measurement systems where fluid properties may change – a point often overlooked by Operators of Measurement Systems.

1.2. OBJECTIVES

The purpose of this paper is to analyse the techniques available to design engineers by Analysing system characteristics and Modelling individual component uncertainties rather than relying solely on the Monte Carlo Simulation approach for evaluation. Whether the engineer is designing Instruments, Measurement Systems or Processing Plants, the fundamental methodology is the same - generate a "Model" of the system identifying each element in the measurement chain, then attribute verifiable uncertainties and sensitivity factors to those elements prior to combining the uncertainties using the "Root-Sum-of-Squares" (RSS) method. Using this technique and following the recommendations laid down in various Measurement Standards & Guidelines the engineer is able to predict probable outcomes in terms of measurement uncertainty.

By analysing the individual component uncertainties and their corresponding sensitivity factors, the relative impact of each element in the chain and its significance with respect to the whole can be established. Modelling in this way allows the engineer to identify those elements of the system that have the greatest impact on the resultant measurement. This Analytical process can be extremely useful in providing guidance to Engineers on the choice of Instruments used in Measurement Systems and the choice of Measurement Systems used in Processing Plants/Allocation Systems. Ultimately, the Modelling could be applied to real time measurement, providing instantaneous diagnosis of instances where measurement systems are failing to meet their agreed Tolerances.

Each of the following Sections of this Paper addresses different levels of measurement Modelling; namely the Instrument, the Metering System Design and finally the Processing Plant/Allocation System.

The ultimately goal being to minimise measurement "Exposure" in terms of the cost that uncertainty represents when assessing the design and operation of measurement systems. Uncertainty Modelling in this way allows the engineer to avoid spending large amounts of money on elements in a system that contributes little to the desired outcome (i.e. concentrating on the elements of the system that give the best return in terms of reducing the financial "Exposure" to the Operator).

Authors Note: Whilst not specifically addressed in this document, often too much emphasis is placed on measurement uncertainty where the Tolerance is based on requiring the same measurement uncertainty over the whole operating range of the measurement system (e.g. ± 1.0% of Point for Gas, ± 0.25% of Point for Oil). But the "Exposure" at the low flow rates may be less than a quarter of that at the maximum flow rates when viewed purely in terms of units measured. I would suggest that Tolerances may be better expressed in terms of unitary or financial "Exposure" rather than ± 1.0%, or ± 0.25% of the flow over the full range – hence it could be argued that it is the "Exposure" that should the defining point, not the relative "Uncertainty".

2.0 INSTRUMENT UNCERTAINTIES

2.1. OBJECTIVES

This Section addresses the design issues associated with obtaining accurate measurement and the problems associated with assessing the uncertainty of that measurement as applied to a Particular Device (i.e. typically the Primary Instrument in a Metering System). Simple case studies looking at the "Physics" of the method of measurement and the impact of those effects on the measurement process will help to establish the anticipated "Uncertainty" of the device when in use. This methodology is especially useful when considering the implications of deviations from recommended practice such as improper installation or changes in Process Fluids that detrimentally effect the measurement (a factor often encountered when reviewing individual Metering Systems – see following Section for examples).

2.2. METHODOLOGY

2.2.1. The METCO Approach

We at Metco have implemented the principles set out in ISO TR 5168 and ISO GUM using the Root Sum or Squares method with sensitivities determined by partial differentiation. However we apply two additional stages of simple mathematical manipulation to significantly simplify the partial differentiation sensitivity coefficients. This manipulation is set-out below.

2.2.2. Fundamental Principals

It is appropriate to initiate a measurement uncertainty analysis with the equations that define the system under analysis. For example; let the "final result" be equal to the value "q", but 'q' may be calculated from other variables "x", "y" & "z" which may be measured quantities OR may in turn be calculated from further sets of variables that are ultimately derived from measured quantities.

In mathematical terms "q" is a Function of x, y & z i.e.

$q = F^n(x, y, z)$ Which means any equation incorporating the variables x, y & z.

But "x" may be a function of a, b, c & d.
And "y" may be a function of e, f & g etc.

To estimate the uncertainty of the value "q" it is necessary to determine the uncertainties of the contributing variables 'x', 'y, & 'z'. The fundamental equation of 'error' is:-

$$\delta q = \pm \frac{\partial(q)}{\partial(x)} \times \delta x \quad \pm \frac{\partial(q)}{\partial(y)} \times \delta y \quad \pm \frac{\partial(q)}{\partial(z)} \times \delta z$$

Where:

δq	Represents the error (in Engineering units) in the value "q"
δx	Represents the error (in Engineering units) in the value "x"
δy	Represents the error (in Engineering units) in the value "y"
δz	Represents the error (in Engineering units) in the value "z"
$\frac{\partial(q)}{\partial(x)}$	Represents the Partial Derivative of "q" by the variable 'x'
$\frac{\partial(q)}{\partial(y)}$	Represents the Partial Derivative of "q" by the variable 'y'
$\frac{\partial(q)}{\partial(z)}$	Represents the Partial Derivative of "q" by the variable 'z'

2.2.3. Relative Uncertainty

The Relative uncertainty is the 'error' (e.g. δq), in engineering units, divided by the measured, or calculated, value of the variable (q).

Relative Uncertainty $U_q = \frac{\delta q}{q}$

To develop the previous equation from an 'Error' equation to a 'Relative Uncertainty' equation both sides must be divided by "q" as follows:

$$\frac{\delta q}{q} = \pm \frac{\partial(q)}{\partial(x)} \times \frac{1}{q} \times \delta x \pm \frac{\partial(q)}{\partial(y)} \times \frac{1}{q} \times \delta y \pm \frac{\partial(q)}{\partial(z)} \times \frac{1}{q} \times \delta z$$

This may be manipulated to express each term on the RHS as a Relative uncertainty as follows:

$$\frac{\delta q}{q} = \pm \left(\frac{\partial(q)}{\partial(x)} \times \frac{x}{q} \times \frac{\delta x}{x} \right) \pm \left(\frac{\partial(q)}{\partial(y)} \times \frac{y}{q} \times \frac{\delta y}{y} \right) \pm \left(\frac{\partial(q)}{\partial(z)} \times \frac{z}{q} \times \frac{\delta z}{z} \right)$$

Where:

$\frac{\partial(q)}{\partial(x)} \times \frac{x}{q}$	Represents the "sensitivity" of Term "x" in the calculation of "q".
$\frac{\partial(q)}{\partial(y)} \times \frac{y}{q}$	Represents the "sensitivity" of Term "y" in the calculation of "q".
$\frac{\partial(q)}{\partial(z)} \times \frac{z}{q}$	Represents the "sensitivity" of Term "z" in the calculation of "q".

2.2.4. Root Sum Square

It is conventional to evaluate an uncertainty equation as a Root Sum Square. The above equation now becomes:

$$\frac{\delta q}{q} = \sqrt{\left(\frac{\partial(q)}{\partial(x)} \times \frac{x}{q} \times \frac{\delta x}{x}\right)^2 + \left(\frac{\partial(q)}{\partial(y)} \times \frac{y}{q} \times \frac{\delta y}{y}\right)^2 + \left(\frac{\partial(q)}{\partial(z)} \times \frac{z}{q} \times \frac{\delta z}{z}\right)^2}$$

Since Percentage Uncertainty is simply Relative Uncertainty expressed in terms of percentage and is normally assigned the symbol "E".

Percentage Uncertainty of "q" is: $E_q = \frac{\delta q}{q} \times 100 \quad \%$

Hence the above equation may be re-written as:

$$E_q = \sqrt{\left(\frac{\partial(q)}{\partial(x)} \times \frac{x}{q} \times E_x\right)^2 + \left(\frac{\partial(q)}{\partial(y)} \times \frac{y}{q} \times E_y\right)^2 + \left(\frac{\partial(q)}{\partial(z)} \times \frac{z}{q} \times E_z\right)^2}$$

2.2.5. Jokers

The above expressions are purely based upon the mathematical expressions used within the measurement. In the real world there are other factors that may degrade the uncertainty of measurement and it is advisable to incorporate these additional terms into the Root Sum Square equation where appropriate. In flow metering these terms may originate from effects such as:

Fluid Swirl
Fluid Profile Distortion
Flow Computer resolution
Etc

The final equation above thus becomes:

$$E_q = \sqrt{\left(\frac{\partial(q)}{\partial(x)} \times \frac{x}{q} \times E_x\right)^2 + \left(\frac{\partial(q)}{\partial(y)} \times \frac{y}{q} \times E_y\right)^2 + \left(\frac{\partial(q)}{\partial(z)} \times \frac{z}{q} \times E_z\right)^2 + E_{swirl}^2 + E_{profile}^2 + E_{fc}^2 + E_{etc}^2}$$

Authors Note: Addressing these "Real World" operating Uncertainties is a huge subject and will be the subject of a further Paper; however, a single example of the uncertainty derivation with respect to measurement instrumentation is illustrated in more detail in the Appendix of this document.

3.0 METER SYSTEM UNCERTAINTIES

3.1. OBJECTIVES

This Section addresses the design issues associated with obtaining an accurate Flow measurement as applied to a Metering System. It addresses the problems associated with assessing the uncertainty of measurement as applied to the collection of Primary & Secondary Instruments forming a Meter Tube or Metering Skid. The methodology follows on from the previous example, applying the same analytical techniques to by establish the sensitivity of each measurement element based on its place in the algorithm used to determine the flow measurement.

We would stress that it is by using these analytical techniques and deriving the sensitivity of each element of the calculation, that the engineer can get a better appreciation of those elements in the calculation that are most significant – and therefore warrant the most attention. This approach also means that those scenarios where the installation or process fluid does not meet the Manufacturers stated requirements (or comply with the relevant Statutory Guidelines) can be accommodated.

By adopting the following Methodology, it is hoped to demonstrate that uncertainty calculations can be generated that address all the elements affecting the measurement system (both instrument and process) and of the uncertainty assessed over the full operating range of the flow meter.

Using differential pressure flow meters as an example, it can be seen that the uncertainty of the flow meter degrades with reducing flow (typical of most types of meter). Hence, once the base condition of the metering system has been established, the turn-down with respect to flow rate can be demonstrated by reducing (adjustable) steps of differential pressure down to minimum applicable differential pressure. A change-over to a reducing step interval and a change-over between high range and low range differential pressure transmitters may be embedded and applied as required.

At METCO, Measurement System Models are be generated that reflect the uncertainty associated with each element of the system. This is done by breaking down the Uncertainty Calculation into a series of modules that encompass all the known uncertainty elements,. Each type of measurement system is then Modelled such that “Templates” are generated for future systems of a similar nature and uncertainty elements added as required by the application.

By assessing each element of the system and any anomalies associated with its installation one can slowly build up an appreciation of their impact and the final Measurement Uncertainty

Listed below are typical examples of anomalies found in flow measurement systems:

- Primary Instrument without adequate Upstream & Downstream Straight Lengths
- Secondary Instruments not located within the prescribed distance with respect to the Primary Element

- Operating too close to the Phase Envelope in Gaseous Systems, or the Vapour Pressure in Liquid Systems
- Inadequate mixing prior to Sample Loop or Manual Sample Point
- Relative Impact of increasing BS&W on corrected Flow Measurements

Some of the above examples are clearly design issues and can only be properly assessed once the condition is remedied by redesigning the Measurement System, Others can be evaluated either by analysis, or by reference to the Standards/Guidelines (as in the case of Orifice Plate deviations from the Standards) and their uncertainty contribution to the system determined.

Also, it is most important to take into account the verifiable uncertainty of measurement (normally deemed its "Calibration Tolerance") under operating conditions. This is particularly relevant to DP based measurement systems where the Manufacturers Specification may state admirable uncertainty tolerances that just cannot be demonstrated in the field. An example of this might be DP Transmitters operating at extremely high static pressures (normally accommodated by "Footprinting" the DP Transmitter at an approved Laboratory), or by a DP Transmitter being required to operated over an extremely low range of DP – say an averaging Pitot Tube operating over 0- 10 mbar (this low an operating range would be extremely difficult to verify in the field using standard test equipment because of resolution/repeatability issues).

Hopefully the above illustrations help to demonstrate the points to be considered when designing (or assessing) a flow measurement system and methods of assessing their significance.

Authors Note: It is only by having an appreciation of the significance of the issues (i.e. their Relative Uncertainty and – most importantly their Sensitivity Coefficient) that the design engineer and/or those assessing the measurement system can have real confidence in the measurements being made. By generating "Models" that analyse every detail associated with the metering system and its components, engineers will gain a better appreciation of the impact of each element of the system and its effect on the whole.

By using the above analytical techniques, it may be possible to avoid unnecessary mis-measurement corrections and if they occur - attribute an uncertainty to that mis-measurement.

3.2. METHODOLOGY

3.2.1. Root Equation

The example chosen to demonstrate the Methodology in this Section is the Orifice Plate Measurement System. The reason for choosing this method of measurement is that there already exist accepted algorithms for quantifying the Sensitivity of the Primary and Secondary Elements and hence allows a traceable demonstration of the Analytical Methods used to achieve those algorithms used in the Model.

Mass Flow: $$q_m = \frac{\pi d^2}{4} \times \frac{C \times \varepsilon}{\sqrt{1-(\beta)^4}} \times \sqrt{2 \times \Delta p \times \rho}$$

3.2.2. Accepted Sensitivity Algorithm

As can be seen, there are several elements within the calculation, each of which will have an associated uncertainty. A sensitivity analysis of the standard equation for Orifice Flow measurement uncertainty gives the following accepted equation:

$$E_{qm} = \sqrt{E_c^2 + E_\varepsilon^2 + \left(\frac{2 \times \beta^4}{1-\beta^4}\right) \times E_D^2 + \left(\frac{2}{1-\beta^4}\right)^2 \times E_d^2 + \frac{E_{\Delta p}^2}{4} + \frac{E_\rho^2}{4}}$$

But how was the above equation derived? The calculations shown below illustrate how each element in flow algorithm can be analysed in turn with respect to the whole and its sensitivity confirmed; the reasoning behind the above equation and (hopefully) demonstrate the value of the Analytical approach to Uncertainty.

3.2.3. Derivation of Algorithm

To derive this uncertainty algorithm the mass flow equation must be considered as being:

$$q_m = \frac{\pi d^2}{4} \times \frac{C \times \varepsilon}{\sqrt{1-\left(\frac{d}{D}\right)^4}} \times \sqrt{2 \times \Delta p \times \rho}$$

If the partial derivative with respect to “d” is taken of this expression it yields:

$$\frac{\partial(q)}{\partial d} = \frac{1}{2} \times \frac{\pi \times d \times C \times \varepsilon}{\sqrt{1-\left(\frac{d}{D}\right)^4}} \times \sqrt{2 \times \Delta p \times \rho} + \frac{1}{2} \times \frac{\pi \times d^5 \times C \times \varepsilon}{\left(1-\left(\frac{d}{D}\right)^4\right)^{\frac{2}{3}}} \times \frac{\sqrt{2 \times \Delta p \times \rho}}{D^4}$$

If this partial derivative is now divided by “q” i.e.:

$$\frac{\frac{\partial(q)}{\partial d}}{q} = \frac{\frac{1}{2} \times \frac{\pi \times d \times C \times \varepsilon}{\sqrt{1-\left(\frac{d}{D}\right)^4}} \times \sqrt{2 \times \Delta p \times \rho} + \frac{1}{2} \times \frac{\pi \times d^5 \times C \times \varepsilon}{\left(1-\left(\frac{d}{D}\right)^4\right)^{\frac{2}{3}}} \times \frac{\sqrt{2 \times \Delta p \times \rho}}{D^4}}{\frac{\pi d^2}{4} \times \frac{C \times \varepsilon}{\sqrt{1-\left(\frac{d}{D}\right)^4}} \times \sqrt{2 \times \Delta p \times \rho}}$$

Simplifying this becomes:- $= \frac{-2 \times D^4}{-D^4 + d^4} \times \frac{1}{d}$

Which may be written as:- $= \frac{2}{1 - \left(\frac{d}{D}\right)^4} \times \frac{1}{d}$

The Relative Uncertainty with Sensitivity becomes:- $\frac{2}{1-(\beta)^4} \times \frac{\delta d}{d}$

If the partial derivative with respect to "D" is taken of this expression it yields:

$$\frac{\partial(q)}{\partial D} = \frac{-1}{2} \times \frac{\pi \times d^6 \times C \times \varepsilon}{\left(1 - \left(\frac{d}{D}\right)^4\right)^{\frac{2}{3}}} \times \frac{\sqrt{2 \times \Delta p \times \rho}}{D^5}$$

If this partial derivative is now divided by "q" i.e.:

$$\frac{\frac{\partial(q)}{\partial d}}{q} = \frac{\frac{-1}{2} \times \frac{\pi \times d^6 \times C \times \varepsilon}{\left(1 - \left(\frac{d}{D}\right)^4\right)^{\frac{2}{3}}} \times \frac{\sqrt{2 \times \Delta p \times \rho}}{D^5}}{\frac{\pi d^2}{4} \times \frac{C \times \varepsilon}{\sqrt{1 - \left(\frac{d}{D}\right)^4}} \times \sqrt{2 \times \Delta p \times \rho}}$$

Simplifying this becomes:- $= \frac{-2 \times d^4}{-D^4 + d^4} \times \frac{1}{D}$

Which may be written as:- $= \frac{2 \times \left(\frac{d}{D}\right)^4}{1 - \left(\frac{d}{D}\right)^4} \times \frac{1}{d}$

The Relative Uncertainty with Sensitivity becomes:- $\frac{-2(\beta)^4}{1-(\beta)^4} \times \frac{\delta D}{D}$

Authors Note: The Relative uncertainties, $\frac{\delta d}{d}$ and $\frac{\delta D}{D}$ *may be expressed in terms of Percentage Uncertainty (*E_d and E_D respectively*). The negative sensitivity coefficient will be positive when squared; hence the negative sign may be "lost". In both cases a 'convenient' "d" and "D" forms on the denominator from the partial derivative to form the denominator of the Relative Uncertainty term.*

In the same way, it can be demonstrated that the partial derivative of the square root for the Δp *(differential pressure) and* ρ *(density) is ½ which when squared becomes ¼. The sensitivity coefficient for the uncertainties of C and* ε *can be seen to be unity.*

As can be seen, some elements are demonstrated to have a much more significant effect than others. It is by applying these techniques to any measurement application that one can assess the importance of each component of the measurement system and determine which element is most critical to the calculation.

Examination of the above example shows that (whilst important) the density uncertainty does not have as marked effect as it would in a linear equation (such as a Turbine, or Ultrasonic measurement system). In the same way, we can see that the uncertainty of the Diameters (in particular the orifice dimension) is mitigated in part by the Beta Ratio.

It is by analysing systems in this way that a true appreciation of the significance of each component is appreciated.

Thus, by breaking down the equation and considering how each element was determined, it is possible to accommodate most scenarios in measurement. As an illustration, the above equation can be further expanded to identify additional uncertainty factors such as orifice the use of drain holes in the plate, plate contamination and the additional uncertainty associated with the Discharge Coefficient as a result of contamination or flow profile distortion.

$$E_{qm} = \sqrt{\left(E_c + E_{add}\right)^2 + E_{drain}^2 + E_{\varepsilon}^2 + \left(\frac{2\times\beta^4}{1\text{-}\beta^4}\right)\times E_D^2 + \left(\frac{2}{1\text{-}\beta^4}\right)^2 \times E_d^2 + \frac{E_{\Delta p}^2}{4} + \frac{E_{\rho}^2}{4} + E_{fc}^2 + E_{jch}^2}$$

4.0 PLANT & ALLOCATION UNCERTAINTIES

4.1. OBJECTIVES

This Section addresses the design issues associated with combining measurement system uncertainties in order to assess their impact on a Processing Plant or Allocation System (i.e. when considering the uncertainty Processing Plant Balance, Allocation Systems, or the uncertainty in Allocation attributed to each participant in complex Pipeline Network). As previous, the methodology relies on building up an Uncertainty Model which is able to take into account all the elements of the network and establish "Group" uncertainties associated with Network Nodes and outputs.

It should also be noted that the uncertainty of measurement systems using the "by difference" approach (where there may be no direct measurement element) and in/out flows are based on the sum of other measurements in the system can be calculated using this modelling approach.

At this level of Modelling, the engineer can run "what if" scenarios looking at the implications of mis-measurement due to instrument failures and/or incorrect base data. Also engineers can establish the "Risk" associated with any element in the network - which is particularly useful when considering allowing new entrants to share pipelines or disputes relating to the measurement uncertainty of participants within an Allocation System.

Authors Note: As Allocation Systems become more complex, there may be pressures on participants to not only provide end of day reports on fluid movements, but also to attribute an uncertainty statement to accompany those figures. By monitoring instruments operating points and having a knowledge of their calibration tolerances, it would be a simple matter to assign uncertainties to measurement statements that can then be utilised to determine the Allocation Reporting "Exposure".

4.2. METHODOLOGY

The Partial Differentiation results illustrated in Section 1.0 can typically be simplified by algebraic manipulation to represent more complex measurement systems. This leads to a number of **"Standard Cases"** that represent common equation formats and common metering configurations. It is therefore not necessary to carry out partial differentiation action at every stage in an uncertainty analysis, where a 'standard' case equation occurs then the uncertainty expression follows logically.

A selection of common formats is given below.

4.2.1. Case 1: Multiples

The Function of "q" may be the multiple of x, y & z:

Multiplication Function:- $q = x \times y \times z$

The uncertainty equation becomes:

$$E_q = \sqrt{(1 \times E_x)^2 + (1 \times E_y)^2 + (1 \times E_z)^2}$$

Division Function:- $q = \dfrac{x}{y \times z}$

The uncertainty equation also becomes:

$$E_q = \sqrt{(1 \times E_x)^2 + (1 \times E_y)^2 + (1 \times E_z)^2}$$

In both cases all sensitivity terms reduce to unity.

4.2.2. Case 2: Powers

The Function of "q" may incorporate power terms of x, y or z:

Square Function:- $q = x \times y \times z^2$

The uncertainty equation becomes:

$$E_q = \sqrt{(1 \times E_x)^2 + (1 \times E_y)^2 + (2 \times E_z)^2}$$

Square root Function:- $q = x \times \sqrt{y \times z}$

The uncertainty equation becomes:-

$$E_q = \sqrt{(1 \times E_x)^2 + \left(\frac{1}{2} \times E_y\right)^2 + \left(\frac{1}{2} \times E_z\right)^2}$$

4.2.3. Case 3: Addition

The Function of "q" may be the summation of x, y & z:

Addition Function: $q = x + y + z$

In general terms: $q = \sum_{i=1}^{n} (x_i)$

Where "x_i" represents a number of variables from x_1 to x_n

The uncertainty of "q" may be calculated by:

$$E_q = \sqrt{\left(\frac{x}{q}\right)^2 \times (E_x)^2 + \left(\frac{y}{q}\right)^2 \times (E_y)^2 + \left(\frac{z}{q}\right)^2 \times (E_z)^2}$$

In general terms: $$E_q = \sqrt{\sum_{i=1}^{n}((\frac{x_i}{q})^2 \times E_{xi}^2)}$$

Though a general result; this is particularly applicable to parallel metered streams commingling into a common un-metered header and to gas composition calculations.

4.2.4. Case 4: By Difference

Case 3 may be considered to be a specific metering configuration where parallel streams are commingled into a metered common stream. All but one (i.e. "x" or in the general case "x_1") are metered. The un-metered stream is "metered by difference".

Metered by difference: $$x = q - (y + z)$$

In general terms: $$x_1 = q - \left(\sum_{i=2}^{n}(x_i)\right)$$

Uncertainty of Total Flow: $$E_x = \sqrt{\left[(E_q)^2 + (\frac{y}{q})^2 \times E_y^2 + (\frac{z}{q})^2 \times E_z^2\right] \times \left[\frac{q}{x}\right]^2}$$

Or in general terms is: $$E_{x1} = \sqrt{\left[E_q^2 + \sum_{i=2}^{n}((\frac{x_i}{q})^2 \times E_{xi}^2)\right] \times \left[\frac{q}{x_1}\right]^2}$$

It can be seen that where the 'metered by difference' stream flow rate is low compared with the flow through the remaining parallel streams then the uncertainty of the un-metered stream will become excessive. For example an uncertainty of 60 % is easily reported.

Authors Note: However, subject to the actual quantities involved, a 60 % uncertainty may translate to a relatively low quantity with respect to the quantities from the other systems. Due to the uncertainties of the other (i.e. metered) systems the stream metered by difference can be attributed with a negative quantity

4.2.5. Case 5: Attributed Quantity.

Case 5 may be considered to be a specific metering case. Multiple parallel metered streams are commingled into a common header and are then metered through a common stream.

This configuration normally relates to parallel Allocation meters commingling then being exported into a Pipeline through Fiscal quality meters. The actual quantity attributed to each input stream is Allocated by ratio against the 'more accurate' Fiscal meter quantity:

It must be assumed that, due to measurement uncertainties, the summation of the parallel Allocation streams does not equal the Fiscal meter reading.

Common Header "Total Produced Quantity" (tpq): $tpq = x + y + z$

Fiscal meter reading i.e. the Export Quantity is represented by: XQ

Due to metering uncertainties:- $tpq \neq XQ$

The Attributed quantities for the parallel streams are:

For meter stream "x": $AQ_x = \frac{x}{tpq} \times XQ$

For meter stream "y": $AQ_y = \frac{y}{tpq} \times XQ$

For meter stream "z": $AQ_z = \frac{z}{tpq} \times XQ$

If the "normal" uncertainty algorithm were to be applied the uncertainty equation would be:

Uncertainty in quantity allocated to stream "x": $E_{aqx} = \sqrt{E_x^2 + E_{tpq}^2 + E_{xq}^2}$

However: "Engineering Logic" indicates that this algorithm would return an excessive uncertainty for this configuration. This "Engineering Logic" leads to reconsidering the format of the attributed quantity equations as follows:-

For meter stream "x": $AQ_x = \zeta_x \times XQ$ Where:- $\zeta_x = \frac{x}{tpq}$

For meter stream "y": $AQ_y = \zeta_y \times XQ$ Where:- $\zeta_y = \frac{y}{tpq}$

For meter stream "z": $AQ_z = \zeta_z \times XQ$ Where:- $\zeta_z = \frac{z}{tpq}$

The uncertainty in the Allocation Fraction (ζ) can be demonstrated to be represented by:

Fraction for "x":- $$E_{\zeta x} = \sqrt{\left(\frac{y+z}{x+y+z}\right)^2 \times (E_x)^2 + \left(\frac{y}{x+y+z}\right)^2 \times (E_y)^2 + \left(\frac{z}{x+y+z}\right)^2 \times (E_z)^2}$$

Therefore the uncertainty in quantity attributed to stream "x" is:-

$$E_{aqx} = \sqrt{(E_{xq})^2 + \left(\frac{y+z}{x+y+z}\right)^2 \times (E_x)^2 + \left(\frac{y}{x+y+z}\right)^2 \times (E_y)^2 + \left(\frac{z}{x+y+z}\right)^2 \times (E_z)^2}$$

This can be expressed as a general case by the expression:

Uncertainty of mass fraction for any stream 'i':

$$E_{\varsigma.i} = \sqrt{\left(\frac{tpq - x_i}{tpq}\right)^2 \times E_{x.i}^2 + \left(E_{tpq}^2 - \left(\frac{x_i}{tpq}\right)^2 \times E_{x.i}^2\right)}$$

Where the uncertainty in the sum of the parallel streams (tpq) using Case 3 is:

$$E_{tpq} = \sqrt{(\frac{x_1}{tpq})^2 \times E_{x1}^2 + (\frac{x_2}{tpq})^2 \times E_{x2}^2 + \ldots + (\frac{x_n}{tpq})^2 \times E_{xn}^2}$$

Then the Attributed Uncertainty for input stream 'i' is: $E_{aq.i} = \sqrt{E_{\varsigma.i}^2 + E_{xq}^2}$

This expression correlates with an uncertainty evaluation using Monte Carlo Simulation (as do the other 'Case' equations given above).

4.2.6. Further Cases

This basic principle can be extended to cover other common metering configurations, for example phase change through separator vessels.

However: care must be exercised in implementing the mathematics to ensure that logical results are obtained. The above cases were checked by running the general root sum square equations against a Monte Carlo Simulation for the same input data. This was done using MathCAD Worksheets with the root sum square and Monte Carlo Simulation equations set-up in matrix format. Data was input with a variable number of meter streams, variable flow quantities and variable uncertainties against each stream. In all cases the uncertainty calculated using the root sum square algorithms correlated with the Monte Carlo Simulation (thus proving agreement between both the Statistical and Analytical Approach).

The applicable Standards require that the random (Type A) and systematic (Type B) elements of uncertainty are expressed individually. In practice, when assessing meter station uncertainty budgets it is extremely difficult differentiate between the random element and the systematic bias. In reality probably every term will have a random vector and a systematic vector and differentiating between these will be largely guesswork.

If the random & systematic uncertainties must, and can, be separated then the Case 3 expression:

$$E_q = \sqrt{\sum_{i=1}^{n}((\frac{x_i}{q})^2 \times E_{xi}^2)}$$

May be expanded to become:

$$E_{qT} = \sqrt{\sum_{i=1}^{n}((\frac{x_i}{q})^2 \times E_{x.random..i}^2) + \sum_{i=1}^{n}((\frac{x_i}{q})^2 \times E_{x.systematic..i}^2)}$$

Taking the particular case of a metering skid with a number of identical parallel meter streams, and where the systematic uncertainties are common to all streams then this expression may be simplified to:-

$$E_q = \sqrt{\sum_{x=1}^{n}((\frac{x_i}{q})^2 \times E_{x.random..i}^2) + E_{q.systematic}^2}$$

Also if the flow through each meter stream is balanced (i.e. equal flow through each stream) then the flow ratio simplifies to the number of meter streams on line:

$$E_q = \sqrt{\left(\frac{E_{random}}{\text{No of Streams}}\right)^2 + E_{systematic}^2}$$

The other “Case” equations would need to be similarly expanded.

4.3. SUB-ALLOCATION UNCERTAINTY

It is not expedient to provide a real case study of a Sub-Allocation or Pipeline Allocation uncertainty analysis as the Allocation methodology and data is confidential. However the principles involved can be illustrated in general terms.

Taking the example of a North Sea Installation producing from multiple reservoirs licensed to different consortiums; the financial exposure of each Partner in each consortium is dependant upon the relative flow rates of each source as well as the measurement point uncertainties.

A typical Sub-Allocation system for a multiple Reservoir production Installation could involve any of the following:-

Multiple Wells from each Reservoir
Multi-phase flow meter prior to commingling Reservoir fluids

Dedicated Separator for a Reservoir
Separation of Crude Oil, Natural Gas and Condensate phases
Commingling of Crude Oil after first stage Separator
Natural Gas commingled after first stage Separator
Natural Gas cross-flow from common second and third stage Separators
Condensate cross-flow from gas treatment plant
Crude Oil metered through common Fiscal Metering Station to Pipeline or shuttle tanker.
Separate Crude Oil Metering Stations
Common Gas Fiscal Metering Station to Pipeline
Common Condensate Fiscal Metering spiked to either Gas or Crude Oil Pipeline
Condensate spiked upstream of Crude Oil Fiscal Metering Station.
Fuel Gas metering
Flare Gas either metered or determined in some other way.
Lift gas metered and distributed to individual wells as required.

The starting point of the Sub-Allocation uncertainty analysis is to identify the primary mass flow paths through the Installation's Plant and to set-up the "Mass Ring-Fence". The Fluid mass flow Inputs into this Ring-Fence and the Fluid mass flow outputs from this Ring-Fence must be identified.

All inputs and all outputs should be measured, though a frequent alternative allows just one stream to be determined "by-difference".

The Mass Inputs into the Ring Fence will be the Natural Gas, Crude Oil and Produced Water from the individual Reservoirs; this measurement may be by multi-phase meter or as separate Gas, Oil and Water meters from a first stage Separator.

The Mass Outputs may include:

- Fiscally Metered Crude Oil
- Fiscally Metered Natural Gas
- Fiscally Metered Condensate
- Metered Fuel
- Metered Lift Gas
- Metered or estimated or "by-difference" Flare Gas.

The Sub-Allocation Procedure will incorporate the principles of the mass flow through the Installation and also equitable distribution of energy (both as produced energy in the exported products and consumed energy in extracting and treating those products).

This analysis is required to set-up a mathematical model of the Sub-Allocation System; once set-up this mathematical model of the mass (and energy) flow may be translated into an uncertainty analysis using the principles set-out in the combined flow "Cases" above.

These "Cases" will primarily involve:

- Parallel metered inputs to un-metered commingled stream (Case 3)
- Parallel metered inputs (excepting one by difference) to metered commingled. (Case 4)
- Allocation by Composition (Case 3)
- Gas / Liquid phase change through a Separator
- Parallel metered outputs (Case 3)
- Parallel metered outputs (excepting one by difference) to metered commingled. (Case 4)
- Attributed metered quantities against metered Commingled (Case 5)

Case 1 and Case 2 may be implemented as required. Gas / Liquid phase change through a Separator may be considered as “Case 6” (not illustrated above) where there is a mass fraction transfer between the vapour and liquid phases.

The final analysis results should be set-up to attribute the “risk” of the uncertainty associated with the Allocation to each Reservoir (and possibly extended to each Partner) in terms of Percentage uncertainty, actual quantity uncertainty and monetary uncertainty.

The attributed uncertainty is dependent upon the relative flow rates as well as the uncertainty of each measuring point; consequently the attributed uncertainty to any individual Reservoir may not directly reflect the quality of the measurements made by that Reservoir.

5.0 SUMMARY & CONCLUSIONS

5.1. GENERAL

The previous Sections were intended to illustrate the benefits of Analysing systems and deriving their associated measurement uncertainties using basic mathematical functions. By breaking down the uncertainty elements in this way, a true understanding of the essential elements of the measurement process may be better appreciated. For Example:

Instrument Design
By looking at the "Physics" of the measurement method and analysing the parameters that make the measurement process possible (as demonstrated in Section 2.0), then it is possible to get a better appreciation of which parts of the process effect the measurement the most. Thus the design engineers can concentrate on the "essential" elements of the measurement system as defined by their Sensitivity Coefficients and their Relative Uncertainty.

Metering Systems
Basic analytical methods can be used in the same way to address the formulae associated with deriving flow measurements in the preferred units (Mass, Standard Volume, Energy, Etc.). Each additional instrument may be assessed in terms of its Specification, its location and any process issues associated with that measurement. By building up "Templates" for different measurement systems and different instrument arrangements, a comprehensive selection of uncertainty measurement tools can be accumulated which can then be applied to Plant and Allocation Systems.

Plant & Allocation
The final (and often the most taxing) element in the uncertainty chain. Inevitably, these systems are unique to each client and must be built up based on the relative disposition of the measurement systems involved and the Process/Allocation issues associated with the particular application. Once built however, the uncertainty Model can be used to assess the respective benefits of using one measurement system as opposed to another and/or the effects of individual Instrument/Process uncertainties on the Plant/Allocation Model.

It should be noted that the Uncertainty Modelling Process can be run independently at each level, but the real strength of the philosophy is that each element of the uncertainty calculation can build on the previous level to provide a comprehensive analysis of "real" systems. It can be demonstrated that large measurement uncertainties can be missed by relying purely on Manufacturers data alone, or assuming that everything is operating within "design" parameters. It is only by Modelling the Equipment and Systems that a real insight into the measurement uncertainty of an Installation or Plant can be appreciated.

Authors Note: The assessment of "Measurement Uncertainty" really should be considered just the first step in assessing any measurement system. What is of more relevance is "what does this uncertainty mean in terms of Units Measured, or Product Value"; it would be very easy to convert the uncertainty measurement into terms that have a real meaning in the commercial world. Only when the measurement uncertainty is expressed in these terms can the real "Risk" associated with the measurement can be appreciated and decisions made as to its relevance.

Final Suggestion – most operators now have the data available and the computing capacity to assess the measurement uncertainty associated with their operating systems on a daily basis (i.e. the Daily Report). There could be great benefits, either internally or externally, in providing a "quality statement" associated with such Reports which may ultimately avoid the costly exercise of redressing "Mis-measurements" based on out of tolerance data.

APPENDICES

Case Study (Gas Ultrasonic Meter)

Uncertainty in Actual Volume

British Standard BS 7965: 2000 incorporates a method of assessing the uncertainty of an ultrasonic meter. The uncertainty terms in this Standard are mainly given as fixed values but this is not entirely appropriate as flow meters do not normally have a constant uncertainty over their operating envelope. However by using the principles of ISO 5168 and ISO GUM it is appropriate to analyse an ultrasonic meter uncertainty from the equations of measurement – the following illustrates the Methodology used.

The Ultrasonic Flow Meter measures fluid velocities at actual operating conditions consequently the initial flow quantity calculated is Gross Observed Volume (the conventional terminology for the Volume at Actual Operating conditions) as follows:

Gross Observed Volume = Mean Fluid Velocity x Pipe Cross Section Area

Or as an Equation: $$q_{gov} = v_{mean} \times \frac{\pi D^2}{4}$$

Where

v_{mean} Represents the mean fluid velocity in the pipe

D Represents the pipe internal diameter.

In a practical ultrasonic meter used for fiscal metering it is necessary to carry out a “Wet Calibration” hence the equation now becomes:

$$q_{gov} = v_{mean} \times \frac{\pi D^2}{4} \times MF$$

Where: “MF” is the Meter Factor determined by the Wet calibration.

The first level uncertainty equation for the Gross Observed Volume is therefore:

Uncertainty in Gross Observed (Actual) Volume: $E_{gov} = \sqrt{E_{vmean}^2 + (2)^2 \times E_D^2 + E_{mf}^2}$

As the Diameter term is squared it has a sensitivity factor of “2”. The remaining sensitivity factors are unity.

Mean Velocity Uncertainty

The mean fluid velocity is the combination of the individual measured path fluid velocities as a weighted summation:

In Daniel terminology:

$$v_{mean} = \sum_{path=1}^{\text{number of paths}} \left(\omega_{path} \times v_{\text{path}}\right)$$

Or expressing it slightly more mathematical where "np" = Number of Paths and "ip" = Individual Path.

$$v_{\text{m}} = \sum_{\text{ip}=1}^{\text{np}} \left(\omega_{\text{ip}} \times v_{\text{ip}}\right)$$

Other manufacturers use proprietary techniques but in terms of the uncertainty analysis this model should hold true.

By Partial Differentiation it can be shown that the 'mathematical' uncertainty of this equation may be expressed by the algorithm:

$$E_{v\text{m}} = \sqrt{\sum_{\text{ip}=1}^{\text{np}} \left[\left(\frac{\omega_{\text{ip}} \times v_{\text{ip}}}{v_{\text{m}}} \right)^2 \times \left(E_{\omega\text{ip}}^2 + E_{v\text{ip}}^2 \right) \right]}$$

Path Fluid Velocity

The individual path (subscript "ip") fluid velocity is calculated by:

Either:

$$v_{\text{ip}} = \frac{L_{\text{ip}}^2}{2\,X_{\text{ip}}} \times \frac{(\tau_{\text{du}} - \tau_{\text{ud}})}{\tau_{\text{du}} \times \tau_{\text{ud}}}$$

Or by:

$$v_{\text{ip}} = \frac{L_{\text{ip}}}{2 \times \cos(\theta)_{\text{ip}}} \times \frac{(\tau_{\text{du}} - \tau_{\text{ud}})}{\tau_{\text{du}} \times \tau_{\text{ud}}}$$

For a line of sight ultrasonic meter:-

Note that: $\frac{1}{X} \approx \frac{L}{\cos(\theta)}$ Hence $\frac{L_{\text{ip}}^2}{2\,X_{\text{ip}}} = \frac{L_{\text{ip}}}{2 \times \cos(\theta)_{\text{ip}}}$

And that: for any ultrasonic meter. $\frac{(\tau_{\text{du}} - \tau_{\text{ud}})}{\tau_{\text{du}} \times \tau_{\text{ud}}} = \frac{1}{\tau_{\text{ud}}} - \frac{1}{\tau_{\text{du}}}$

Where:

L_{ip} Represents the calibrated distance between a transducer pair.

X_{ip} Represents the 'Axial Beam Traverse'. That is the distance along the pipe that the ultrasonic beam sweeps as projected onto the pipe centreline.

θ Represents the Transducer inclination to the pipe wall.

τ_{du} Represents the Measured time period (second) of the ultrasonic pulse traverse from "down" to "up" (travelling UP stream against the fluid flow)

τ_{ud} Represents the Measured time period (second) of the ultrasonic pulse traverse from "up" to "down" (travelling DOWN stream with the fluid flow)

(The actual time measurement will be in micro second or nano second)

NOTE: The ultrasonic transducers are normally embedded into ports. Therefore the ultrasonic beam traverses a small DEAD SPACE prior to penetrating into the pipe cross section and hence the fluid flow.

For this reason the relationship: $\frac{1}{X} \approx \frac{L}{\cos(\theta)}$

Is not entirely true, it should be: $\frac{1}{X} = \frac{FPL}{\cos(\theta)}$

Where "FPL" represents the "Fluid Path Length", that is the actual ultrasonic beam length as it traverses the pipe cross section within the fluid flow.

$$FPL = L - 2 \times \text{Port Bay depth}$$

Basing the uncertainty analysis on the equation:

Measured Path Fluid Velocity: $v_{ip} = \frac{L_{ip}^2}{2\,X_{ip}} \times \frac{(\tau_{du} - \tau_{ud})}{\tau_{du} \times \tau_{ud}}$

Leads to an uncertainty algorithm:-

$$E_{vip} = \sqrt{(2)^2 \times E_{Lip}^2 + E_{Xip}^2 + \left(\frac{\tau_{ud}}{\tau_{du} - \tau_{ud}}\right)^2 \times E_{\tau du}^2 + \left(\frac{-\tau_{du}}{\tau_{du} - \tau_{ud}}\right)^2 \times E_{\tau ud}^2}$$

Where:

E_{Lip} Represents the uncertainty in the Ultrasonic Path

E_{Xip} Represents the uncertainty in the "Axial Beam Traverse"

$E_{\tau du}$ Represents the timing uncertainty in measuring the beam traversing UP stream

$E_{\tau ud}$ Represents the timing uncertainty in measuring the beam traversing DOWN stream

Further analysis, and evaluation, of these terms must consider the Geometry, the Physics and the Correction Factors applied to individual commercially available ultrasonic meters.

As the Transit Timing is carried out by the same electronic circuitry in both directions the ISO / GUM covariance term should be included with a correlation factor determined either by experiment or from manufacturer's data. The covariance term will be:-

$$\text{covariance} = \left(\frac{\tau_{ud}}{\tau_{du} - \tau_{ud}}\right) \times \left(\frac{-\tau_{du}}{\tau_{du} - \tau_{ud}}\right) \times v_m \times E_{\tau du} \times E_{\tau ud} \times \delta Cf$$

Where: δCf Is the Correlation Factor between the timing circuits.

Authors Note: The above example is intended to illustrate the analytical approach used to identify method used for calculating Flow Velocity. By identifying each element in the calculation, a greater understanding of the implications of incorrect equipment configuration and/or process effects can be seen, thereby allowing a more accurate determination of their effects on the measurement made (e.g. Path Length, Flow Profile, Etc.).