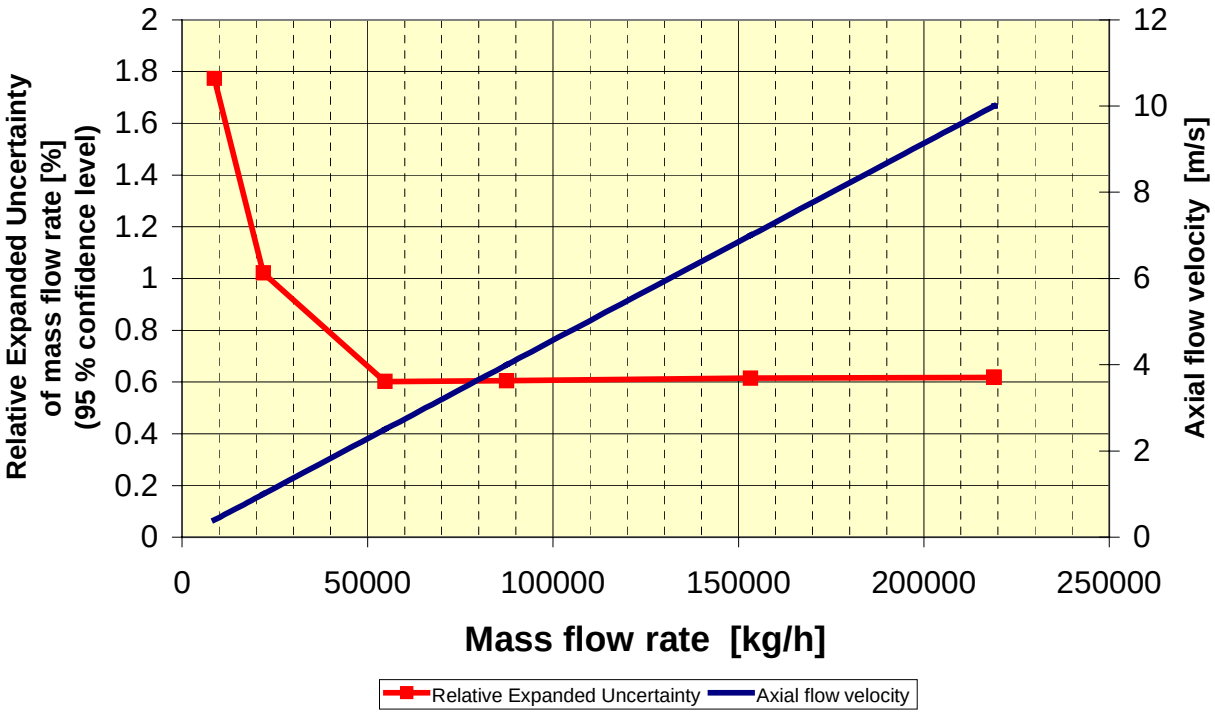
NORWEGIAN SOCIETY FOR OIL AND GAS MEASUREMENT
NORSK FORENING FOR OLJE OG GASSMÅLING

Christian Michelsen Research

Research for Industrial Development

HANDBOOK OF UNCERTAINTY CALCULATIONS

Ultrasonic fiscal gas metering stations



December 2001

Handbook of Uncertainty Calculations
Ultrasonic Fiscal Gas Metering Stations

December 2001

Prepared for

The Norwegian Society for Oil and Gas Measurement (NFOGM)
The Norwegian Petroleum Directorate (NPD)

by

Per Lunde and Kjell-Eivind Frøysa

Christian Michelsen Research AS

The *Handbook* and the Excel program *EMU - USM Fiscal Gas Metering Stations, Version 1.0,* are freeware and can be downloaded from the NFOGM web pages

Norwegian Petroleum Directorate (NPD)	**Norwegian Society for Oil and Gas Measurement (NFOGM)**	**Christian Michelsen Research (CMR)**
P.O. Box 600, N-4003 Stavanger, Norway Phone: (+47) 51 87 60 00 Fax: (+47) 51 55 15 71 Web-page: www.npd.no	*Secretariat:* Norwegian Society of Chartered Engineers, P.O. Box 2312 Solli, N-0201 Oslo, Norway Phone: (+47) 22 94 75 48 / 61 Fax: (+47) 22 94 75 02 Web-page: www.nfogm.no	P.O. Box 6031 Postterminalen, N-5892 Bergen, Norway Phone: (+47) 55 57 40 40 Fax: (+47) 55 57 40 41 Web-page: www.cmr.no

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PREFACE

Norwegian regulations relating to fiscal measurement of oil and gas require that the overall measurement uncertainty is documented to be within defined limits. However, the different methods used have given different results. A consistent and standardised method of uncertainty evaluation has been required, so that different measurement systems could be directly and reliably compared.

In 1993 the ISO report "*Guide to the expression of uncertainty in measurement*" (commonly referred to as the "*Guide*" or the "*GUM*") was published, with a revision in 1995. This report is providing general rules for evaluating and expressing uncertainty in measurement, intended for a broad scope of measurement areas. The *GUM* was jointly developed by the International Organisation of Standardization (ISO), the International Electrotechnical Commision (IEC), the International Organization of Legal Metrology (OIML) and the International Bureau of Weights and Measurement (BIPM).

In 1999 a "*Handbook of uncertainty calculations - Fiscal metering stations*" was developed by the Norwegian Petroleum Directorate (NPD), the Norwegian Society for Oil and Gas Measurement (NFOGM) and Christian Michelsen Research (CMR), addressing fiscal metering of oil using turbine meters, and fiscal metering of gas using orifice meters.

The intention of this initiative was that a user-friendly handbook together with an Excel program, based upon the principles laid down in the *GUM*, would satisfy the need for a modern method of uncertainty evaluation in the field of fiscal oil and gas measurement.

As a further development with respect to fiscal metering of gas, a follow-up project was initiated by the same partners to develop a handbook addressing the uncertainty of fiscal gas metering stations using ultrasonic meters (USM).

A reference group consisting of nine persons with a broad and varied competence from oil and gas measurement has evaluated and commented the handbook. The reference group has consisted of:

Reidar Sakariassen, MetroPartner,
Erik Malde, Phillips Petroleum Company Norway,
Trond Folkestad, Norsk Hydro,
Endre Jacobsen, Statoil,
Tore Løland, Statoil,
John Eide, JME Consultants and Holta-Haaland,
Jostein Eide, Kongsberg Fimas,
Håkon Moestue, Norsk Hydro,
Hans Arne Frøystein, The Norwegian Metrology and Accreditation Service.

The reference group concludes that the "*Handbook of uncertainty calculations - Ultrasonic fiscal gas metering stations*" is reliable and in conformity with the *GUM*.

We wish to express our thanks to the project leader at CMR, Per Lunde, and to the members of the reference group for their contribution to this handbook.

December 2001

Norwegian Petroleum Directorate (NPD)
Einar Halvorsen

Norwegian Society for Oil and Gas Measurement (NFOGM)
Phillips Petroleum Company Norway
Svein Neumann

PART A

USER'S GUIDE

1. INTRODUCTION

1.1 Background

In a cooperation between the Norwegian Society of Oil and Gas Measurement (NFOGM), the Norwegian Petroleum Directorate (NPD) and Christian Michelsen Research AS (CMR), there has earlier been worked out a "Handbook on uncertainty calculations - Fiscal metering stations" [Dahl *et al.*, 1999]. That handbook concentrated on fiscal oil metering stations based on a turbine flow meter, and fiscal gas metering stations based on an orifice flow meter. As a further development with respect to fiscal gas metering stations, a follow-up project has been initiated between the same partners for developing a handbook on uncertainty calculations for gas metering stations which are based on a flow calibrated multipath ultrasonic gas flow meter [Lunde, 2000].

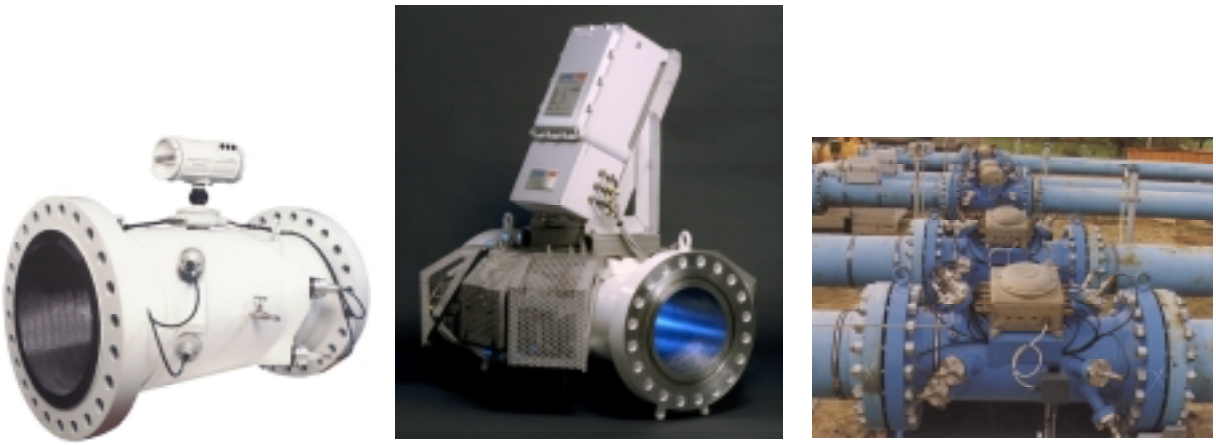
1.1.1 USM fiscal metering of gas

Multipath transit-time ultrasonic gas flow meters (USMs) are today increasingly taken into use for fiscal metering of natural gas, and have already been developed to a level where they are competitive alternatives to more conventional technology as turbine and orifice meters.

USM technology offers significant operational advantages such as no moving parts, no obstruction of flow, no pressure loss, and bi-directional operation (reducing need for pipework). Compact metering stations can be constructed on basis of the large turn-down ratio of USMs (35:1 or larger, tentatively), reducing the need for a multiplicity of meters to cover a wide flow range, and the short upstream/downstream requirements with respect to disturbances. Measurement possibilities are offered which have not been available earlier, such as flow monitoring (e.g. pulsating flow, flow velocity profile, velocity of sound (VOS) profile), and self-checking capabilities (from sound velocity, signal level, etc.). There are also potentials of utilizing additional information such as for gas density and calorific value determination.

The first generation of USMs has been on the market for about a decade. Three manufacturers offer such meters for gas fiscal metering today [Daniel, 2000], [Kongsberg, 2000], [Instromet, 2000], cf. Fig. 1.1. USMs have demonstrated their capability to provide metering accuracy within national regulation requirements [NPD, 2001], [AGA, 1998]. Better than ±1 % uncertainty of mass flow rate (measured value) is being reported, as required for custody transfer [NPD, 2001]. In ap-

propriate applications, multi-path ultrasonic meters can offer significant cost benefits. Compared to conventional turbine and orifice meters, USM technology is increasingly gaining acceptance throughout the industry, and is today in use in gas metering stations onshore and offshore. The world market for precision metering of gas is significant, and several hundred meters are today delivered each year on a world basis (including fiscal and "check" meters).



(a) (b) (c)

Fig. 1.1 Three types of multipath ultrasonic flow meters for gas available today: (a) Daniel Senior-Sonic [Daniel, 2000, 2001], (b) Kongsberg MPU 1200 [Kongsberg, 2000, 2001], (c) Instromet Q-Sonic [Instromet, 2000]. Used by permission of Daniel Industries, FMC Kongsberg Metering and Instromet.

In Norway the use of USMs is already included in the national regulations and standards for fiscal metering of gas [NPD, 1997; 2001], [NORSOK, 1998a]. In the USA the AGA report no. 9 [AGA-9, 1998] provides guidelines for practical use of USMs for fiscal gas metering. Internationally, work has been initiated for ISO standardization of ultrasonic gas flow measurement, based e.g. on the reports ISO/TR 126765:1997 [ISO, 1997] and AGA-9.

In spite of the considerable interest from oil and gas industry users, USMs still represent "new" technology, which in Europe has been tried out over a period of about 10 years. In the USA, Canada, etc., users were to a large extent awaiting the AGA-9 document, issued in 1998. A USM is an advanced "high-technology" electronics instrument, which requires competence both in production and use. USM technology is relatively young compared with more traditional flow metering technologies, and potentials and needs exist for further robustness and development. The technology is expected to mature over time, and the need for standardization and improved traceability is significant.

For example, methods and tools to evaluate the accuracy of USMs will be important in coming years. As there is a great number of ways to determine the flow of natural gas, the seller and buyer of a gas product often chooses to measure the gas with two different methods, involving different equipment. This will often result in two different values for the measured gas flow, creating a dispute as to the correctness of each value. Although it is common knowledge that each value of the measured gas flow has an uncertainty, it often causes contractual disputes. The many different approaches to calculating the uncertainty is also a source of confusion; - varying practice in this respect has definitely been experienced among members in the group of USM manufacturers, engineering companies (metering system designers) and operating companies.

In practical use of USMs in fiscal gas metering stations, the lack of an accepted method for evaluating the uncertainty of such metering stations has thus represented a "problem". Along with the increasing use of USMs in gas metering stations world wide, the need for developing standardized and accepted uncertainty evaluation methods for such metering stations is significant.

For example, in the NPD regulations entering into force in January 2002 [NPD, 2001] it is stated that “it shall be possible to document the total uncertainty of the measurement system. An uncertainty analysis shall be prepared for the measurement system within a 95 % confidence level” (cf. Section C.1). In the ongoing work on ISO standardization of ultrasonic gas flow meters, an uncertainty analysis is planned to be included.

Even within such high accuracy figures as referred to above (±1 % of mass flow rate), demonstrated in flow testing, systematic errors may over time accumulate to significant economic values. Moreover, in service, conditions may be different from the test situation, and practical problems may occur so that occasionally it may be difficult to ensure that such accuracy figures are actually reached. A great challenge is now to be able to be confident of the in-service performance over a significant period of time and changing operational conditions.

For example, the uncertainty figure found in flow calibration of a USM does not necessarily reflect the real uncertainty of the meter when placed in field operation. Methods and tools to evaluate the consequences for the uncertainty of the metering station due to e.g. deviation in conditions from flow calibration to field operation are needed. This concerns both installation conditions (bends, flow conditioners, flow velocity profiles, meter orientation re. bends, wall corrosion, wear, pitting, etc.) and

gas conditions (pressure, temperature, etc.). Such tools should be based on internationally recognized and sound measurement practice in the field, also with respect to uncertatinty evaluation[1].

Another perspective in future development of USM technology which also affects evaluation of the measurement uncertainty, is the role of "dry calibration" and flow calibration[2]. Today, a meter is in general subject to both "dry calibration" and flow calibration prior to installing the USM in field operation. However, some manufacturers have argued that "dry calibration" is sufficient (i.e. without flow calibration of individual meters), and AGA-9 does also in principle open up for such a practice, provided the USM manufacturer can document sufficient accuracy[3]. However, “dry calibration” is not a calibration, and it is definitely a question whether such a practice would be sufficiently traceable, and whether it could be accepted by national authorities for fiscal metering applications[4]. In any case, analysis using an accepted

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1 For instance, actual upstream lengths are in practice included in flow calibration, and flow conditioner used and included in the flow calibration. (The NORSOK I-104 standard states that “flow conditioner of a recognized standard shall be installed, unless it is verified that the ultrasonic meter is not influenced by the layout of the piping upstream or downstream, in such a way that the overall uncertainty requirements are exceeded” [NORSOK, 1998a].)

2 For an explanation of the terms "dry calibration" and flow calibration, cf. Appendix A.

3 A possible approach with reduced dependence on flow calibration in the future would impose higher requirements to the USM technology and to the USM manufacturer. There are several reasons for that, such as:

- Installation effects would not be calibrated “away” (i.e. the USM technology itself would have to be sufficiently robust with respect to the range of axial and transversal flow profiles met in practice, cf. Table 1.4),
- Systematic transit time effects (cf. Table 1.4) would not be calibrated “away” to the same extent, unless the production of the USM technology is extremely reproducible (with respect to transit time contributions).
- With respect to pressure and temperature correction of the meter body dimensions, the reference pressure and temperature would be the “dry calibration” P and T (e.g. 1 atm. and 20 ^{o}C) instead of the flow calibration P and T, which might impose larger and thus more important corrections.
- Traceability aspects (see below),
- Documentation of sufficient accuracy in relation to the national regulations.

4 A possible reduced dependency on flow calibration in the future would necessitate the establishment of a totally new chain of traceability to national and international standards for the USM measurement. Today, the traceability is achieved through the accreditation of the flow calibration laboratory. With a possible reduced dependence on flow calibration, and an increased dependency on "dry calibration", the traceability of the individual meter manufacturer’s "dry calibration" procedures would become much more important and critical than today, especially for transit time "dry calibration". This involves both (1) the measurement uncertainty of the "dry calibration" methods, (2) change of the "dry calibration" parameters with operational conditions (pressure, temperature, gas composition and transducer distance, relative to at "dry calibration" conditions), and (3) the contributions of such "dry calibration" uncertainties to the total USM measurement uncertainty. Today USM measurement technology is not at a level where the traceability of the "dry calibration" methods has been proved.

uncertainty evaluation method would be central. However, use of "dry calibration" only is quite another scenario than the combined use of "dry calibration" and flow calibration, also with respect to uncertainty evaluation. The two approaches definitely require two different uncertainty models[5, 6].

1.1.2 Contributions to the uncertainty of USM fiscal gas metering stations

In the present *Handbook* an uncertainty model for fiscal gas metering stations based on a flow calibrated USM is proposed and implemented in an Excel uncertainty calculation program. A possible future sceneario with “dry calibrated" USMs only is not covered by the present *Handbook* and calculation program. The model is to some extent simplified relative to the physical effects actually taking place in the USM. This is by no means a necessity, but has been done mainly to simplify the user interface, to avoid a too high “user treshold” in the Excel program (with respect to specification of input uncertainties), and with the intention to formulate the model in a best possible meter independent way which preferably meets the input uncertainty terms commonly used in the field of USM technology.

The type of fiscal gas metering stations considered in the present *Handbook* is described and motivated in Sections 1.2 and 2.1. It consists basically of a USM, a flow computer, and instrumentation such as pressure transmitter, temperature element and transmitter, a vibrating element densitometer, compressibility factors calculated from

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5 In this context it is important to distinguish between an uncertainty model for a USM which is only ”dry calibrated”, and an uncertainty model for a USM which is both ”dry calibrated” and flow calibrated.

The uncertainty model *GARUSO* [Lunde *et al.*, 1997; 2000a] represents an uncertainty model for USMs which have *not* been flow calibrated, only "dry calibrated".

In the present *Handbook*, the USM is assumed to be "dry calibrated" *and* flow calibrated, and then operated in a metering station. The two uncertainty models are related but different, and are developed for different use.

The uncertainty model developed here for fiscal gas metering stations which are based on a flow calibrated USM, uses the *GARUSO* model as a basis for the development, cf. Appendix E. Among others, it represents an adaptation and extension of the *GARUSO* model to the scenario with flow calibration of the USM.

6 If the USM is flow calibrated in a flow calibration facility, the AGA-9 report [AGA, 1998] recommends that the USM shall meet specific minimum measurement performance requirements before the application of any correction factor adjustment. These requirements (deviation limits) therefore in practice represent "dry calibration" requirements.

If the USM is *not* flow calibrated (only "dry calibrated"), AGA-9 recommends that the manufacturer shall provide sufficient test data confirming that each meter shall meet the minimum performance requirements. In such contexts an uncertainty model of the *GARUSO* type is relevant.

gas chromatography (GC) analysis, and a calorimeter for measurement of the calorific value.

Flow calibration of USMs is used to achieve a traceable comparison of the USM with a traceable reference measurement, and to "eliminate" or reduce a number of systematic effects in the USM. That is, eliminate or reduce certain systematic effects related to the meter body dimensions, the measured transit times and the integration method (installation effects).

Tables 1.1-1.4 give an overview of some effects which may influence on a USM fiscal gas metering station, assumed that the meter and instruments otherwise function according to manufacturer recommendations. Table 1.1 gives contributions to the uncertainty of the instruments used for gas measurement. Table 1.2 gives uncertainty contributions related to flow calibration of the USM. Table 1.3 gives uncertainty contributions due to the signal communication and flow computer calculations. Table 1.4 gives some uncertainty contributions related to the USM in field operation. Flow calibration of the USM may eliminate or reduce a number of the systematic effects, but, as indicated in Table 1.4, several effects may still be influent, despite flow calibration. Uncertainty contributions such as those listed in these tables are accounted for in the uncertainty model for USM gas metering stations described in Chapter 3[7].

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7 It should be noted that some of the effects listed in Table 1.4 are not sufficiently well understood today. Improved control and corrections could be achieved if better understanding and a more solid theoretical basis for the USM methodology was available. The expressions forming the basis for present-day USMs are based on a number of assumptions which are not fulfilled in practice, such as uniform axial flow (i.e. infinite Reynolds number, *Re*), uniform or no transversal flow, interaction of infinitely thin acoustic beams (rays) with the flow, and simplified (if any) treatment of diffraction effects. In reality, the axial flow profile will change both with *Re* and with the actual installation conditions (such as bend configurations, use of flow conditioner, wall corrosion, wear, pitting, deposits, etc.). Transversal flow is usually significant and non-uniform (swirl, cross-flow, etc.). Moreover, in reality the acoustic beam has a finite beam width, interacting with the flow over a finite volume, and with acoustic diffraction effects (due to the finite transducer aperture). All of these factors influence on the USM integration method as well as the measured transit times, as systematic effects.

Therefore, although the USM technology has definitely proven to be capable of measuring at very high accuracy already, as required for fiscal measurement of gas, there clearly exist potentials for further improved accuracy of this technology, by improved understanding and correction for such systematic effects. In this context, it is worth mentioning that even a relatively “small” reduction of a meter’s systematic error by, say, 0.1-0.2 %, may over time transfer to sicnific economic values [NPD, 2001].

Table 1.1. Uncertainty contributions related to measurement / calculation of gas perameters in the USM fiscal gas metering station.

Measurand	**Uncertainty contribution (examples)**
Pressure	• Transmitter uncertainty (calibration) • Stability of pressure transmitter • RFI effects on pressure transmitter • Ambient temperature effects on pressure transmitter • Atmospheric pressure • Vibration • Power supply effects • Mounting effects
Temperature	• Transmitter/element uncertainty (calibrated as a unit) • Stability of temperature transmitter • Stability of temperature element • RFI effects on temperature transmitter • Ambient temperature effects on temperature transmitter • Vibration • Power supply effects • Lead resistance effects
Compressibility	• Model uncertainty (uncertainty of the equation of state itself, and uncertainty of basic data unerlying the equation of state) • Analysis uncertainty (GC measurement uncertainty, variation in gas composition)
Density	• Densitometer uncertainty (calibrated) • Repeatability • Temperature correction (line and calibration temperatures) • VOS correction (line and densitometer sound velocities, calibration constant, periodic time) • Installation (by-pass) correction (line and densitometer *P* and *T*) • Miscellaneous effects (e.g. stability, pressure effect, deposits, corrosion, condensation, viscosity, vibration, power supply effects, self induced heat, flow rate in by-pass density line, sampling representativity, etc.)
Calorific value	• Calorimeter uncertainty

Table 1.2. Uncertainty contributions related to flow calibration of the USM.

Source	**Uncertainty contribution (examples)**
USM flow calibration	• Calibration laboratory • Deviation factor • USM repeatability in flow calibration, including repeatability of the flow calibration laboratory

Table 1.3. Uncertainty contributions related to signal communication and flow computer calculations.

Source	**Uncertainty contribution (examples)**
Flow computer	• Signal communication (analog (frequency) or digital signal) • Flow computer calculations

Table 1.4. Uncertainty contributions to an ultrasonic gas flow meter (USM) in field operation.

Ultrasonic gas flow meter (USM) in field operation			
Uncertainty term	**Type of effect**	**Uncertainty contribution (examples)**	**Eliminated by flow calibration?** [a)]
Meter body	Systematic	• Measurement uncertainty of dimensional quantities (at "dry calibration" conditions) • Out-of-roundness • *P & T* effects on dimensional quantities (after possible *P & T* corrections)	Eliminated Eliminated
Transit times	Random (repeatability)	• Turbulence (transit time fluctuations due to random velocity and temperature fluctuations) • Incoherent noise (RFI, pressure control valves, pipe vibrations, etc.) • Coherent noise (acoustic cross-talk through meter body, electromagnetic cross-talk, acoustic reverberation in gas, etc.) • Finite clock resolution • Electronics stability (possible random effects) • Possible random effects in signal detection/processing (e.g. erronous signal period identification). • Power supply variations	
	Systematic	• Cable/electronics/transducer/diffraction time delay, including finite-beam effects (line *P* and *T* effects, ambient temperat. effects, drift, transducer exchange) • Δt-correction (line *P* and *T* effects, ambient temperature effects, drift, reciprocity, effects of possible transducer exchange) • Possible systematic effects in signal detection /processing (e.g. erronous signal period identification) • Possible cavity delay correction • Possible deposits at transducer front (lubricant oil, liquid, wax, grease, etc.) • Sound refraction (flow profile effects on transit times) • Possible beam reflection at the meter body wall	Eliminated
Integration method (installation effects)	Systematic	• Pipe bend configurations upstream of USM (possible difference rel. flow calibration) • In-flow profile to upstream pipe bend (possible difference rel. flow calibration) • Meter orientation relative to pipe bends (possible difference rel. flow calibration) • Initial wall roughness (influence on flow profiles) • Changed wall roughness over time: corrosion, wear, pitting (influence on flow profiles) • Possible wall deposits (lubricant oil, liquid, wax, grease, etc.) (influence on flow profiles) • Possible use of flow conditioner	Eliminated [b)] Eliminated [c)]
Miscellaneous	Systematic	• Inaccuracy of the functional relationship (mathematical model), with respect to transit times, and integration method	Possibly eliminated, to some extent

[a)] Only uncertainties related to *changes of conditions from flow calibration to field operation* are in question here. That means, systematic USM uncertainty contributions which are practically eliminated by flow calibration, are *not* to be included in the uncertainty evaluation.

[b)] If the USM is flow calibrated together with the upstream pipe section to be used in field operation, which is relatively common practice.

[c)] If the USM is flow calibrated together with the flow conditioner to be used in field operation.

1.1.3 Uncertainty evaluation of USMs and USM gas flow metering stations

For fiscal gas metering stations, a measured value is to be accompanied with a statement of the uncertainty of the measured value. In general, an uncertainty analysis is needed to establish the measurement uncertainty of the metering station [NPD, 2001]. The uncertainty analysis is to account for the propagation of all input uncertainties which influence on the uncertainty of the station. These are the uncertainty of the flow meter in question (in the present case the USM, cf. Table 1.4), the uncertainty of the reference meter used by the flow calibration laboratory at which the gas meter was calibrated, and uncertainties of additional measurements and models used (e.g. pressure, temperature and density measurements, Z-factor measurement/calculation, calorific value measurement), etc.

Today there exists no established and widely accepted uncertainty model for USMs, nor an uncertainty model for a USM used as part of a fiscal gas metering station, derived from the basic functional relationship of such stations. It has thus been considered necessary to develop such a model in the present work, as a part of the scope of work. In the *Handbook*, some more space has thus been necessary to use for description of the model itself, than would have been needed if a more established and accepted uncertainty model of USM fiscal gas metering stations was available.

The uncertainty model for USM fiscal gas metering stations presented here has been developed on basis of earlier developments in this field. In the following, a brief historic review is given.

In 1987-88, a sensitivity study of multipath USMs used for fiscal gas metering was carried out by CMR for Statoil and British Petroleum (BP) [Lygre *et al.*, 1988]. An uncertainty model for Fluenta's FGM 100 single-path flare gas meter was prepared by CMR in 1993 [Lunde, 1993]. In 1995 an uncertainty model for Fluenta's multipath ultrasonic gas flow meter FMU 700[8] was developed [Lunde *et al.*, 1995].

Based e.g. on recommendations given in the GERG TM 8 [Lygre *et al.*, 1995], a group of 9 participants in the GERG Project on Ultrasonic Gas Flow Meters[9] initi-

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8 Fluenta's FMU 700 technology was in 1996 sold to Kongsberg Offshore (now FMC Kongsberg Metering), and is from 1998 marketed as MPU 1200 by FMC Kongsberg Metering. From 2001 Fluenta AS, Bergen, Norway, is a part of Roxar Flow Measurement AS.

9 Participating GERG (Groupe Européen de Recherches Gazières) companies in the project were BG plc (UK), Distrigaz (Belgium), ENAGAS (Spain), N. V. Nederlandse Gasunie (Holland), Gaz de France DR (France), NAM (Holland), Ruhrgas AG (Germany), SNAM (Italy) and Statoil (Norway).

ated in 1996 the development of a relatively comprehensive uncertainty model for USMs, named *GARUSO* [Lunde *et al.*, 1997; 2000a] [Sloet, 1998], [Wild, 1999][10]. One major intention was to develop an uncertainty model in conformity with accepted international standards and recommendations on uncertainty evaluation, such as the *GUM* [ISO, 1995a]. Uncertainty contributions such as most of those listed in Table 1.4 were accounted for. As described in a footnote of Section 1.1.1, the *GARUSO* model relates to USMs which are “dry calibrated” but not flow calibrated.

On basis of an initiative from the Norwegian Society of Oil and Gas Metering (NFOGM) and the Norwegian Petroleum Directorate (NPD), a "Handbook on uncertainty calculations - Fiscal metering stations" was developed in 1999 [Dahl *et al.*, 1999]. That handbook concentrated on fiscal oil metering stations based on a turbine flow meter, and fiscal gas metering stations based on an orifice flow meter. For an orifice gas metering station, the secondary instrumentation (pressure, temperature, density, compressibility and calorific value) is often the same as in USM fiscal gas metering stations.

In UK a British Standard BS 7965:2000 [BS, 2000] was issued in 2000, containing an uncertainty analysis of an ultrasonic flow meter used in a gas metering station (for mass flow rate measurement), combined with secondary instrumentation (pressure and temperature measurements, and density obtained from GC analysis). A number of uncertainty contributions were identified and combined in the uncertainty model by a root-mean-square approach, using sensitivity coefficients apparently set equal to 1 for the USM part (or not described). All uncertainty contributions were thus assumed to be uncorrelated, and apparently (for the USM part) to contribute with equal weights to the total uncertainty. The connection between the USM functional relationship and the USM uncertainty model was not described. This model has not been found to be sufficiently well documented and accurate to be used as a basis for the present *Handbook*.

In this *Handbook*, the theoretical basis for the *GARUSO* model [Lunde *et al.*, 1997] has been used as a fundament for development of an uncertainty model and a handbook on uncertainty calculations for fiscal gas metering stations which are based on

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10 In later years (1998-2000) the *GARUSO* model has beeen further developed at CMR e.g. to enable use of CFD (“computational fluid dynamics”) - calculations of 3-dimensional flow velocity profiles as input to the uncertainty calculations, to study installation effects [Lunde *et al.*, 2000b], [Hallanger *et al.*, 2001].

use of flow calibrated multipath USMs[11], cf. Chapter 3. However, in comparison with the *GARUSO* model, the present description is adapted to a less detailed level, with respect to user input. The present description is also combined with the uncertainty evaluation of the secondary instrumentation[12] (pressure, temperature, density measurements and *Z*-factor evaluation). Also the calorific value uncertainty is accounted for here.

In relation to the uncertainty model for the USM described in the British Standard BS 7965:2000 [BS, 2000], a different model is proposed here. Essentially, the same types of basic uncertainty contributions may be described in the two models[13], but in different manners, for a number of the uncertainty contributions. The present model is based on an approach where the various contributions are derived from the metering station's functional relationship. That is, sensitivity coefficients are derived and documented, providing a traceable weighting and propagatation of the various uncertainty contributions to the overall uncertainty. Also, correlated as well as uncorrelated effects are described and documented, for traceability purposes. The resulting expressions for the uncertainty model are different from the ones proposed in [BS, 2000].

1.2 About the *Handbook*

The *Handbook of uncertainty calculations - USM fiscal gas metering stations* consists of

- The *Handbook* (the present document),
- The Microsoft Excel program *EMU - USM Fiscal Gas Metering Station* for performing uncertainty calculations of fiscal gas metering stations based on USM flow meters, and individual instruments of such stations.

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11 Cf. the footnote on the *GARUSO* model in Section 1.1.1.

12 The description of the pressure, temperature and density measurements, and the calculation of their expanded uncertainties, are similar to the descriptions given in [Dahl *et al.*, 1999], with some modifications.

13 In the uncertainty model for the USM proposed here, the same types of input uncertainty contributions are accounted for as in the BS 7965:2000 model, in addition to some others, cf. Tables 1.2 - 1.4.

The *Handbook* and the Excel program address calculation of the uncertainty of fiscal metering stations for natural gas which are based on use of a flow calibrated multipath ultrasonic transit-time flow meter (USM).

For fiscal gas metering stations, four flow rate figures are normally to be calculated [NORSOK, 1998a]:

- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard reference conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e (application specific).

The *Handbook* was originally intended to address two of these four flow rate figures [Lunde, 2000]: the actual volume flow and the mass flow rate. However, also the standard volume flow and the energy flow rate have been addressed in the present *Handbook*, so that the expanded uncertainty can be calculated for all four flow rate figures using the program *EMU - USM Fiscal Gas Metering Station*[14].

The equipment and instruments considered are, cf. Section 2.1.2:

- USM (flow calibrated),
- Flow computer,
- Pressure transmitter,
- Temperature element and transmitter,
- Densitometer (on-line installed vibrating element),
- Compressibility factors calculated from gas chromatography (GC) analysis,
- Calorific value measurement (calorimeter).

By propagation of input uncertainties the model calculates among others the metering station's "expanded uncertainty" and "relative expanded uncertainty".

The USM fiscal gas metering stations addressed in the present *Handbook* are assumed to be designed, constructed and operated according to NPD regulations [NPD, 2001]. For USM fiscal metering of gas, the NPD regulations refer to e.g. the

14 For the energy flow rate, a simplified approach has been used here, in which the calorific value measurement has been assumed to be uncorrelated with the standard volume flow measurement, cf. Sections 3.1 and 3.2.5.

NORSOK I-104 national standard [NORSOK, 1998a] and the AGA Report No. 9 [AGA-9, 1998] as recognised standards ("accepted norm"). Both the NPD regulations and the NORSOK I-104 standard refer to the *GUM* (*Guide to the expression of uncertainty in measurement)* [ISO, 1995a] as the "accepted norm" with respect to uncertainty analysis (cf. Appendix C).

Consequently, the present *Handbook* and the computer program *EMU - USM Fiscal Gas Metering Station* are based primarily on the recommended procedures in the *GUM*. They are also considered to be in consistence with the proposed revision of ISO 5168 [ISO/CD 5168, 2000] (which is based on the *GUM*).

With respect to uncertainty evaluation and documentation, the NPD regulations and the NORSOK I-104 standard state that the expanded uncertainty of the measurement system shall be specified at a 95 % confidence level, using a coverage factor $k = 2$ (cf. Appendix C and Section B.3). This has consequently been adopted here and used in the *Handbook* and the program *EMU - USM Fiscal Gas Metering Station*.

The uncertainty model for the USM gas metering station developed here is based on an analytical approach. That is, the uncertainty models for the USM, pressure transmitter, temperature element/transmitter, densitometer, calculation of compressibility factors, and calorimeter, are fully analytical, with expressions given and documented for the model and the sensitivity coefficients (cf. Chapter 3, which is based on Appendices E, F and G).

It has been chosen [Ref Group, 2001] to calculate the uncertainty of the metering station only in the "flow calibration points", i.e. at the M test flow rate figures described in Section 2.1 (typically $M = 5$ or 6)[15], with a possibility to draw a curve between these points. An example of such output is given in Fig. 5.20.

The *Handbook* and the accompanying Excel program provides a practical approach to the field of uncertainty calculations of ultrasonic fiscal gas metering stations, and is primarily written for experienced users and operators of fiscal gas metering stations, manufacturers of ultrasonic gas flow meters, engineering personnel, as well as others with interests within the field.

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15 In the AGA-9 report [AGA-9, 1998] and in [OIML, 1989], 6 calibration test flow rates ("calibration points") have been recommended. In the NORSOK I-104 industry standard [NORSOK, 1998a], 5 calibration points are recommended, as a compromise between cost and performance [Ref Group, 2001].

1.3 About the program EMU - USM Fiscal Gas Metering Station

As a part of the present *Handbook*, an Excel program *EMU - USM Fiscal Gas Metering Station* has been developed for performing uncertainty evaluations of the fiscal gas metering station[16]. The program is implemented in Microsoft Excel 2000 and is opened as a normal workbook in Excel. The program file is called "*EMU - USM Fiscal Gas Metering Station.xls*". The abbreviation *EMU* is short for "Evaluation of Metering Uncertainty" [Dahl *et al.*, 1999].

It has been the intention that the Excel program *EMU - USM Fiscal Gas Metering Station* may be run without needing to read much of the *Handbook*. However, Chapter 5 which gives an overview of the program, as well as Chapter 4 which - through an uncertainty evaluation example - is intended to provide some guidelines for specifying input parameters and uncertainties to the program, may be useful to read together with running the program for the first time. At each "input cell" in the program a comment is given, with reference to the relevant section(s) of the Handbook in which some information and help about the required input can be found. As delivered, the program is "loaded" with the input parameters and uncertainties used for the example calculations given in Chapter 4.

As the Excel program *EMU - USM Fiscal Gas Metering Station* is described in more detail in Chapter 5, only a brief overview is given here. It is organized in 24 worksheets, related to: gas parameters, USM setup, pressure, temperature, compressibility factors, density, calorific value, flow calibration, USM field operation, flow computer etc., various worksheets for plotting of output uncertainty data (graphs, bar-charts), a worksheet for graph and bar-chart set-up, a summary report, two additional worksheets listing the plotted data and the calculated USM transit times, and two program information worksheets.

As described in Section 1.2, the program calculates the expanded and relative expanded uncertainties of a gas metering station which is based on a flow calibrated USM, for the four measurands in questions, q_v, Q, q_m and q_e.

The theoretical basis for the uncertainty calculations is described in Chapters 2 and 3. A calculation example is given in Chapter 4, including discussion of input uncertain-

16 In the earlier *Handbook* [Dahl *et al.*, 1999], the two Excel programs were named *EMU - Fiscal Gas Metering Station* (for gas metering stations based on orifice plate), and *EMU - Fiscal Oil Metering Station* (for oil metering stations based on a turbine meter).

ties and some guidelines to the use of the program. An overall description of the program is given in Chapter 5.

In addition to calculation/plotting/reporting of the expanded uncertainty of the gas metering station (cf. Figs. 5.20 and 5.27-5.31), and the individual instruments of the station, the Excel program can be used to calculate, plot and analyse the relative importance of the various contributions to the uncertainty budget for the actual instruments of the metering station (using bar-charts), such as:

- Pressure transmitter (a number of contributions) (cf. Fig. 5.21),
- Temperature element / transmitter (a number of contributions) (cf. Fig. 5.22),
- Compressibility ratio evaluation (a number of contributions) (cf. Fig. 5.23),
- Densitometer (including temperature, VOS and installation corrections) (Fig. 5.24),
- USM flow calibration (laboratory uncertainty, meter factor, repeatability) (Fig. 5.25),
- USM field operation in the the metering station (deviation from flow calibration, with respect to *P* & *T* corrections of meter body dimensions, repeatability, systematic transit time effects and integration/installation effects) (cf. Fig. 5.26),
- The gas metering station in total (cf. Fig. 5.27).

In the program the uncertainties of the gas density, pressure and temperature measurements can each be specified at two levels (cf. Tables 1.5, 3.1, 3.2 and 3.4; see also Sections 5.4, 5.5 and 5.7):

(1) ***"Overall level":*** The user specifies the combined standard uncertainty of the gas density, pressure or temperature estimate, $u_c(\hat{\rho})$, $u_c(\hat{P})$ or $u_c(\hat{T})$, respectively, directly as input to the program. It is left to the user to calculate and document $u_c(\hat{\rho})$, $u_c(\hat{P})$ or $u_c(\hat{T})$ first. This option is general, and covers any method of obtaining the uncertainty of the gas density, pressure or temperature estimate (measurement or calculation)[17].

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17 The "overall level" options may be of interest in several cases, such as e.g.:

- If the user wants a "simple" and quick evaluation of the influence of $u_c(\hat{\rho})$, $u_c(\hat{P})$ or $u_c(\hat{T})$ on the expanded uncertainty of the gas metering station,
- In case of a different installation of the gas densitometer (e.g. in-line),
- In case of a different gas densitometer functional relationship than Eq. (2.25),
- In case of density measurement using GC analysis and calculations instead of densitometer measurement.
- In case the input used at the "detailed level" does not fit sufficiently well to the type of input data / uncertainties which are relevant for the pressure transmitter or temperature element/transmitter at hand.

(2) ***"Detailed level":*** $u_c(\hat{\rho})$, $u_c(\hat{P})$ or $u_c(\hat{T})$, respectively, is calculated in the program, from more basic input for the gas densitometer, pressure transmitter and temperature element transmitter, provided by the instrument manufacturer and calibration laboratory. Such input is at the level described in Table 1.1.

For the input uncertainty of the compressibility factors, the user input is specified at the level described in Table 1.1. See Tables 1.5, 3.3 and Section 5.6.

For the input uncertainty of the calorific value measurement, only the "overall level" is implemented in the present version of the program, cf. Tables 1.5, 3.5 and Section 5.8[18].

With respect to USM flow calibration, the user input is specified at the level described in Table 1.2. See Tables 1.5, 3.6 and Sections 4.3, 5.9.

Table 1.5. Uncertainty model contributions, and optional levels for specification of input uncertainties to the program *EMU - USM Fiscal Gas Metering Station*.

Uncertainty contribution	Overall level	Detailed level
Pressure measurement uncertainty	✓	✓
Temperature measurement uncertainty	✓	✓
Compressibility factor uncertainties		✓
Density measurement uncertainty	✓	✓
Calorific value measurement uncertainty	✓	
USM flow calibration uncertainty		✓
USM field uncertainty	✓	✓
Signal communication and flow computer calculations	✓	

With respect to USM field operation, a similar strategy as above with "overall level" and "detailed level" is used for specification of input uncertainties, see Table 1.5 and Section 5.10. For the "detailed level", the level for specification of input uncertainties is adapted to data from "dry calibration" / flow calibration / testing of USMs to be provided by the USM manufacturer (cf. Chapter 6). In particular this concerns:

- **Repeatability.** The user specifies either (1) the repeatability of the indicated USM flow rate measurement, or (2) the repeatability of the measured transit times (cf. Tables 1.4, 3.8 as well as Sections 4.4.1 and 5.10.1). Both can be given as flow rate dependent.

18 Improved descriptions to include a calculation of the calorific value uncertainty (with input at the "detailed level") may be implemented in a possible future revision of the *Handbook*, cf. Chapter 7.

- **Meter body parameters.** The user specifies whether correction for pressure and temperature effects is used by the USM manufacturer, and the uncertainties of the pressure and temperature expansion coefficients. Cf. Tables 1.4 and 3.8 as well as Sections 4.4.1 and 5.10.2.

- **Systematic transit time effects.** The user specifies the uncertainty of uncorrected systematic effects on the measured upstream and downstream transit times. Examples of such effects are given in Table 1.4, cf. Table 3.8 and Sections 4.4.3, 5.10.2.

- **Integration method (installation effects).** The user specifies the uncertainty due to installation effects. Examples of such are given in Table 1.4, cf. Table 3.8 and Sections 4.4.4, 5.10.2.

It should be noted that for all of these USM field uncertainty contributions, only uncertainties related to *changes of installation conditions from flow calibration to field operation* are in question here. That means, systematic USM uncertainty contributions which are practically eliminated by flow calibration, are *not* to be included in the uncertainty model (cf. Table 1.4).

Consequently, with respect to USM technology, the program *EMU - USM Fiscal Gas Metering Station* can be run in two modes:

(A) Completely meter independent, and
(B) Weakly meter dependent[19].

Mode (A) corresponds to choosing the "overall level" in the "*USM*" worksheet (both for the repeatability and the systematic deviation re. flow calibration), as described above. Mode (B) corresponds to choosing the "detailed level". These options are further described in Section 5.3. See also Section 5.10 and Chapter 6.

It is required to have Microsoft Excel 2000 installed on the computer[20] in order to run the program *EMU - USM Fiscal Gas Metering Station*. Some knowledge about

19 By "weakly meter dependent" is here meant that the diameter, number of paths and the number of reflections for each path need to be known. However, actual values for the inclination angles, lateral chord positions and integration weights do not need to be known. Only very approximate values for these quantities are needed (for calculation of certain sensitivity coefficients), as described in Chapter 6 (cf. Table 6.3).

Microsoft Excel is useful, but by no means necessary. The layout of the program is to a large extent self-explaining and comments are used in order to provide online help in the worksheets, with reference to the corresponding sections in the *Handbook*.

In the NPD regulations it is stated that “it shall be possible to document the total uncertainty of the measurement system. An uncertainty analysis shall be prepared for the measurement system within a 95 % confidence level” [NPD, 2001] (cf. Section C.1). The *GUM* [ISO, 1995a] put requirements to such documentation, cf. Appendix B.4. The expanded uncertainties calculated by the present program may be used in such documentation of the metering station uncertainty, with reference to the *Handbook*. That means, provided the user of the program (on basis of manufacturer information or another source) can document the numbers used for the input uncertainties to the program, the *Handbook* and the program gives procedures for propagation of these input uncertainties.

It is emphasised that for traceability purposes the inputs to the program (quantities and uncertainties) must be documented by the user, cf. Section B.4. The user must also document that the calculation procedures and functional relationships implemented in the program (cf. Chapter 2) are in conformity with the ones actually applied in the fiscal gas metering station[21].

1.4 Overview of the *Handbook*

The *Handbook* has been organized in two parts, "Part A - User's Guide" and "Part B - Appendices".

Part A constitutes the main body of the *Handbook*. The intention has been that the reader should be able to read and use Part A without needing to read Part B. In Part A one has thus limited the amount of mathematical details. However, to keep Part A

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20 The program is optimised for “small fonts” and 1152 x 864 screen resolution (set in the Control Panel by entering “Display” and then “ Settings”). For other screen resolutions, it is recommended to adapt the program display to the screen by the Excel zoom functionality, and saving the Excel file using that setting.

21 If the “overall level” options of the program are used, the program should cover a wide range of situations met in practice.

However, note that in this case possible correlations between the estimates which are specified at the “overall level” are not accounted for (such as e.g. between the calorific value and the *Z*-factors, when these are all obtained from GC analysis). In cases where such correlations are important, the influence of the covariance term on the expanded uncertainty of the metering station should be investigated.

"self-contained", all the expressions which are implemented in the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station* have been included and described also in Part A. It is believed that this approach simplifies the practical use of the *Handbook*.

Part B has been included as a more detailed documentation of the theoretical basis for the uncertainty model, giving necessary and essential details, for completeness and traceability purposes.

1.4.1 Part A - User's Guide

Part A is organized in Chapters 1-7. Chapter 1 gives an introduction to the *Handbook*, including background for the work and a brief overall description of the *Handbook* and the accompanying uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*. The type of instrumentation of the USM fiscal gas metering stations addressed in the *Handbook* is described in Chapter 2, including description of and necessary functional relationships for the instruments involved. That is, the USM itself as well as pressure, temperature and density measurements. The gas compressibility factor calculations and calorific value measurement/calculation are also addressed.

These descriptions and functional relationships serve as a basis for the development of the uncertainty model of the USM fiscal gas metering station, which is described in Chapter 3. The model is derived in detail in Appendix E, on basis of the functional relationships given in Chapter 2. Only those expressions which are necessary for documentation of the model and the Excel program *EMU - USM Fiscal Gas Metering Station* have been included in Chapter 3. Also definitions introduced in Appendix E have - when relevant - been included in Chapter 3, to make Chapter 3 "self contained" so that it can be read independently, without needing to read the appendices.

In Chapter 4 the uncertainty model is used in a calculation example, which to some extent may also serve as a guideline to use of the program *EMU - USM Fiscal Gas Metering Station*. The program itself is described in Chapter 5, with an overview of the various worksheets involved. Chapter 6 summarizes proposed uncertainty data to be specified by USM manufacturers. In Chapter 7 some concluding remarks are given.

1.4.2 Part B - Appendices

Part B is organized in Appendices A-G, giving definitions, selected national regulations and the theoretical basis for the uncertainty model described in Chapter 3.

Some definitions and abbreviations related to USM technology are given in Appendix A. For reference, selected definitions and procedures for evaluation of uncertainty as recommended by the *GUM* [ISO, 1995a] are given in Appendix B. Selected national regulations for USM fiscal gas metering are included in Appendix C.

Appendix D addresses the pressure and temperature correction of the USM meter body, and different approaches used by USM meter manufacturers are discussed and compared, as a basis for Chapter 2. The theoretical basis of the uncertainty model for the USM fiscal gas metering station is given in Appendix E. This constitutes the main basis for Chapter 3. In Appendix F three alternative approaches for evaluation of partially correlated quantities are discussed, and it is shown that the "decomposition method"[22] approach used in Appendix E is equivalent to the "covarance method" approach recommended by the *GUM* [ISO, 1995a]. In Appendix G details on the uncertainty model of the vibrating element gas densitometer are given. Literature references are given at the end of Part B.

1.5 Acknowledgements

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22 To the knowledge of the authors, the method which is here referred to as the "decomposition method" (with decomposition of partially correlated quantities into correlated and uncorrelated parts) has been introduced by the authors, cf. Appendices E and F.

2. USM FISCAL GAS METERING STATION

The present chapter gives a description of typical USM fiscal gas metering stations, serving as a basis for the uncertainty model of such metering stations described in Chapter 3, the uncertainty evaluation example given in Chapter 4, and the Excel program *EMU - USM Fiscal Gas Metering Station* described in Chapter 5.

This includes a brief description of metering station methods and equipment (Section 2.1), as well as the functional relationships of the metering station (Section 2.2), the USM instrument (Section 2.3), the gas densitometer (Section 2.4) and the pressure and temperature instruments (Sections 2.5 and 2.6).

2.1 Description of USM fiscal gas metering station

2.1.1 General

Fiscal measurement is by [NPD, 2001] defined as measurement used for sale or calculation of royalty and tax. By [NORSOK, 1998a] this includes

- sales and allocation measurement of gas,
- measurement of fuel and flare gas,
- sampling, and
- gas chromatograph (GC) measurement.

For fiscal gas metering stations, four flow rate figures are normally to be calculated [NORSOK, 1998a]:

- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard reference conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e (application specific).

The USM fiscal gas metering stations to be evaluated in the present *Handbook* are assumed to be designed, constructed and operated according to NPD regulations [NPD, 2001]. For USM fiscal metering of gas, the NPD regulations refer to e.g. the

NORSOK I-104 national standard [NORSOK, 1998a] and the AGA Report No. 9 [AGA-9, 1998] as recognised standards ("accepted norm")[23].

As a basis for the uncertainty calculations of the present *Handbook*, thus, a selection of NPD regulations and NORSOK I-104 requirements which apply to USM metering stations for sales and allocation metering of gas, have been summarized in Appendix C. Only regulations which influence on the uncertainty model and calculations of USM metering stations are included. (The selection is not necessarily complete.)

With respect to instrumentation of gas sales metering stations, the NPD regulations [NPD, 2001] state that (cf. Section C.1): "On sales metering stations the number of parallel meter runs shall be such that the maximum flow of hydrocarbons can be measured with one meter run out of service, whilst the rest of the meter runs operate within their specified operating range." In practice, this means that on USM gas sales metering stations a minimum of two parallel meter runs shall be used, each with at least one USM. For allocation metering stations there is no such requirement of two parallel meter runs.

The NORSOK I-104 standard states that "flow conditioner of a recognized standard shall be installed, unless it is verified that the ultrasonic meter is not influenced by the layout of the piping upstream or downstream, in such a way that the overall uncertainty requirements are exceeded" [NORSOK, 1998a]. In practice, actual upstream lengths and flow conditioner are included in the flow calibration.

The NPD regulations further state that "pressure, temperature, density and composition analysis shall be measured in such way that representative measurements are achieved as input signals for the fiscal calculations".

As an "accepted norm", the NORSOK I-104 standard [NORSOK, 1998a] state that (cf. Section C.2): "Pressure and temperature shall be measured in each of the meter runs. Density shall be measured by at least two densitometers in the metering station." It is further stated that "The density shall be measured by the vibrating element technique." In practice, this means that density *can* be measured using densitometers in each meter run, or by two densitometers located at the gas inlet and outlet of the metering station.

23 In the revised NPD regulations [NPD, 2001] the requirements are to a large extent stated as *functional* requirements. In the comments to the requirements, it is referred to "accepted norms" including industry standards, which provide *possible* (but not necessary) ways of fulfilling the stated requirements. Cf. Appendix C.

On the other hand, the NPD regulations also open up for calculation of density from GC measurements: "density may be determined by continuous gas chromatography, if such determination can be done within the uncertainty requirements applicable to density measurement. If only one gas chromatograph is used, a comparison function against for example one densitometer should be carried out. This will provide independent control of the density value and that density is still measured when GC is out of operation" (cf. Section C.1).

With respect to measurement of energy contents, the NPD regulations state that "Gas composition from continuous flow proportional gas chromatography or from automatic flow proportional sampling shall be used for determination of energy content. With regard to sales gas metering stations two independent systems shall be installed" (cf. Section C.1). That is, for such metering stations two GCs in parallel are required, while for allocation stations use of a single GC may be sufficient.

Such relatively complex instrument configurations may be further complicated by the fact that the multiple measurements may be used differently by different companies: they may be averaged (i.e., they are both used in the reported measurement), or used only as back-up or redundancy measurements (i.e., not used in the reported measurement). Moreover, in some stations two USMs are used in series. The implications of such relatively complex instrument configurations and varying company practice on the evaluation of the metering station's measurement uncertainty, is addressed in Section 2.1.2.

The fiscal gas metering stations considered here are based on flow rate measurements using one or several flow calibrated multipath ultrasonic gas flow meters (USM), cf. Fig. 1.1. The USM measures basically the average axial gas flow velocity, which when multiplied with the pipe's cross-sectional area yields the volumetric flow rate at line conditions, q_v.

Several methods are in use for measurement of Q, the volumetric flow rate at standard reference condition, cf. Table 2.1[24]. Q can be calculated from q_v, the line density, ρ, and the density at standard reference conditions, ρ_0. Each of ρ and ρ_0 can either be measured by densitometers, or calculated from GC measurement of the gas composition. Three such combination methods are indicated in Table 2.1 [Sakariassen, 2001]. In the North Sea, a common approach is to measure ρ using a densi-

24 Symbols used in Tables 2.1-2.3 are explained in Section 2.2.1.

tometer and to calculate ρ_0 from the gas composition output of a GC (method 2 in Table 2.1). Measurement of ρ_0 using a densitometer is less common (method 1 in Table 2.1).

Several methods can be used for measurement of the line density ρ [Tambo and Søgaard, 1997] and thus the mass flow rate, q_m. According to the NPD regulations [NPD, 2001], the line denstity ρ can either be measured using densitometer(s), or calculated from GC measurement of the gas composition, cf. Table 2.2.

Table 2.1. Different methods of measuring the volumetric flow rate at standard reference conditions, Q.

		Primary measurement	
Method no.	**Functional relationship**	**Line conditions**	**Std. reference conditions**
1	$Q = \frac{\rho}{\rho_0} q_v$	ρ measured using **densitometer**	ρ_0 measured using **densitometer**
2	$Q = \frac{\rho Z_0 R T_0}{m P_0} q_v$	ρ measured using **densitometer**	Z_0, m calculated from **GC** analysis
3	$Q = \frac{P T_0 Z_0}{P_0 T Z} q_v$	Z calculated from **GC** analysis	Z_0 calculated from **GC** analysis

Table 2.2. Different methods of measuring the mass flow rate, q_m.

		Primary measurement
Method no.	**Functional relationship**	**Line conditions**
1	$q_m = \rho q_v$	ρ measured using **densitometer**
2	$q_m = \frac{mP}{ZRT} q_v$	Z, m calculated from **GC** analysis

Table 2.3. Different methods of measuring the energy flow rate, q_e.

		Primary measurement		
Method no.	**Functional relationship**	**Line conditions**	**Std. reference conditions**	**Calorific value**
1	$q_e = H_S \frac{\rho}{\rho_0} q_v$	ρ measured using **densitometer**	ρ_0 measured using **densitometer**	H_S measured using **calorimeter**
2		ρ measured using **densitometer**	ρ_0 measured using **densitometer**	H_S calculated from **GC** analysis
3	$q_e = H_S \frac{\rho Z_0 R T_0}{m P_0} q_v$	ρ measured using **densitometer**	Z_0, m calculated from **GC** analysis	H_S calculated from **GC** analysis
4	$q_e = H_S \frac{P T_0 Z_0}{P_0 T Z} q_v$	Z calculated from **GC** analysis	Z_0 calculated from **GC** analysis	H_S calculated from **GC** analysis
5		Z calculated from **GC** analysis	Z_0 calculated from **GC** analysis	H_S measured using **calorimeter**

Several methods are in use also for measurement of q_e, the energy flow rate, cf. Table 2.3. q_e can be calculated from Q and the superior (gross) calorific value H_s, both specified at standard reference conditions. The conversion from q_v therefore involves ρ, ρ_0 and H_s. According to the NPD regulations [NPD, 2001], the superior (gross) calorific value H_s can either be calculated from the gas composition measured using GC analysis, or measured directly on basis of gas combustion (using a calorimeter) (cf. Section C.1). Five combination methods for measurement of q_e are indicated in Table 2.3 [Sakariassen, 2001], correspondig to the three methods for measurement of Q shown in Table 2.1. In the North Sea, a common method is to measure ρ using a densitometer and to calculate ρ_0 and H_s using the gas composition output from a GC (method 3 in Table 2.3).

2.1.2 Gas metering station equipment considered in the *Handbook*

From the more general description of USM gas metering stations given in Section 2.1.1, the gas metering station equipment considered in the present Handbook is addressed in the following.

With respect to measurement of the volumetric flow rate at standard reference conditions, Q, Method 3 of Table 2.1 is considered here [Lunde, 2000], [Ref. Group, 2001]. That is, P and T are measured, and Z and Z_0 are calculated from GC analysis. This has been done to reduce the complexity of the uncertainty model. A more general treatment of all three approaches, and the implications of these for the uncertainty model of the gas metering station, would be beyond the scope of the present *Handbook* [Ref. Group, 2001][25].

For measurement of the mass flow rate, q_m, Method 1 in Table 2.2 is considered here [Ref. Group, 2001], since the NORSOK standard I-104 state that "the density shall be measured using the vibrating element technique", i.e. by densitometer[26], cf. Section C.2.

With respect to measurement of the energy flow rate, q_e, Method 5 in Table 2.3 is considered in the present *Handbook* [Ref. Group, 2001]. That is, P and T are meas-

25 Other methods for measurement of Q listed in Table 2.1 may be included in a possible later revision of the *Handbook*, cf. Chapter 7.

26 Note that the NORSOK I-104 industry standard represents an “accepted norm” in the NPD regulations (i.e. gives *possible ways* of fulfilling the NPD requirements), and that the NPD regulations also open up for use of calculated density from GC analysis, cf. Sections 2.1.1 and C.1.

ured, Z and Z_0 are calculated from GC analysis, and a calorimeter is used to measure H_s. This has been done to reduce the complexity of the uncertainty model, without treating possible correlations between Z, Z_0 and H_s. A more general treatment of all five methods, and the implications of these, would be beyond the scope of this *Handbook* [Ref. Group, 2001][27].

Consequently, the type of fiscal gas metering stations considered in the present *Handbook* consist basically of one or several USMs (q_v), a flow computer, and instrumentation such as pressure transmitter (P), temperature element and transmitter (T), vibrating element densitometer (ρ), compressibility factors calculated from GC analysis (Z and Z_0), and a calorimeter (H_s) [Ref. Group, 2001], cf. Table 2.4.

Table 2.4. USM fiscal gas metering station equipment considered in the *Handbook*. Included is also example instrumentation used for uncertainty evaluation of a fiscal gas metering station.

Measurement	**Instrument**
Ultrasonic meter (USM)	Multipath, flow calibrated USM. Otherwise not specified.
Flow computer	Not specified.
Pressure (static), P	Not specified.
	Example: Rosemount 3051P Reference Class Pressure Transmitter [Rosemount, 2000].
Temperature, T	Not specified.
	Example: Pt 100 element (EN 60751 tolerance A) [NORSOK, 1998a].
	Rosemount 3144 Smart Temperature Transmitter [Rosemount, 2000].
Density, ρ	On-line installed vibrating element densitometer. Otherwise not specified.
	Example: Solartron 7812 Gas Density Transducer [Solartron, 1999].
Compressibility, Z and Z_0	Calculated from GC measurements. Otherwise not specified
Calorific value, H_s	Calorimeter (combustion method). Otherwise not specified

Only flow calibrated USMs are considered here, which means that the *Handbook* typically addresses sales and allocation metering stations, as well as fuel gas metering stations in which a flow calibrated USM is employed. Ultrasonic flare gas meters and fuel gas meters which are not flow calibrated, are not covered by the present *Handbook* [Ref. Group, 2001]. Sampling and the GC measurement itself are not addressed.

In general, the flow computer, USM, pressure transmitter, temperature element/transmitter, densitometer, gas chromatograph and calorimeter are not limited to

27 Other methods for measurement of q_e listed in Table 2.3 may be included in a possible later revision of the *Handbook*, cf. Chapter 7.

specific manufacturers or models, cf. Table 2.4. The temperature element and transmitter are assumed to be calibrated together. For the densitometer only on-line installed vibrating element density transducers are considered [NORSOK; 1998a]. The calorific value is assumed to be measured using a calorimeter (since possible correlation between Z, Z_0 and H_s is not accounted for). Within these limitations, the *Handbook* and the program *EMU - USM Fiscal Gas Metering Station* should cover a relatively broad range of instruments.

For uncertainty evaluation of such USM gas metering stations, a conservative "worst-case" approach may be sound and useful, in which only one instrument of each type is considered. Such an approach should give the highest uncertainty of the metering station. That means, one USM, one pressure transmitter, one temperature element/transmitter, one densitometer and one calorimeter. This instrument configuration should cover the relevant applications, and is used here for the purpose of uncertainty evaluation [Ref. Group, 2001].

With respect to measurement of P, T and ρ, Table 2.4 also gives some equipment chosen by NPD, NFOGM and CMR [Ref. Group, 2001] for the example uncertainty evaluation of a USM fiscal gas metering station described in Chapter 4. These example instruments represent typical equipment commonly used today when upgrading existing fiscal metering stations and designing new fiscal metering stations. The example temperature transmitter evaluated is the Rosemount Model 3144 Smart Temperature Transmitter [Rosemount, 2000], in combination with a Pt 100 4-wire RTD (calibrated together). The example pressure transmitter evaluated is the Rosemount Model 3051P Reference Class Pressure Transmitter [Rosemount, 2000]. The example densitometer evaluated is the Solartron 7812 Gas Density Transducer [Solartron, 1999].

2.2 Functional relationship - USM fiscal gas metering station

The functional relationships of the USM fiscal gas metering station which are needed for expressing the uncertainty model of the metering station are discussed in the following. First, the functional relationships which are used in practice are given. These are then re-written to an equivalent form which is more convenient for the purpose of accounting (in the uncertainty model) for the relevant uncertainties and systematic effects related to flow calibration of the USM.

2.2.1 Basic operational functional relationships

The measurement of natural gas using a flow calibrated ultrasonic gas flow meter (USM) can be described by the equations (cf. Section 2.1)

$$\boxed{q_v = 3600 \cdot K \cdot q_{USM}} \qquad [\mathrm{m^3/h}] , \tag{2.1}$$

$$\boxed{Q = \frac{PT_0Z_0}{P_0TZ} q_v} \qquad [\mathrm{Sm^3/h}] , \tag{2.2}$$

$$\boxed{q_m = \rho q_v} \qquad [\mathrm{kg/h}] , \tag{2.3}$$

$$\boxed{q_e = H_S Q} \qquad [\mathrm{MJ/h}] , \tag{2.4}$$

for the axial volumetric flow rate at line conditions, q_v, the axial volumetric flow rate at standard reference conditions, Q, the axial mass flow rate, q_m, and the energy flow rate, q_e, respectively[28].

Here, the correction factor[29]

$$K \equiv f(K_1, K_2, \ldots, K_M, q_v) \tag{2.5}$$

is some function, f, of the M meter factors, K_j, measured at different nominal test flow rates ("calibration points"),

---

28 It should be noted that the conversion from
- Volumetric flow rate at line conditions (q_v) to volumetric flow at standard ref. conditions (Q),
- Volumetric flow rate at line conditions (q_v) to mass flow rate (q_m), and
- Volumetric flow rate at line conditions (q_v) to energy flow rate (q_e),

can be made using different methods, involving different types of instrumentation, cf. Tables 2.1-2.3. In the present *Handbook*, method 3 of Table 2.1, method 1 of Table 2.2 and method 5 of Table 2.3 are used, respectively, as discussed in Section 2.1.2.

29 By [AGA-9, 1998; p. A-4] the function K is referred to as "calibration factor". Here this terminology will be avoided, for reasons explained in the following.

By [IP, 1987, paragraph 2.3.4] a meter factor is defined as "the ratio of the actual volume of liquid passing through the meter to the volume indicated by the meter", cf. also [API, 1987; paragraph 5.2.8.1.2].

By the same IP reference, the K-factor (meter output factor) is defined as "the number of signal pulses emitted by the meter while the unit volume is delivered". The K-factor thus has the unit of pulses per volume. For flow meters the term "calibration factor" is essentially covering what is meant by the "K-factor" [Eide, 2001b].

In this terminology (mainly originating from use of turbine meters), thus, the numbers K_j, $j = 1, \ldots, M$, are meter factors. The curve K established on basis of these M meter factors (e.g. using the methods described in Section 2.2.2) will here be referred to as the "correction factor". The terms "K-factor" and "calibration factor" will be avoided in the present document, to reduce possible confusion between USM and turbine meter terminologies.

$$K_j \equiv \frac{q_{ref,j}}{q_{USM,j}}, \qquad j = 1, \ldots, M \quad , \tag{2.6}$$

and

q_v : axial volumetric flow rate at actual pressure (P), temperature (T) and gas compositional conditions ("line conditions"),

Q: axial volumetric flow rate at standard reference conditions (P_0 and T_0), for the actual gas composition,

q_m : axial mass flow rate,

q_e : axial energy flow rate,

$q_{ref,j}$: reference value for the axial volumetric flow rate under flow calibration of the USM (axial volumetric flow rate measured by the flow calibration laboratory), at test flow rate no. j, $j = 1, \ldots, M$, at flow calibration pressure and temperature, P_{cal} and T_{cal}.

$q_{USM,j}$: axial volumetric flow rate measured by the USM under flow calibration, at test flow rate no. j, $j = 1, \ldots, M$, at flow calibration pressure and temperature, P_{cal} and T_{cal}.

q_{USM} : axial volumetric flow rate measured by the USM under field operation (at line conditions), i.e. at P, T, actual gas and actual flow rate, before the correction factor K is applied.

P : static gas pressure in the meter run ("line pressure"),

T : gas temperature in the meter run ("line temperature"),

Z : gas compressibility factor in the meter run ("line compressibility factor"),

P_0 : static gas pressure at standard reference conditions[30]: P_0 = 1 atm. = 1.01325 bara,

T_0: gas temperature at standard reference conditions: T_0 = 15 ^{o}C = 288.15 K,

Z_0 : gas compressibility factor at standard reference conditions, P_0, T_0 (for the actual gas composition),

P_{cal} : static gas pressure at USM flow calibration,

T_{cal}: gas temperature at USM flow calibration,

ρ : gas density in the meter run ("line density"), for the actual gas, P and T,

H_s : superior (gross) calorific value per unit volume [MJ/Sm^3], at standard reference conditions, P_0, T_0, and at combustion reference temperature 25 ^{o}C [ISO 6976, 1995c],

m: molar mass [kg/mole].

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30 In the present *Handbook*, Q and q_e and their uncertainties are specified at standard reference conditions, P_0 = 1 atm. and T_0 = 15 ^{o}C. However, as the acual values for P_0 and T_0 are used only for calculation of Q and q_e, their calculated uncertainties at standard reference conditions will be representative also for *normal* reference conditions, P_0 = 1 atm. and T_0 = 0 ^{o}C.

Since $q_{USM,j}$ and q_{USM} are measurement values obtained using *the same meter* (at the flow laboratory and in the field, respectively), $q_{USM,j}$ and q_{USM} are partially correlated. That is, some contributions to $q_{USM,j}$ and q_{USM} are correlated, while others are uncorrelated.

2.2.2 Flow calibration issues

The correction factor is normally calculated by the USM manufacturer or the operator of the gas metering station. As discussed e.g. in the AGA-9 report [AGA-9, 1998], there are in general several ways to calculate the correction factor K from the M meter factors K_j, $j = 1, \ldots, M$. Some suggested methods of establishing the correction factor from the meter factors are[31]:

(a) *Single-factor correction.* In this case, a single (constant) correcion value may be used over the meter's expected flow range, calculated e.g. as:
- the flow-weighted mean error (FWME) [AGA-9, 1998][32],
- the weighted mean error (WME) [OIML, 1989][33], or
- the average meter factor.

Single-factor correction is effective especially when the USM's flow measurement output is linear (or close to linear) over the meter's flow rate operational range.

(b) *Multi-factor correction.* In this case, multiple correcion values are used over the meter's expected flow range, calculated using a more sophisticated error correction scheme over the meter's range of flow rates, e.g:

---

31 By [AGA-9, 1998], the "single factor correction" is referred to as "single calibration-factor correction". As explained in the footnote accompanying Eq. (2.5), this terminology will be avoided here.

32 The (constant) correction factor calculated on basis of the FWME is given as [AGA-9, 1998]

$$K = \frac{100}{100 + FWME}, \qquad \text{where} \qquad FWME = \frac{\sum_{j=1}^{M} \left(q_{USM,j}/q_{max}\right) Dev_{U,j}}{\sum_{j=1}^{M} q_{USM,j}/q_{max}}$$

is the flow-weighted mean error, and

$$Dev_{U,j} = \frac{q_{USM,j} - q_{ref,j}}{q_{ref,j}} = \frac{1}{K_j} - 1$$

is the (uncorrected) deviation at test flow rate no. j, $j = 1, \ldots, M$.

33 According to the "OIML method" [OIML, 1989], the weighted mean error (WME) is calculated as the flow-weighted mean error (FWME) (see the footnote above), with the exception that at the highest flow rate, $q_{USM,j}/q_{max} \approx 1$, the weight factor is 0.4 instead of 1.0.

- a piecewise linear interpolation method [AGA-9, 1998],
- a multi-point (higher-order) polynomial algorithm [AGA-9, 1998], or
- a regression analysis method.

Multi-factor correction is useful when the USM's flow measurement output is nonlinear over the meter's flow rate operational range.

The correction method formulated in Eqs. (2.1)-(2.4), and used below, covers both method (a) (single factor correction) and the more general method (b) (multi-factor correction).

2.2.3 Alternative functional relationship

Eqs. (2.1)-(2.4) are the expressions used in practice, for the USM field measurements in the type of gas metering stations addressed here (cf. Section 2.1.2). However, Eq. (2.1) as it stands is not very well suited as a functional relationship to be used as the basis for deriving an uncertainty model of the USM gas metering station. In the uncertainty model one needs to account (among others) for the uncertainty related to use of the correction factor, K, the uncertainty of the flow calibration laboratory, the partial correlation between the USM field measurement and the USM flow calibration measurement, and the cancelling of systematic effects by performing the flow calibration. To model such effects by the functional relationship, Eq. (2.1) is rearranged to some extent in the following.

Let test flow rate point no. j be the calibration point closest to the actually measured flow rate. In the vicinity of this test point no. j, the correction factor K may conveniently be written as

$$K \equiv f(K_1, K_2, \ldots, K_M, q_v) = K_j \left[\frac{f(K_1, K_2, \ldots, K_M, q_v)}{K_j} \right] \equiv K_j \cdot K_{dev,j} \qquad (2.7)$$

where the deviation factor at test flow rate no. j, $K_{dev,j}$, is defined as the ratio of the correction factor and the meter factor no. j,

$$K_{dev,j} \equiv \frac{K}{K_j} \ . \qquad (2.8)$$

K, K_j and $K_{dev,j}$ are all very close to unity. From Eqs. (2.6)-(2.8) the functional relationship (2.1) becomes

$$\boxed{q_v = 3600 \ K_{dev,j} \frac{q_{ref,j}}{q_{USM,j}} q_{USM}} \ . \qquad (2.9)$$

It should be noted that Eqs. (2.9) and (2.1) are equivalent. While Eq. (2.1) is the expression used in practice for the field measurement, Eq. (2.9) is better suited for deriving the uncertainty model for the USM gas metering station, as motivated above.

An interpretation of the deviation factor $K_{dev,j}$ is needed as a basis for specifying the necessary uncertainty of $K_{dev,j}$ as input to the uncertainty calculations (cf. Section 3.3.2). For this purpose an example of a USM flow calibration correction discussed by [AGA-9, 1998] serves to be convenient, cf. Fig. 2.1. The figure shows the percentage relative deviation[34] for a set of flow calibration data (uncorrected, open circles) at six test rate points, j = 1, …, 6 (M = 6 here), defined as $Dev_{U,j} = (q_{USM,j} - q_{ref,j})/q_{ref,j}$. Shown is also the percentage corrected relative deviation at test point j after multiplication of $q_{USM,j}$ by a constant correction factor, K (in this example calculated on basis of the FWME[35]) (shown with triangles). The corrected relative deviation is defined as

$$Dev_{C,j} = \frac{Kq_{USM,j} - q_{ref,j}}{q_{ref,j}} = \frac{K}{K_j} - 1 , \qquad j = 1, \ldots, M \quad . \tag{2.10}$$

Now, from Eqs. (2.10) and (2.8), the deviation factor can be written as

$$K_{dev,j} = 1 + Dev_{C,j} \quad . \tag{2.11}$$

For example, at q/q_{max} = 0.05, 0.1 and 1.0, one finds $K_{dev,1}$ = 1.01263, $K_{dev,2}$ = 1.00689 and $K_{dev,10}$ = 0.99943, respectively, in this example (Fig. 2.1). The close relationship between $Dev_{C,j}$ and $K_{dev,j}$ is the background for referring to $K_{dev,j}$ as the "deviation factor".

It should be noted that in general, the correction factor K is not necessarily equal to the meter factor K_j at test flow rate no. j. That will depend on the method used for calculation of K (cf. e.g. the above FWME example, where $K \neq K_j$). However, *if* K is calculated so that $K = K_j$ at test flow rate no. j, one will have $Dev_{C,j}$ = 0, and $K_{dev,j}$ = 1. So will be the case if e.g. a piecewise linear interpolation method is used for calculation of K.

---

34 By [AGA-9, 1998], the ordinate of Fig. 2.1 is called "percent error". Here, the term "error" will be avoided in this context, since error refers to comparison with the (true) value of the flow rate. Fig. 2.1 refers to comparison with the reference measurement of the flow calibration laboratory (cf. Eq. (2.10)), and hence the term "percentage relative deviation" is preferred here.

35 In the present example (cf. Fig. 2.1) the applied correction factor is constant and equal to K = 1.0031 [AGA-9, 1998].

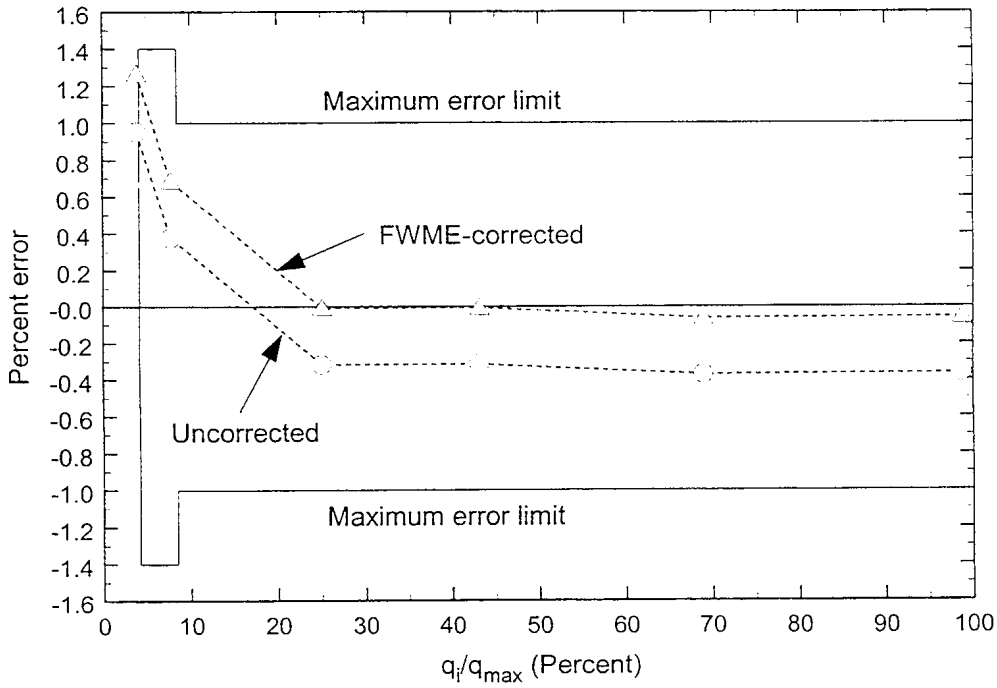


Fig. 2.1 Example of uncorreced ($Dev_{U,j}$) and FWME-corrected ($Dev_{C,j}$) flow-calibration data for a 8" diameter USM. Copied from the AGA Report No. 9, *Measurement of Gas by Multipath Ultrasonic Meters* [AGA-9, 1998], with the permission of the copyright holder, American Gas Association. (The figure was based on data provided by Soutwest Research Institute, San Santonio, Texas.)

2.3 Functional relationship - Multipath ultrasonic gas flow meter

The functional relationship of the USM gas metering station, given by Eqs. (2.9) and (2.2)-(2.4), involves the volumetric flow rates measured by the USM in the field and at the flow calibration laboratory, q_{USM} and $q_{USM,j}$, respectively. To account for the systematic effects which are eliminated by flow calibration, as well as correlated and uncorrelated effects between q_{USM} and $q_{USM,j}$, the functional relationship of the USM is needed.

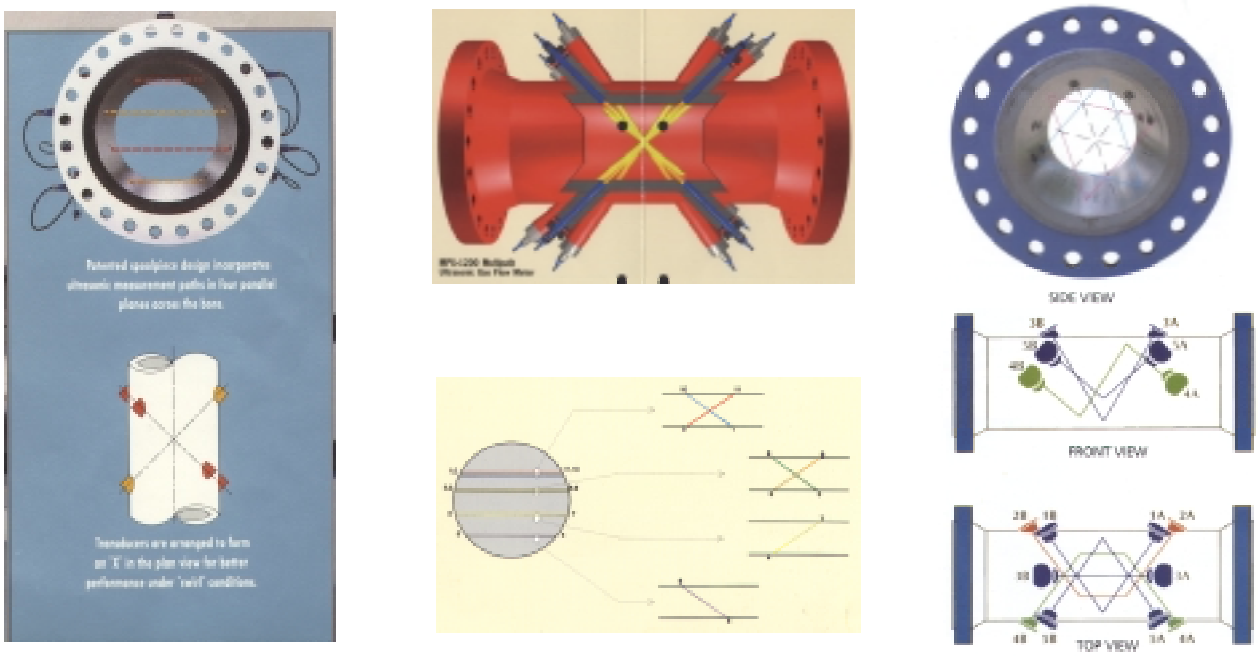
2.3.1 Basic USM principles

A multipath ultrasonic transit time gas flow meter is a device consisting basically of a cylindrical meter body ("spoolpiece"), ultrasonic transducers typically located along the meter body wall, an electronics unit with cables and a flow computer [ISO, 1997], [AGA-9, 1998], [Lunde *et al.*, 2000a], cf. Figs. 1.1, 2.2 and 2.3 The transducers are usually mounted in transducer ports and in direct contact with the gas stream, using gas-tight seals (o-rings) to contain the pressure in the pipe.

USMs derive the volumetric gas flow rate by measuring electronically the transit times of high frequency sound pulses. Transit times are measured for sound pulses propagating across the pipe, at an angle with respect to the pipe axis, downstream with the gas flow, and upstream against the gas flow (cf. Figs. 2.2 and 2.3). Multiple paths are used to improve the measurement accuracy in pipe configurations with complex axial flow profiles and transversal flow.

Different types of path configurations are used in USMs available today, including non-reflecting and reflecting path configurations. Fig. 2.2 illustrates some concepts used, corresponding to Fig. 1.1. The present *Handbook* accounts for USMs with reflecting paths as well as USMs with non-reflecting paths.

For each of the N acoustic paths, the difference between the upstream and downstream propagating transit times is proportional to the average gas flow velocity along the acoustic path. The gas flow velocities along the N acoustic paths are used to calculate the average axial gas flow velocity in the meter run, which (in the USM) is multiplied with the pipe cross-sectional area to give the volumetric axial gas flow rate at line conditions.



(a) (b) (c)

Fig. 2.2 Three different types of USM path configurations used today:
(a) Daniel *SeniorSonic* 4-path non-reflecting-path configuration [Daniel, 2000],
(b) Kongsberg *MPU 1200* 6-path non-reflecting-path configuration [Kongsberg, 2000],
(c) Instromet *Q-Sonic* 5-path reflecting-path configuration [Instromet, 2000].
Copied from the respective sales brochures with the permission of Daniel Industries, FMC Kongsberg Metering and Instromet.

Normally, USMs used for sales metering of gas are (1) “dry calibrated” in the factory and (2) flow calibrated in an accredited flow calibration laboratory[36], before installation for duty in the gas line [AGA-9, 1998].

The USM metering principle is described elsewhere (cf. e.g. [AGA-9, 1998], [Lunde *et al.*, 2000a]), and details of that are not further outlined here, except for the functional relationship described in the following, which is needed for the uncertainty analysis of Chapter 3.

2.3.2 Alternative USM functional relationships

A schematic illustration of a single path in a multipath ultrasonic gas flow meter (USM) is shown in Fig. 2.3. Four different formulations of the functional relationship of the USM measurement are relevant. These are here referred to as Formulations A, B, C and D. The four formulations are equivalent, but differ with respect to which input quantities that are used for describing the geometry of the USM[37].

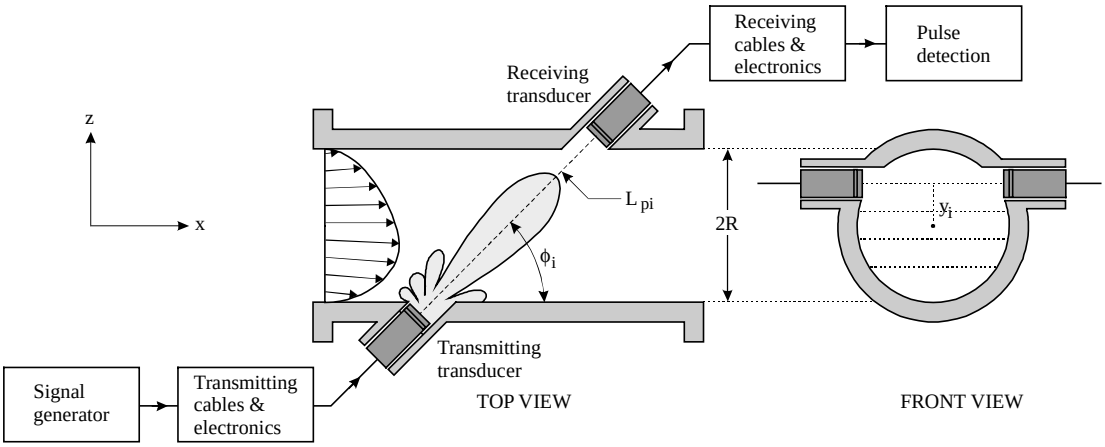


Fig. 2.3 Schematic illustration of a single path in a multipath ultrasonic transit time gas flow meter with non-reflecting paths (for downstream sound propagation). (Left: centre path example (y_i = 0); Right: path at lateral chord position y_i.)

36 Note that due to limitations in pressure and temperature of current accredited flow calibration laboratories (e.g. 10 °C and 50 bar maximum pressure), the deviations between line conditions and flow calibration conditions may be significant, especially for line applications with temperatures of 50-60 °C, or pressures of 150 - 200 bar. This has implications for the meter body uncertainty (cf. Sections 2.3.4 and 3.4.1), as well as for systematic effects on transit times (cf. Section 3.4.2).

37 With respect to the meter independency of the uncertainty model for the USM fiscal gas metering station, cf. Sections 3.3 and 5.3.

The expressions for the four formulations A, B, C and D are summarized briefly in Table 2.5, along with the geometrical quantities involved in the respective formulations.

Table 2.5. Alternative and equivalent functional relationships for multipath ultrasonic transit time flow meters.

Formulation	**Functional relationship**	**Geometrical quantities involved *(i = 1, …, N)*.**
A	$q_{USM} = 2\pi R^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i}+1)\sqrt{R^2 - y_i^2}\,(t_{1i} - t_{2i})}{t_{1i} t_{2i} \lvert \sin 2\phi_i \rvert}$	R, y_i and ϕ_i
B	$q_{USM} = \pi R^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i}+1) L_i (t_{1i} - t_{2i})}{2 t_{1i} t_{2i} \cos\phi_i}$	R, L_i and ϕ_i
C	$q_{USM} = \pi R^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i}+1) L_i^2 (t_{1i} - t_{2i})}{2 x_i t_{1i} t_{2i}}$	R, L_i and x_i
D	$q_{USM} = \pi R^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i}+1) \left\lvert x_i^2 + 4\left(R^2 - y_i^2\right) \right\rvert (t_{1i} - t_{2i})}{2 x_i t_{1i} t_{2i}}$	R, y_i and x_i

Here, the following terminology is used:

R: inner radius of the USM meter body,

y_i: lateral distance from the pipe center (lateral chord position) for path no. i, $i = 1, \ldots, N$,

ϕ_i: inclination angle (relative to the pipe axis) of path no. i, $i = 1, \ldots, N$,

L_i: (a) *For non-reflecting paths ($N_{refl,i} = 0$):* interrogation length of path no. i, $i = 1, \ldots, N$ (i.e., the length of the portion of the inter-transducer centre line lying inside a cylinder formed by the meter body's inner diameter).

(b) *For reflecting paths ($N_{refl,i} > 0$):* the portion of the distance from the transmitting transducer front to the first reflection point, which is lying inside a cylinder formed by the meter body's inner diameter, for path no. i, $i = 1, \ldots, N$. This length is $1/(N_{refl,i}+1)$ of the total propagation distance (interrogation length) of the acoustic path inside the pipe.

x_i: length of the projection of L_i along the pipe axis (x-direction), for path no. i, $i = 1, \ldots, N$.

w_i: integration weight factor for path no. i, $i = 1, \ldots, N$,

t_{1i}: transit time for upstream sound propagation of path no. i, $i = 1, \ldots, N$,

t_{2i}: transit time for downstream sound propagation of path no. i, $i = 1, \ldots, N$,

$N_{refl,i}$: number of wall reflections for path no. i, $i = 1, \ldots, N$,

N: number of acoustic paths.

With respect to $(N_{refl,i}+1)$, this factor has been introduced here to account both for USMs with reflecting paths and USMs with non-reflecting paths (cf. [Frøysa et al., 2001]). For example, in the 4-path Daniel SeniorSonic [Daniel, 2000] and the 6-path Kongsberg MPU 1200 [Kongsberg, 2000] meters, no reflecting paths are used, and $N_{refl,i} = 0$. For the 5-path Instromet Q-sonic [Instromet, 2000], three single- and two double reflecting paths are used, and $N_{refl,i} = 1$ and 2, respectively.

Formulation A was used in [Lunde *et al.*, 1997] and [Lunde *et al.*, 2000a], with the exception that the factor $(N_{refl,i}+1)$ has been included here to account also for USMs with reflecting paths. Formulation C corresponds to the expression given by [ISO, 1997] (Eq. (23) of that document), except for the factor $(N_{refl,i}+1)$.

For the flow calibration measurement of the USM, $q_{USM,j}$, similar expression as given in Table 2.5 apply.

2.3.3 Choice of USM functional relationship and meter independency aspects

It should be highly emphasized that for the uncertainty model of the USM gas metering station presented in Chapter 3, and for evaluation of the expanded uncertainty of the metering station using the Excel program *EMU - USM Fiscal Gas Metering Station* described in Chapter 5, the actual choice of USM formulation A, B, C or D is *not* essential or critical.

The formulations A, B, C, or D differ only by which meter body geometry parameters that are used as input quantities, cf. Table 2.5. By the flow calibration, measurement uncertainties and out-of roundness effects on the meter body geometry (systematic effects) are eliminated (cf. Table 1.4), so that only dimensional changes due to possible pressure and temperature differences are left to be accounted for in the model. The description of such dimensional changes is independent of USM formulation A, B, C or D, and can be described similarly using any of these. For the uncertainty model of a *flow calibrated* USM, it is thus not important which geometry parameters that have actually been *measured* on the USM meter body at hand, in the "dry calibration"[38].

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38 For a USM for which flow calibration has *not* been made, i.e. the USM has been subject only to "dry calibration", the situation would be somewhat different. In this case, the measurement uncertainties and out-of roundness effects on the meter body geometry (systematic effects) would have to be accounted for in the model (such as in the *GARUSO* model, cf. [Lunde et al., 1997,

The main reason for introducing the four formulations, as done in Section 2.3.2, and for making a choice of one of them for the following description, is threefold:

(1) Firstly, a specific USM formulation is needed *in the process* of deriving a mathematically and formally valid uncertainty model, to sort out which uncertainty terms that are to be accounted for and not, and how they shall appear in the model (correlation aspects, etc.). This process is described in Appendix E. However, *after* this process, uncertainty terms are associated and assembled in groups, so that the resulting uncertainty model, Eq. (3.6), becomes essentially independent of USM formulation. In fact, this meter independent uncertainty model, Eq. (3.6), would be the result irrespective of which USM formulation A, B, C or D that were used for the formal derivation of the model.

(2) With respect to USM related input uncertainties, the user has the option to run the program *EMU - USM Fiscal Gas Metering Station* at two levels (cf. Sections 1.3, 3.4 and 5.10):

(a) An "overall level", where the relative standard uncertainties E_{rept} and $E_{USM,\Delta}$ (cf. Eq. (3.19)) are given directly. In this operational mode the uncertainty model is *completely meter independent* (does not involve formulations A, B, C, or D at all).

(b) A more "detailed level", where E_{rept}, $E_{USM,\Delta}$ or both, are calculated from more basic input (cf. Eqs. (3.20)-(3.47)). In this operational mode, sensitivity coefficients have to be calculated in the program. Therefore, one of the formulations A, B, C and D has to be chosen, for calculation of these coefficients. However, the choice of formulation is arbitrary; - i.e. the uncertainty model does not depend on *which* of the formulations A, B, C and D that is actually used[39].

2000]). The choice of formulation A, B, C and D would then be more important. This complication may be overcome e.g. as follows:

As an example, assume formulation A is used. For USMs for which other geometry quantities than R, y_i and ϕ_i are used as input quantites, such as one of the sets $\{R, L_i, \phi_i\}$, $\{R, L_i, x_i\}$ or $\{R, y_i, x_i\}$, the uncertainty of such USMs may be evaluated in terms of formulation A by calculating the combined standard uncertainties of R, y_i and ϕ_i from the standard uncertainties of the sets $\{R, L_i, \phi_i\}$, $\{R, L_i, x_i\}$ or $\{R, y_i, x_i\}$, respectively, and using these combined standard uncertainties of R, y_i and ϕ_i as input to the uncertainty calculations.

39 Sensitivity coefficients involving the USM functional relationship formulation appear in the uncertainty terms related to (1) repeatability of the USM in field operation, E_{rept}, and (2) the USM field operation of the USM, $E_{USM,\Delta}$, cf. Eqs. (3.19)-(3.20).

(3) Thirdly, for description of uncertainties related to dimensional changes of the meter body, caused by possible pressure and temperature differences from flow calibration to field operation, one of the formulations A, B, C or D is needed, since Eqs. (2.20)-(2.22) are not generally valid, cf. Section 2.3.4.

For these three reasons, one has to choose one of the four formulations A, B, C or D, given by Eqs. (2.12)-(2.15), as the USM functional relationship.

In the present document, Formulation A is used, i.e.

$$\boxed{q_{USM} = 2\pi R^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i}+1)\sqrt{R^2 - y_i^2}\,(t_{1i} - t_{2i})}{t_{1i} t_{2i} \left|\sin 2\phi_i\right|}} \quad , \tag{2.12}$$

involving the geometrical quantities R, y_i and ϕ_i, i = 1, ..., N, i.e., the meter body inner radius, the lateral chord positions and the inclination angles of the paths.

2.3.4 Pressure and temperature corrections of USM meter body

Pressure and temperature effects on the meter body dimensions (expansion/contraction, cf. Fig. 2.4) act as systematic effects and appear directly as errors in the flow rate measurement result, q_v, if not corrected for. Such errors in q_v become significant, e.g. about 0.1 %, for a temperature difference of 24 ^{o}C, or a pressure difference of 36 bar (for a 12” USM example with wall thickness 8.4 mm), relative to the P and T conditions at flow calibration[40]. If such temperature and pressure effects

With respect to E_{rept}, the sensitivity coefficients are completely independent of which formulation A, B, C and D that is used, cf. Eqs. (3.43)-(3.44) and the text accompanying Eq. (3.28).

With respect to $E_{USM,\Delta}$, sensitivity coefficients appear in connection with the uncertainty terms $E_{time,\Delta}$ and $E_{body,\Delta}$, cf. Eq. (3.20). For $E_{time,\Delta}$, the sensitivity coefficients are completely independent of formulation, for the same reason as given in connection with E_{rept} above.

For $E_{body,\Delta}$, the sensitivity coefficients are given by Eqs. (3.25)-(3.27). $E_{body,\Delta}$ accounts for uncertainties related to meter body quantities. In the case studied here, with flow calibration of the USM, only uncertainties related to pressure and temperature expansison/contraction of the meter body are to be accounted for in $E_{body,\Delta}$ (cf. Table 1.4). In this case, since the P and T effects on the geometrical quantities are correlated (cf. Eq. (3.21)), it can be shown that $E_{body,\Delta}$ is independent of *which* of the formulations A, B, C and D that is actually used.

40 These results can be calculated e.g. using Eqs. (2.20)-(2.22) below, with USM meter body material data as given in Table 4.3, and Eq. (2.19) used for the radial pressure expansion coefficient, β. The results are consistent with the analysis and results given in the AGA-9 report [AGA-9, 1998, Section 5.1.1]. Note that other models for β (cf. Table 2.6) would give other figures for the pressure expansion/contraction.

work in opposite "directions" (e.g. expansion due to pressure increase and contraction due to a temperatur decrease, relative to the reference conditions), they will cancel each other. However, if they work in the same "direction", that would result in an error of 0.2 %. Over time a systematic error of this magnitude may accumulate to significant economic values, so correcting for the error is often recommended.

For example, the NPD regulations [NPD, 2001] state that "Correction shall be made for documented measurement errors. Correction shall be carried out if the deviation is larger than 0.02 % of the total volume" (cf. Section C.1). Pressure and temperature correction is also addressed by the AGA-9 report[41].

To correct for the small dimensional changes (expansion/contraction) of the meter body caused by the operational pressure and temperature in the field (cf. Fig. 2.4), some meter manufacturers have implemented correction factors for such dimensional changes. Note that today such corrections address the dimensional changes of the meter body, but not necessarily the effect of dimensional changes of the transducers[42].

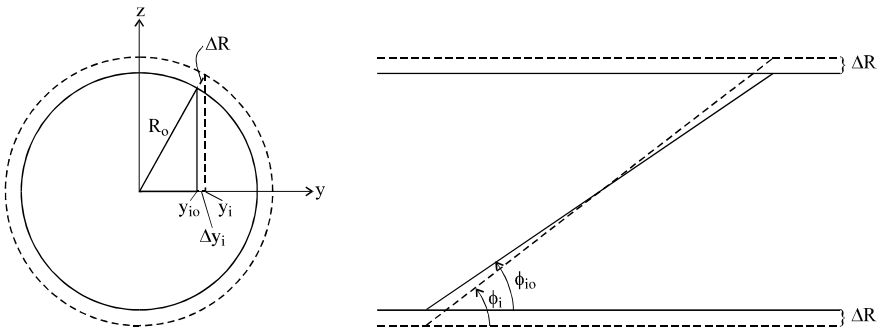

Fig. 2.4 Geometry for acoustic path no. *i* in the USM, showing lateral change with *T* and *P* (schematically).

At a temperature *T* and pressure *P*, the meter body radius (*R*), the lateral chord positions (y_i), and the inclination angles (ϕ_i), are (cf. Appendix D)

41 For each individual USM, the AGA-9 report recommends measurement and documentation of relevant dimensions of the meter at atmospheric conditions and a temperature 20 °C, as a part of the "dry calibration" of the USM [AGA-9, 1998]. This concerns the average inner diameter of the meter body, the length of each acoustic path between transducer faces, the axial distance between transducer pairs, and inclination angles. Some recommendations for measurements of these quantities in the factory are given in the AGA-9 report.

42 In the program *EMU - USM fiscal gas metering stations*, the effects of transducer expansion/contraction due to pressure and temperature effects can be accounted for by including such effects in the standard uncertainties of the coefficients of linear thermal and pressure expansion, $u(\alpha)$ and $u(\beta)$, respectively. Cf. Section 3.4.1.

An alternative approach would be to account for such effects in the functional relationship of the USM, and that may be included in possible future upgrades of the *Handbook*.

$$R \approx K_T K_P R_0 \tag{2.13}$$

$$y_i \approx K_T K_P y_{i0} \tag{2.14}$$

$$\phi_i \approx tan^{-1}\left(\frac{tan(\phi_{i0})}{1-(1-\beta^*/\beta)(K_P-1)}\right), \qquad i = 1, \ldots, N \tag{2.15}$$

where subscript "*0*" is used to denote the relevant geometrical quantity at "dry calibration" conditions, i.e. R_0, y_{i0} and ϕ_{i0}. The correction factors for the inner radius of the meter body due to dimensional changes caused by temperature and pressure changes relative to "dry calibration" conditions, are given as (cf. e.g. [API, 1981; 1995], [IP, 1989], [AGA-9, 1998])

$$K_T \equiv 1+\alpha\Delta T_{dry}\ , \qquad \Delta T_{dry} \equiv T\text{-}T_{dry}\ , \tag{2.16}$$

$$K_P \equiv 1+\beta\Delta P_{dry}\ , \qquad \Delta P_{dry} \equiv P\text{-}P_{dry}\ , \tag{2.17}$$

respectively, where P_{dry} and T_{dry} are the pressure and temperature at "dry calibration" conditions, e.g. P_{dry} = 1 atm. and T_{dry} = 20 ^{o}C [43]. α is the coefficient of linear thermal expansion of the meter body material. β and β* are the *radial* and *axial* linear pressure expansion coefficients for the meter body, respectively.

For convenience, K_P and K_T are here referred to as the *radial* pressure and temperature correction factors for the USM meter body, respectively[44]. β and β* depend on the type of support provided for the meter body installation (i.e. the model used for the meter body pressure expansion / contraction). Table 2.6 gives different models in use for β, and corresponding expressions for β* are given in Appendix D. Note that all models in use represent simplifications. For thin-walled cylindrical and isotropically elastic meter bodys, β and β* are releated as (cf. Appendix D)

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43 Note that the actual values of the temperature and pressure at "dry calibration" (P_{dry} and T_{dry}) are never used in the uncertainty model of the gas metering station, i.e. they are not used at all in the program *EMU - USM Fiscal Gas Metering Station*.

This is so because the relevant reference temperature and pressure with respect to meter body expansion / contraction are not the "dry calibration" temperature and pressure, but the temperature and pressure at flow calibration of the USM, cf. Section 3.4.1.

44 The *radial* pressure and temperature correction factors for the USM meter body, K_P and K_T, should not be confused with the corresponding *volumetric* pressure and temperature correction factors of the meter body, C_{psm} and C_{tsm}, cf. e.g. Eqs. (2.21)-(2.22).

$$\frac{\beta^*}{\beta} \approx \begin{cases} -\sigma = -0.3 & \text{for the cylindrical pipe section model (ends free),} \\ 0 & \text{for the ininite-length cylindrical pipe model (ends clamped),} \\ \dfrac{1-2\sigma}{2-\sigma} = 0.235 & \text{for the cylindrical tank model (ends capped),} \end{cases} \quad (2.18)$$

where the values inserted for β^*/β apply to steel (σ = 0.3).

Table 2.6. Models used by USM manufaturers etc. for linear pressure expansion of the inner radius of the USM meter body (isotropic material assumed), under uniform internal pressure.

Reference / USM manufacturer	**Models for the coefficient of linear radial pressure expansion, β**	**USM meter body assumptions**
[AGA-9, 1998], [Roark, 2001, p. 592]	$\beta = \frac{R_0}{wY}$	• Cylindrical pipe section model (ends free) • Thin wall, $w < R_0/10$
Daniel Industries [Daniel, 1996, 2001]	$\beta = \frac{1}{Y}\frac{1.3(R_0+w)^2 + 0.4R_0^{\ 2}}{(R_0+w)^2 - R_0^{\ 2}}$ ($\beta \approx 0.85\frac{R_0}{wY}$ for $w << R_0$)	• Cylindrical tank model (pipe with ends capped) • Thick wall • Steel material (σ = 0.3)
FMC Kongsberg Metering [Kongsberg, 2001], [Roark, 2001, p. 593]	$\beta = \frac{R_0}{wY}\left(1-\frac{\sigma}{2}\right)$ ($\beta = 0.85\frac{R_0}{wY}$ for σ = 0.3 (steel))	• Cylindrical tank model (pipe with ends capped) [45] • Thin wall, $w < R_0/10$
Instromet [Autek, 2001]	No P or T correction used. Pressure expansion analysis based on: $\beta = 0.5\frac{R_0}{wY}$	• Infinitely long pipe model (ends clamped, no axial displacement) • Radial expansison assumed to be = 0.5 · radial expansion for ends-free model • Thin wall, $w < R_0/10$

45 In [API, 1981], [API, 1995], [IP, 1989] the volumetric pressure correction factor C_{psp} for a cylindrical container (such as a conventional pipe prover, a tank prover, or a test measure), is given as $C_{psp} \approx 1 + (2R_0/wY)\Delta P$.

This is relatively close to the volumetric pressure correction factor C_{psp} for a cylindrical tank (pipe with ends capped) which is given as $C_{psp} = (1+\beta\Delta P)^2(1+\beta^*\Delta P) \approx 1+(2\beta+\beta^*)\Delta P$, where β^* is the coefficient of linear pressure expansion for the axial displacement of the USM meter body. For a thin-walled cylindrical tank one has [Roark, 2001, p. 593] $\beta \approx (R_0/wY)(1-\sigma/2)$ and $\beta^* \approx (R_0/wY)(0.5-\sigma)$, giving $C_{psp} \approx 1+(2R_0/wY)(1.25-\sigma)\Delta P$. For steel ($\sigma$ = 0.3) this gives $C_{psp} \approx 1+(1.95R_0/wY)\Delta P$.

[IP, 1989] state that their expression is based upon a number of approximations that may not hold in practical cases. For values of $(C_{psp}-1)$ that do not exceed 0.00025, the value of $(C_{psp}-1)$ may sometimes be subject to an uncertainty of ±20 %. For higher values of $(C_{psp}-1)$ (such as for high pressure differences), the uncertainty may be larger.

Here the following terminology has been used,

w	=	Average wall thickness of the meter body [m],
R_0	=	Average inner radius of the meter body, at "dry calibration" conditions [m],
Y	=	Young's modulus (modulus of elasticity) of the meter body material,
σ	=	Poisson's ratio of the meter body material (dimensionless).

The "cylindrical pipe section model" [Roark, 2001, p. 592] applies to a finite-length pipe section with free ends and does not account for flanges, bends, etc. The "cylindrical tank model" (pipe with ends capped) [Roark, 2001, p. 593] may to some extent apply to installation in a pipe section between bends, and does not account for flanges or very long pipe installations. Both models allow for displacement of the pipe in axial direction, but in different directions [Roark, 2001, pp. 592-3]. The infinitely-long-pipe model (ends clamped) used by Instromet [Autek, 2001] assumes no axial displacement and that the radial displacement is at maximum half the value of the ends-free model[46]. The ends-clamped model may be relevant for installation of the USM in a long straight pipeline. Relative to the ends-free model, the ends-capped model and the ends-clamped model used by Instromet differ by about 15 % and 50 %, for thin-walled steel pipes, as can easily be seen from the table.

For simplicity (to limit the number of "cases")[47], and as a possible "worst case" approach (since it gives about 15 % larger pressure expansion than the cylindrical tank model), it has for the present *Handbook* been chosen to use the expression for β which has been applied by [AGA-9, 1998], namely

$$\beta = \frac{R_0}{wY} \ , \tag{2.19}$$

i.e. the expression corresponding to the "cylindrical pipe section model (ends free)".

It should also be noted that for USMs where all inclination angles are equal to $\pm 45^{o}$, i.e, $\phi_{i0} = \pm 45^{o}$, $i = 1, \ldots, N$, Eqs. (2.12)-(2.17) can be written as (cf. Appendix D)

$$q_{USM} \approx q_{USM,0} \cdot C_{tsm} \cdot C_{psm} \ , \tag{2.20}$$

46 The assumption that the radial displacement is at maximum half the of the value of the ends-free model was a tentative estimate [Autek, 2001].

47 In a possible future revision of the *Handbook*, various models for β may be implemented, cf. Chapter 7.

where

$$C_{tsm} = K_T^3 = (1 + \alpha \Delta T_{dry})^3 \approx 1 + 3\alpha \Delta T_{dry} \quad , \tag{2.21}$$

$$C_{psm} = K_P^3 = (1 + \beta \Delta P_{dry})^3 \approx 1 + 3\beta \Delta P_{dry} \quad , \tag{2.22}$$

are the *volumetric* thermal and pressure correction factors of the USM meter body[48], and $q_{USM,0}$ is given by Eqs. (2.12)-(2.15), with the "dry calibration" quantities R_0, y_{i0}, L_{i0}, x_{i0} and ϕ_{i0} inserted instead of the quantities R, y_i, L_i, x_i and ϕ_i, $i = 1, \ldots, N$.

The relationship (2.20) has been shown in Appendix D for all formulations A, B, C and D, fully consistent with the less general discussion on this topic given in [AGA-9, 1998; p. C-29].

Hence, for such meters Eqs. (2.12)-(2.17) and Eqs. (2.20)-(2.22) are equivalent[49, 50]. That means, the P and T corrections of the geometrical quantities of the meter body can be separated from the basic USM functional relationship and put outside of the summing over paths.

It should be noted that Eqs. (2.20)-(2.22) is strictly not valid expressions for meters involving inclination angles different from 45°, for which they represents an approximation (cf. Appendix D). Eq. (D.28) gives the relative error by using this approximation. It turns out that for moderate pressure deviations ΔP_{dry} (a few tens of bars), the errors made by using Eqs. (2.20)-(2.22) may be neglected, and these equa-

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48 For the correction factor of the *meter body*, a notation is used according to "common" flow metering terminology, where subsripts *t*, *p*, *s* and *m* refer to "temperature", "pressure", "steel" and "meter", respectively, cf. e.g. [IP, 1989], [API, 1981], [API, 1995], [Dahl *et al.*, 1999].

49 Eqs. (2.20)-(2.22) are used by Daniel Industries for the *SeniorSonic* USM [Daniel, 2001], for which it is a valid expression since all inclination angles in the meter are 45°. FMC Kongsberg Metering are using a correction approach of the type given by Eqs. (2.13)-(2.17) [Kongsberg, 2001].

50 By one manufacturer [Autek, 2001], alternative expressions for the temperature and pressure correction factors of the USM meter body (alternatives to Eqs. (2.21)-(2.22)) have been presented, which in the notation used here can be written as

$$C_{tsm} \approx 1 + 4\alpha \Delta T_{dry} \quad , \quad C_{psm} \approx 1 + 4\beta \Delta P_{dry} \, .$$

On basis of these expressions (which in fact predict larger dimensional changes than Eqs. (2.21)-(2.22), if the same β-model is used), it is argued [Autek, 2001] that the errors due to pressure and temperature expansion / contraction are negligible except under extreme situations (such as when the errors approach the uncertainty of the best flow calibration laboratories, about 0.3 %).

tions may be used for inclination angles in the range of relevance for current USMs, 40° to 60°. However, for larger pressure deviations, and especially for inclination angles approaching 60°, the error introduced by using Eqs. (2.20)-(2.22) increases. For example, for the 12" USM data given in Table 4.3 and angles equal to 60°, pressure deviations of e.g. ΔP_{dry} = 10 and 100 bar yield relative errors in flow rate of about $6 \cdot 10^{-5}$ = 0.006 % and $6 \cdot 10^{-4}$ = 0.06 %, respectively, by using Eqs. (2.20)-(2.22) (cf. Appendix D).

Today, the only USM manufacturer using Eqs. (2.20)-(2.22), Daniel Industries [Daniel, 1996, 2001], employs $\pm 45^o$ inclination angles, for which the two formulations (2.13)-(2.15) and (2.20)-(2.22) have been shown here to be practically equivalent (Appendix D). For this reason, and since both formulations are in use, the more generally valid equations (2.13)-(2.15) are used to describe *P* and *T* effects on the meter body in the uncertainty model of Chapter 3.

2.3.5 Transit time averaging and corrections

In practice, the transit times t_{1i} and t_{2i} for upstream and downstream sound propagation appearing in Eq. (2.12) are *time averaged* transit times.

Moreover, the measured upstream and downstream transit times t_{1i} and t_{2i} contain possible time delays due to the electronics, cables, transducers and diffraction effects (including finite beam effects), and possible cavities in front of the transducers, cf. Fig. 2.3 [Lunde *et al.*, 1997; 2000a]. To achieve sufficient accuracy of the USM, these additional time delays may have to be corrected for in the USM.

However, such time averagings and time corrections have been implemented in different ways by the different USM manufacturers, and a description of these would be meter dependent.

In order to avoid meter dependent functional relationships in the uncertainty model of the gas metering station, possible averagings and corrections of the measured transit times are not adressed here. In the uncertainty model described in Chapter 3, it is left to the USM manufacturer to specify and document the following input uncertainties:

(1) With respect to ***repeatability (random transit time effects)***: One specifies either E_{rept} (the relative standard uncertainty of the repeatability of the measured flow rate, at a particular flow rate), or $u(\hat{t}_{1i}^{random})$ (the standard uncertainty of the tran-

sit time t_{1i}, at a particular flow rate, after possible time correction) (cf. Section 3.4.2). Both approaches are meter independent, cf. Section 2.3.3[51].

(2) With respect to ***systematic transit time effects***: $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$, one specifies the standard uncertainties related to systematic effects on the transit times t_{1i} and t_{2i}, respectively (cf. Section 3.4.2). This approach is meter independent, cf. Section 2.3.3[52].

2.3.6 USM integration method

Different USM manufacturers use different integration methods, i.e. different integration weights, w_i, $i = 1, \ldots, N$. A description of these would thus be meter dependent.

In order to avoid meter dependent functional relationships in the uncertainty model of the gas metering station, the specific USM integration method is not adressed here[53].

In the uncertainty model described in Chapter 3, it is left to the USM manufacturer to specify and document $E_{I,\Delta}$, the relative standard uncertainty of the USM integration method due to change of installation conditions from flow calibration to field operation (cf. Section 3.4.3).

2.4 Functional relationship - Gas densitometer

As described in Section 1.3, the uncertainty of the gas densitometer can in the program *EMU - USM Fiscal Gas Metering Station* be specified at two levels (cf. also Chapter 5):

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51 If specific time averaging algorithms were accounted for in the repeatability of the transit times, that might be meter dependent.

52 If transit time corrections used in USMs were accounted for (such as correction for one or several time delays due to transducers, diffraction, elecronics, transducer cavities, and nonzero transit time difference (Δt) at zero flow), that would be meter dependent.

53 Very approximate values for the integration weights w_i, $i = 1, \ldots, N$, are used only to calculate certain sensitivity coefficients in the "weakly meter dependent" mode of the program, cf. Section 5.3 and Chapter 6 (Table 6.3).

(1) ***"Overall level":*** The user specifies the relative combined standard uncertainty of the density measurement, E_ρ, directly as input to the program. It is left to the user to calculate and document E_ρ first. This option is completely general, and covers any method of obtaining the uncertainty of the gas density estimate (measurement or calculation)[54].

(2) ***"Detailed level":*** E_ρ is calculated in the program, from more basic input uncertainties for the vibrating element gas densitometer, provided by the instrument manufacturer and calibration laboratory.

The following discussion concerns the "Detailed level". In this case a functional relationship of the gas densitometer is needed.

Gas densitometers considered in the "Detailed level" are based on the vibrating cylinder principle, vibrating in the cylinder's Hoop vibrational mode, cf. Fig. 2.5[55]. They consist of a measuring unit and an amplifier unit. The vibrating cylinder is situated in the measuring unit and is activated at its natural frequency by the amplifier unit. The output signal is a frequency or a periodic time (τ), which is primarily dependent upon density, and secondarily upon other parameters, such as pressure, temperature and gas composition [Tambo and Søgaard, 1997]. Any change in the natural frequency will represent a density change in the gas that surrounds the vibrating cylinder.

Here, only on-line installation of the densitometer is considered, using a by-pass gas sample line, cf. e.g. [ISO/CD 15970, 1999]. By this method, gas is extracted (sampled) from the pipe and introduced into the densitometer. From the densitometer the sample flow can either be returned to the pipe (to the sample probe or another low-pressure point) or sent to the atmosphere (by the flare system). To reduce the temperature differences between the densitometer and the line, the density transducer is

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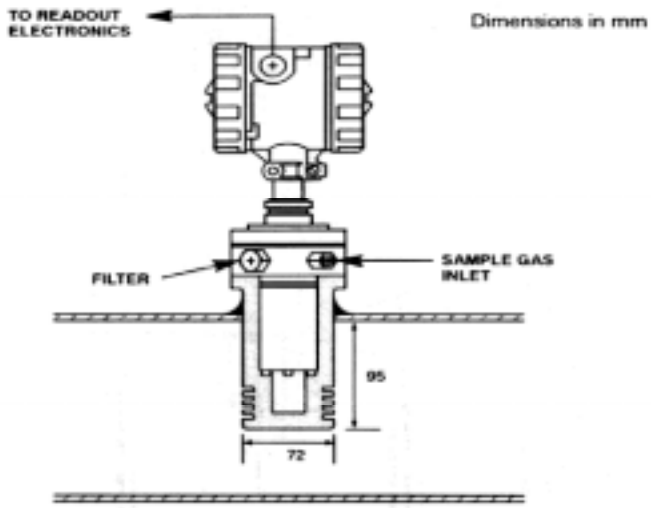
54 The "overall level" option may be of interest in several cases, such as e.g.:
- If the user wants a "simple" and quick evaluation of the influence of $u_c(\hat{\rho})$ on the expanded uncertainty of the gas metering station,
- In case of a different installation of the gas densitometer (e.g. in-line),
- In case of a different gas densitometer functional relationship than Eq. (2.28),
- In case of density measurement using GC analysis and calculation instead of densitometer measurement(s).

55 The NORSOK regulations for fiscal measurement of gas [NORSOK, 1998a, §5.2.3.7] state that "the density shall be measured by the vibrating element technique".

installed in a pocket in the main line, and the whole density transducer installation including the sampling line is thermally insulated from the ambient.

(a)



(b)

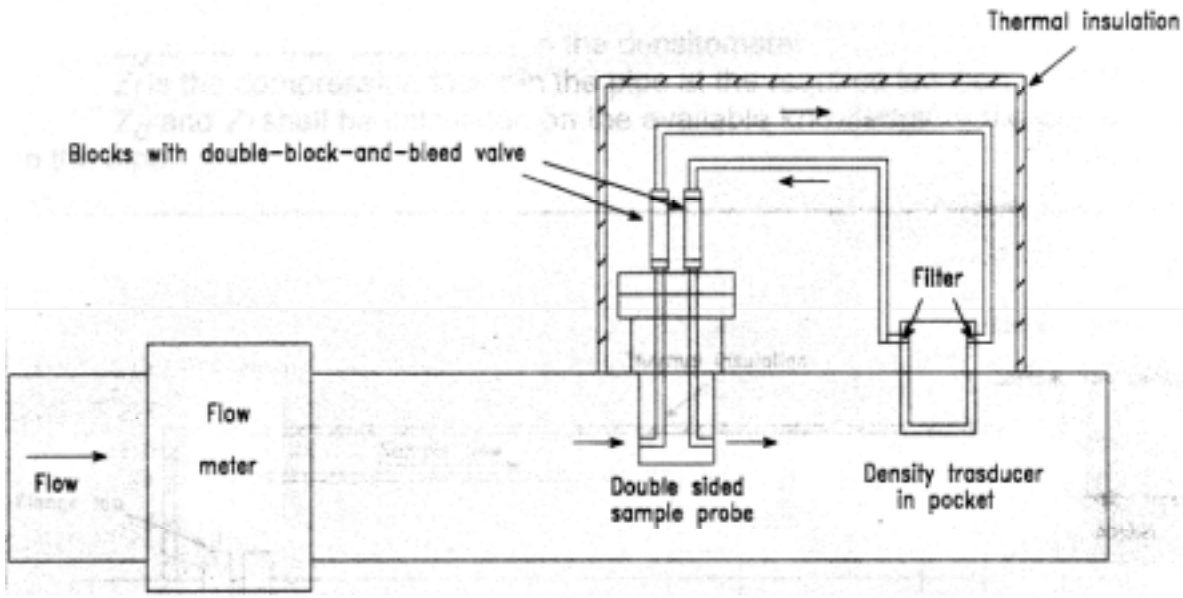


Fig. 2.5 (a) The Solartron 7812 gas density transducer [Solartron, 1999] (example). (b) Principle sketch of possible on-line installation of a gas densitometer on a gas line (figure taken from [ISO/CD 15970, 1999]).

For USM metering stations where the flow meter causes no natural pressure drop in the pipe, the sampling device (probe) may be designed to form a pressure drop, so that the pressure difference between the sample inlet hole and the sample return hole can create sufficient flow through the sample line / densitometer to be continuously representative with respect to gas, pressure and temperature [ISO/CD 15970, 1999].

The functional relationship involves a set of calibration constants, as well as temperature correction, velocity of sound (VOS) correction, and installation correction (see below).

In the following, reference will be made to the Solartron 7812 Gas Density Transducer [Solartron, 1999], a commonly used densitometer in North Sea fiscal gas metering stations. This is also the densitometer used for example calculations in Chapter 4. However, it should be emphasized that the functonal relationship described in the following is relatively general, and should apply to any on-line installed vibrating element gas density transducer.

2.4.1 General density equation (frequency relationship regression curve)

For gas density transducers based on the vibrating cylinder principle, the output is the periodic time of the resonance frequency of the cylinder's Hoop vibrational mode. The relation between the density and the periodic time is obtained through calibration of the densitometer at a given calibration temperature (normally 20 °C), on a known pure reference gas (normally nitrogen, argon or methane, due to their acknowledged properties), and at several points along the densitometer's measuring range. The calibration results are then fitted with a regression curve, $\rho_u = f(\tau, c, T, P)$ [Tambo and Søgaard, 1997]. One common regression curve is [ISO/CD 15970, 1999], [Solartron, 1999; §6.4]

$$\rho_u = K_0 + K_1\tau + K_2\tau^2, \tag{2.23}$$

where

ρ_u	- indicated (uncorrected) density, in density transducer [kg/m^3],
K_0, K_1, K_2	- regression curve constants (given in the calibration certificate),
τ	- periodic time (inverse of the resonance frequency, output from the densitometer) [μs].
c	- sound velocity of the gas surrounding the vibrating element [m/s].

The periodic time, τ, is a function of density and varies typically in the range 200 - 900 μs [Tambo and Søgaard, 1997].

The form of the regression curve can vary from manufacturer to manufacturer, and Eq. (2.23) is one example of such a curve. However, note that the form of the regression curve is actually not used in the densitometer uncertainty model, and that K_0, K_1 and K_2 are not needed as input to the uncertainty model, cf. Section 3.2.4. The present uncertainty model is thus independent of the type of regression curve used.

2.4.2 Temperature correction

When the densitometer operates at temperatures other than the calibration temperature, a correction to the density calculated using Eq. (2.23) should be made for best accuracy. In the Solartron 7812 gas density transducer, a 4-wire Pt 100 temperature element is incorporated, for installation and check purposes [Solartron, 1999]. The equation for temperature correction uses coefficient data given on the calibration certificate, and is given as [ISO/CD 15970, 1999], [Solartron, 1999; §6.5]

$$\rho_T = \rho_u \left[1 + K_{18}\left(T_d - T_c\right)\right] + K_{19}\left(T_d - T_c\right), \tag{2.24}$$

where

ρ_T	- temperature corrected density, in density transducer [kg/m^3],
K_{18} , K_{19}	- constants from the calibration certificate[56],
T_d	- gas temperature in density transducer [K],
T_c	- calibration temperature [K].

2.4.3 VOS correction

The periodic time, τ, of the vibrating cylinder is influenced by the gas compressibility (or, in other words, the gas composition), and thus on the VOS in the gas. Eqs. (2.23)-(2.24) do not account for such effects. Consequently, when the vibrating element gas densitometer is used on gases other than the calibration gases (normally nitrogen or argon), a small calibration offset may be experienced. This offset is predictable, and it may be desirable to introduce VOS corrections to maintain the accuracy of the transducer [Solartron, 1999; §6.6 and Appendix E][57].

The basic relationship for VOS correction is [ISO/CD 15970, 1999], [Solartron, 1999; §6.6, Appendix E]

$$\rho_d = \rho_T \cdot \left[\frac{1 + \left(\frac{K_d}{\tau c_c}\right)^2}{1 + \left(\frac{K_d}{\tau c_d}\right)^2} \right], \tag{2.25}$$

56 Here, the notation of [Solartron, 1999] for the calibration constants K_{18} and K_{19} is used.

57 It is stated in [Solartron, 1999; §E.1] that “the 7812 Gas Density Transducer is less sensitive to VOS influence than previous models of this instrument and, in consequence, the need to apply VOS correction is less likely. However, when it is necessary, one of the correction methods are suggested”.

where

ρ_d - temperature and VOS corrected density, in density transducer [kg/m^3],

K_d - transducer constant [μm] (characteristic length for the Hoop mode resonance pattern of the vibrating element [Eide, 2001a]), equal to $2.10 \cdot 10^4$ μm for 7812, $1.35 \cdot 10^4$ μm for 7810 and $2.62 \cdot 10^4$ μm for 7811 sensors [Solartron, 1999].

c_c - VOS for the calibration gas, at calibration temperature and pressure conditions [m/s].

c_d - VOS for the measured gas, in the density transducer [m/s].

There are several well established methods of VOS correction, and four common methods are:

1. For metering stations involving a USM, the VOS measured by the USM (averaged over the paths) is often used for c_d. This method is here referred to as the "**USM method**", and may be useful for measurement of different gases at varying operating conditions.

2. The "**Pressure/Density method**" [ISO/CD 15970, 1999], [Solartron, 1999; Appendix E] calculates the VOS (c_d) based on the line pressure and density and applies the required correction. This method has been recommended for measurement of different gases at varying operating conditions.

3. The **"User Gas Equation" method** [Solartron, 1999; Appendix E] calculates the VOS (c_d) based on the specific gravity and the line temperature, and applies a correction based on two coefficients that define the VOS characteristic. This equation is shown on nitrogen or argon calibration certificates. The User Gas Equation is an approximate correction for a typical mixture of the calibration gas (normally nitrogen or argon) and methane. This correction method is recommended by Solartron for applications where pressure data is not available, but where gas composition and temperature do change. For this method, a different (and approximate) expression for the VOS correction than Eq. (2.25) is used.

4. For measurement of gas which has a reasonably well defined composition, Solartron can supply a "**User Gas Calibration Certificate**" [Solartron, 1999; Appendix E]. This specifies modified values of K_0, K_1, K_2, K_{18} and K_{19}, in order to include the effects of VOS for the given gas composition.

In the following, VOS correction methods based *directly* on Eq. (2.25) are considered. This includes the "USM method" and the "pressure/density method"[58].

2.4.4 Installation correction

The vibrating element density transducer is assumed to be installed in a by-pass line (on-line installation), downstream of the USM. Despite thermal insulation of the by-pass density line, and precautions to avoid pressure loss, the gas conditions at the density transducer may be different from the line conditions (at the USM), especially with respect to temperature (due to ambient temperature influence), but also possibly with respect to pressure (cf. e.g. [Geach, 1994]). There may thus be need for an installation correction of the density[59]. Temperature is a critical installation consideration as a 1 ^{o}C temperature error represents a 0.3 % density error [Matthews, 1994], [Tambo and Søgaard, 1997] or more [Sakariassen, 2001][60].

In this connection it is worth remembering that the densitometer will always give the density for the gas in the density transducer. Installation errors result from the sample gas in the density transducer not being at the same temperature or pressure as the gas in the line, and hence its density is different.

With respect to temperature deviation between the density transducer and the main flow due to ambient temperature effects, [Geach, 1994] state that "The pipework should be fully insulated between these two points to reduce temperature changes and, where possible, external loop pipework should be in direct contact with the main line. Unfortunately, this can be difficult to achieve. To aid density equalization, density transducers should be installed in a thermal pocket in the main line". Temperature measurement is available in the density transducer since a Pt 100 element is integrated in the 7812 densitometer [Solartron, 1999]. The temperature

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58 The VOS correction algorithm given by Eq. (2.25) was chosen by [Ref. Group, 2001] for use in the present Handbook. Other VOS correction algorithms may be included in later possible revisions of the *Handbook*.

59 The NORSOK I-104 industry standard for fiscal measurement of gas [NORSOK, 1998a, §5.2.3.7] state that (1) "The density shall be corrected to the conditions at the fiscal measurement point", and (2) "if density is of by-pass type, temperature compensation shall be applied".

60 A tempreature change of 1 ^{o}C can correspond to much more than 0.3 % in density change, since the temperature also changes the compressibility, *Z*. In some cases the change can be as large as 0.9 % (e.g. in dry gas at 110-150 bar and 10 ^{o}C) [Sakariassen, 2001].

transmitter for this Pt 100 element may be located close to the densitometer[61] or further away, in the flow computer.

With respect to possible pressure deviation, it is emphasized by [Geach, 1994] that "careful consideration should be given to any flow control valves, filters (including transducer in-built filters), etc., installed in the external loop. These devices, if installed between the flow element measuring point and the density transducer, are liable to cause unacceptable pressure drops". The flow through the densitometer must be kept low enough to ensure that the pressure change from the main line is negligible, but fast enough to represent the changes in gas composition [Tambo and Søgaard, 1997]. Normally, pressure measurement is *not* available in the density transducer [Geach, 1994], [ISO/CD 15970, 1999]. [Geach, 1994] state that "such instrumentation should only be used as a last resort where it is not possible to ensure good pressure equalization with the meter stream". Procedures for pressure shift tests are discussed by [ISO/CD 15970, 1999], and resorts to overcome the problem of satisfying pressure and temperature equilibrium are discussed by [Geach, 1994].

From the real gas law, correction for deviation in gas conditions at the densitometer (in the by-pass line) and at the USM (line conditions) is made according to [ISO, 1999]

$$\rho = \rho_d \left(\frac{T_d}{T} \right) \left(\frac{P}{P_d} \right) \left(\frac{Z_d}{Z} \right) , \tag{2.26}$$

where

- T - gas temperature in the pipe, at the USM location (line conditions) [K],
- P - gas pressure in the pipe, at the USM location (line conditions) [bara],
- P_d - pressure in the density transducer [bara],
- Z_d - gas compressibility factor for the gas in the density transducer,
- Z - gas compressibility factor for the gas in the pipe, at USM location (line conditions)

For the uncertainty analysis of the densitometer described in Sections 3.2.4, 4.2.4 and 5.7, the following instrumentation is considered:

61 In practice, the densitometer's temperature transmitter is usually located in the densitometer, and the temperature element and transmitter in the densitometer are calibrated together (at the same time as the densitometer), to minimize the uncertainty of the densitometer's temperature reading.

For a densitometer of the by-pass type, only one pressure transmitter is here assumed to be installed: in the meter run (close to the USM, for measurement of the line pressure P). That is, pressure measurement is not available in the density transducer, i.e. P_d is not measured. In practice, then, the operator of the metering station typically assumes that the densitometer pressure is equal to the line pressure, $\hat{P} \approx \hat{P}_d$. However, there will be an uncertainty associated with that assumption. To account for this situation, let $P_d = P + \Delta P_d$, where ΔP_d is the relatively small and unknown pressure difference between the line and the densitometer pressures (usually negative). ΔP_d may be estimated empirically, from pressure shift tests, etc., or just taken as a "worst case" value. In this description, ΔP_d represents the uncertainty of assuming that $P_d = P$, cf. Sections 3.2.4 and 4.2.4.[62]

Two temperature transmitters are assumed to be installed: in the meter run (close to the USM, for measurement of the line temperature T), and in the density transducer (for measurement of the temperature at the densitometer, T_d).

In practice, the gas composition is the same at the USM as in the densitometer, and the pressure deviation is relatively small[63]. However, the temperatures in the densitometer and in the line can vary by several ^{o}C , so that the gas compressibility factors in the line and in the densitometer (Z and Z_d) can differ significantly. Correction for deviation in gas compressibility factors is thus normally made.

Consequently, with negligible loss of accuracy, the expression Eq. (2.26) for installation correction is here replaced by

$$\rho = \rho_d \left(\frac{T_d}{T} \right) \left(\frac{1}{1 + \Delta P_d / P} \right) \left(\frac{Z_d}{Z} \right) . \tag{2.27}$$

62 Note that by one gas USM manufacturer, USMs are available today for which the meter body is coned towards the ends, i.e. the inner diameter decreases slightly over some centimeters from the ends towards the metering volume (introduced for flow profile enchancement purposes). The line pressure P is the pressure in the metering volume. Consequently, if the density sampling probe is located in the cone, or outside the cone (outside the meter body), the additional pressure difference between the line pressure and the pressure at the density sampling point has to be included in ΔP_d.

63 Tests with densitometers have indicated a pressure difference between the densitometer and the line of up to 0.02 % of the line pressure [Eide, 2001a], which for a pressure of 100 bar corresponds to 20 mbar. Differences in pressure will have more influence on low pressure systems than high-pressure systems.

2.4.5 Corrected density

By combining Eqs. (2.24)-(2.27), the functional relationship of the corrected density measurement becomes

$$\rho = \left\{\rho_u\left[1+K_{18}\left(T_d-T_c\right)\right]+K_{19}\left(T_d-T_c\right)\right\}\left[\frac{1+\left(\frac{K_d}{\tau c_c}\right)^2}{1+\left(\frac{K_d}{\tau c_d}\right)^2}\right]\left(\frac{T_d}{T}\right)\left(\frac{1}{1+\Delta P_d/P}\right)\left(\frac{Z_d}{Z}\right), \qquad (2.28)$$

in which all three corrections (the temperature correction, the VOS correction and the installation correction) are accounted for in a single expression.

Note that in Eq. (2.28), the *indicated* (uncorrected) density ρ_u has been used as the input quantity related to the densitometer reading instead of the periodic time τ. That has been done since $u(\hat{\rho}_u)$ is the uncertainty specified by the manufacturer [Solartron, 1999], and not $u(\hat{\tau})$, cf. Sections 3.2.4 and 4.2.4.

Eq. (2.28) is a relatively general functional relationship for on-line installed vibrating element gas densitometers, cf. e.g. [ISO/CD 15970, 1999], which apply to the Solartron 7812 Gas Density Transducer [Solartron, 1999] (used in the example calculations in Chapter 4), as well as other densitometers of this type[64].

2.5 Functional relationship - Pressure measurement

As described in Section 1.3, the uncertainty of the pressure transmitter can in the program *EMU - USM Fiscal Gas Metering Station* be specified at two levels (cf. also Chapter 5):

(1) ***"Overall level":*** The user gives $u_c(\hat{P})$ directly as input to the program. It is left to the user to calculate and document $u_c(\hat{P})$ first. This option is completely general, and covers any method of obtaining the uncertainty of the pressure measurement[65].

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64 Note that alternative (but practically equivalent) formulations of the VOS correction may possibly be used in different densitometers, as mentioned in Section 2.4.3.

65 The "overall level" option may be of interest in several cases, such as e.g.:

- If the user wants a "simple" and quick evaluation of the influence of $u_c(\hat{P})$ on the expanded uncertainty of the gas metering station,

(2) ***"Detailed level":*** $u_c(\hat{P})$ is calculated in the program, from more basic input uncertainties for the pressure transmitter, provided by the instrument manufacturer and calibration laboratory.

The following discussion concerns the "Detailed level". It has been found convenient to base the user input to the program on the type of data which are typically specified for common pressure transmitters used in North Sea fiscal gas metering stations.

The example pressure transmitter chosen by NFOGM, NPD and CMR [Ref. Group, 2001] to be used in the present *Handbook* for the uncertainty evaluation example of Chapter 4 is the Rosemount 3051P Reference Class Pressure Transmitter [Rosemount, 2000], cf. Table 2.4 and Fig. 2.6. This transmitter is also chosen for the layout of the presssure transmitter user input to the program *EMU - USM Fiscal Gas Metering Station*. The Rosemount 3051P is a widely used pressure transmitter when upgrading existing North Sea fiscal gas metering stations and when designing new metering stations. The pressure transmitter output is normally the overpressure (gauge pressure), i.e. the pressure relative to the atmospheric pressure [barg].

Fig. 2.6 The Rosemount 3051P Reference Class Pressure Transmitter (example). Published in the Rosemount Comprehensive Product Catalog, Publication No. 00822-0100-1025 [Rosemount, 2000] © 2000 Rosemount Inc. Used by permission.

Measurement principles of gauge pressure sensors and transmitters are described e.g. in [ISO/CD 15970, 1999]. However, as the transmitter is calibrated and given a specific "accuracy" in the calibration data sheet, no functional relationship is actually

- In case the input used at the "detailed level" does not fit sufficiently well to the type of input data / uncertainties which are relevant for the pressure transmitter at hand.

used here for calculation of the uncertainty of the pressure measurements (cf. also [Dahl *et al.*, 1999]). The functional relationship is only internal to the pressure transmitter, and the uncertainty due to the functional relationship is included in the calibrated "accuracy" of the transmitter[66].

2.6 Functional relationship - Temperature measurement

As described in Section 1.3, the uncertainty of the temperature transmitter can in the program *EMU - USM Fiscal Gas Metering Station* be specified at two levels (cf. also Chapter 5):

(1) ***"Overall level":*** The user gives $u_c(\hat{T})$ directly as input to the program. It is left to the user to calculate and document $u_c(\hat{T})$ first. This option is completely general, and covers any method of obtaining the uncertainty of the gas temperature measurement[67].

(2) ***"Detailed level":*** $u_c(\hat{T})$ is calculated in the program, from more basic input uncertainties for the temperature element / transmitter, provided by the instrument manufacturer and calibration laboratory

The following discussion concerns the "Detailed level". As for the pressure measurement, it has been found convenient to base the user input to the program on the type of data which are typically specified for common temperature transmitters used in North Sea fiscal gas metering stations.

The temperature loop considered here consists of a Pt 100 or 4-wire RTD element and a smart temperature transmitter, installed either as two separate devices, or as one unit [NORSOK, 1998a; §5.2.3.5]. The Pt 100 temperature element is required as a minimum to be in accordance with EN 60751 tolerance A, cf. Section C.2. By [NORSOK, 1998a; §5.2.3.5], the temperature transmitter and the Pt 100 element shall be calibrated as one system (cf. Section 2.1 and Appendix C.2). A 3-wire tem-

66 Cf. the footnote accompanying Eq. (3.11).

67 The "overall level" option may be of interest in several cases, such as e.g.:
- If the user wants a "simple" and quick evaluation of the influence of $u_c(\hat{T})$ on the expanded uncertainty of the gas metering station,
- In case the input used at the "detailed level" does not fit sufficiently well to the type of input data / uncertainties which are relevant for the temperature element / transmitter at hand.

perature element may be used if the temperature element and transmitter are installed as one unit, where the Pt 100 element is screwed directly into the transmitter.

The signal is transferred from the temperature transmitter using a HART protocol, i.e. the “digital accuracy” is used.

The temperature transmitter chosen by NFOGM, NPD and CMR [Ref. Group, 2001] to be used in the present *Handbook* for the example uncertainty evaluation of Chapter 4 is the Rosemount 3144 Smart Temperature Transmitter [Rosemount, 2000], cf. Table 2.4 and Fig. 2.7. The Rosemount 3144 transmitter is widely used in the North Sea when upgrading existing fiscal gas metering stations and when designing new metering stations. This transmitter is also chosen for the layout of the temperature transmitter user input to the program *EMU - USM Fiscal Gas Metering Station*.

Fig. 2.7 The Rosemount 3144 Temperature Transmitter (example). Published in the Rosemount Comprehensive Product Catalog, Publication No. 00822-0100-1025 [Rosemount, 2000] © 2000 Rosemount Inc. Used by permission.

The measurement principle and functional relationship of RTDs is described e.g. in [ISO/CD 15970, 1999]. However, as the element/transmitter is calibrated and given a specific “accuracy” in the calibration data sheet, no functional relationship is actually used here for calculation of the uncertainty of the temperature measurements (cf. also [Dahl *et al.*, 1999]). The functional relationship is only internal to the temperature element/transmitter, and the uncertainty due to the functional relationship is included in the calibrated “accuracy” of the element/transmitter[68].

68 Cf. the footnote accompanying Eq. (3.12).

3. UNCERTAINTY MODEL OF THE METERING STATION

The present chapter summarizes the uncertainty model of the USM fiscal gas metering station, as a basis for the uncertainty calculations of Chapter 4, and the Excel program *EMU - USM Fiscal Gas Metering Station* described in Chapter 5. The uncertainty model is based on the description and functional relationships for the metering station given in Chapter 2. The model is developed in accordance with the terminology and procedures described in Appendix B.

As outlined in Section 1.4, the detailed mathematical derivation of the uncertainty model is given in Appendix E. The main expressions of the model are summarized in the present chapter. That means, - the expressions which are necessary for a full documentation of the calculations made in the Excel program have been included here.

The intention has been that the reader should be able to use the present *Handbook* without needing to read Appendix E. On the other hand, Appendix E has been included as a documentation of the theoretical basis for the uncertainty model, for completeness and traceability purposes. Hence, definitions introduced in Appendix E have - when relevant - been included also in the present chapter, to make Chapter 3 self-consistent so that it can be read independently.

The chapter is organized as follows: The expressions for the uncertainty of the USM fiscal gas metering station are given first, for the four measurands in question, q_V, Q, q_m and q_e (Section 3.1). Detailed expressions are then given for the gas measurement uncertainties (related to P, T, ρ, Z/Z_0 and H_s, Section 3.2), the flow calibration uncertainties (Section 3.3), the USM field uncertainty (Section 3.4) and the signal communication and flow computer calculatations (Section 3.5). For convenience, a summary of the input uncertainties to be specified for the program *EMU - USM Fiscal Gas Metering Station* is given in Section 3.6.

3.1 USM gas metering station uncertainty

For USM measurement of natural gas, the basic functional relationships are given by Eqs. (2.9) and (2.2)-(2.4), for the axial volumetric flow rate at line conditions, q_V, the axial volumetric flow rate at standard reference conditions, Q, the axial mass flow rate, q_m, and the axial energy flow rate, q_e, respectively.

For these four measurands, the relative expanded uncertainties are given as

$$\boxed{\frac{U(\hat{q}_v)}{|\hat{q}_v|} = k\frac{u_c(\hat{q}_v)}{|\hat{q}_v|} = kE_{q_v}} \quad , \tag{3.1}$$

$$\boxed{\frac{U(\hat{Q})}{|\hat{Q}|} = k\frac{u_c(\hat{Q})}{|\hat{Q}|} = kE_Q} \quad , \tag{3.2}$$

$$\boxed{\frac{U(\hat{q}_m)}{|\hat{q}_m|} = k\frac{u_c(\hat{q}_m)}{|\hat{q}_m|} = kE_{q_m}} \quad , \tag{3.3}$$

$$\boxed{\frac{U(\hat{q}_e)}{|\hat{q}_e|} = k\frac{u_c(\hat{q}_e)}{|\hat{q}_e|} = kE_{q_e}} \quad , \tag{3.4}$$

respectively, where k is the coverage factor[69] (cf. Section B.3). Here, E_{q_v}, E_Q, E_{q_m} and E_{q_e} are the relative combined standard uncertainties of q_V, Q, q_m and q_e, defined as

$$E_{q_v} \equiv \frac{u_c(\hat{q}_v)}{|\hat{q}_v|} \quad , \quad E_Q \equiv \frac{u_c(\hat{Q})}{|\hat{Q}|}, \quad E_{q_m} \equiv \frac{u_c(\hat{q}_m)}{|\hat{q}_m|}, \quad E_{q_e} \equiv \frac{u_c(\hat{q}_e)}{|\hat{q}_e|} \tag{3.5}$$

respectively. It can be shown (Appendix E) that these may be expressed as

$$\boxed{E_{q_v}^2 = E_{cal}^2 + E_{USM}^2 + E_{comm}^2 + E_{flocom}^2} \tag{3.6}$$

$$\boxed{E_Q^2 = E_P^2 + E_T^2 + E_{Z/Z_0}^2 + E_{q_v}^2} \tag{3.7}$$

$$\boxed{E_{q_m}^2 = E_\rho^2 + E_{q_v}^2} \tag{3.8}$$

$$\boxed{E_{q_e}^2 = E_{H_S}^2 + E_Q^2} \tag{3.9}$$

69 In the present *Handbook* k = 2 is used [NPD, 2001], corresponding to a 95 % confidence level (approximately) and a normal probability distribution (cf. Section B.3).

respectively. The various terms involved in Eqs. (3.6)-(3.9) are:

E_{cal} ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, related to flow calibration of the USM.

E_{USM} ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, related to field operation of the USM.

E_{comm} ≡ relative standard uncertainty of the estimate $\hat{q}_v$, due to signal communication between USM field electronics and flow computer (e.g. use of analog frequency or digital signal output), in flow calibration and field operation (assembled in on term).

E_{flocom} ≡ relative standard uncertainty of the estimate $\hat{q}_v$, due to flow computer calculations, in flow calibration and field operation (assembled in on term).

E_P ≡ relative combined standard uncertainty of the line pressure estimate, $\hat{P}$.

E_T ≡ relative combined standard uncertainty of the line temperature estimate, $\hat{T}$.

$E_{Z/Z0}$ ≡ relative combined standard uncertainty of the estimate of the compressibility ratio between line and standard reference conditions, $\hat{Z}/\hat{Z}_0$.

E_ρ ≡ relative combined standard uncertainty of the density estimate, $\hat{\rho}$.

E_{H_S} ≡ relative standard uncertainty of the superior (gross) calorific value estimate, $\hat{H}_S$.

Note that Eqs. (3.6)-(3.9) as they stand are completely meter independent, and thus independent of the USM functional relationship (Formulations A, B, C or D, cf. Section 2.3.2).

Note also that to obtain Eqs. (3.6)-(3.9), the following assumption have been made:

- The deviation factor estimate $\hat{K}_{dev,j}$ is assumed to be uncorrelated with the other quantities appearing in Eq. (2.9).

- Possible correlation between $\hat{H}_S$ and $\hat{Q}$ has been neglected, cf. Sections 2.1.2 and 3.2.5.

As an example, the expanded uncertainty of a USM fiscal gas metering station is evaluated according to Eqs. (3.6)-(3.9) in Section 4.6.

3.2 Gas measurement uncertainties

Gas parameter uncertainties are involved in Eq. (3.7) for E_Q (involving *P, T, Z,* and Z_0), Eq. (3.8) for E_{q_m} (involving ρ) and Eq. (3.9) for E_{q_e} (involving H_s).

The relative combined standard uncertainties of these gas parameters are defined as

$$\boxed{\begin{aligned} &E_P \equiv \frac{u_c(\hat{P})}{\hat{P}}, \qquad E_T \equiv \frac{u_c(\hat{T})}{\hat{T}}, \qquad E_{Z/Z_0} \equiv \frac{u_c(\hat{Z}/\hat{Z}_0)}{\hat{Z}/\hat{Z}_0}, \\ &E_\rho \equiv \frac{u_c(\hat{\rho})}{\hat{\rho}}, \qquad E_{H_S} \equiv \frac{u(\hat{H}_S)}{|\hat{H}_S|} \end{aligned}} \tag{3.10}$$

respectively, where

$u_c(\hat{P}) \equiv$ combined standard uncertainty of the line pressure estimate, $\hat{P}$.

$u_c(\hat{T}) \equiv$ combined standard uncertainty of the line temperature estimate, $\hat{T}$.

$u_c(\hat{Z}/\hat{Z}_0) \equiv$ combined standard uncertainty of the estimate of the compressibility ratio between line and standard reference conditions, $\hat{Z}/\hat{Z}_0$.

$u_c(\hat{\rho}) \equiv$ combined standard uncertainty of the line density estimate, $\hat{\rho}$,

$u(\hat{H}_S) \equiv$ standard uncertainty of the superior (gross) calorific value estimate, $\hat{H}_S$.

3.2.1 Pressure measurement

The combined standard uncertainty of the static gas pressure measurement, $u_c(\hat{P})$, can be given as input to the program *EMU - USM Fiscal Gas Metering Station* at two levels: “Overall level” and “Detailed level”, cf. Sections 1.3, 2.5 and 5.4.

As the "Overall level" is straightforward, only the "Detailed level" is discussed in the following. The description is similar to that given in [Dahl *et al.*, 1999] (pp. 84-89), for the static gas pressure measurement. The uncertainty model for the pressure transmitter is quite general, and applies to e.g. the Rosemount 3051P Pressure Transmitter, and similar transmitters (cf. Section 2.5).

At the "Detailed level", $u_c(\hat{P})$ is assumed to be given as[70]

$$\boxed{\begin{aligned} u_c^2(\hat{P}) = {} & u^2(\hat{P}_{transmitter}) + u^2(\hat{P}_{stability}) + u^2(\hat{P}_{RFI}) + u^2(\hat{P}_{temp}) \\ & + u^2(\hat{P}_{atm}) + u^2(\hat{P}_{vibration}) + u^2(\hat{P}_{power}) + u^2(\hat{P}_{misc}) \end{aligned}} \tag{3.11}$$

where [Rosemount, 2000]

$u(\hat{P}_{transmitter}) \equiv$ standard uncertainty of the pressure transmitter, including hysteresis, terminal-based linearity, repeatability and the standard uncertainty of the pressure calibration laboratory.

$u(\hat{P}_{stability}) \equiv$ standard uncertainty of the stability of the pressure transmitter, with respect to drift in readings over time.

$u(\hat{P}_{RFI}) \equiv$ standard uncertainty due to radio-frequency interference (RFI) effects on the pressure transmitter.

$u(\hat{P}_{temp}) \equiv$ standard uncertainty of the effect of ambient gas temperature on the pressure transmitter, for change of ambient temperature relative to the temperature at calibration.

$u(\hat{P}_{atm}) \equiv$ standard uncertainty of the atmospheric pressure, relative to 1 atm. ≡ 1.01325 bar, due to local meteorological effects.

$u(\hat{P}_{vibration}) \equiv$ standard uncertainty due to vibration effects on the pressure measurement.

$u(\hat{P}_{power}) \equiv$ standard uncertainty due to power supply effects on the pressure transmitter.

$u(\hat{P}_{misc}) \equiv$ standard uncertainty due to other (miscellaneous) effects on the pressure transmitter, such as mounting effects, etc.

70 Here, the sensitivity coefficients have been assumed to be equal to 1 throughout Eq. (3.11), as a simplified approach, and in accordance with common company practice [Dahl *et al.*, 1999], [Ref Group, 2001]. An alternative and more correct approach would have been to start from the functional relationship of the pressure measurement, and derive the uncertainty model according to the recommendations of the *GUM* [ISO, 1995a].

$u(\hat{P})$ needs to be traceable to national and international standards. It is left to the calibration laboratory and the manufacturer to specify $u(\hat{P}_{transmitter})$, $u(\hat{P}_{stability})$, $u(\hat{P}_{temp})$, $u(\hat{P}_{RFI})$, $u(\hat{P}_{vibration})$ and $u(\hat{P}_{power})$, and document their traceability. (Cf. also Table 3.1.)

As an example, the uncertainty of the Rosemount 3051P pressure transmitter is evaluated in Section 4.2.1.

3.2.2 Temperature measurement

The combined standard uncertainty of the temperature measurement, $u_c(\hat{T})$, can be given as input to the program *EMU - USM Fiscal Gas Metering Station* at two levels: "Overall level" and "Detailed level", cf. Sections 1.3, 2.6 and 5.5.

As the "Overall level" is straightforward, only the "Detailed level" is discussed in the following. The description is similar to that given in [Dahl *et al.*, 1999] (pp. 79-83). The uncertainty model for the temperature element/transmitter is quite general, and applies to e.g. the Rosemount 3144 Temperature Transmitter used with a Pt 100 element, and similar transmitters (cf. Section 2.6).

At the "Detailed level", $u_c(\hat{T})$ is assumed to be given as[71]

$$\boxed{\begin{aligned} u_c^2(\hat{T}) = {} & u^2(\hat{T}_{elem,transm}) + u^2(\hat{T}_{stab,transm}) + u^2(\hat{T}_{RFI}) + u^2(\hat{T}_{temp}) + u^2(\hat{T}_{stab,elem}) \\ & + u^2(\hat{T}_{vinbration}) + u^2(\hat{T}_{power}) + u^2(\hat{T}_{cable}) + u^2(\hat{T}_{misc}) \end{aligned}} \qquad (3.12)$$

where [Rosemount, 2000]

$u(\hat{T}_{elem,transm}) \equiv$ standard uncertainty of the temperature element and temperature transmitter, calibrated as a unit.

$u(\hat{T}_{stab,transm}) \equiv$ standard uncertainty of the stability of the temperature transmitter, with respect to drift in the readings over time.

$u(\hat{T}_{RFI}) \equiv$ standard uncertainty due to radio-frequency interference (RFI) effects on the temperature transmitter.

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71 In accordance with common company practice [Dahl *et al.*, 1999], [Ref Group, 2001], the sensitivity coefficients have been assumed to be equal to 1 throughout Eq. (3.12). Note that this is a simplified approach. An alternative and more correct approach would have been to start from the full functional relationship of the temperature measurement, and derive the uncertainty model according to the recommendations of the *GUM* [ISO, 1995a].

$u(\hat{T}_{temp}) \equiv$ standard uncertainty of the effect of temperature on the temperature transmitter, for change of gas temperature relative to the temperature at calibration.

$u(\hat{T}_{stab,elem}) \equiv$ standard uncertainty of the stability of the Pt 100 4-wire RTD temperature element. Instability may relate e.g. to drift during operation, as well as instability and hysteresis effects due to oxidation and moisture inside the encapsulation, and mechanical stress during operation.

$u(\hat{T}_{vibration}) \equiv$ standard uncertainty due to vibration effects on the temperature transmitter.

$u(\hat{T}_{power}) \equiv$ standard uncertainty due to power supply effects on the temperature transmitter.

$u(\hat{T}_{cable}) \equiv$ standard uncertainty of lead resistance effects on the temperature transmitter.

$u(\hat{T}_{misc}) \equiv$ standard uncertainty of other (miscellaneous) effects on the temperature transmitter.

$u_c(\hat{T})$ needs to be traceable to national and international standards. It is left to the calibration laboratory and the manufacturer to specify $u(\hat{T}_{elem,transm})$, $u(\hat{T}_{stab,transm})$, $u(\hat{T}_{RFI})$, $u(\hat{T}_{temp})$, $u(\hat{T}_{stab,elem})$, $u(\hat{T}_{vibration})$, $u(\hat{T}_{power})$ and $u(\hat{T}_{cable})$, and document their traceability. (Cf. also Table 3.2.)

As an example, the uncertainty of the Rosemount 3144 temperature transmitter used with a Pt 100 element is evaluated in Section 4.2.2.

3.2.3 Gas compressibility factors *Z* and Z_0

For natural gas, empirical equations of state can be used to calculate *Z* and Z_0, i.e. the gas compressibility factor at line and standard reference conditions, respectively, cf. Table 2.1. Input to these equations are e.g. pressure, temperature and gas composition. Various equations of state are available for such calculations, such as the AGA-8 (92) equation [AGA-8, 1994] and the GERG / ISO method [ISO 12213-3, 1997]. In the program *EMU - USM Fiscal Gas Metering Station*, *Z* and Z_0 are given manually.

For each of the estimates $\hat{Z}$ and $\hat{Z}_0$, two kinds of uncertainties are accounted for here (cf. e.g. [Tambo and Søgaard, 1997]): the *model uncertainty* (i.e. the uncertainty of the model (equation of state) used for calculation of $\hat{Z}$ and $\hat{Z}_0$), and the *analysis uncertainty* (due to the inaccurate determination of the gas composition). For conversion of q_v to Q according to Eq. (2.2), the model uncertainties are assumed to be mutually uncorrelated[72], whereas the analysis uncertainties act as systematic effects, and are taken to be mutually correlated. That means,

$$\boxed{E^2_{Z/Z_0} = E^2_{Z,\mathrm{mod}} + E^2_{Z0,\mathrm{mod}} + (E_{Z,ana} - E_{Z0,ana})^2} \quad , \tag{3.13}$$

where

$E_{Z,mod} \equiv$ relative standard uncertainty of the estimate $\hat{Z}$ due to *model uncertainty* (the uncertainty of the equation of state itself, and the uncertainty of the "basic data" underlying the equation of state),

$E_{Z0,mod} \equiv$ relative standard uncertainty of the estimate $\hat{Z}_0$ due to *model uncertainty* (the uncertainty of the equation of state itself, and the uncertainty of the "basic data" underlying the equation of state),

$E_{Z,ana} \equiv$ relative standard uncertainty of the estimate $\hat{Z}$ due to *analysis uncertainty* (measurement uncertainty of the gas chromatograph used to determine the line gas composition, and variation in gas composition),

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72 In the derivation of Eq. (3.13), the model uncertainties of the *Z*-factor estimates $\hat{Z}$ and $\hat{Z}_0$ have been assumed to be uncorrelated.

If $\hat{Z}$ is calculated from the AGA-8 (92) equation [AGA-8, 1994] or the ISO / GERG method [ISO 12213-3, 1997], and $\hat{Z}_0$ is calculated from ISO 6976 [ISO, 1995c] (as is often made), this is clearly a reasonable and valid approach.

If $\hat{Z}$ and $\hat{Z}_0$ are estimated using *the same* equation of state (such as e.g. the AGA-8 (92) equation or the ISO 12213-3 method), some comments should be given.

The argumentation is then as follows: In many cases $\hat{Z}$ and $\hat{Z}_0$ relate to highly different pressures. Since the equation of state is empirical, it may not be correct to assume that the error of the equation is systematic over the complete pressure range (e.g.: the error may be positive at one pressure, and negative at another pressure). That means, $\hat{Z}$ (at high pressure) and $\hat{Z}_0$ (at 1 atm.) are not necessarily correlated. A similar argumentation applies if the line temperature is significantly different from 15 ^{o}C. As a conservative approach thus, $\hat{Z}$ and $\hat{Z}_0$ are here treated as being uncorrelated.

However, this choice may be questionable, especially in cases where the USM is operated close to standard reference conditions (close to 1 atm. and 15 ^{o}C). That means, in a *P-T* range so narrow that the error of the equation may possibly be expected to be more systematic. In such cases E_{Z/Z_0} may possibly be overestimated by using Eq. (3.13).

$E_{Z0,ana} \equiv$ relative standard uncertainty of the estimate $\hat{Z}_0$ due to *analysis uncertainty* (measurement uncertainty of the gas chromatograph used to determine the gas composition, and variation in gas composition),

3.2.3.1 Model uncertainties

For the AGA-8 (1992) equation of state [AGA-8, 1994], the relative model uncertainty of the *Z*-factor calculations have been specified to, cf. Fig. 3.1:

- 0.1 % in the range *P* = 0-120 bar and *T* = 265-335 K (-8 to +62 ^{o}C),
- 0.3 % in the range *P* = 0-172 bar and *T* = 213-393 K (-60 to +120 ^{o}C),
- 0.5 % in the range *P* = 0-700 bar and *T* = 143-473 K (-130 to +200 ^{o}C),
- 1.0 % in the range *P* = 0-1400 bar and *T* = 143-473 K (-130 to +200 ^{o}C).

For input of the relative standard uncertainties $E_{Z,\mathrm{mod}}$ and $E_{Z0,\mathrm{mod}}$ in the program *EMU - USM Fiscal Gas Metering Station*, the user of the program has four choices:

Option (1): $E_{Z,mod}$ and $E_{Z0,mod}$ are given directly as input to the program,
Option (2): $E_{Z,mod}$ is automatically determined from uncertainties specified in [AGA-8, 1994],
Option (3): $E_{Z0,mod}$ is automatically determined from uncertainties specified in [AGA-8, 1994], or
Option (4): $E_{Z0,mod}$ is automatically determined from uncertainties specified in ISO 6976 [ISO, 1995c].

Option (1) is straigthforward, so only Options (2)-(4) are discussed in the following.

In Options (2) or (3), $E_{Z,mod}$ or $E_{Z0,mod}$ are calculated from uncertainties specified for the AGA-8 (92) equation of state [AGA-8, 1994], by assuming that the expanded uncertainties specified in the AGA-8 report refer to a Type A evaluation of uncertainty, a 95 % level of confidence and a normal probability distribution (*k* = 2, cf. Section B.3). That means:

- $E_{Z,\mathrm{mod}} = 0.1\%/2 = 5.0 \cdot 10^{-4} = 0.05\%$, in the range 0-120 bar and -8 to +62 ^{o}C,
- $E_{Z,\mathrm{mod}} = 0.3\%/2 = 1.5 \cdot 10^{-3} = 0.15\%$, in the range 0-172 bar and -60 to +120 ^{o}C,
- $E_{Z,\mathrm{mod}} = 0.5\%/2 = 2.5 \cdot 10^{-3} = 0.25\%$, in the range 0-700 bar -130 to +200 ^{o}C,
- $E_{Z,\mathrm{mod}} = 1.0\%/2 = 5 \cdot 10^{-3} = 0.5\%$, in the range 0-1400 bar and -130 to +200 ^{o}C.

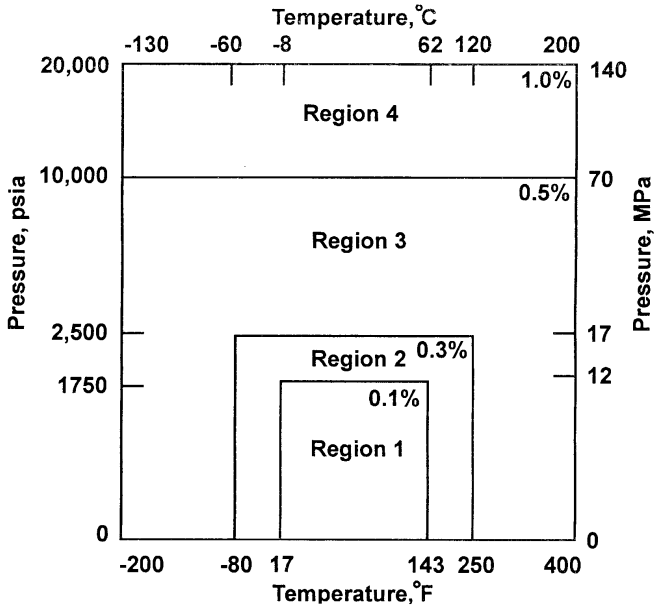

Fig. 3.1 Targeted uncertainty for natural gas compressibility factors using the detail characterization method. Copied from the AGA Report No. 8, *Compressibility Factors of Natural Gas and Other Related Hydrocarbon Gases* [AGA-8, 1994], with the permission of the copyright holder, American Gas Association.

At standard reference conditions, ISO 6976 can be used for calculation of $\hat{Z}_0$ (Option 4). The contributions to $E_{Z0,mod}$ are then 0.05 % (basic data) and 0.015 % (equation of state) [ISO, 1995c], [Sakariassen, 2001]. By assuming a 100 % level of confidence and a rectangular probability distribution ($k = \sqrt{3}$, cf. Section B.3), this yields $E_{Z0,\mathrm{mod}} = \sqrt{0.05^2 + 0.015^2}\,\%/\sqrt{3} \approx 0.0522\,\%/\sqrt{3} \approx 0.030\,\%$.

3.2.3.2 Analysis uncertainties

With respect to the uncertainty contributions $E_{Z,ana}$ and $E_{Z0,ana}$, both of these are to be entered manually by the user of the program. The suggested method for establishing these input uncertainty contributions involves numerical analysis (Monte Carlo type of simulations). Based on knowledge of the metering station, variation limits can be established for each of the gas components as input to the Monte Carlo simulations.

Such limits must take into account the uncertainty of the GC - measurement and the natural variations of the gas composition (at least when an online GC is not used). Next, a number of gas compositions within such variation limits for each gas component can then be established (where of course the sum of the gas components must add up to 100 %), and the *Z*-factor is calculated for each of these gas compositions. The spread of the *Z*-factors (for example the standard deviation) calculated in this way, will give information about the analysis uncertainty. This method is used in Section 4.2.3. Ideally, a large number of gas compositions generated randomly

within the limits of each gas component, should be used in this calculation. In practice, however, a smaller number (less than 10) will often provide useful information about the analysis uncertainty.

Other (less precise) methods may also be used, e.g based on the decision of the uncertainty of the molecular weight [Sakariassen, 2001].

3.2.4 Density measurement

The relative combined standard uncertainty of the gas density measurement, E_ρ, can be given as input to the program *EMU - USM Fiscal Gas Metering Station* at two levels: "Overall level" and "Detailed level", cf. Sections 1.3, 2.4 and 5.7.

As the "Overall level" is straightforward, only the "Detailed level" is discussed in the following. The uncertainty model for the gas densitometer is quite general, and should apply to any on-line installed vibrating-element densitometer, such as e.g. the Solartron 7812 gas density transducer (cf. Section 2.4)[73]. It represents an extension of the uncertainty model for gas densitometers presented by [Tambo and Søgaard, 1997].

At the "Detailed level", the relative combined standard uncertainty E_ρ is given as (cf. Appendix G)

$$\boxed{\begin{aligned} u_c^2(\hat{\rho}) = {} & s_{\rho_u}^2 u^2(\hat{\rho}_u) + u^2(\hat{\rho}_{rept}) + s_{\rho,T}^2 u_c^2(\hat{T}) + s_{\rho,T_d}^2 u^2(\hat{T}_d) + s_{\rho,T_c}^2 u^2(\hat{T}_c) \\ & + s_{\rho,K_d}^2 u^2(\hat{K}_d) + s_{\rho,\tau}^2 u^2(\hat{\tau}) + s_{\rho,c_c}^2 u^2(\hat{c}_c) + s_{\rho,c_d}^2 u^2(\hat{c}_d) \\ & + s_{\rho,\Delta P_d}^2 u^2(\Delta\hat{P}_d) + s_{\rho,P}^2 u_c^2(\hat{P}) + u^2(\hat{\rho}_{temp}) + u^2(\hat{\rho}_{misc}) \end{aligned}} \quad (3.14)$$

where

$u(\hat{\rho}_u) \equiv$ standard uncertainty of the indicated (uncorrected) density estimate, $\hat{\rho}_u$, including the calibration laboratory uncertainty, the reading error during calibration, and hysteresis,

$u(\hat{\rho}_{rept}) \equiv$ standard uncertainty of the repeatability of the indicated (uncorrected) density estimate, $\hat{\rho}_u$,

73 The extension of the present densitometer uncertainty model in relation to the model presented in [Tambo and Søgaard, 1997, Annex 2 and 3], relates mainly to the more detailed approach which has been used here with respect to the temperature, VOS and installation corrections. Here, the uncertainty model includes sensitivity coefficients derived from the function relationship, Eq. (2.28), instead of taking them to be equal to 1.

$u(\hat{T}_d) \equiv$ standard uncertainty of the gas temperature estimate in the densitometer, $\hat{T}_d$,

$u(\hat{T}_c) \equiv$ standard uncertainty of the densitometer calibration temperature estimate, $\hat{T}_c$,

$u(\hat{K}_d) \equiv$ standard uncertainty of the VOS correction densitometer constant estimate, $\hat{K}_d$,

$u(\hat{c}_c) \equiv$ standard uncertainty of the calibration gas VOS estimate, $\hat{c}_c$,

$u(\hat{c}_d) \equiv$ standard uncertainty of the densitometer gas VOS estimate, $\hat{c}_d$,

$u(\hat{\tau}) \equiv$ standard uncertainty of the periodic time estimate, $\hat{\tau}$,

$u(\Delta\hat{P}_d) \equiv$ standard uncertainty of assuming that $\hat{P}_d = \hat{P}$, due to possible deviation of gas pressure from densitometer to line conditions,

$u(\hat{\rho}_{temp}) \equiv$ standard uncertainty of the temperature correction factor for the density estimate, $\hat{\rho}$ (represents the *model uncertainty* of the temperature correction model used, Eq. (2.24)).

$u(\hat{\rho}_{misc}) \equiv$ standard uncertainty of the indicated (uncorrected) density estimate, $\hat{\rho}_u$, accounting for miscellaneous uncertainty contributions[74], such as due to:

- stability (drift, shift between calibrations[75]),
- reading error during measurement (for digital display instruments)[76],
- possible deposits on the vibrating element,
- possible corrosion of the vibrating element,
- possible liquid condensation on the vibrating element,

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74 In accordance with common company practice [Dahl *et al.*, 1999], [Ref Group, 2001], various "miscellaneous uncertainty contributions" listed in the text have been accounted for in the uncertainty model (Eq. (3.14) by a "lumped" term, $u(\hat{\rho}_{misc})$, with a weight (sensitivity coefficient) equal to one. Note that this is a simplified approach. An alternative and more correct approach would have been to start from the full functional relationship of the uncorrected density measurement ρ_u, Eq. (2.23), and derive the influences of such miscellaneous uncertainty contributions on the total uncertainty according to the recommendations of the *GUM* [ISO, 1995a], i.e. with derived sensitivity coefficients.

75 For guidelines with respect to uncertainty evaluation of shift between calibrations, cf. [Tambo and Søgaard, 1997, Annex 2].

76 For guidelines with respect to uncertainty evaluation of reading error during measurement, cf. [Tambo and Søgaard, 1997, Annex 2].

- mechanical (structural) vibrations on the gas line,
- variations in power supply,
- self-induced heat,
- flow in the bypass density line,
- possible gas viscosity effects,
- neglecting possible pressure dependency in the regression curve, Eq. (2.23),
- model uncertainty of the VOS correction model, Eq. (2.25).

In this model, the estimates $\hat{T}$, $\hat{T}_d$ and $\hat{T}_c$ are assumed to be uncorrelated (since random effects contribute significantly to the uncertainty of the temperature measurement, cf. Table 4.8 and Fig. 5.22), and so are also the estimates $\hat{P}$ and $\Delta\hat{P}_d$.

The sensitivity coefficients appearing in Eq. (3.14) are defined as (cf. Appendix G)

$$s_{\rho_u} = \frac{\hat{\rho}\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}, \tag{3.15a}$$

$$s_{\rho,T} = -\frac{\hat{\rho}}{\hat{T}}, \tag{3.15b}$$

$$s_{\rho,T_d} = \left[1+\frac{\hat{T}_d\left[\hat{\rho}_u\hat{K}_{18}+\hat{K}_{19}\right]}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right]\frac{\hat{\rho}}{\hat{T}_d}, \tag{3.15c}$$

$$s_{\rho,T_c} = -\left[\frac{\hat{T}_c\left[\hat{\rho}_u\hat{K}_{18}+\hat{K}_{19}\right]}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right]\frac{\hat{\rho}}{\hat{T}_c} \tag{3.15d}$$

$$s_{\rho,K_d} = \left[\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_c)^2}-\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_d)^2}\right]\frac{\hat{\rho}}{\hat{K}_d}, \tag{3.15e}$$

$$s_{\rho,\tau} = -\left[\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_c)^2}-\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_d)^2}\right]\frac{\hat{\rho}}{\hat{\tau}} \tag{3.15f}$$

$$s_{\rho,c_c} = -\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_c)^2}\frac{\hat{\rho}}{\hat{c}_c}, \qquad s_{\rho,c_d} = \frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_d)^2}\frac{\hat{\rho}}{\hat{c}_d}, \tag{3.15g}$$

$$s_{\rho,\Delta P_d} = -\frac{\hat{\rho}}{\hat{P}+\Delta\hat{P}_d}, \qquad s_{\rho,P} = \frac{\Delta\hat{P}_d}{\hat{P}+\Delta\hat{P}_d}\frac{\hat{\rho}}{\hat{P}}, \tag{3.15h}$$

respectively.

$u_c(\hat{\rho})$ needs to be traceable to national and international standards. It is left to the calibration laboratory and the manufacturer to specify $u(\hat{\rho}_{\rho_u})$, $u(\hat{\rho}_{rept})$, $u(\hat{T}_c)$, $u(\hat{\rho}_{temp})$, $u(\hat{\tau})$ *and* $u(\hat{K}_d)$, and document their traceability, cf. Table 3.4. It is left to the user of the program *EMU - USM Fiscal Gas Metering Station* to specify $u(\hat{c}_c)$, $u(\hat{c}_d)$, $u(\Delta\hat{P}_d)$ and $u(\hat{\rho}_{misc})$. $u_c(\hat{T})$ and $u_c(\hat{T}_d)$ are in the program set to be equal and are given by Eq. (3.12). $u_c(\hat{P})$ is given by Eq. (3.11).

In [Tambo and Søgaard, 1997, Annex 2 and 3], $u(\hat{\rho}_{rept})$ is referred to as "the standard uncertainty of type A component", to be obtained by determining the density (at stable conditions) at least 10 times and deriving the standard deviation of the mean.

With respect to $u(\hat{c}_d)$, there are (as described in Section 2.4.3) at least two methods in use today to obtain the VOS at the density transducer, c_d: the "USM method" and the "pressure/density method". For the "USM method", there are basically two contributions to the uncertainty of c_d: (1) the uncertainty of the USM measurement of the line VOS, and (2) the deviation of the line VOS from the VOS at the densitometer. For the "pressure/density method", the uncertainty of c_d is to be calculated from the expressions used to calculate c_d and the input uncertainties to these. Evaluation of $u(\hat{c}_d)$ according to these (or other) methods is not a part of the present *Handbook*, - $u(\hat{c}_d)$ is to be calculated and given by the user of the program. In this approach, the uncertainty model is independent of the particular method used to estimate c_d in the metering station.

Note that the uncertainty of the *Z*-factor correction part of the installation correction described in Section 2.4.4, $u(\hat{Z}_d/\hat{Z})$, is negligible (Appendix G), and has thus been neglected here.

As an example of density uncertainty evaluation, the Solartron 7812 Gas Density Transducer is evaluated in Section 4.2.4.

3.2.5 Calorific value

As described in Section 2.1, the calorific value is usually either (1) calculated from the gas composition measured using GC analysis, or (2) measured directly by combustion (using a calorimeter), cf. Table 2.3 ([NPD, 2001], Section C.1).

In the first case, ISO 6976 is used [ISO, 1995c], and uncertainties in the calorific value can be due to uncertainties in the gas composition and model uncertainties of the ISO 6976 procedure. In the second case, the uncertainties of the actual calorimeter and measurement method will contribute.

In the present *Handbook* no detailed analysis is carried out for the relative standard uncertainty of the calorific value, E_{H_S}. Such an analysis would be outside the scope of work for the *Handbook* [Lunde, 2000], [Ref Group, 2001]. The user of the program is to specify the relative standard uncertainty E_{H_S} directly as input to the program (i.e. at the "overall level"), cf. Table 1.5 and Section 5.8.

Further, as a simplification, it has been assumed that the uncertainty of the calorific value estimate, $\hat{H}_s$, is uncorrellated to the uncertainty of the volumetric flow rate at standard reference conditions, $\hat{Q}$, cf. Eq. (3.9). As the conversion from line conditions to standard reference conditions for the volumetric flow is assumed here to be carried out using a gas chromatograph (calculation of Z and Z_0, cf. Table 2.1), the calorific value is thus implicitly assumed to be measured using a method which is uncorrelated with gas chromatography (such as e.g. a calorimeter), cf. a footnote accompanying Eq. (2.4), and Table 2.3 (method 5).

3.3 Flow calibration uncertainty

The relative combined standard uncertainty of the flow calibration which appears in Eq. (3.6) is given as (cf. Appendix E)

$$\boxed{E_{cal}^2 \equiv E_{q_{ref,j}}^2 + E_{K_{dev,j}}^2 + E_{rept,j}^2}\ , \tag{3.16}$$

where

$E_{q_{ref,j}} \equiv$ relative standard uncertainty of the reference measurement, $\hat{q}_{ref,j}$, at test flow rate no. j, j = 1, …, M (representing the uncertainty of the flow calibration laboratory, including reproducability),

$E_{K_{dev,j}} \equiv$ relative standard uncertainty of the deviation factor estimate, $\hat{K}_{dev,j}$,

$E_{rept,j} \equiv$ repeatability (relative standard uncertainty, i.e. relative standard deviation) of the USM flow calibration measurement (volumetric flow rate), at test flow rate no. j, j = 1, …, M (due to random transit time effects on the N acoustic paths of the USM, including repeatability of the flow laboratory reference measurement).

The relative standard uncertainties in question here are defined as

$$\boxed{E_{q_{ref,j}} \equiv \frac{u(\hat{q}_{ref,j})}{\left|\hat{q}_{ref,j}\right|}\ , \qquad E_{K_{dev,j}} \equiv \frac{u(\hat{K}_{dev,j})}{\left|\hat{K}_{dev,j}\right|}\ , \qquad E_{rept,j} \equiv \frac{u(\hat{q}^{rept}_{USM,j})}{\left|\hat{q}_{USM,j}\right|}\ ,} \qquad (3.17)$$

respectively, where

$u(\hat{q}_{ref,j}) \equiv$ standard uncertainty of the reference measurement, $\hat{q}_{ref,j}$, at test flow rate no. j, j = 1, …, M (representing the uncertainty of the flow calibration laboratory, including reproducability),

$u(\hat{K}_{dev,j}) \equiv$ standard uncertainty of the deviation factor estimate, $\hat{K}_{dev,j}$,

$u(\hat{q}^{rept}_{USM,j}) \equiv$ repeatability (standard uncertainty, i.e. standard deviation) of the USM flow calibration measurement (volumetric flow rate), at test flow rate no. j, j = 1, …, M (due to random transit time effects on the N acoustic paths of the USM, including repeatability of the flow laboratory reference measurement).

As an example, E_{cal} is evaluated in Section 4.3.4.

3.3.1 Flow calibration laboratory

The relative standard uncertainty of the flow calibration laboratory, $E_{q_{ref,j}}$, will serve as an input uncertainty to the program *EMU - USM Fiscal Gas Metering Station*, and is to be given by the program user, cf. Sections 4.3.1 and 5.9, and Table 3.6.

It is left to the flow calibration laboratory to specify $E_{q_{ref,j}}$ and document its traceability to national and international standards. As an example, $E_{q_{ref,j}}$ is discussed in Section 4.3.1.

3.3.2 Deviation factor

With respect to the deviation factor $K_{dev,j}$, the task here is the following: For a *given* correction factor, K, and *after* correction of the USM measurement data using K (cf. Section 2.2 and Fig. 2.1), the uncertainty of the resulting deviation curve is to be determined.

This uncertainty is the standard uncertainty of the deviation factor, $u(\hat{K}_{dev,j})$. It is calculated by the program *EMU - USM Fiscal Gas Metering Station* on basis of the

deviation data $Dev_{C,j}$, $j = 1, \ldots, M$ (cf. Eq. (2.10)), at the M test flow rates for which flow calibration has been made ("calibration points"), cf. Sections 4.3.2 and 5.9, and Table 3.6.

$u(\hat{K}_{dev,j})$ is determined by the "span" of the deviation factor $\hat{K}_{dev,j}$, which ranges from 1 to $Dev_{C,j}$, cf. Eq. (2.11). By assuming a Type A uncertainty, a 100 % confidence level and a rectangular probability distribution within the range $\pm Dev_{C,j}$ ($k = \sqrt{3}$, cf. Section B.3), the standard uncertainty and the relative standard uncertainty of the deviation factor are here calculated as

$$u(\hat{K}_{dev,j}) = \frac{\left|Dev_{C,j}\right|}{\sqrt{3}}, \qquad E_{K_{dev,j}} = \frac{u(\hat{K}_{dev,j})}{\left|\hat{K}_{dev,j}\right|} = \frac{1}{\sqrt{3}}\left|\frac{Dev_{C,j}}{\hat{K}_{dev,j}}\right|, \quad j = 1, \ldots, M, \tag{3.18}$$

respectively. It is left to the USM manufacturer to specify and document the deviation data $Dev_{C,j}$, $j = 1, \ldots, M$, at the M test flow rates, cf. Tables 4.12 and 6.4. As an example, $E_{K_{dev,j}}$ is evaluated in Section 4.3.2.

3.3.3 USM repeatability in flow calibration

The relative combined standard uncertainty of the USM repeatability in flow calibration, at test flow rate no. j, $E_{rept,j}$, will serve as an input uncertainty to the program *EMU - USM Fiscal Gas Metering Station*, and is to be given by the program user, cf. Sections 4.3.3 and 5.9, and Table 3.6.

Note that in the uncertainty model of the USM gas metering station, the repeatability in flow calibration and in field operation have both been accounted for, by different symbols, $E_{rept,j}$ and E_{rept}, respectively (cf. above and Section 3.4). The two uncorrelated repeatability terms are not assembled into one term, since in general they may account for different effects and therefore may be different in magnitude, as explained in the following: The repeatability in flow calibration, $E_{rept,j}$, accounts for random transit time effects on the N acoustic paths of the USM in flow calibration (standard deviation of the spread), and the repeatability of the flow laboratory itself, cf. Table 1.2. The repeatability in field operation, E_{rept}, accounts for random transit time effects on the N acoustic paths of the USM in field operation (standard deviation of the spread), which may also include effects not present in flow calibration, such as incoherent noise from pressure reduction valves (PRV noise), etc., cf. Table 1.4.

It is left to the USM manufacturer to specify and document $E_{rept,j}$, cf. Table 6.4. As an example, $E_{rept,j}$ is discussed in Section 4.3.3.

3.4 USM field uncertainty

The relative combined standard uncertainty of the USM in field operation, appearing in Eq. (3.6), is given as (cf. Appendix E)

$$\boxed{E_{USM}^2 \equiv E_{rept}^2 + E_{USM,\Delta}^2 + E_{misc}^2} \ , \tag{3.19}$$

where

$E_{rept} \equiv$ repeatability (relative standard uncertainty, i.e. relative standard deviation) of the USM measurement in field operation, $\hat{q}_v$, at the flow rate in question (due to random transit time effects on the *N* acoustic paths),

$E_{USM,\Delta} \equiv$ relative combined standard uncertainty of the estimate $\hat{q}_v$, related to field operation of the USM (due to change of conditions from flow calibration to field operation),

$E_{misc} \equiv$ relative standard uncertainty of miscellaneous effects on the USM field measurement $\hat{q}_v$, which are not eliminated by flow calibration, and which are not covered by other uncertainty terms accounted for here (e.g. inaccuracy of the USM functional relationship (the underlaying mathematical model), etc.).

The subscript "Δ" used in Eq. (3.19) (and elsewhere) denotes that *only deviations relative to the conditions at the flow calibration* are to be accounted for in the expressions involving this subscript. That means, uncertainty contributions which are practically eliminated at flow calibration, are *not* to be included in these expressions.

The term $E_{USM,\Delta}$ is further given as (cf. Appendix E)

$$\boxed{E_{USM,\Delta}^2 \equiv E_{body,\Delta}^2 + E_{time,\Delta}^2 + E_{I,\Delta}^2 \ ,} \tag{3.20}$$

where

$E_{body,\Delta} \equiv$ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to possible uncorrected change of the USM meter body dimensions from flow calibration to field operation. That is, uncertainty of the meter body inner radius, $\hat{R}$, the lateral chord positions of the *N* acoustic paths, $\hat{y}_i$, and the inclination angles of the *N* acoustic paths, ϕ_i, $i = 1, \ldots, N$, caused by possible deviation in pressure and/or tem-

perature between flow calibration and field operation. Only those systematic meter body effects which are not corrected for in the USM or not eliminated by flow calibration are to be accounted for in $E_{body,\Delta}$.

$E_{time,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to possible uncorrected systematic effects on the transit times of the *N* acoustic paths, $\hat{t}_{1i}$ and $\hat{t}_{2i}$, *i* = 1, …, *N*, caused e.g. by possible deviation in conditions from flow calibration to field operation (*P*, *T*, transducer deposits, transducer ageing, etc). Only those systematic transit time effects which are not corrected for in the USM or not eliminated by flow calibration are to be accounted for in $E_{time,\Delta}$.

$E_{I,\Delta}$ ≡ relative standard uncertainty of the USM integration method due to possible change of installation conditions from flow calibration to field operation.

Note that Eqs. (3.19)-(3.20) as they stand here are completely meter independent, and thus independent of the choice of USM functional relationship (Formulations A, B, C or D, cf. Section 2.3.2).

The following subsections address in more detail the contributions to the USM meter body uncertainty (Section 3.4.1), the USM transit time uncertainties (Section 3.4.2), and the USM integration method uncertainty (accounting for installation condition effects) (Section 3.4.3).

As an example, E_{USM} is evaluated in Section 4.4.6.

3.4.1 USM meter body uncertainty

The relative combined standard uncertainty related to meter body dimensional changes which appears in Eq. (3.20), is given as (cf. Appendix E)

$$\boxed{E_{body,\Delta} \equiv E_{rad,\Delta} + E_{chord,\Delta} + E_{angle,\Delta}}\,, \tag{3.21}$$

where

$E_{rad,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to uncertainty of the meter body inner radius, $\hat{R}$, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{chord,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to uncertainty of the lateral chord positions of the *N* acoustic paths, $\hat{y}_i$, *i* =

1, …, *N*, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{angle,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to uncertainty of the inclination angles of the *N* acoustic paths, ϕ_i, *i* = 1, …, *N*, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

These relative combined standard uncertainties are defined as (cf. Appendix E)

$$\boxed{E_{rad,\Delta} \equiv s_R^* E_{R,\Delta}}, \tag{3.22}$$

$$\boxed{E_{chord,\Delta} \equiv \sum_{i=1}^{N} sign(\hat{y}_i) s_{yi}^* E_{yi,\Delta}}, \tag{3.23}$$

$$\boxed{E_{angle,\Delta} \equiv \sum_{i=1}^{N} sign(\hat{\phi}_i) s_{\phi i}^* E_{\phi i,\Delta}}, \tag{3.24}$$

respectively, where

$E_{R,\Delta}$ ≡ relative combined standard uncertainty of the meter body inner radius, $\hat{R}$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{yi,\Delta}$ ≡ relative combined standard uncertainty of the lateral chord position of acoustic path no. *i*, $\hat{y}_i$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{\phi i,\Delta}$ ≡ relative combined standard uncertainty of the inclination angle of acoustic path no. *i*, $\hat{y}_i$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

Here, s_R^*, s_{yi}^* and $s_{\phi i}^*$ are the *relative (non-dimensional) sensitivity coefficients* for the sensitivity of the estimate $\hat{q}_{USM}$ to the input estimates $\hat{R}, \hat{y}_i$ and $\hat{\phi}_i$, respectively, given as [Lunde *et al.*, 1997; 2000a]

$$s_R^* = \frac{1}{\left|\hat{Q}\right|} \sum_{i=1}^{N} \hat{Q}_i \left(2 + \frac{1}{1 - \left(\hat{y}_i / \hat{R}\right)^2} \right), \tag{3.25}$$

$$s_{yi}^* = -sign(\hat{y}_i) \frac{\hat{Q}_i}{\left|\hat{Q}\right|} \frac{\left(\hat{y}_i / \hat{R}\right)^2}{1 - \left(\hat{y}_i / \hat{R}\right)^2}, \tag{3.26}$$

$$s^*_{\phi i} = -\frac{\hat{Q}_i}{\left|\hat{Q}\right|}\frac{2\left|\hat{\phi}_i\right|}{tan\,2\hat{\phi}_i}, \tag{3.27}$$

respectively, where for convenience in notation, the definititions

$$\hat{Q} \equiv \sum_{i=1}^{N}\hat{Q}_i\ , \qquad \hat{Q}_i \equiv 7200\pi\hat{R}^2\,\frac{\hat{P}T_0\hat{Z}_0}{P_0\hat{T}\hat{Z}}\,w_i\,\frac{(N_{refl,i}+1)\sqrt{\hat{R}^2-\hat{y}_i^2}\,(\hat{t}_{1i}-\hat{t}_{2i})}{\hat{t}_{1i}\hat{t}_{2i}\left|sin\,2\hat{\phi}_i\right|}\ , \tag{3.28}$$

have been used.

Note that in the program *EMU - USM Fiscal Gas Metering Station*, the ratio $\hat{Q}_i\big/\left|\hat{Q}\right|$ is for convenience set equal to (calculated as) the weight factor of path no. *i*, w_i, without much loss of generality. For the uniform flow velocity profile this is an exact identity, whereas for more realistic (non-uniform) profiles it represents an approximation.

The subscript "Δ" used in Eqs. (3.22)-(3.24) denotes that *only deviations relative to the conditions at the flow calibration (with respect to pressure and temperature)* are to be accounted for in these expressions. That means, uncertainty contributions such as measurement uncertainties of $\hat{R}_0$, $\hat{y}_{i0}$ and $\hat{\phi}_{i0}$, i = 1, …, N, and out-of-roundness of $\hat{R}_0$, are eliminated at flow calibration (cf. Table 1.4), and are *not* to be included in these expressions. The main contributions to $E_{rad,\Delta}$, $E_{chord,\Delta}$ and $E_{angle,\Delta}$ are thus due to possible change of pressure and temperature from flow calibration conditions to field operation (line) conditions.

Consequently, the relative uncertainty terms involved at the right-hand side of Eqs. (3.22)-(3.24) are given as [Lunde *et al.*, 1997; 2000a]

$$\boxed{E^2_{R,\Delta} = E^2_{KP} + E^2_{KT}}, \tag{3.29}$$

$$\boxed{E^2_{yi,\Delta} = E^2_{KP} + E^2_{KT}}, \tag{3.30}$$

$$\boxed{E_{\phi i,\Delta} = \frac{B\sin 2\hat{\phi}_{i0}}{2\hat{\phi}_{i0}}E_{KP}}, \tag{3.31}$$

respectively, where the definitions

$$E_{KP} \equiv \frac{u_c(\hat{K}_P)}{\left|\hat{K}_P\right|}\ , \qquad E_{KT} \equiv \frac{u_c(\hat{K}_T)}{\left|\hat{K}_T\right|} \tag{3.32}$$

have been used and

$u_c(\hat{K}_P)$ ≡ combined standard uncertainty of the estimate of the radial pressure correction factor for the USM meter body, $\hat{K}_P$,

$u_c(\hat{K}_T)$ ≡ combined standard uncertainty of the estimate of the radial temerature correction factor for the USM meter body, $\hat{K}_T$.

These uncertainty contributions are evaluated in the following.

<u>Meter body pressure effects</u>

For deviation in gas pressure between line and flow calibration conditions, the radial pressure correction factor for the USM meter body is given from Eq. (2.17) as

$$K_P = 1 + \beta \Delta P_{cal}\ , \qquad \Delta P_{cal} = P - P_{cal}\ , \tag{3.33}$$

where P_{cal} is the gas pressure at flow calibration conditions. From Eq. (3.33) one has

$$u_c^2(\hat{K}_P) = (\Delta\hat{P}_{cal})^2 u^2(\hat{\beta}) + \hat{\beta}^2 u_c^2(\Delta\hat{P}_{cal})\ , \tag{3.34}$$

where

$u(\hat{\beta})$ ≡ standard uncertainty of the estimate of the coefficient of linear pressure expansion for the meter body material, β. This includes propagation of uncertainties of the input quantities to the β model used (e.g. Eq. (2.19)), and the uncertainty of the β model *itself* (cf. Table 2.6).

$u_c(\Delta\hat{P}_{cal})$ ≡ combined standard uncertainty of the estimate of the difference in gas pressure between line and flow calibration conditions, $\Delta\hat{P}_{cal}$.

For calculation of $u_c(\Delta\hat{P}_{cal})$, two options need to be discussed:

(1) **Meter body pressure correction *not* used.** In situations where no pressure correction of the dimensional quantities of the meter body (R, y_i, ϕ_i, L_i and x_i) is used by the the USM manufacturer, $u_c(\Delta\hat{P}_{cal})$ is determined by the *"span"* of the pressure difference, equal to $\Delta\hat{P}_{cal}$. By assuming a Type B uncertainty, a 100 % confidence level and a rectangular probability distribution within the range $\pm\Delta\hat{P}_{cal}$ ($k = \sqrt{3}$, cf. Section B.3), the standard uncertainty of the pressure difference is thus calculated as

$$u_c(\Delta\hat{P}_{cal}) = \frac{\left|\Delta\hat{P}_{cal}\right|}{\sqrt{3}} \quad . \tag{3.35}$$

(2) **Meter body pressure correction *is* used.** In situations where pressure correction of the dimensional quantities of the meter body (R, y_i, ϕ_i, L_i and x_i) *is* used by the USM manufacturer, $u_c(\Delta\hat{P}_{cal})$ is determined by the measurement uncertainty of the pressure difference estimate $\Delta\hat{P}_{cal}$ itself, given as

$$u_c^2(\Delta\hat{P}_{cal}) = u_c^2(\hat{P}) + u_c^2(\hat{P}_{cal}) \approx 2u_c^2(\hat{P}) \quad . \tag{3.36}$$

where the pressure measurements in the field and at the flow calibration have been assumed to be uncorrelated, and their combined standard uncertainties approximately equal. $u_c(\hat{P})$ is given by Eq. (3.11).

<u>Meter body temperature effects</u>

For deviation in gas temperature between line and flow calibration conditions, the radial temperature correction factor of the USM meter body is given from Eq. (2.16) as

$$K_T = 1 + \alpha\Delta T_{cal} \,, \qquad \Delta T_{cal} = T - T_{cal} \,, \tag{3.37}$$

where T_{cal} is the gas temperature at flow calibration conditions. Eq. (3.37) yields

$$u_c^2(\hat{K}_T) = (\Delta\hat{T}_{cal})^2 u^2(\hat{\alpha}) + \hat{\alpha}^2 u_c^2(\Delta\hat{T}_{cal}) \,, \tag{3.38}$$

where

$u(\hat{\alpha})$ ≡ standard uncertainty of the estimate of the coefficient of linear temperature expansion for the meter body material, $\hat{\alpha}$.

$u_c(\Delta\hat{T}_{cal})$ ≡ combined standard uncertainty of the estimate of the difference in gas temperature between line and flow calibration conditions, $\Delta\hat{T}_{cal}$.

For calculation of $u_c(\Delta\hat{T}_{cal})$, two options need to be discussed:

(1) **Meter body temperature correction *not* used.** In situations where no temperature correction of the dimensional quantities of the meter body (R, y_i, ϕ_i, L_i and x_i) is used by the the USM manufacturer, $u_c(\Delta\hat{T}_{cal})$ is determined by the *"span"* of the temperature difference, equal to $\Delta\hat{T}_{cal}$. By assuming a Type B uncertainty, a 100 % confidence level and a rectangular probability distribution

within the range $\pm\Delta\hat{T}_{cal}$ ($k = \sqrt{3}$, cf. Section B.3), the standard uncertainty of the temperature difference is calculated as

$$u_c(\Delta\hat{T}_{cal}) = \frac{\left|\Delta\hat{T}_{cal}\right|}{\sqrt{3}} \quad . \tag{3.39}$$

(2) **Meter body temperature correction *is* used.** In situations where temperature correction of the dimensional quantities of the meter body (R, y_i, ϕ_i, L_i and x_i) *is* used by the USM manufacturer, $u_c(\Delta\hat{T}_{cal})$ is determined by the measurement uncertainty of the temperature difference estimate $\Delta\hat{T}_{cal}$ itself, given as

$$u_c^2(\Delta\hat{T}_{cal}) = u_c^2(\hat{T}) + u_c^2(\hat{T}_{cal}) \approx 2u_c^2(\hat{T}) \quad . \tag{3.40}$$

where the temperature measurements in the field and at the flow calibration have been assumed to be uncorrelated, and their combined standard uncertainties approximately equal. $u_c(\hat{T})$ is given by Eq. (3.12).

In the Excel program *EMU - USM Fiscal Gas Metering Station*, the user specifies the relative standard uncertainties $u(\hat{\alpha})/\left|\hat{\alpha}\right|$ and $u(\hat{\beta})/\left|\hat{\beta}\right|$, and has the choice between the two Options (1) and (2) for calculation of $u_c(\Delta\hat{T}_{cal})$ *and* $u_c(\Delta\hat{P}_{cal})$. The USM manufacturer must specify whether temperature and pressure correction are made or not.

As an example, the meter body uncertainty is evaluated in Section 4.4.2.

3.4.2 USM transit time uncertainties

Uncertainties related to transit times measured by the USM are involved in Eqs. (3.19) and (3.20). These are the relative combined standard uncertainties related to random transit time effects (representing the USM repeatability in field operation), E_{rept}, and to systematic transit time effects (representing those parts of the systematic change of transit times from flow calibration to line conditions which are not corrected for in the USM), $E_{time,\Delta}$, given as (cf. Appendix E)

$$\boxed{E_{rept}^2 \equiv 2\sum_{i=1}^{N}(s_{t1i}^{*}E_{t1i,U})^2} \quad , \tag{3.41}$$

$$\boxed{E_{time,\Delta} \equiv \sum_{i=1}^{N} \left(s_{t1i}^{*} E_{t1i,C}^{\Delta} + s_{t2i}^{*} E_{t2i,C}^{\Delta} \right)} \,, \tag{3.42}$$

respectively, where

$E_{t1i,U}$ ≡ relative standard uncertainty of those contributions to the transit time estimates $\hat{t}_{1i}$ and $\hat{t}_{2i}$ which are *uncorrelated* with respect to upstream and downstream propagation, such as turbulence, noise (coherent and incoherent), finite clock resolution, electronics stability (possible random effects), possible random effects in signal detection/processing (e.g. erronous signal period identification), and power supply variations.

$E_{t1i,C}^{\Delta}$ ≡ relative standard uncertainty of uncorrected systematic transit time effects on upstream propagation of acoustic path no. *i*, $\hat{t}_{1i}$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{t2i,C}^{\Delta}$ ≡ relative standard uncertainty of uncorrected systematic transit time effects on downstream propagation of acoustic path no. *i*, $\hat{t}_{2i}$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

Here, s_{t1i}^{*} and s_{t2i}^{*} are the *relative (non-dimensional) sensitivity coefficients* for the sensitivity of the estimate $\hat{Q}$ to the input estimates $\hat{t}_{1i}$ and $\hat{t}_{2i}$, respectively, given as [Lunde *et al*., 1997; 2000a] (cf. Appendix E)

$$s_{t1i}^{*} = \frac{\hat{Q}_i}{\left|\hat{Q}\right|} \frac{\hat{t}_{2i}}{\hat{t}_{1i} - \hat{t}_{2i}} \,, \tag{3.43}$$

$$s_{t2i}^{*} = -\frac{\hat{Q}_i}{\left|\hat{Q}\right|} \frac{\hat{t}_{1i}}{\hat{t}_{1i} - \hat{t}_{2i}} \,, \tag{3.44}$$

respectively. The coefficients s_{t1i}^{*} and s_{t2i}^{*} have opposite sign and are almost equal in magnitude, but not exactly equal, which is important especially for the magnitude of the term $E_{time,\Delta}$ given by Eq. (3.42).

3.4.2.1 USM repeatability in field operation (random transit time effects)

The relative combined standard uncertainty related to random transit time effects, representing the USM repeatability in field operation, is given by Eq. (3.41). Here, the definition

$$E_{t1i,U} \equiv \frac{u(\hat{t}_{1i}^{random})}{\hat{t}_{1i}} \tag{3.45}$$

has been used (cf. Appendix E), where

$u(\hat{t}_{1i}^{random}) \equiv$ standard uncertainty (i.e. standard deviation) due to in-field random effects on the transit time $\hat{t}_{1i}$ (and $\hat{t}_{2i}$), at a specific flow rate, and after possible time averaging (representing the repeatability of the transit times), such as (cf. Table 1.4):

- Turbulence effects (from temperature and velocity fluctuations, especially at high flow velocities).
- Incoherent noise (from pressure reduction valves (PRV), electromagnetic noise (RFI), pipe vibrations, etc.).
- Coherent noise (from acoustic cross-talk (through meter body), electromagnetic cross-talk, acoustic reflections in meter body, possible other acoustic interference, etc.).
- Finite clock resolution.
- Electronics stability (possible random effects).
- Possible random effects in signal detection/processing (e.g. randomly occuring erronous signal period identification, giving alternating measured transit times over the time averaging period, by one or several half periods), Such random effects could occur in case of e.g. (1) insufficiently robust signal detection solution, and (2) noise and turbulence influence.
- Power supply variations.

Such effects are typically uncorrelated, and will contribute to $u(\hat{t}_{1i}^{random})$ by a root-sum-square calculation (if that needs to be calculated). In practice, at a given flow rate, $u(\hat{t}_{1i}^{random})$ may be taken as the standard uncertainty of the spread of transit times.

Note that in the uncertainty model of the USM gas metering station, the repeatability in flow calibration and in field operation have both been accounted for, by different symbols, $E_{rept,j}$ and E_{rept}, respectively. This is motivated as described in Section 3.3.3.

For user input / calculation of E_{rept}, user input at different levels may be convenient and useful. Two options are discussed:

(1) **Specification of E_{rept} directly.** In this case the relative standard uncertainty of the repeatability of the in-field USM *flow rate reading* (E_{rept}) is given directly,

from information specified by the USM manufacturer. Eqs. (3.41) and (3.45) are not used in this case.

(2) **Specification of $u(\hat{t}_{1i}^{random})$, and calculation of E_{rept}.** In this case the repeatability of the *transit times* of the in-field USM flow rate reading $(u(\hat{t}_{1i}^{random}))$ is given, from information which may preferably be specified by the USM manufacturer[77]. E_{rept} is then calculated using Eqs. (3.41), (3.43) and (3.45).

In this context, it should be noted that in case of option (1), i.e. if E_{rept} is the input uncertainty specified, and if E_{rept} is given to be constant over the flow velocity range (as a relevant example, on basis of USM manufacturer information provided today, cf. Table 6.1), this will give a uncertainty contribution to the USM measurement from the USM repeatability which is constant over the flow velocity range. This result is simplified, and may be incorrect.

On the other hand, in case of option (2), i.e. if $u(\hat{t}_{1i}^{random})$ is the input specified, the situation is more complex. In the lower end of the flow rate range, turbulence effects may be small, so that $u(\hat{t}_{1i}^{random})$ is normally dominated by the background noise and/or time detection uncertainties, which are relatively constant with respect to flow rate. At lower flow rates, thus, this will give an uncertainty contribution to the USM measurement (E_{rept}) which increases at low velocities, due to the increasing relative sensitivity coefficient s_{1i}^{*}, cf. Eq. (3.43).

At higher flow rates, however, turbulence effects, flow noise and possible PRV noise become more dominant so that $u(\hat{t}_{1i}^{random})$ may increase by increasing flow rate. This will give an uncertainty contribution to the USM measurement (E_{rept}) which increases by increasing flow rate.

In practice, thus, E_{rept} may have a minimum at intermediate flow rates, with an increase both at the low and high flow rates.

In spite of such “complications”, both Options (1) and (2) in the program *EMU - USM Fiscal Gas Metering Station* may be useful, as a user’s choice. Today, the repeatability information available from USM manufacturers is related directly to E_{rept}, and where E_{rept} in practice often is given to be a constant (cf. Table 6.1).

77 The repeatability of the transit times (e.g. the standard deviation) is information which should be readily available in USM flow computers today, for different flow rates.

However, it is a hope for the future that also information on $u(\hat{t}_{1i}^{random})$, and its variation with flow rate, may be available from USM manufacturers (cf. Chapter 6)[78]. Alternatively, the USM manufacturer might provide information on the variation of E_{rept} with flow rate. In both cases, the variation of E_{rept} with flow rate would be described by the present uncertainty model.

Consequently, in the program, E_{rept} or $u(\hat{t}_{1i}^{random})$ serve as optional input uncertainties, cf. Sections 4.4.1 and 5.10.1 (Figs. 5.12 and 5.13). It is left to the USM manufacturer to specify and document E_{rept} or $u(\hat{t}_{1i}^{random})$. As an example, the USM repeatability in field operation is evaluated in Section 4.4.1.

3.4.2.2 Systematic transit time effects

The relative combined standard uncertainty related to those parts of the systematic change of transit times from flow calibration to line conditions which are not corrected for in the USM, is given by Eq. (3.42). In this expression the definitions

$$E_{t1i,C}^{\Delta} \equiv \frac{u(\hat{t}_{1i}^{systematic})}{\hat{t}_{1i}}, \qquad E_{t2i,C}^{\Delta} \equiv \frac{u(\hat{t}_{2i}^{systematic})}{\hat{t}_{2i}}, \tag{3.46}$$

have been used, where

$u(\hat{t}_{1i}^{systematic}) \equiv$ standard uncertainty of uncorrected systematic effects on the upstream transit time, $\hat{t}_{1i}$, due to possible deviation in conditions between flow calibration and field operation,

$u(\hat{t}_{2i}^{systematic}) \equiv$ standard uncertainty of uncorrected systematic effects on the downstream transit time, $\hat{t}_{2i}$, due to possible deviation in conditions between flow calibration and field operation.

Such systematic effects may be due to (cf. Table 1.4):

- Cable/electronics/transducer/diffraction time delay, including finite-beam effects (due to line pressure and temperature effects, ambient temperature effects, drift, effects of possible transducer exchange).
- Possible Δt-correction (line pressure and temperature effects, ambient temperature effects, drift, reciprocity, effects of possible transducer exchange).

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78 In fact, as possible additional information to USM manufacturer repeatability, information about $u(\hat{t}_{1i}^{random})$, $u(\hat{t}_{1i}^{random})$ might actually be *calculated* from more basic input, such as signal-to-noise ratio (coherent and incoherent noise), clock resolution, etc. [Lunde *et al.*, 1997]. However, this is not considered here.

- Possible systematic effects in signal detection/processing (e.g. systematic erronous signal period identification, giving systematically shifted measured transit times over the time averaging period, by one or several half periods). Such systematic effects could occur in case of (1) insufficiently robust signal detection solution, or (2) significantly changed pulse form, due to e.g. (a) noise and turbulence influence, (b) transducer change or failure (changed transducer properties, for one or several transducers), (c) transducer exchange (different transducer properties relative to original transducers).
- Possible cavity time delay correction (e.g. sound velocity effects).
- Possible transducer deposits (lubricant oil, grease, wax, etc.).
- Sound refraction (flow profile effects ("ray bending"), especially at high flow velocities, and by changed installation conditions) [Frøysa *et al.*, 2001].

Note that in Eqs. (3.42) and (3.46), the symbol "Δ" denotes that *only deviations relative to the conditions at the flow calibration* are to be accounted for in these expressions.

In the program *EMU - USM Fiscal Gas Metering Station*, $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$ serve as input uncertainties, cf. Sections 4.4.4 and 5.10.2.2 (Fig. 5.17). These may be meter specific, and it is therefore left to the USM manufacturer to specify and document $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$. The individual effects described above are typically uncorrelated, and will contribute to each of $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$ by a root-sum-square calculation.

As an example, the uncertainty due to systematic transit time effects is discussed in Section 4.4.3.

3.4.3 USM integration uncertainties (installation conditions)

In Eq. (3.20), the relative standard uncertainty $E_{I,\Delta}$ accounts for installation effects on the uncertainty of the USM fiscal gas metering station, and is here defined as

$$E_{I,\Delta} \equiv \frac{u(\hat{q}_{USM,I}^{\Delta})}{\left|\hat{q}_{USM}\right|} \quad , \tag{3.47}$$

where

$u(\hat{q}^{\Delta}_{USM,I})$ ≡ standard uncertainty of the USM integration method due to change of installation conditions from flow calibration to field operation.

Such installation effects on the USM integration uncertainty may be due to:

- Change of *axial flow velocity profile* (from flow calibration to field operation), and
- Change of *transversal flow velocity profiles* (from flow calibration to field operation), both due to e.g. (cf. Table 1.4):

  - possible different pipe bend configuration upstream of the USM,
  - possible different in-flow profile to the upstream pipe bend,
  - possible change of meter orientation relative to pipe bends,
  - possible changed wall roughness over time (corrosion, wear, pitting, etc.), in the pipe and meter body,
  - possible wall deposits / contamination in the pipe and meter body (grease, liquid, lubricants, etc.).

Note that the subscript "Δ" denotes that *only changes of installation conditions from flow calibration to field operation* are to be accounted for in $u(\hat{q}^{\Delta}_{USM,I})$ and $E_{I,\Delta}$.

In the program *EMU - USM Fiscal Gas Metering Station*, $E_{I,\Delta}$ serves as an input uncertainty, cf. Chapters 4 and 5. It is left to the USM manufacturer to specify and document $E_{I,\Delta}$. The individual effects described above may typically be uncorrelated, and will then contribute to $u(\hat{q}^{\Delta}_{USM,I})$ by a root-sum-square calculation.

As an example, the uncertainty of the integration method is discussed in Section 4.4.4.

3.4.4 Miscellaneous USM effects

In Eq. (3.19), the relative uncertainty term E_{misc} accounts for possible miscellaneous uncertainty contributions to the USM measurement which have not been accounted for by the other uncertainty terms involved in the uncertainty model of the gas metering station. Cf. the definition of E_{misc} accompanying Eq. (3.19). Such contributions could be e.g. inaccuracy of the USM functional relationship (cf. Table 1.4), or other uncertainty contributions.

Such miscellaneous uncertainty contributions are not adressed further here, but in the program *EMU - USM Fiscal Gas Metering Station* the user has the possibility to specify a value accounting for such uncertainty contributions, in case that is found to be useful, cf. Section 5.10.2.2 (Fig. 5.17).

3.5 Signal communication and flow computer calculations

In Eq. (3.6), the relative uncertainty term E_{comm} accounts for the uncertainties due to the signal communication between the USM field electronics and the flow computer, in the uncertainty model of the gas metering station (e.g. the flow computer calculation of frequency in case of analog frequency output). E_{flocom} accounts for the uncertainty of the flow computer calculations, and is normally relatively small. Cf. the definition of these terms accompanying Eq. (3.6).

These two uncertainty contributions are not adressed further here, but in the program *EMU - USM Fiscal Gas Metering Station* the user has the possibility to specify such uncertainty contributions, in case that is found to be necessary, cf. Section 5.11 (Fig. 5.19).

3.6 Summary of input uncertainties to the uncertainty model

In Tables 3.1-3.8, the input uncertainties to be given as input to the program *EMU - USM Fiscal Gas Metering Station* are specified. The various uncertainties are defined in Sections 3.1-3.5, and are here organized in eight groups (following the structure of the worksheets in the program, cf. Chapter 5):

- Pressure measurement uncertainty (Table 3.1),
- Temperature measurement uncertainty (Table 3.2),
- Compressibility factor uncertainty (Table 3.3),
- Density measurement uncertainty (Table 3.4),
- Calorific value measurement uncertainty (Table 3.5),
- Flow calibration uncertainties (Table 3.6),
- Signal communication and flow computer calculations (Table 3.7),
- USM field uncertainty (Table 3.8).

For some of the quantities, input uncertainties can be specified at two levels, (1) “overall level” and (2) “detailed level”, as discussed in Section 1.3 (cf. Table 1.5), and in Chapters 3 and 5. Examples of input uncertainties given for the “detailed level” are described in Chapter 4.

Table 3.1. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the static gas pressure measurement.

Gas pressure measurement, *P*				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
(1) Overall level	$u_c(\hat{P})$	Combined standard uncertainty of the pressure estimate, $\hat{P}$.	Program user	3.2.1 and 5.4
(2) Detailed level	$u(\hat{P}_{transmitter})$	Standard uncertainty of the pressure transmitter.	Manufacturer / Calibration lab.	3.2.1, 4.2.1 and 5.4
	$u(\hat{P}_{stability})$	Standard uncertainty of the stability of the pressure transmitter (drift).	Manufacturer	----- “ -----
	$u(\hat{P}_{RFI})$	Standard uncertainty due to RFI effects on the pressure transmitter.	Manufacturer	----- “ -----
	$u(\hat{P}_{temp})$	Standard uncertainty of the effect of ambient temperature on the pressure transmitter.	Manufacturer	----- “ -----
	$u(\hat{P}_{atm})$	Standard uncertainty of the atmospheric pressure.	Program user	----- “ -----
	$u(\hat{P}_{vibration})$	Standard uncertainty due to vibration effects on the pressure transmitter.	Manufacturer	----- “ -----
	$u(\hat{P}_{power})$	Standard uncertainty due to power supply effects on the pressure transmitter.	Manufacturer	----- “ -----
	$u(\hat{P}_{misc})$	Standard uncertainty due to other (miscellaneous) effects on the pressure transmitter.	Manufacturer / Program user	----- “ -----

Table 3.2. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the gas temperature measurement.

Gas temperature measurement, *T*				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
(1) Overall level	$u_c(\hat{T})$	Combined standard uncertainty of the temperature estimate, $\hat{T}$.	Program user	3.2.2 and 5.5
(2) Detailed level	$u(\hat{T}_{elem,transm})$	Standard uncertainty of the temperature element and temperature transmitter, calibrated as a unit.	Manufacturer / Calibration lab.	3.2.2, 4.2.2 and 5.5
	$u(\hat{T}_{stab,transm})$	Standard uncertainty of the stability of the temperature transmitter (drift).	Manufacturer	----- “ -----
	$u(\hat{T}_{RFI})$	Standard uncertainty due to RFI effects on the temperature transmitter.	Manufacturer	----- “ -----
	$u(\hat{T}_{temp})$	Standard uncertainty of the effect of ambient temperature on the temperature transmitter.	Manufacturer	----- “ -----
	$u(\hat{T}_{stab,elem})$	Standard uncertainty of the stability of the Pt 100 4-wire RTD temperature element	Program user	----- “ -----
	$u(\hat{T}_{vibration})$	Standard uncertainty due to vibration effects on the temperature transmitter.	Manufacturer	----- “ -----
	$u(\hat{T}_{power})$	Standard uncertainty due to power supply effects on the temperature transmitter.	Manufacturer	----- “ -----
	$u(\hat{T}_{cable})$	Standard uncertainty of lead resistance effects on the temperature transmitter.	Manufacturer / Program user	----- “ -----
	$u(\hat{T}_{misc})$	Standard uncertainty due to other (miscellaneous) effects on the pressure transmitter.	Manufacturer / Program user	----- “ -----

Table 3.3. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the gas compressibility factor ratio, Z/Z_0.

Gas compressibility factors, Z and Z_0				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
(1) Overall level	Not available			
(2) Detailed level	$E_{Z,mod}$	Relative standard uncertainty due to *model uncertainty* (uncertainty of the model used for calculation of $\hat{Z}$)	Program user	3.2.3, 4.2.3 and 5.6
	$E_{Z0,mod}$	Relative standard uncertainty due to *model uncertainty* (uncertainty of the model used for calculation of $\hat{Z}_0$)	Program user	----- " -----
	$E_{Z,anal}$	Relative standard uncertainty due to *analysis uncertainty* for $\hat{Z}$ (inaccurate determ. of gas composition)	Manufacturer / Program user	----- " -----
	$E_{Z0,anal}$	Relative standard uncertainty due to *analysis uncertainty* for $\hat{Z}_0$ (inaccurate determ. of gas composition)	Manufacturer / Program user	----- " -----

Table 3.4. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the gas density measurement.

Gas density measurement, ρ				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
(1) Overall level	$u_c(\hat{\rho})$	Combined standard uncertainty of the density estimate, $\hat{\rho}$.	Program user	3.2.4 and 5.7
(2) Detailed level	$u(\hat{\rho}_u)$	Standard uncertainty of the indicated (uncorrected) density esitmate, $\hat{\rho}_u$	Manufacturer	3.2.4, 4.2.4 and 5.7
	$u(\hat{\rho}_{rept})$	Standard uncertainty of the repetatility of the indicated (uncorrected) density esitmate, $\hat{\rho}_u$	Manufacturer	----- " -----
	$u(\hat{T}_c)$	Standard uncertainty of the calibration temperature estimate, $\hat{T}_c$	Manufacturer	----- " -----
	$u_c(\hat{T})$	Combined standard uncertainty of the line temperature estimate, $\hat{T}$	Calculated by program	----- " -----
	$u_c(\hat{T}_d)$	Combined standard uncertainty of the densitometer temperature estimate, $\hat{T}_d$	Calculated: $u_c(\hat{T}_d) = u_c(\hat{T})$	----- " -----
	$u(\hat{P})$	Standard uncertainty of the line pressure estimate, $\hat{P}$	Calculated by program	----- " -----
	$u(\Delta\hat{P}_d)$	Standard uncertainty of the pressure difference estimate, $\Delta\hat{P}_d$ (from densitometer to line conditions)	Calculated by program	----- " -----
	$u(\hat{c}_c)$	Standard uncertainty of the calibration gas VOS estimate, $\hat{c}_c$	Manufacturer / Program user	----- " -----
	$u(\hat{c}_d)$	Standard uncertainty of the densitometer gas VOS estimate, $\hat{c}_d$	Manufacturer / Program user	----- " -----
	$u(\hat{\tau})$	Standard uncertainty of the periodic time estimate, $\hat{\tau}$	Manufacturer	----- " -----
	$u(\hat{K}_d)$	Standard uncertainty of the VOS correction transducer constant estimate, $\hat{K}_d$	Manufacturer	----- " -----
	$u(\hat{\rho}_{temp})$	Standard uncertainty representing the model uncertainty of the temperature correction	Manufacturer	----- " -----
	$u(\hat{\rho}_{misc})$	Standard uncertainty of the indicated density esitmate, $\hat{\rho}_u$, due to other (miscellaneous) effects	Program user	----- " -----

Table 3.5. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the calorific value measurement.

USM flow calibration				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
Overall level	E_{H_S}	Relative standard uncertainty of the calorific value estimate, $\hat{H}_S$.	Program user	3.2, 4.2.5, 5.8

Table 3.6. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the USM flow calibration.

USM flow calibration				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
(1) Overall level	Not available			
(2) Detailed level	$E_{q_{ref,j}}$	Relative standard uncertainty of the reference measurement, $\hat{q}_{ref,j}$, at test flow rate no. *j*, *j* = 1, …, *M* (representing the uncertainty of the flow calibration laboratory).	Flow calibration laboratory	3.3.1, 4.3.1 and 5.9
	$E_{K_{dev,j}}$	Relative standard uncertainty of the deviation factor estimate, $\hat{K}_{dev,j}$, at test flow rate no. *j*, *j* = 1, …, *M*.	Calculated by program	3.3.2, 4.3.2 and 5.9
	$E_{rept,j}$	Relative standard uncertainty of the repeatability of the USM at flow calibration, at test flow rate no. *j*, *j* = 1, …, *M*.	USM manufacturer	3.3.3, 4.3.3 and 5.9

Table 3.7. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, with respect to signal communication and flow computer calculations.

Signal communication and flow computer calculations				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
	E_{comm}	Relative standard uncertainty of the estimate $\hat{q}_v$ due to signal communication with flow computer (flow computer calculation of frequency, in case of analog frequency output)	USM manufacturer	3.5 , 4.5 and 5.11
	E_{flocom}	Relative standard uncertainty of the estimate $\hat{q}_v$ due to flow computer calculations	USM manufacturer	----- “ -----

Table 3.8. Input uncertainties to the uncertainty calculation program *EMU - USM Fiscal Gas Metering Station*, for calculation of the expanded uncertainty of the USM in field operation (uncorrected deviations re. flow calibration conditions).

USM field operation				
Input level	**Input uncertainty**	**Description**	**To be specified and documented by:**	**Ref. to Handbook Section**
Specified in both cases, (1) and (2)	E_{rept} or $u(\hat{t}_{1i}^{random})$	Relative standard uncertainty of the repeatability of the USM in field operation, at actual flow rate. Standard uncertainty (i.e. standard deviation) due to in-field random effects on the transit times, after possible time averaging. It represents the repeatability of the transit times, at the actual flow rate. Such random effects may be due to e.g.: • Turbulence effects (from temperature and velocity fluctuations, especially at high flow velocities), • Incoherent noise (from pressure reduction valves (PRV), electromagnetic noise (RFI), pipe vibrations, etc.) • Coherent noise (from acoustic cross-talk (through meter body), electromagnetic cross-talk, acoustic reflections in meter body, possible other acoustic interference, etc.), • Finite clock resolution. • Electronics stability (possible random effects). • Possible random effects in signal detection/ processing (e.g. erronous signal period identification). • Power supply variations.	USM manufacturer USM manufacturer	3.4.2, 4.4.1 and 5.10.1 3.4.2, 4.4.1 and 5.10.1
	E_{misc}	Relative standard uncertainty of the estimate $\hat{q}_v$ due to miscellaneous effects on the USM measurement.	USM manufacturer/ Program user	3.4.4 and 4.4.5
(1) Overall level	$E_{USM,\Delta}$	Relative combined standard uncertainty of the estimate $\hat{q}_v$ at actual flow rate, related to field operation of the USM, due to *change* of conditions from flow calibration to field operation.	USM manufacturer	3.4 and 5.10.2
(2) Detailed level	$u(\hat{\alpha})/\lvert\hat{\alpha}\rvert$	Rel. standard uncertainty of the coefficient of linear temperature expansion for the meter body material, $\hat{\alpha}$	Program user / USM manufacturer	3.4.1, 4.4.2 and 5.10.2
	$u(\hat{\beta})/\lvert\hat{\beta}\rvert$	Rel. standard uncertainty of the coefficient of linear radial pressure expansion for the meter body, $\hat{\beta}$	Program user / USM manufacturer	----- " -----
	$u_c(\Delta\hat{T}_{cal})$	Combined standard uncertainty of the temperature difference estimate, $\Delta\hat{T}_{ref}$ (flow calibration to line cond.)	Calculated by program	----- " -----
	$u_c(\Delta\hat{P}_{cal})$	Combined standard uncertainty of the pressure difference estimate , $\Delta\hat{P}_{ref}$ (flow calibration to line condit.)	Calculated by program	----- " -----
	$u(\hat{t}_{1i}^{systematic})$	Standard uncertainty of uncorrected systematic effects on the *upstream* transit times, due to *change* of conditions from flow calibration to field operation. Such systematic effects may be due to e.g.: • Cable/electronics/transducer/diffraction time delay (line *P* & *T* effects, ambient temperature effects, drift, effects of possible transducer exchange), • Possible *Δt*-correction (line *P* & *T* effects, ambient temperature effects, drift, reciprocity, effects of possible transducer exchange), • Possible systematic effects in signal detection / processing (e.g. erronous signal period identification), • Possible cavity time delay correction (e.g. sound	USM manufacturer	3.4.2, 4.4.3 and 5.10.2

		velocity effects), • Possible deposits at transducer faces (lubricant oil, liquid, waxm, grease, etc.), • Sound refraction (flow profile effects on transit times, especially at high flow velocities, and by changed installation conditions).		
	$u(\hat{t}_{2i}^{systematic})$	Standard uncertainty of uncorrected systematic effects on the *downstream* transit times, due to *change* of conditions from flow calibration to field operation (see above).	USM manufacturer	----- " -----
	$E_{I,\Delta}$	Relative standard uncertainty of the USM integration method, due to *change* of installation conditions from flow calibration to field operation. Such installation effects on the USM integration uncertainty may be due to e.g.: • Change of *axial flow velocity profile* (from flow calibration to field operation), and • Change of *transversal flow velocity profiles* (from flow calibration to field operation), due to e.g.: ▹ possible different upstream pipe bend configuration, ▹ possible different in-flow profile to upstr. pipe bend, ▹ possible different meter orientation rel. to pipe bends ▹ possible changed wall roughness over time (corrosion, wear, pitting), ▹ possible wall deposits, contamination (grease, etc.).	USM manufacturer	3.4.3, 4.4.4 and 5.10.2

4. UNCERTAINTY EVALUATION EXAMPLE

In the present chapter the uncertainty model of the USM gas metering station described in Chapter 3, is used to evaluate the relative expanded uncertainty of a gas metering station at given operating conditions, as an example. Input uncertainties are evaluated for the different instruments of the metering station, and propagated in the uncertainty model. The input uncertainties discussed here are the same as those used as input to the program *EMU - USM Fiscal Gas Metering Station*, cf. Chapters 3 and 5. Hence, to some extent the present chapter also serves as a guideline to using the program, as a basis for Chapter 5.

4.1 Instrumentation and operating conditions

The USM fiscal gas metering station evaluated in the present *Handbook* consists of the equipment listed in Table 4.1, as specified by NFOGM, NPD and CMR for this example [Ref Group, 2001], cf. Section 2.1. The pressure, temperature and density instruments specified in the table are the same as those used for uncertainty evaluation of an orifice fiscal gas metering station by [Dahl *et al.*, 1999][79]. With respect to the USM, flow computer, gas chromatograph and calorimeter, no specific equipments are considered.

Operating conditions, etc., used for the present uncertainty evaluation example are given in Table 4.2. Meter body data are given in Table 4.3, where for simplicity the data used in the AGA-9 report have been used also here. Data for the densitometer (Solartron 7812 example) are given in Table 4.4.

79 The uncertainty models for the pressure transmitter and the temperature element/transmitter are similar to the ones used in [Dahl *et al.*, 1999], with some modifications.

However, the uncertainty model for the densitometer is somewhat different here, both with respect to (a) the functional relationship used (different formulations used for the VOS and installation corrections), and (b) the uncertainty model itself. For instance, the densitometer uncertainty model developed and used here is fully analytical, with analytical expressions for the sensitivity coefficients, whereas in [Dahl *et al.*, 1999] the sensitivity coefficients were calculated using a more numerical approach. Also, here the input uncertainties to the densitometer uncertainty model are somewhat different.

Table 4.1. The evaluated USM fiscal gas metering station instrumentation (cf. Table 2.4).

Measurement	**Instrument**
Ultrasonic meter (USM)	Not specified.
Flow computer	Not specified.
Pressure (static), P	Rosemount 3051P Reference Class Smart Pressure Transmitter [Rosemount, 2000].
Temperature, T	Pt 100 element: according to EN 60751 tolerance A [NORSOK, 1998a]. Rosemount 3144 Smart Temperature Transmitter [Rosemount, 2000].
Density, ρ	Solartron Model 7812 Gas Density Transducer [Solartron, 1999].
Compressibility, Z and Z_0	Not specified (calculated from GC measurements).
Calorific value, H_s	Not specified (calorimeter measurement).

Table 4.2. Gas parameters of the USM fiscal gas metering station being evaluated (example). (Corresponds to Fig. 5.1.)

Conditions	**Quantity**	**Value**
Operating	Gas composition	North Sea example[80]
	Line pressure, P (static, absolute)	100 bara
	Line temperature, T	50 ^{o}C (= 323.15 K)
	Gas compressibility, Z	0.8460
	Sound velocity, c	417.0 m/s
	Gas density, ρ	81.62 kg/m^3
	Ambient (air) temperature, T_{air}	0 ^{o}C
Densitometer	Temperature, T_d	48 ^{o}C [81]
	Sound velocity, c_d	415.24 m/s
	Indicated (uncorrected) gas density in densitometer, ρ_u	82.443 kg/m^3
	Calibration temperature, T_c	20 ^{o}C
	Calibration sound velocity, c_c	350 m/s
Flow calibration	Pressure, P_{cal} (static, absolute)	50 bara
	Temperature, T_{cal}	10 ^{o}C
Pressure transmitter	Ambient (air) temperature at calibration	20 ^{o}C
Temperature transm.	Ambient (air) temperature at calibration	20 ^{o}C
Standard ref. cond.	Gas compressibility, Z_0	0.9973
	Superior (gross) calorific value, H_s	41.686 MJ/Sm^3

80 Example of dry gas composition, taken from a North Sea pipeline: C_1: 83.98 %, C_2: 13.475 %, C_3: 0.943 %, i-C_4: 0.040 %, n-C_4: 0.067 %, i-C_5: 0.013 %, n-C_5: 0.008 %, CO_2: 0.756 %, N_2: 0.718 %.

81 Temperature deviation between line and densitometer conditions may be as large as 7-8 ^{o}C [Sakariassen, 2001]. A representative value may be about 10 % of the temperature difference between densitometer and ambient (air) conditions. Here, 2 ^{o}C deviation is used as a moderate example.

Table 4.3. Meter body data for the USM being evaluated (AGA-9 example). (Data used in Section 4.4.2 and Fig. 5.2.)

Meter body	Quantity	Value	Reference
Dimensions	Inner diameter, $2R_0$ ("dry calibration" value)	308 mm	[AGA-9, 1998]
	Average wall thickness, w	8.4 mm	[AGA-9, 1998]
Material data	Temperature expansion coeff., α	$14 \cdot 10^{-6}$ K^{-1}	[AGA-9, 1998]
	Young's modulus (or modulus of elasiticity), Y	$2 \cdot 10^{5}$ MPa	[AGA-9, 1998]

Table 4.4. Gas densitometer data used in the uncertainty evaluation (Solartron 7812 example). (Data used in Section 4.2.4 and Fig. 5.9.)

Symbol	Quantity	Value	Reference
τ	Periodic time	650 μs	[Eide, 2001a]
K_{18}	Constant	$-1.360 \cdot 10^{-5}$	[Solartron, 1999]
K_{19}	Constant	$8.440 \cdot 10^{-4}$	[Solartron, 1999]
K_d	Constant (characteristic length)	21000 μm	[Solartron, 1999]

4.2 Gas measurement uncertainties

In the present subsection, the expanded uncertainties of the gas pressure, temperature and density measurements are evaluated, as well as the *Z*-factor and calorific value estimates.

4.2.1 Pressure measurement

The combined standard uncertainty of the pressure measurement, $u_c(\hat{P})$, is given by Eq. (3.11). This expression is evaluated in the following.

Performance specifications for the Rosemount Model 3051P Reference Class Pressure Transmitter are given in Table 4.5[82], as specified in the data sheet [Rosemount, 2000], etc.

The contributions to the combined standard uncertainty of the pressure measurement are described in the following.

82 Note that the expanded uncertainties given in the transmitter data sheet [Rosemount, 2000] are specified at a 99 % confidence level (k = 3).

Table 4.5. Performance specifications of the Rosemount Model 3051P Reference Class Pressure Transmitter [Rosemount, 2000], used as input to the uncertainty calculations given in Table 4.6.

Quantity or Source	**Value or Expanded uncertainty**	**Coverage factor, *k***	**Reference**
Calibration ambient temperature (air)	20 °C	-	Calibration certificate, *(NA)*
Time between calibrations	12 months	-	Example
Span (calibrated)	70 bar	-	Calibration certificate, *(NA)*
URL (upper range limit)	138 barG	-	[Rosemount, 2000]
Transmitter uncertainty, $U(\hat{P}_{transmitter})$	±0.05 % of span	3	[Rosemount, 2000]
Stability, $U(\hat{P}_{stability})$	±0.125 % of URL for 5 years for 28 °C temperature changes, and up to 69 bar line pressure.	3	[Rosemount, 2000]
	For fiscal gas metering: 0.1 % of URL for 1 year (used here).	-	[Rosemount, 1999]
RFI effects, $U(\hat{P}_{RFI})$	±0.1 % of span from 20 to 1000 MHz and for field strength up to 30 V/m.	3	[Rosemount, 2000]
Ambient temperature effects (air), $U(\hat{P}_{temp})$	±(0.006% URL + 0.03% span) per 28°C	3	[Rosemount, 2000]
Vibration effects, $U(\hat{P}_{vibration})$	*Negligible* (except at resonance frequencies, see text below).	3	[Rosemount, 2000]
Power supply effects, $U(\hat{P}_{power})$	*Negligible* (less than ±0.005 % of calibrated span per volt).	3	[Rosemount, 2000]
Mounting position effect	*Negligible* (influence only on differential pressure measurement, not static pressure measurement)	3	[Dahl *et al.*, 1999]
Static pressure effect	*Negligible* (influence only on differential pressure measurement, not static pressure measurement)	3	[Dahl *et al.*, 1999]

1. ***Pressure transmitter uncertainty,*** $U(\hat{P}_{transmitter})$***:*** If the expanded uncertainty specified in the calibration certificate is used for the uncertainty evaluation, the transmitter uncertainty is to include the uncertainty of the temperature calibration laboratory (which shall be traceable to international standards). The confidence level and the probability distribution of the reported expanded uncertainty shall be specified.

   Alternatively, if the calibration laboratory states that the transmitter uncertainty (including the calibration laboratory uncertainty) is within the “reference accuracy” given in the manufacturer data sheet [Rosemount, 2000], one may - as a

conservative approach - use the latter uncertainty value in the calculations. This approach is used here.

The "reference accuracy" of the 3051P pressure transmitter accounts for hysteresis, terminal-based linearity and repeatability, and is given in the manufacturer data sheet as ±0.05 % of span at a 99 % confidence level (cf. Table 4.5), i.e. with k = 3 (Section B.3). It is assumed here that this figure refers to the calibrated span. As an example, the calibrated span is here taken to be 50 - 120 bar, i.e. 70 bar (Table 4.5), giving $u(\hat{P}_{transmitter}) = U(\hat{P}_{transmitter})/3 = [70 \cdot 0.0005]bar/3 = 0.035\,bar/3 = 0.012\,bar$ [83].

2. ***Stability - pressure transmitter,*** $u(\hat{P}_{stability})$***:*** The stability of the pressure transmitter represents a drift (increasing/decreasing offset) in the readings with time. This contribution is zero at the time of calibration, and is specified as a maximum value at a given time.

The stability of the 3051P pressure transmitter is given in the manufacturer data sheet [Rosemount, 2000] as ±0.125 % of URL for 5 years for 28 °C temperature changes and up to 69 barg line pressure (Table 4.5). However, this uncertainty becomes artificially low when considering normal calibration intervals at fiscal metering stations of two or three months [Dahl *et al.*, 1999]. Furthermore, the uncertainty is limited to line pressures below 69 barg.

In a dialog with the manufacturer, the manufacturer has therefore provided a more applicable uncertainty specification when it comes to stability of the 3051P pressure transmitter regarding use in fiscal metering stations (the uncertainty specified in the data sheet is still valid). The alternative stability uncertainty is given to be 0.1 % of URL for 1 year [Rosemount, 1999]. (Note that this uncertainty specification does not include limitations with respect to temperature changes and line pressure.) This alternative uncertainty specification is therefore used for the 3051P pressure transmitter in the present evaluation example.

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83 The manufacturer's uncertainty specification is used here. By calibration of the pressure transmitter in an accredited calibration laboratory, the transmitter uncertainty may be further reduced. An example of a calibration certificate specification for the expanded uncertainty $U(\hat{P}_{transmitter})$ may be in the range 0.018-0.022 bar, at a 95 % confidence level (k = 2) [Eide, 2001a], i.e. 0.009-0.011 bar for the standard uncertainty. This includes linearity, hysteresis, repeatability, reading uncertainty, and reference instruments uncertainty.

The time dependency of the stability uncertainty is not necessarily linear. However, for simplicity, a linear time dependency has been assumed here[84].

The confidence level is not specified, but is assumed here to be 95 %, at a normal probability distribution (k = 2, cf. Section B.3). Consequently, if the transmitter is calibrated every 12 months, the uncertainty specified by [Rosemount, 1999] due to stability effects is divided by 12 and multiplied with 12. That is, $u(\hat{P}_{stability}) = U(\hat{P}_{stability})/2 = [138 \cdot 0.001 \cdot (12/12)]bar/2 \approx$ 0.138 bar / 2 ≈ 0.069 bar.

3. ***RFI effects - pressure transmitter,*** $u(\hat{P}_{RFI})$***:*** Radio-frequency interference, effects (RFI) is given in the manufacturer data sheet [Rosemount, 2000] as ±0.1 % of span for frequencies from 20 to 1000 MHz, and for field strength up to 30 V/m, cf. Table 4.5.

It is noted that the specified RFI uncertainty is atually twice as large as the uncertainty of the transmitter itself. In practice, this uncertainty contribution may be difficult to evaluate, and the RFI electric field at the actual metering station should be measured in order to document the actual electric field at the pressure transmitter. I.e. the RFI electric field must be documented in order to evaluate if, and to what extent, the uncertainty due to RFI effects may be reduced.

However, as long as the RFI electric field at the pressure transmitter is not documented by measurement, the uncertainty due to RFI effects must be included in the uncertainty evaluation as given in the data sheet. Consequently, $u(\hat{P}_{RFI}) = U(\hat{P}_{RFI})/3 = [70 \cdot 0.001]bar/3 = 0.07\,bar/3 = 0.023\,bar$.

4. ***Ambient temperature effects - pressure transmitter,*** $u(\hat{P}_{temp})$***:*** The ambient temperature effect on the Rosmount 3051P pressure transmitter is given in the manufacturer data sheet [Rosemount, 2000] as ±(0.006 % URL + 0.03 % span) per 28 °C temperature change, cf. Table 4.5. The temperature change referred to is the change in ambient temperature relative to the ambient temperature at calibration (to be specified in the calibration certificate).

84 In a worst case scenario, the uncertainty due to stability may be used directly without using the time division specified.

Consequently, for a possible "worst case" example of ambient North Sea temperature taken as 0 ^{o}C (Table 4.2), and a calibration temperature equal to 20 ^{o}C (Table 4.5), i.e. a max. temperature change of 20 ^{o}C, one obtains $u(\hat{P}_{temp}) = U(\hat{P}_{temp})/3 \quad = [(138 \cdot 0.006 + 70 \cdot 0.03) \cdot 10^{-2} \cdot (20/28)]bar/3$ $= [0.0059 + 0.0150]bar/3 \approx 0.0209\,bar/3 \approx 0.007\,bar$.

5. ***Atmospheric pressure,*** $u(\hat{P}_{atm})$***:*** The Rosemount 3051P pressure transmitter measures the excess pressure, relative to the atmospheric pressure. The uncertainty of the absolute static pressure $u_c(\hat{P})$ is thus to include the uncertainty of the atmospheric pressure, due to day-by-day atmospheric pressure variations.

   In the North Sea, the average atmospheric pressure is about 1008 and 1012 mbar for the winter and summer seasons, respectively (averaged over the years 1955-1991) [Lothe, 1994]. For convenience, 1 atm. ≡ 1013.25 mbar is taken as the average value. On a world-wide basis, the observed atmospheric pressure range includes the range 920 - 1060 mbar, - however, the upper and lower parts of this range (beyond about 940 and 1040 mbar) are very rare (not observed every year) [Lothe, 2001].

   The variation of the atmospheric pressure around the value 1 atm. ≡ 1013.25 mbar is here taken to be 90 mbar, as a conservative approach. Assuming a 99 % confidence level, and a normal probability distribution for the variation range of the atmospheric pressure (k = 3, cf. Section B.3), one obtains $u(\hat{P}_{atm}) = U(\hat{P}_{atm})/3 = 90\,mbar/3 = 0.09\,bar/3 = 0.03\,bar$.

6. ***Vibration effects - pressure transmitter,*** $u(\hat{P}_{vibration})$***:*** According to the manufacturer data sheet [Rosemount, 2000], "measurement effect due to vibrations is negligible except at resonance frequencies. When at resonance frequencies, vibration effect is less than 0.1 % of URL per *g* when tested from 15 to 2000 Hz in any axis relative to pipe-mounted process conditions" (Table 4.5).

   Based on communication with the manufacturer [Rosemount, 1999] and a calibration laboratory [Fimas, 1999], the vibration level at fiscal metering stations is considered to be very low (and according to recognised standards). Hence, the uncertainty due to vibration effects may be neglected.

   In the program *EMU - USM Fiscal Gas Metering Station*, the uncertainty due to vibration effects is neglected for the 3051P transmitter: $u(\hat{P}_{vibration}) = 0$.

7. ***Power supply effects - pressure transmitter,*** $u(\hat{P}_{power})$***:*** The power supply effect is quantified in the manufacturer data sheet [Rosemount, 2000] as less than ±0.005 % of the calibrated span per volt (Table 4.5). According to the supplier [Rosemount, 1999] this uncertainty is specified to indicate that the uncertainty due to power supply effects is negligible for the 3051P transmitter, which was not always the case for the older transmitters [Dahl *et al.*, 1999].

   Hence, in the program, the uncertainty due to power supply effects is neglected for the 3051P transmitter: $u(\hat{P}_{power}) = 0$.

8. ***Static pressure effect - pressure transmitter:*** The static pressure effect [Rosemount, 2000] will only influence on a differential pressure transmitter, and not on static pressure measurements, as considered here [Dahl *et al.*, 1999][85].

9. ***Mounting position effects - pressure transmitter:*** The mounting position effect [Rosemount, 2000] will only influence on a differential pressure transmitter, and not on static pressure measurements, as considered here [Dahl *et al.*, 1999][86].

A sample uncertainty budget is given in Table 4.6 for evaluation of the expanded uncertainty of the pressure measurement according to Eq. (3.11). The figures used for the input uncertainties are those given in the discussion above.

The calculated expanded and relative expanded uncertainties (specified at a 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) are 0.16 bar and 0.15 %, respecively. These values are further used in Section 4.6.

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85 The static pressure effect influencing on 3051P differential pressure transmitters consists of (a) the *zero error*, and (b) the *span error* [Rosemount, 2000]. The zero error is given in the data sheet [Rosemount, 2000] as ±0.04 % of URL per 69 barg. The zero error can be calibrated out at line pressure. The span error is given in the data sheet [Rosemount, 2000] as ±0.10 % of reading per 69 barG.

86 Mounting position effects are due to the construction of the 3051P differential pressure transmitter with oil filled chambers [Dahl *et al.*, 1999]. These may influence the measurement if the transmitter is not properly mounted. The mounting position error is specified in the data sheet [Rosemount, 2000] as "zero shifts up to ±1.25 inH_2O (0.31 kPa = 0.0031 bar), which can be calibrated out. No span effect".

Table 4.6. Sample uncertainty budget for the measurement of the absolute static gas pressure using the Rosemount Model 3051P Pressure Transmitter [Rosemount, 2000], calculated according to Eq. (3.11). (Corresponds to Figs. 5.4 and 5.21.)

Source	**Input uncertainty**				**Combined uncertainty**	
	Expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Standard uncertainty**	**Sens. coeff.**	**Variance**
Transmitter uncertainty	0.035 bar	99 % (norm)	3	0.012 bar	1	$1.361 \cdot 10^{-4}$ bar^2
Stability, transmitter	0.138 bar	95 % (norm)	2	0.069 bar	1	$4.761 \cdot 10^{-3}$ bar^2
RFI effects	0.070 bar	99 % (norm)	3	0.023 bar	1	$5.443 \cdot 10^{-4}$ bar^2
Ambient temperature effects, transmitter	0.021 bar	99 % (norm)	3	0.007 bar	1	$4.853 \cdot 10^{-5}$ bar^2
Atmospheric pressure	0.090 bar	99 % (norm)	3	0.030 bar	1	$9.0 \cdot 10^{-4}$ bar^2
Vibration	Negligible			0	1	0
Power supply	Negligible			0	1	0
Sum of variances				$u_c^2(\hat{P})$		$6.39 \cdot 10^{-3}$ bar^2
Combined standard uncertainty				$u_c(\hat{P})$		0.0799 bar
Expanded uncertainty (95 % confidence level, k = 2)				$U(\hat{P})$		**0.16 bar**
Operating pressure				$\hat{P}$		100 bara
Relative expanded uncertainty (95 % confidence level)				$U(\hat{P})/\hat{P}$		**0.16 %**

4.2.2 Temperature measurement

The combined standard uncertainty of the temperature measurement, $u_c(\hat{T})$, is given by Eq. (3.12). This expression is evaluated in the following.

Performance specifications for the Rosemount Model 3144 Smart Temperature Transmitter and the Pt 100 4-wire RTD element are given in Table 4.7[87], as specified in the data sheet [Rosemount, 2000], etc.

The contributions to the combined standard uncertainty of the temperature measurement are described in the following. The discussion is similar to that given in [Dahl *et al.*, 1999].

87 Note that the expanded uncertainties given in the transmitter data sheet [Rosemount, 2000] are specified at a 99 % confidence level (k = 3).

Table 4.7. Performance specifications of the Rosemount Model 3144 Temperature Transmitter [Rosemount, 2000] and the Pt 100 4-wire RTD element, used as input to the uncertainty calculations given in Table 4.8.

Quantity or Source	**Value or Expanded uncertainty**	**Coverage factor, *k***	**Reference**
Calibration ambient temperature (air)	20 °C	-	Calibration certificate *(NA)*
Time between calibrations	12 months	-	Example
Transmitter/element uncertainty (not calibrated as a unit), $U(\hat{T}_{elem,transm})$	"Digital accuracy": 0.10 °C	3	[Rosemount, 2000]
	"D/A accuracy": ±0.02 % of span.	3	
Transmitter/element uncertainty (calibrated as a unit), $U(\hat{T}_{elem,transm})$	*NA*	*NA*	Calibration certificate *(NA)*
Stability - temperature transmitter, $U(\hat{T}_{stab,transm})$	±0.1 % of reading or 0.1 °C, whichever is greater, for 24 months.	3	[Rosemount, 2000]
RFI effects - transmitter, $U(\hat{T}_{RFI})$	Worst case, with unshielded cable: equivalent to the transmitter "accuracy".	3	[Rosemount, 2000]
Ambient temperature effects - transmitter, $U(\hat{T}_{temp})$	"Digital accuracy": 0.0015 °C per 1 °C.	3	[Rosemount, 2000]
	D/A effect: 0.001 % of span, per 1 °C.	3	
Stability - temperature element, $U(\hat{T}_{stab,elem})$	0.050 °C	-	[BIPM, 1997]
Vibration effects, $U(\hat{T}_{vibration})$	*Negligible* (tested to given specifications with no effect on performance).	3	[Rosemount, 2000]
Power supply effects, $U(\hat{T}_{power})$	*Negligible* (less than ±0.005 % of span per volt).	3	[Rosemount, 2000]
Lead resistance effects, $U(\hat{T}_{cable})$	*Negligible* (no effect, independent on lead resistance).	3	[Rosemount, 1998]

1. ***Transmitter/element uncertainty (calibrated as a unit),*** $U(\hat{T}_{elem,transm})$***:*** The temperature element and the temperature transmitter are assumed to be calibrated as a unit [NORSOK, 1998a].

   If the expanded uncertainty specified in the calibration certificate is used for the uncertainty evaluation, the transmitter/element uncertainty (calibrated as a unit) will include the uncertainty of the temperature calibration laboratory (to be traceable to international standards). The confidence level of the reported expanded uncertainty is to be specified. When first recording the characteristics of the temperature element and then loading this characteristic into the transmitter prior to the final calibration, the uncertainty due to the element can be minimised [Fimas, 1999].

Alternatively, if the calibration laboratory states that the transmitter/element uncertainty (calibrated as a unit, and including the calibration laboratory uncertainty) is within the "accuracy" given in the manufacturer data sheet [Rosemount, 2000], one may - as a conservative approach - use the latter uncertainty value in the calculations. This approach is used here.

The "accuracy" of the 3144 temperature transmitter used together with a Pt 100 4-wire RTD element is tabulated in the data sheet [Rosemount, 2000, Table 1]. The output signal is accessed using a HART protocol, i.e. only the "digital accuracy" is used here (cf. Table 4.7). The expanded uncertainty is then given as 0.10 °C at a 99 % confidence level (k = 3, cf. Section B.3). That is, $u(\hat{T}_{elem,transm}) = U(\hat{T}_{elem,transm})/3 = 0.10\ ^oC/3 = 0.033\ ^oC$ [88].

2. ***Stability - temperature transmitter,*** $u(\hat{T}_{stab,transm})$***:*** The stability of the temperature transmitter represents a drift in the readings with time. This contribution is zero at the time of calibration, and is specified as a maximum value at a given time.

For use in combination with RTD elements, the stability of the 3144 temperature transmitter is given in the manufacturer data sheet [Rosemount, 2000] as 0.1 % of reading (measured value), or 0.1 °C, whichever is greater for 24 months, cf. Table 4.7. The time dependency is not necessarily linear. However, for simplicity, a linear time dependency is assumed here[89].

The value "0.1 % of reading for 24 months" corresponds to $[(273+50)\cdot 0.001]\,^oC \approx 0.323\ ^oC$. As this is greater than 0.1 °C, this uncertainty value is used.

88 The manufacturer's uncertainty specification is used here, for temperature element and transmitter combined. By calibration of the the element and transmitter in an accredited calibration laboratory, the element/transmitter uncertainty may be significantly reduced. As an example, the calibration certificate specification for the element/transmitter's expanded uncertainty $U(\hat{T}_{elem,transm})$ may be 0.03 ^{o}C, at a 95 % confidence level (k = 2) [Eide, 2001a], corresponding to 0.015 ^{o}C for the standard uncertainty.

89 In a worst case scenario, the uncertainty due to stability may be used directly without using the time division specified.

Consequently, if the transmitter is calibrated every 12 months, the uncertainty given in the data sheet due to stability effects is divided by 24 and multiplied with 12. That is, $u(\hat{T}_{stab,transm}) = U(\hat{T}_{stab,transm})/3 = [(273+50)\cdot 0.001\cdot(12/24)]\,^{o}C/3 = 0.1615\,^{o}C/3 \approx 0.054\,^{o}C$.

3. ***RFI effects - temperature transmitter,*** $u(\hat{T}_{RFI})$***:*** Radio-frequency interference, effects (RFI) may cause a worst case uncertainty equivalent to the transmitter's nominal uncertainty, when used with an unshielded cable [Rosemount, 2000]. For fiscal metering stations all cables are shielded, i.e. the RFI effects should be less than the worst case specified in the data sheet. Nevertheless, RFI effects (and also effects due to bad instrument earth) may cause additional uncertainty to the temperature measurement that is hard to quantify.

   It is time consuming to predict or measure the actual RFI effects at the metering station, and difficult to evaluate correctly the influence on the temperature measurement.

   It is therefore recommended to use the worst case uncertainty specified in the data sheet for the uncertainty due to RFI effects. For the "digital acuracy" of the 3144 transmitter, the expanded uncertainty is specified to be 0.10 ^{o}C, cf. Table 4.7. That is, $u(\hat{T}_{RFI}) = U(\hat{T}_{RFI})/3 = 0.10\,^{o}C/3 = 0.033\,^{o}C$.

4. ***Ambient temperature effects - temperature transmitter,*** $u(\hat{T}_{temp})$***:*** The Rosemount 3144 temperature transmitters are individually characterised for the ambient temperature range -40 °C to 85 °C , and automatically compensate for change in ambient temperature [Rosemount, 2000].

   Some uncertainty still arises due to the change in ambient temperature. This uncertainty is tabulated in the data sheet as a function of changes in the ambient temperature (in operation) from the ambient temperature when the transmitter was calibrated, cf. Table 4.7

   The ambient temperature uncertainty for Rosemount 3144 temperature transmitters used together with Pt-100 4-wire RTDs is given in the data sheet as 0.0015 °C per 1 °C change in ambient temperature relative to the calibration ambient temperature (the "digital accuracy").

   Consequently, for a possible "worst case" ambient North Sea temperature taken as 0 ^{o}C (Table 4.2), and a calibration temperature equal to 20 ^{o}C (Table 4.7, i.e. a

max. temperature change of 20 °C, one obtains $u(\hat{T}_{temp}) = U(\hat{T}_{temp})/3 = 0.0015 \cdot 20\,^{o}C/3 = 0.03\,^{o}C/3 = 0.01\,^{o}C$.

5. ***Stability - temperature element,*** $u(\hat{T}_{stab,elem})$***:*** The Pt-100 4-wire RTD element will cause uncertainty to the temperature measurement due to drift during operation. Oxidation, moisture inside the encapsulation and mechanical stress during operation may cause instability and hysteresis effects [EN 60751, 1995], [BIPM, 1997].

   BIPM [BIPM, 1997] has performed several tests of the stability of temperature elements which shows that this uncertainty is typically of the order of 0.050 °C, cf. Table 4.7. The confidence level of this expanded uncertainty is not given, however, and a 95 % confidence level and a normal probability distribution is assumed here (k = 2, cf. Section B.3). That is, $u(\hat{T}_{stab,elem}) = U(\hat{T}_{stab,elem})/2$ = 0.050 °C / 2 = 0.025 °C.

6. ***Vibration effects - temperature transmitter,*** $u(\hat{T}_{vibration})$***:*** According to the manufacturer data sheet [Rosemount, 2000], "transmitters are tested to the following specifications with no effect on performance: 0.21 mm peak displacement for 10-60 Hz; 3*g* acceleration for 60-2000 Hz". Moreover, in communication with the manufacturer [Rosemount, 1999] and a calibration laboratory [Fimas, 1999], and considering that the vibration level at fiscal metering stations shall be very low (and according to recognised standards), the uncertainty due to vibration effects may be neglected.

   Hence, in the program *EMU - USM Fiscal Gas Metering Station*, the uncertainty due to vibration effects is neglected for the Rosemount 3144 temperature transmitter, $u(\hat{T}_{vibration}) = 0$.

7. ***Power supply effects - temperature transmitter,*** $u(\hat{T}_{power})$***:*** The power supply effect is quantified in the manufacturer data sheet [Rosemount, 2000] as being less than ±0.005 % of span per volt. According to the supplier [Rosemount, 1999] this uncertainty is specified to indicate that the uncertainty due to power supply effects is negligible for the 3144 transmitter, which was not always the case for the older transmitters [Dahl *et al.*, 1999].

Hence, in the program *EMU - USM Fiscal Gas Metering Station*, the uncertainty due to power supply effects is neglected for the Rosemount 3144 temperature transmitter, $u(\hat{T}_{power}) = 0$.

8. ***Sensor lead resistance effects - temperature transmitter,*** $u(\hat{T}_{cable})$***:*** According to the manufacturer data sheet for the 3144 transmitter [Rosemount, 1998], the error due to lead resistance effects is "none" (independent of lead resistance) for 4-wire RTDs. 4-wire RTDs are normally used in fiscal metering stations.

   Hence, in the program *EMU - USM Fiscal Gas Metering Station*, the uncertainty due to lead resistance effects is neglected for the 3144 transmitter: $u(\hat{T}_{cable}) = 0$.

A sample uncertainty budget is given in Table 4.8 for evaluation of the expanded uncertainty of the temperature measurement according to Eq. (3.12). The figures used for the input uncertainties are those given in the discussion above.

Table 4.8 Sample uncertainty budget for the temperature measurement using the Rosemount Model 3144 Temperature Transmitter [Rosemount, 2000] with a Pt 100 4-wire RTD element, calculated according to Eq. (3.12). (Corresponds to Figs. 5.6 and 5.22.)

Source	**Input uncertainty**				**Combined uncertainty**	
	Expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Standard uncertainty**	**Sens. coeff.**	**Variance**
Transmitter/element uncertainty	0.10 °C	99 % (norm)	3	0.033 °C	1	$1.111 \cdot 10^{-3}$ °C^2
Stability, transmitter	0.1615 °C	99 % (norm)	3	0.054 °C	1	$2.900 \cdot 10^{-3}$ °C^2
RFI effects	0.10 °C	99 % (norm)	3	0.033 °C	1	$1.111 \cdot 10^{-3}$ °C^2
Ambient temperature effects, transmitter	0.03 °C	99 % (norm)	3	0.010 °C	1	$1.0 \cdot 10^{-4}$ °C^2
Stability, element	0.050 °C	95 % (norm)	2	0.025 °C	1	$6.250 \cdot 10^{-4}$ °C^2
Vibration	Negligible			0	1	0
Power supply	Negligible			0	1	0
Lead resistance	Negligible			0	1	0
Sum of variances				$u_c^2(\hat{T})$		$5.848 \cdot 10^{-3}$ °C^2
Combined standard uncertainty				$u_c(\hat{T})$		0.0765 °C
Expanded uncertainty (95 % confidence level, *k* = 2)				$U(\hat{T})$		**0.15 °C**
Operating temperature				$\hat{T}$		50 °C (≈323 K)
Relative expanded uncertainty (95 % confidence level)				$U(\hat{T})/\hat{T}$		**0.047 %**

The calculated expanded and relative expanded uncertainties (specified at 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) are 9.15 ^{o}C and 0.048 %, respecively. These values are further used in Section 4.6.

4.2.3 Gas compressibility factors

The relative combined standard uncertainty of the ratio of the gas compressibility factors, E_{Z/Z_0}, is given by Eq. (3.13), and is evaluated in the following.

In the present example the AGA-8 (92) equation of state [AGA-8, 1994] is used for calculation of $\hat{Z}$, and ISO 6976 [ISO, 1995c] is used for $\hat{Z}_0$. It is here assumed that the corresponding uncertainty figures correspond to rectangular probability distributions and 100 % confidence levels ($k = \sqrt{3}$, cf. Section B.3). The relative standard uncertainty of the estimate $\hat{Z}$ due to *model uncertainty* is then $E_{Z,\mathrm{mod}} = 0.1\%/\sqrt{3} \approx$ 0.0577 %, cf. Fig. 3.1. The corresponding relative standard uncertainty of the estimate $\hat{Z}_0$ due to *model uncertainty* becomes $E_{Z0,mod} = \sqrt{0.05^2 + 0.015^2}\%/\sqrt{3} = \approx 0.0522\%/\sqrt{3} \approx$ 0.030 %, cf. Section 3.2.3.1.

The relative standard uncertainty of the estimate $\hat{Z}$ due to *analysis uncertainty* (inaccurate determination of the line gas composition), is in general more complicated to estimate. The uncertainty will depend on (a) the uncertainty of the GC measurement, and (b) the actual variations in the gas composition. Both uncertainty contributions (a) and (b) will depend on the specific gas quality in question, and also on the pressure and temperature in question. To give a typical value to be representative in all cases is not possible. Examples have shown that this uncertainty can be all from negligible to around 1 %. As described in Section 3.2.3.2, it can be determined e.g. by using a Monte Carlo type of simulation where the gas composition is varied within its uncertainty limits.

For the specific example discussed here, it has - in a simplified approach - been assumed that the C1-component varies with ±0.5 %, the C2-component with ±0.4 % and the C3-component with ±0.1 % (of the total gas content). Such variation ranges can be observed in practice [Sakariassen, 2001] as natural variations over a time scale of months, and can be of relevance here if the gas composition data are fed manually to the flow computer e.g. monthly instead of being measured online. The variations in the other gas components are smaller, and have been neglected here for simplicity. In the case of online measurements (using GC analysis), the variation limits may be smaller, especially for the C2 and C3 components. 10 gas composi-

tions within the limits selected for C1, C2 and C3 have been used, and the *Z*-factor has been calculated for each of them, at line conditions and at standard reference conditions. The resulting calculated standard deviation for the *Z* factor at line condition is about 0.16 % [90]. This number has been used here as the relative standard uncertainty $E_{Z,ana}$ (k = 1, cf. Section B.3).

At standard reference conditions, the resulting calculated standard deviation using this method is less than 0.01 % [91]. Therefore, $E_{Z0,ana}$ has been neglected in the current example, so that $E_{Z0,ana}$ = 0 is used here.

A sample uncertainty budget is given in Table 4.9 for evaluation of the expanded uncertainty of the ratio of the gas compressibility factors according to Eq. (3.13). The figures used for the input uncertainties are those given in the discussion above.

The calculated relative expanded uncertainty (specified at 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) is 0.339 %. This value is further used in Section 4.6.

Table 4.9 Sample uncertainty budget for the ratio of compressibility factors, Z_0/Z, calculated according to Eq. (3.13). (Corresponds to Fig. 5.7.)

Source	**Input uncertainty**				**Combined uncertainty**	
	Relative expanded uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative variance**
Model uncertainty, Z	0.1 %	95 % (norm)	2	0.05 %	1	$2.50 \cdot 10^{-7}$
Model uncertainty, Z_0	0.052 %	95 % (norm)	2	0.026 %	1	$6.76 \cdot 10^{-8}$
Analysis uncertainty, Z	0.16 %	67 % (norm)	1	0.16 %	1	$2.56 \cdot 10^{-6}$
Analysis uncertainty, Z_0	0	67 % (norm)	1	0		
Sum of relative variances				$E^2_{Z/Z0}$		$2.878 \cdot 10^{-6}$
Relative combined standard uncertainty				$E_{Z/Z0}$		0.170 %
Relative expanded uncertainty (95 % confidence level)				$k \cdot E_{Z/Z0}$		**0.339 %**

90 It should be noted that the selected variation limits are larger than what can often be found in practice, especially when an online GC is used. The value $E_{Z,ana}$ = 0.16 % is therefore far from being a best case value.

91 It can be shown (by a Monte Carlo type of statistical analysis, as used above) that in spite of possible significant uncertainty in the gas compostion (such as e.g. due to GC measurement uncertainty, or natural variations in gas composition), the influence of such uncertainty on the analysis uncertainty of Z is quite small at low pressures when Z is very close to unity, $\hat{Z} \approx 1$, such as for $\hat{Z}_0$.

4.2.4 Density measurement

The combined standard uncertainty of the density measurement, $u_c(\hat{\rho})$, is given by Eq. (3.14). This expression is evaluated in the following, for the example considered here.

Performance specifications for the Solartron 7812 gas density transducer are given in Table 4.10 as specified in the data sheet [Solartron, 1999], etc.

Table 4.10 Performance specifications of the Solartron 7812 Gas Density Transducer [Solartron, 1999], used as input to the uncertainty calculations given in Table 4.11.

Quantity or Source	**Value or Expanded uncertainty**	**Coverage factor, *k***	**Reference**
Calibration temperature (air)	20 °C	-	[Solartron, 1999; §A.1]
Full scale density range	1 - 400 kg/m^3	-	[Solartron, 1999, §A.1]
Densitometer "accuracy", $U(\hat{\rho}_u)$	< ±0.1 % of m.v. (nitrogen) < ±0.15 % of m.v. (nat. gas)		[Solartron, 1999, §A.1]
Repeatability, $U(\hat{\rho}_{rept})$	Within ±0.01 % of full scale density		[Solartron, 1999, §1.3.2]
Calibration temperature, $U(\hat{T}_c)$	0.1 ^{o}C (at 20 ^{o}C)		[Solartron, 1999, §7.2.2]
Temperature correction model, $U(\hat{\rho}_{temp})$	< 0.001 kg/m^3/^{o}C		[Solartron, 1999, §A.1]
VOS correction model, $U(\hat{\rho}_{misc})$	Not specified (see text)		[Solartron, 1999]
Pressure effect, $U(\hat{\rho}_{misc})$	Negligible (see text)		[Solartron, 1999, §A.1]
Stability - element, $U(\hat{\rho}_{misc})$	Negligible (see text)		[Solartron, 1999, §1.3.3]
Deposits, $U(\hat{\rho}_{misc})$	Not specified (see text)		[Solartron, 1999, §1.3.3; §3.8]
Condensation, $U(\hat{\rho}_{misc})$	Not specified (see text)		[Solartron, 1999, §3.8]
Corrosion, $U(\hat{\rho}_{misc})$	Not specified (see text)		[Solartron, 1999, §1.3.3; §3.8]
Gas viscosity, $U(\hat{\rho}_{misc})$	Negligible (see text)		[Matthews, 1994]
Vibration effects, $U(\hat{\rho}_{misc})$	Not specified (see text)		[Solartron, 1999, §1.3.1; §3.8]
Power supply effects, $U(\hat{\rho}_{misc})$	Negligible (see text)		[Solartron, 1999, §1.3.1]
Self induced heat effects, $U(\hat{\rho}_{misc})$	Negligible (see text)		[Solartron, 1999, §1.3.1]
Sample flow effects, $U(\hat{\rho}_{misc})$	Negligible (see text)		[Matthews, 1994]

The contributions to the combined standard uncertainty of the density measurement are described in the following. As the confidence level of the expanded uncertainties

is not specified in [Solartron, 1999], this is here assumed to be 95 %, with a normal probability distribution (k = 2, cf. Section B.3).

1. ***Densitometer "accuracy",*** $u(\hat{\rho}_u)$***:*** The densitometer "accuracy" is specified in the manufacturer instrument manual [Solartron, 1999, §A.1] as being less than ±0.1 % of reading in nitrogen, and less than ±0.15 % of reading in natural gas. This includes uncertainties of Eq. (2.23). That is, $u(\hat{\rho}_u) = U(\hat{\rho}_u)/2 = \left[0.15 \cdot 10^{-2} \cdot 82.443\ kg/m^3\right]/2$ = 0.0618 kg/m^3 [92]. The relative standard uncertainty is $u(\hat{\rho}_u)/\hat{\rho}_u$ = 0.15%/2 = 0.075 %.

2. **Repeatability**, $u(\hat{\rho}_{rept})$***:*** In the manufacturer instrument manual [Solartron, 1999, §1.3.2], the repeatability is specified to be within ±0.01 % of full scale density. The density range of the 7812 densitometer is given to be 1 to 400 kg/m^3. That is, $u(\hat{\rho}_{rept}) = U(\hat{\rho}_{rept})/2 = 0.01 \cdot 10^{-2} \cdot 400/2 = 0.02\ kg/m^3$.

3. **Calibration temperature**, $u(\hat{T}_c)$***:*** The uncertainty of the calibration temperature is specified in the manufacturer instrument manual [Solartron, 1999, §7.2.2] as 0.1 ^{o}C at 20 ^{o}C. That is, $u(\hat{T}_c) = U(\hat{T}_c)/2 = 0.1\ ^oC/2 = 0.05\ ^oC$.

4. **Line temperature**, $u_c(\hat{T})$***:*** The expanded uncertainty of the line temperature is taken from Table 4.8.

5. **Densitometer temperature**, $u_c(\hat{T}_d)$***:*** The expanded uncertainty of the densitometer temperature is taken from Table 4.8.

6. **Line pressure**, $u_c(\hat{P})$***:*** The expanded uncertainty of the line pressure is taken from Table 4.6.

7. **Pressure difference, densitometer to line**, $u(\Delta\hat{P}_d)$***:*** The densitometer pressure $\hat{P}_d$ is not measured, and assumed to be equal to the line pressure $\hat{P}$. In practice the density sampling system is designed so that the pressure deviation

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92 The manufacturer's uncertainty specification is used here. By calibration of the the densitometer in an accredited calibration laboratory, the densitometer "accuracy" may be significantly reduced. Example of a calibration certificate specification for the densitometer "accuracy" $U(\hat{\rho}_u)$ may be e.g. 0.027-0.053 %, for the density range 25-250 kg/m^3, at a 95 % confidence level (k = 2) [Eide, 2001a]. Such values correspond to 0.014-0.027 % for the relative standard uncertainty of the densitometer "accuracy". This includes linearity, hysteresis, repeatability, reading uncertainty, and reference instruments uncertainty.

between the densitometer and line, $\Delta\hat{P}_d$, is relatively small. Tests with densitometers have indicated a pressure deviation $\Delta\hat{P}_d$ of up to 0.02 % of the line pressure, $\hat{P}$ [Eide, 2001a]. $\Delta\hat{P}_d$ can be positive or negative, depending on the actual installation [Sakariassen, 2001].

For the present case ($\hat{P}$ = 100 bara, cf. Table 4.2), $\Delta\hat{P}_d$ = 20 mbar is used as a representative example. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±20 mbar ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\Delta\hat{P}_d) = U(\Delta\hat{P}_d)/\sqrt{3} = 0.02\ bar/\sqrt{3} = 0.0115\ bar$.

8. **VOS, calibration gas**, $u(\hat{c}_c)$**:** The uncertainty of the sound velocity estimate of the calibration gas is tentatively taken to be 1 m/s. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±1 m/s ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{c}_c) = U(\hat{c}_c)/\sqrt{3} = 1/\sqrt{3} \approx 0.577$ m/s.

9. **VOS, densitometer gas**, $u(\hat{c}_d)$**:** As described in Section 2.4.3, when using Eq. (2.25) for VOS correction, there are at least two methods in use today to obtain the VOS at the density transducer, c_d: the "USM method" and the "pressure/density method". As explained in Section 3.2.4, evaluation of $u(\hat{c}_d)$ according to these (or other) methods is not a part of the present *Handbook*. Hence, one does not rely on the particular method used to estimate c_d in the metering station.

   The uncertainty of the VOS estimate in the density transducer is here taken to be 1 m/s, tentatively. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±1 m/s ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{c}_d) = U(\hat{c}_d)/\sqrt{3} = 1/\sqrt{3} \approx 0.577$ m/s.

10. **Periodic time**, $u(\hat{\tau})$**:** The uncertainty of the periodic time τ involved in the VOS correction depends on the time resolution of the flow computer, which is here set to 0.1 μs, tentatively (10 MHz oscillator) [Eide, 2001a]. Assuming a Type A uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±0.1 μs ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{\tau}) = U(\hat{\tau})/\sqrt{3} = 0.1\ \mu s/\sqrt{3} = 0.0577$ μs.

11. **VOS correction constant**, $u(\hat{K}_d)$**:** A figure for the uncertainty of the dimensional constant $\hat{K}_d$ used in the VOS correction has not been available for the present study. A tentative uncertainty figure of 10 % is used here, as a reasonable example [Eide, 2001a]. For $\hat{K}_d$ = 21000 µm (cf. Table 4.4), that gives $U(\hat{K}_d)$ = 2100 µm. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±2100 µm ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{K}_d) = U(\hat{K}_d)/\sqrt{3} = 2100\ \mu m/\sqrt{3}$ = 1212 µm.

12. **Temperature correction model**, $u(\hat{\rho}_{temp})$**:** Temperature changes affect both the modulus of elasticity of the vibrating element, and its dimensions. Both of these affect the resonance frequency [Matthews, 1994]. For high-accuracy densitometers like the Solartron 7812, this effect is largely eliminated using Ni-span C stainless steel[93], and the temperature correction model given by Eq. (2.24). However, the temperature correction model *itself* is not perfect, and will have an uncertainty.

The uncertainty of the temperature correction model itself, Eq. (2.24), is specified in the manufacturer instrument manual [Solartron, 1999, §A.1] as being less than 0.001 kg/m^3/^{o}C. That is, $u(\hat{\rho}_{temp}) = U(\hat{\rho}_{temp})/2 = [0.001 \cdot 48]/2 = 0.024\, kg/m^3$.

13. **VOS correction model**, $u(\hat{\rho}_{misc})$**:** For gas densitometers the fluids are very compressible (low VOS), and VOS correction is important [Solartron, 1999], [Matthews, 1994]. For high-accuracy densitometers like the Solartron 7812, this effect is largely eliminated using the VOS correction model given by Eq. (2.25). However, the VOS correction model *itself* is not perfect (among others due to use of a calibration gas, with another VOS than the line gas in question), and will have an uncertainty.

The uncertainty of the VOS correction model itself, Eq. (2.25), is not specified in the manufacturer instrument manual [Solartron, 1999]. In the present calculation example the uncertainty of the VOS correction model is neglected for the Solartron 7812 densitometer. That is, the VOS correction model contribution to $u(\hat{\rho}_{misc})$ is set to zero.

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93 For densitometers with vibrating element made from other materials than Ni-span C, the temperature effect may be considerably larger [Matthews, 1994].

14. **Pressure effect**, $u(\hat{\rho}_{misc})$**:** The uncertainty of the pressure effect is not specified in the manufacturer instrument manual [Solartron, 1999]. According to [Matthews, 1994], "for vibrating cylinders there is no pressure effect on the resoncance frequency because the fluid surrounds the vibrating element, so all forces are balanced". Consequently, in the present calculation example this uncertainty contribution is assumed to be negligible for the Solartron 7812 densitometer. That is, the pressure effect contribution to $u(\hat{\rho}_{misc})$ is set to zero.

15. **Stability - element**, $u(\hat{\rho}_{misc})$**:** The instrument manual states that [Solartron, 1999; §1.3.3] "The long term stability of this density sensor is mainly governed by the stability of the vibrating cylinder sensing element. This cylinder is manufactured from one of the most stable metals, and being unstressed, will maintain its properties for many years". In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the long time stability effects contribution to $u(\hat{\rho}_{misc})$ is set to zero.

16. **Deposits**, $u(\hat{\rho}_{misc})$**:** The instrument manual states that [Solartron, 1999; §1.3.3] "Deposition on the cylinder will degrade the long term stability, and care should be taken to ensure that the process gas is suitable for use with materials of construction. The possibility of deposition is reduced by the use of filters, but, should deposition take place, the sensing element can be removed and cleaned"[94]. According to [Campbell and Pinto, 1994], "another problem with the gas transducers can be the presence of black dust like particles on the walls of the sensing element. These particles can often cause pitting on the sensing element which renders the cylinder as scrap". In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to deposits is set to zero.

17. **Condensation and liquid contamination**, $u(\hat{\rho}_{misc})$**:** In the instrument manual it is stated that [Solartron, 1999; §3.8] "Condensation of water or liquid vapours on the sensing element will cause effects similar to deposition of solids except that the effects will dissappear if re-evaporation takes place." Cf. also [Geach, 1994].

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94 The risk of damaging the element in case of dismantling and clening offshore by unexperienced personnel may be large [Campbell and Pinto, 1994]. Scratches or denting during the cleaning procedure reduces the element to scrap.

According to [Campbell and Pinto, 1994], "transducers which are returned for calibration have been found on many occasions to contain large quantities of lubricating type oil, which has the effect of stopping the transducer vibrating. The presence of this liquid usually indicates a problem with the lub oil seals of the export compressors".

In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to condensation is set to zero.

18. **Corrosion**, $u(\hat{\rho}_{misc})$**:** In the instrument manual it is stated that [Solartron, 1999; §1.3.3] "Corrosion will degrade the long term stability, and care should be taken to ensure that the process gas is suitable for use with materials of construction." In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to corrosion is set to zero.

19. **Gas viscosity**, $u(\hat{\rho}_{misc})$**:** Viscosity has the effect of damping all vibrating-element transducers which causes a small over-reading in density. For gas densitometers the effect of viscosity is so small that it is virtually impossible to measure at anything but very low densities [Matthews, 1994]. In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to gas viscosity is set to zero.

20. **Vibration effects**, $u(\hat{\rho}_{misc})$**:** In the instrument manual it is stated that [Solartron, 1999; §3.8] "The 7812 can tolerate vibration up to 0.5*g*, but levels in excess of this may affect the accuracy of the readings. Use of anti-vibration gasket will reduce the effects of vibration by at least a factor of 3, at levels up to 10g and 2200 Hz". In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to vibration effects is set to zero.

21. **Power supply effects**, $u(\hat{\rho}_{misc})$**:** In the instrument manual it is stated that [Solartron, 1999; §3.8] the 7812 is insensitive to variations in power supply. In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to power supply effects is set to zero.

22. **Self induced heat effects**, $u(\hat{\rho}_{misc})$**:** In the instrument manual it is stated that [Solartron, 1999; §3.8] "since the power consumption is extremely small, the self induced heat may be neglected". In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to heating is set to zero.

Table 4.11. Sample uncertainty budget for the measurement of the gas density using the Solartron 7812 Gas Density Transducer [Solartron, 1999], calculated according to Eq. (3.14). (Corresponds to Figs. 5.9 and 5.23.)

Source	**Input uncertainty**				**Combined uncertainty**	
	Expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Standard uncertainty**	**Sens. coeff.**	**Variance**
Densitometer "accuracy", $U(\hat{\rho}_u)$	0.11 kg/m^3	95 % (norm)	2	0.0618 kg/m^3	0.989734	$3.745\cdot10^{-3}$ (kg/m^3)^2
Repeatability, $U(\hat{\rho}_{rept})$	0.04 kg/m^3	95 % (norm)	2	0.02 kg/m^3	1	$4.0\cdot10^{-4}$ (kg/m^3)^2
Calibration temperature, $U(\hat{T}_c)$	0.1 ^{o}C	95 % (norm)	2	0.05 ^{o}C	-0.000274	$1.88\cdot10^{-10}$ (kg/m^3)^2
Line temperature, $U(\hat{T})$	0.15 ^{o}C	95 % (norm)	2	0.0765 ^{o}C	0.252576	$3.73\cdot10^{-4}$ (kg/m^3)^2
Densitometer temperature, $U(\hat{T}_d)$	0.15 ^{o}C	95 % (norm)	2	0.0765 ^{o}C	0.253875	$3.77\cdot10^{-4}$ (kg/m^3)^2
Line pressure, $U(\hat{P})$	0.16 bar	95 % (norm)	2	0.08 bar	-0.000163	$1.7\cdot10^{-10}$ (kg/m^3)^2
Pressure difference, densitometer-line, $U(\Delta\hat{P}_d)$	20 mbar	100 % (rect)	$\sqrt{3}$	0.0115 bar	-0.816037	$8.88\cdot10^{-5}$ (kg/m^3)^2
VOS, calibration gas, $U(\hat{c}_c)$	1 m/s	100 % (rect)	$\sqrt{3}$	0.577 m/s	-0.00394	$5.18\cdot10^{-6}$ (kg/m^3)^2
VOS, densitometer gas, $U(\hat{c}_d)$	1 m/s	100 % (rect)	$\sqrt{3}$	0.577 m/s	0.002365	$1.87\cdot10^{-6}$ (kg/m^3)^2
Periodic time, $U(\hat{\tau})$	0.1 µs	100 % (rect)	$\sqrt{3}$	0.0577 µs	-0.000611	$1.24\cdot10^{-9}$ (kg/m^3)^2
Model constant, $U(\hat{K}_d)$	2100 µm	100 % (rect)	$\sqrt{3}$	1212 µm	$1.89\cdot10^{-5}$	$5.25\cdot10^{-4}$ (kg/m^3)^2
Temperature correction model, $U(\hat{\rho}_{temp})$	0.048 kg/m^3	95 % (norm)	2	0.024 kg/m^3	1	$5.76\cdot10^{-4}$ (kg/m^3)^2
Miscell. uncertainty contributions, $U(\hat{\rho}_{misc})$	Neglected			0 kg/m^3	1	0 (kg/m^3)^2
Sum of variances				$u_c^2(\hat{\rho})$		$6.092\cdot10^{-3}$ (kg/m^3)^2
Combined standard uncertainty				$u_c(\hat{\rho})$		0.07805 kg/m^3
Expanded uncertainty (95 % confidence level, k = 2)				$U(\hat{\rho})$		0.1561 kg/m^3
Operating density				ρ		81.62 kg/m^3
Relative expanded uncertainty (95 % confidence level)				$U(\hat{\rho})/\hat{\rho}$		**0.19 %**

23. **Sample line flow effects**, $u(\hat{\rho}_{misc})$**:** According to [Matthews, 1994], "All resonant element sensors will be affected by flow rate in some way. As flow rate increases, the output will generally give a positive over-reading of density and the readings will become more unstable. However this effect is very small and providing the manufacturers recommendations are followed then the effects can be ignored". In the present calculation example this uncertainty contribution is neglected for the Solartron 7812 densitometer. That is, the uncertainty contribution to $u(\hat{\rho}_{misc})$ which is related to flow in the density sample line is set to zero.

A sample uncertainty budget is given in Table 4.11 for evaluation of the expanded uncertainty of the pressure measurement according to Eq. (3.14). The figures used for the input uncertainties are those given in the discussion above.

The calculated relative expanded uncertainty (specified at a 95 % confidence level and a normal probilility distribution, with k = 2, cf. Section B.3) is 0.19 %. Note that this value is calculated under the assumption that the input uncertainties taken from the [Solartron, 1999] data sheet corresponds to a 95 % confidence level (k = 2). As Solartron has not specified k, this is an assumption, and another coverage factor k would alter the densitometer uncertainty. The relative expanded uncertainty value calculated above is further used in Section 4.6.

4.2.5 Calorific value

The relative expanded uncertainty of the superior (gross) calorific value, $k \cdot E_{H_S}$, is here taken to be 0.15 %, at a 95 % confidence level and a normal probability distribution (k = 2, cf. Section B.3). This figure is not based on any uncertainty analysis of the calorific value estimate, but is taken as a tentative example [Sakariassen, 2001], at a level corresponding to NPD regulation requirements [NPD, 2001] (cf. Section C.1). This value is further used in Section 4.6.

4.3 Flow calibration uncertainty

The relative combined standard uncertainty of the USM flow calibration, E_{cal}, is given by Eq. (3.16). This expression is evaluated in the following. As described in Section 3.3, it accounts for the contributions from the flow calibration laboratory, the deviation factor, and the USM repeatability in flow calibration.

4.3.1 Flow calibration laboratory

The expanded uncertainty of current high-precision gas flow calibration laboratories is taken to be typically 0.3 % of the measured value[95], at a 95 % confidence level and a normal probability distribution (k = 2, cf. Section B.3). Assuming that this figure applies to all the M test flow rates, as an example, one obtains $E_{ref,j} = \left[U(\hat{q}_{ref,j})/\left|\hat{q}_{ref,j}\right|\right]/2 = 0.3\ \%/2 = 0.15\ \%$, $j = 1, \ldots, M$.

4.3.2 Deviation factor

For a given application, the deviation data $Dev_{C,j}$, $j = 1, \ldots, M$, are to be specified by the USM manufacturer.

As an example evaluation of the uncertainty of the deviation factor estimate $\hat{K}_{dev,j}$, the AGA-9 report results given in Fig. 2.1 are used here. The example serves to illustrate the procedure used for evaluation of $E_{K_{dev,j}}$.

Table 4.12. Evaluation of $E_{K_{dev,j}}$ for the AGA-9 example of Fig. 2.1. (Corresp. to parts of Fig. 5.11.)

Test flow rate no.	Test flow rate	$\hat{K}_{dev,j}$	$Dev_{C,j}$	$E_{K_{dev,j}}$
1	q_{min}	1.01263	0.01263	0.720 %
2	0.10 q_{max}	1.00689	0.00689	0.395 %
3	0.25 q_{max}	0.99991	- 0.00009	0.005 %
4	0.40 q_{max}	0.99995	- 0.00005	0.003 %
5	0.70 q_{max}	0.99937	- 0.00063	0.036 %
6	q_{max}	0.99943	- 0.00057	0.033 %

Table 4.12 gives $\hat{K}_{dev,j}$, $Dev_{C,j}$ and $E_{K_{dev,j}}$ for this example, calculated for the M = 6 test flow rates discussed in the AGA-9 report, according to Eqs. (2.8), (2.10) and (3.18), respectively. Input data for these calculations are given in [AGA-9, 1998; Appendix D]. As described in Section 3.3.2, the expanded uncertainty $U(\hat{K}_{dev,j})$ is taken to be equal to $\left|Dev_{C,j}\right|$, and a Type B uncertainty is assumed, at a 100 % confidence level and a rectangular probability distribution within the range $\pm Dev_{C,j}$ ($k = \sqrt{3}$, cf. Section B.3).

95 Some flow calibration laboratories operate with flow rate dependent uncertainty figures, such as Bernoulli/Westerbork and Advantica [Sakariassen, 2001].

4.3.3 USM repeatability in flow calibration

The repeatability of current USMs is typically given to be 0.2 % of measured value, cf. e.g. [Daniel, 2000], [Kongsberg, 2000], cf. Table 6.1. In practice, the repeatability may be expected to be flow rate dependent, - however any dependency on flow rate is not specified by manufacturers.

The repeatability in flow calibration is also to include the repeatability of the flow laboratory reference measurement. As this value may be difficult to quantify, and is expected to be significantly less than 0.2 %, the 0.2 % figure is here taken to represent both repeatabilities.

Confidence level and probability distribution are unfortunately not available from USM manufacturer data sheets, cf. Chapter 6. Assuming that this repeatability figure corresponds to a 95 % confidence level at a normal probability distribution (k = 2, cf. Section B.3)[96], and to all the M test flow rates, one obtains $E_{rept,j}$ = $\left[U(\hat{q}^{rept}_{USM,j})/\left|\hat{q}_{USM,j}\right|\right]/2 = 0.2\ \%/2 = 0.1\ \%,\ j = 1, \ldots, M.$

4.3.4 Summary - Expanded uncertainty of flow calibration

Sample uncertainty budgets are given in Tables 4.13 and 4.14, for evaluation of the expanded uncertainty of the flow calibration according to Eq. (3.16). The figures used for the input uncertainties are those given in the discussion above. Note that in each table only a single test flow rate is evaluated[97]. Tables 4.13 and 4.14 apply to test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively, cf. Table 4.12.

The calculated relative expanded uncertainties (specified at a 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) are 0.87 % and 0.36 %, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively. These values are further used in Section 4.6.

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96 If the repeatability figure of 0.2 % corresponds to the *standard deviation* of the flow rate reading, a coverage factor k = 1 is to be used.

97 In the program *EMU - USM Fiscal Gas Metering Station*, uncertainty evaluation is made for all test flow rates, cf. Section 5.9.

Table 4.13. Sample uncertainty budget for USM flow calibration (example), calculated according to Eq. (3.16), for a *single test flow rate*, $0.10q_{max}$ (cf. Table 4.12). (Corresponds to Fig. 5.11.)

Source	Input uncertainty				Combined uncertainty	
	Relative expanded uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Flow calibration laboratory (Section 4.3.1)	0.3 %	95 % (norm)	2	0.15 %	1	$2.25 \cdot 10^{-6}$
Deviation factor (Table 4.12)	0.69 %	100 % (rect)	$\sqrt{3}$	0.395 %	1	$15.60 \cdot 10^{-6}$
USM repeatability in flow calibration (Section 4.3.3)	0.2 %	95 % (norm)	2	0.1 %	1	$1.00 \cdot 10^{-6}$
Sum of relative variances				E_{cal}^2		$18.85 \cdot 10^{-6}$
Relative combined standard uncertainty				E_{cal}		0.434 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{cal}$		**0.87 %**

Table 4.14. Sample uncertainty budget for USM flow calibration (example), calculated according to Eq. (3.16), for a *single test flow rate*, $0.70q_{max}$ (cf. Table 4.12).

Source	Input uncertainty				Combined uncertainty	
	Relative expanded uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Flow calibration laboratory (Section 4.3.1)	0.3 %	95 % (norm)	2	0.15 %	1	$2.25 \cdot 10^{-6}$
Deviation factor (Table 4.12)	0.072 %	100 % (rect)	$\sqrt{3}$	0.036 %	1	$1.30 \cdot 10^{-7}$
USM repeatability in flow calibration (Section 4.3.3)	0.2 %	95 % (norm)	2	0.1 %	1	$1.00 \cdot 10^{-6}$
Sum of relative variances				E_{cal}^2		$3.38 \cdot 10^{-6}$
Relative combined standard uncertainty				E_{cal}		0.183 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{cal}$		**0.37 %**

4.4 USM field uncertainty

The relative combined standard uncertainty of the USM in field operation, E_{USM}, is given by Eq. (3.19), and is evaluated in the following. As described in Section 3.4, it accounts for the contributions from the USM repeatability in field operation, and effects of changes in conditions from flow calibration to field operation (systematic effects related to meter body dimensions, transit times and integration method).

4.4.1 Repeatability (random transit time effects)

The relative combined standard uncertainty representing the repeatability of the USM in field operation, E_{rept}, is given by Eqs. (3.41) and (3.45), and is due to random

transit time effects. In this description, one has the option of specifying either E_{rept} or $u(\hat{t}_{1i}^{random})$, cf. Section 3.4.2 and Table 3.8. Both types of input are addressed here.

Alternative (1), Specification of flow rate repeatability: In the first and simplest approach, the repeatability of the flow measured rate, E_{rept}, is specified direcly, using the USM manufacturer value, typically 0.2 % of measured value, cf. e.g. [Daniel, 2000], [Kongsberg, 2000], Table 6.1. In practice, the repeatability of the flow rate may be expected to be flow rate dependent, cf. Section 3.4.2, - however a possible dependency on flow rate is not specified in these USM manufacturer data sheets.

Confidence level and probability distribution for such repeatability figures are unfortunately not available in USM manufacturer data sheets, cf. Chapter 6. Assuming that this repeatability figure corresponds to a 95 % confidence level at a normal probability distribution (k = 2, cf. Section B.3), and to all the M test flow rates, one obtains $E_{rept} = 0.2\ \%/2 = 0.1\ \%$, at all flow rates, cf. Table 4.15 [98].

Table 4.15. Sample uncertainty budget for the USM repeatability in field operation (example), for the simplified example of constant E_{rept} over the flow rate range. (Corresponds to Fig. 5.12.)

Test rate no.	**Test flow velocity**	**Test flow rate**	**Rel. exp. uncertainty** kE_{rept} **(repeatability)**	**Conf. level & Distribut.**	**Cov. fact.,** k	**Relative standard uncertainty,** E_{rept}	**Rel. exp. uncertainty (95 % c.l.)** $2E_{rept}$
1	0.4 m/s	q_{min}	0.2 %	95 % (norm)	2	0.1 %	0.2 %
2	1.0 m/s	0.10 q_{max}	"	"	"	"	"
3	2.5 m/s	0.25 q_{max}	"	"	"	"	"
4	4.0 m/s	0.40 q_{max}	"	"	"	"	"
5	7.0 m/s	0.70 q_{max}	"	"	"	"	"
6	10.0 m/s	q_{max}	"	"	"	"	"

Alternative (2), Specification of transit time repeatability: In the second approach, E_{rept} is calculated from the specified typical repeatability of the transit times, represented by $u(\hat{t}_{1i}^{random})$, the standard deviation of the measured transit times (after possible time averaging), at the flow rate in question. Normally, data for $u(\hat{t}_{1i}^{random})$ are today not available from USM manufaturerer data sheets. However, such data should be readily available in USM flow computers, and may hopefully be available also to customers in the future.

98 Note that the simplification used in this example calculation is not a limitation in the program *EMU - USM Fiscal Gas Metering Station.* In the program, E_{rept} can be specified individually at each test flow rate, cf. Section 5.10.1. If only a single value for E_{rept} is available (which may be a typical situation), this value is used at all flow rates.

To illustrate this option, a simplifed example is used here where the standard deviation $u(\hat{t}_{1i}^{random})$ is taken to be 2.5 ns, at all paths and flow rates. Note that in practice, the repeatability of the measured transit times may be expected to be flow rate dependent (increasing at high flow rates, with increasing turbulence). The repeatability may also be influenced by the lateral chord position. Consequently, this example value represents a simplification, as discussed in Section 3.4.2 [99].

For this example then, Table 4.16 gives E_{rept} at the same test flow rates as used in Table 4.12, calculated according to Eqs. (3.41), (3.43) and (3.45).

Table 4.16. Sample uncertainty budget for the USM repeatability in field operation (example), calculated according to Eqs. (3.41), (3.43) and (3.45), for the simplified example of constant $U(\hat{t}_{1i}^{random})$ = 5 ns over the flow rate range. (Corresponds to Fig. 5.13.)

Test rate no.	**Test flow velocity**	**Test flow rate**	**Expanded uncertainty,** $U(\hat{t}_{1i}^{random})$ **(repeatability)**	**Conf. level & Distribut.**	**Cov. fact., *k***	**Standard uncertainty,** $u(\hat{t}_{1i}^{random})$	**Rel. comb. standard uncertainty,** E_{rept}	**Rel. exp. uncertainty (95 % c.l.),** $2E_{rept}$
1	0.4 m/s	q_{min}	5 ns	95 % (norm)	2	2.5 ns	0.158 %	0.316 %
2	1.0 m/s	0.10 q_{max}	"	"	"	"	0.063 %	0.126 %
3	2.5 m/s	0.25 q_{max}	"	"	"	"	0.025 %	0.050 %
4	4.0 m/s	0.40 q_{max}	"	"	"	"	0.016 %	0.031 %
5	7.0 m/s	0.70 q_{max}	"	"	"	"	0.009 %	0.018 %
6	10.0 m/s	q_{max}	"	"	"	"	0.006 %	0.012 %

In Sections 4.4.6 and 4.6, Alternative (1) above is used (Table 4.15), i.e. with specification of the manufacturer value E_{rept} = 0.2 %.

4.4.2 Meter body

The relative combined standard uncertainty of the meter body, $E_{body,\Delta}$, is given by Eq. (3.21), and is evaluated in the following. Only uncorrected changes of meter body dimensions from flow calibration to field operation are to be accounted for in $E_{body,\Delta}$.

The user input uncertainties to $E_{body,\Delta}$ are the standard uncertainties of the temperature and pressure expansion coefficients, $u(\hat{\alpha})$ and $u(\hat{\beta})$, respectively, cf. Section 3.4.1. With respect to pressure and temperature correction of the meter body dimen-

99 Note that the simplification used in this example calculation is not a limitation in the program *EMU - USM Fiscal Gas Metering Station.* In the program, $u(\hat{t}_{1i}^{random})$ can be specified individually at each test flow rate, cf. Section 5.10.

sions from flow calibration to field operation, two options have been implemented in the program *EMU - USM Fiscal Gas Metering Station*: (1) *P* and *T* corrections of the meter body are *not* used by the USM manufacturer, and (2) *P* and *T* corrections *are* used. Both cases are described in the calculation examples addressed here.

Alternative (1), P & T correction of meter body not used: When pressure and temperature corrections are not used, the dimensional change (expansion or contraction) of the meter body itself becomes the uncertainty, cf. Section 3.4.1. The contributions to the combined standard uncertainty of the meter body are described in the following.

1. ***Pressure expansion coefficient,*** $u(\hat{\beta})$***:*** Eq. (2.19) is based on a number of approximations that may not hold in practical cases. First, there is the question of which model for β that is most correct for a given installation (pipe support), cf. Section 2.3.4 and Table 2.6. Secondly, there is the question of the validity of the linear model used for K_P, cf. Eq. (2.17). Here a tentative figure of 20 % for $U(\hat{\beta})$ is taken, as an example. β is there given from Eq. (2.19) and Table 4.3 as $\hat{\beta} = 0.154/(8.4 \cdot 10^{-3} \cdot 2 \cdot 10^{6})$ bar^{-1} = 9.166·10^{-6} bar^{-1}. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±20 % ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{\beta}) = U(\hat{\beta})/\sqrt{3} = 0.2 \cdot 9.166 \cdot 10^{-6}\ bar^{-1}/\sqrt{3}$ = 1.05848·10^{-6} bar^{-1}.

2. ***Pressure difference,*** $u_c(\Delta\hat{P}_{cal})$***:*** In the present example, the pressure difference between flow calibration and line conditions is $\Delta\hat{P}_{cal}$ = 50 bar, cf. Table 4.2. Without meter body pressure correction, and assuming 100 % confidence level and a rectangular probability distribution within the range ±50 bar ($k = \sqrt{3}$, cf. Section B.3), the standard uncertainty due to the pressure difference is given from Eq. (3.35) as $u_c(\Delta\hat{P}_{cal}) = \left|\Delta\hat{P}_{cal}\right|/\sqrt{3} = 50\ bar/\sqrt{3} \approx 28.8675$ bar.

3. ***Temperature expansion coefficient,*** $u(\hat{\alpha})$***:*** For the uncertainty of the temperature expansion coefficient (including possible inaccuracy of the linear temperature expansion model) a tentative figure of 20 % is taken here. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±20 % ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{\alpha}) = U(\hat{\alpha})/\sqrt{3} = 0.2 \cdot 14 \cdot 10^{-6}\ K^{-1}/\sqrt{3}$ = 1.61658·10^{-6} K^{-1}.

4. ***Temperature difference,*** $u_c(\Delta\hat{T}_{cal})$***:*** In the present example, the temperature difference between flow calibration and line conditions is $\Delta\hat{T}_{cal}$ = 40 ^{o}C, cf. Ta-

ble 4.2. Without meter body temperature correction, and assuming a 100 % confidence level and a rectangular probability distribution within the range ±40 ^{o}C ($k = \sqrt{3}$, cf. Section B.3), the standard uncertainty due to the temperature difference is given from Eq. (3.39) as $u_c(\Delta\hat{T}_{cal}) = \left|\Delta\hat{T}_{cal}\right| / \sqrt{3} = 40\ ^{o}C / \sqrt{3} \approx 23.0940$ ^{o}C.

From these basic input uncertainties, one finds, from Eqs. (3.34) and (3.38) respectively,

$$\begin{aligned} u_c(\hat{K}_P) &= \sqrt{(\Delta\hat{P}_{cal})^2 u^2(\hat{\beta}) + \hat{\beta}^2 u_c^2(\Delta\hat{P}_{cal})} \\ &= \sqrt{50^2 \cdot 1.058^2 + 9.166^2 \cdot 28.8675^2} \cdot 10^{-6} = 2.6983 \cdot 10^{-4} \end{aligned} \tag{4.1}$$

$$\begin{aligned} u_c(\hat{K}_T) &= \sqrt{(\Delta\hat{T}_{cal})^2 u^2(\hat{\alpha}) + \hat{\alpha}^2 u_c^2(\Delta\hat{T}_{cal})} \\ &= \sqrt{40^2 \cdot 1.616^2 + 14^2 \cdot 23.0940^2} \cdot 10^{-6} = 3.2971 \cdot 10^{-4} \end{aligned} \tag{4.2}$$

Since $\hat{K}_P = 1 + 9.166 \cdot 10^{-6} \cdot 50$ = 1.00004583 and $\hat{K}_T = 1 + 14 \cdot 10^{-6} \cdot 40$ = 1.00056 (cf. Eqs. (3.33) and (3.37), one finds $E_{KP} = u_c(\hat{K}_P) / \left|\hat{K}_P\right|$ $= 2.6983 \cdot 10^{-4} / 1.00004583$ = $2.6971 \cdot 10^{-4}$ and $E_{KT} = u_c(\hat{K}_T) / \left|\hat{K}_T\right|$ $= 3.2971 \cdot 10^{-4} / 1.00056$ = $3.2952 \cdot 10^{-4}$ (cf. Eqs. (3.32).

Table 4.17. Sample uncertainty budget for USM meter body, calculated according to Eqs. (3.21)-(3.40) and Table 4.2 for meter body data given in Table 4.3, and *no temperature and pressure correction* of the meter body dimensions. (Corresponds to Fig. 5.15.)

Input uncertainty	**Relative expanded uncert.**	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**
Coefficient of linear temperature expansion, α	20 %	100 % (rect)	$\sqrt{3}$	11.5 %
Coefficient of linear pressure expansion, β	20 %	100 % (rect)	$\sqrt{3}$	11.5 %
Rel. comb. standard uncertainty, temperature correction, E_{KT}				$3.30 \cdot 10^{-2}$ %
Rel. comb. standard uncertainty, pressure correction, E_{KP}				$2.70 \cdot 10^{-2}$ %

Contributions to meter body uncertainty	**Rel. comb. standard uncertainty**	**Rel. sens. coeff.**	**Rel. comb. standard uncertainty**
Meter body inner radius, $E_{rad,\Delta}$	$4.26 \cdot 10^{-2}$ %	3.6	0.1533 %
Lateral chord positions, $E_{chord,\Delta}$	$4.26 \cdot 10^{-2}$ %	- 0.6	- 0.0256 %
Inclination angles, $E_{angle,\Delta}$			$-2.1 \cdot 10^{-18}$ %
Relative combined standard uncertainty (sum)	$E_{body,\Delta}$		0.1278 %
Relative expanded uncertainty (95 % confidence level, k = 2)	$k \cdot E_{body,\Delta}$		**0.26 %**

Eqs. (3.29)-(3.30) yield $E_{R,\Delta} = E_{yi,\Delta} = \sqrt{E_{KP}^2 + E_{KT}^2} = \sqrt{2.6971^2 + 3.2952^2} \cdot 10^{-4}$ = $4.2582 \cdot 10^{-4}$. Insertion into Eqs. (3.31) and (3.22)-(3.24) then leads to $E_{rad,\Delta}$ = 0.1533 %, $E_{chord,\Delta}$ = -0.0256 % and $E_{angle,\Delta}$ = $-2.1 \cdot 10^{-18}$ %, giving $E_{body,\Delta}$ = 0.1278 % from Eq. (3.21). Table 4.17 summarizes these calculations.

Alternative (2), P & T correction of meter body used: In the present description, pressure and temperature corrections are assumed to be done according to Eqs. (2.13)-(2.19), cf. Section 2.3.4. The contributions to the combined standard uncertainty of the meter body are described in the following.

1. ***Pressure expansion coefficient,*** $u(\hat{\beta})$***:*** The uncertainty of the pressure expansion coefficient is the same as for Alternative (1) above: $u(\hat{\beta})$ = $1.05848 \cdot 10^{-6}$ bar^{-1}.

2. ***Pressure difference,*** $u_c(\Delta\hat{P}_{cal})$***:*** When meter body pressure correction is used, the standard uncertainty due to the pressure difference is given from Eq. (3.36) and Table 4.6 as $u_c(\Delta\hat{P}_{cal}) = \sqrt{2} u_c(\hat{P})$ = $\sqrt{2} \cdot 0.078$ = 0.110 bar.

3. ***Temperature expansion coefficient,*** $u(\hat{\alpha})$***:*** The uncertainty of the temperature expansion coefficient is the same as for Alternative (1) above: $u(\hat{\alpha})$ = $1.61658 \cdot 10^{-6}$ K^{-1}.

4. ***Temperature difference,*** $u_c(\Delta\hat{T}_{cal})$***:*** When meter body temperature correction is used, the standard uncertainty due to the temperature difference is given from Eq. (3.40) and Table 4.8 as $u_c(\Delta\hat{T}_{cal}) = \sqrt{2} u_c(\hat{T})$ = $\sqrt{2} \cdot 0.076$ = 0.107 ^{o}C.

From these basic input uncertainties, one finds, from Eqs. (3.34) and (3.38) respectively,

$$\begin{aligned} u_c(\hat{K}_P) &= \sqrt{(\Delta\hat{P}_{cal})^2 u^2(\hat{\beta}) + \hat{\beta}^2 u_c^2(\Delta\hat{P}_{cal})} \\ &= \sqrt{50^2 \cdot 1.05848^2 + 9.166^2 \cdot 0.110^2} \cdot 10^{-6} = 5.2934 \cdot 10^{-5} \end{aligned} \tag{4.3}$$

$$\begin{aligned} u_c(\hat{K}_T) &= \sqrt{(\Delta\hat{T}_{cal})^2 u^2(\hat{\alpha}) + \hat{\alpha}^2 u_c^2(\Delta\hat{T}_{cal})} \\ &= \sqrt{40^2 \cdot 1.61658^2 + 14^2 \cdot 0.107^2} \cdot 10^{-6} = 6.4681 \cdot 10^{-5} \end{aligned} \tag{4.4}$$

Since $\hat{K}_P = 1 + 9.166 \cdot 10^{-6} \cdot 50$ = 1.0004583 and $\hat{K}_T = 1 + 14 \cdot 10^{-6} \cdot 40$ = 1.00056 (cf. Eqs. (3.33) and (3.37)), one finds $E_{KP} = u_c(\hat{K}_P)/|\hat{K}_P|$ $= 5.2934 \cdot 10^{-5}/1.0004583$ = $5.2909 \cdot 10^{-5}$ and $E_{KT} = u_c(\hat{K}_T)/|\hat{K}_T|$ $= 6.4681 \cdot 10^{-5}/1.00056$ = $6.4644 \cdot 10^{-5}$ (cf. Eqs. (3.32)).

Eqs. (3.29)-(3.30) yield $E_{R,\Delta} = E_{yi,\Delta} = \sqrt{E_{KP}^2 + E_{KT}^2} = \sqrt{5.2909^2 + 6.4644^2} \cdot 10^{-5}$ = $8.3536 \cdot 10^{-5}$. Insertion into Eqs. (3.31) and (3.22)-(3.27) then leads to $E_{rad,\Delta}$ = 0.0301 %, $E_{chord,\Delta}$ = -0.0050 % and $E_{angle,\Delta}$ = $-4.2 \cdot 10^{-19}$ %, giving $E_{body,\Delta}$ = 0.025 % from Eq. (3.21). Table 4.18 summarizes these calculations.

Table 4.18. Sample uncertainty budget for USM meter body, calculated according to Eqs. (3.21)-(3.40) and Table 4.2 for meter body data given in Table 4.3, and *with temperature and pressure correction* of the meter body dimensions (according to Eqs. (2.13)-(2.19)).

Input uncertainty	**Relative expanded uncert.**	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**
Coefficient of linear temperature expansion, α	20 %	100 % (rect)	$\sqrt{3}$	11.5 %
Coefficient of linear pressure expansion, β	20 %	100 % (rect)	$\sqrt{3}$	11.5 %
Rel. comb. standard uncertainty, temperature correction, E_{KT}				$6.46 \cdot 10^{-3}$ %
Rel. comb. standard uncertainty, pressure correction, E_{KP}				$5.29 \cdot 10^{-3}$ %

Contributions to meter body uncertainty	**Rel. comb. standard uncertainty**	**Rel. sens. coeff.**	**Rel. comb. standard uncertainty**
Meter body inner radius, $E_{rad,\Delta}$	$8.35 \cdot 10^{-3}$ %	3.6	0.0301 %
Lateral chord positions, $E_{chord,\Delta}$	$8.35 \cdot 10^{-3}$ %	- 0.6	- 0.0050 %
Inclination angles, $E_{angle,\Delta}$			$-4.2 \cdot 10^{-19}$ %
Relative combined standard uncertainty (sum)	$E_{body,\Delta}$		0.0251 %
Relative expanded uncertainty (95 % confidence level, k = 2)	$k \cdot E_{body,\Delta}$		**0.05 %**

In Section 4.4.6 and 4.6, Alternative (1) above is used (Table 4.17), i.e. with no *P* and *T* correction of the meter body dimensions used by the USM manufacturer.

4.4.3 Systematic transit time effects

The relative combined standard uncertainty $E_{time,\Delta}$ is given by Eqs. (3.42)-(3.44) and (3.46). Only systematic deviations in transit times from flow calibration to field operation are to be accounted for in $E_{time,\Delta}$ (due to e.g. *P* and *T* effects, drift/ageing, deposits at transducer fronts, etc.). Since such timing errors may in practice not be cor-

rected for in current USMs, they are to be accounted for as uncertainties. Examples of possible contributions are listed in Section 3.4.2.2.

Data for $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$ are today unfortunately not available from USM manufaturerer data sheets, and the knowledge on systematic transit time effects is insufficient today. It is hoped that, as improved knowledge on such effects is being developed, such uncertainty data will be available in USM manufacturers data sheets in the future, including documentation of the methods by which they are obtained.

Consequenty, only a tentative calculation example is taken here. The contributions to the combined standard uncertainty of the meter body are described in the following, for the example of flow calibration at 10 ^{o}C and 50 bara (cf. Table 4.2).

1. ***Upstream transit time,*** $u(\hat{t}_{1i}^{systematic})$***:*** $u(\hat{t}_{1i}^{systematic})$accounts for change in the upstream transit time correction relative to the value at flow calibration conditions. This includes electronics/cable/transducer/diffraction time delay, possible transducer cavity delay, possible deposits at transducer front, sound refraction effects, and change of such delays with *P*, *T* and time (drift), cf. Table 3.8.

   In the present example, it is assumed that all other effects than shift in transducer delay can be neglected. It is assumed that the transducers have been "dry calibrated" at the same transducer distances as used in the USM meter body[100]. In this example, the pressure and temperature differences from flow calibration to line conditions are 50 bar and 40 ^{o}C. Measurement data from [Lunde *et al.*, 1999; 2000] indicate that for a temperature change of 35 ^{o}C a shift in transducer delay of about 1-2 µs may not be unrealistic e.g. for transducers with epoxy front[101]. Here, a tentative value $U(\hat{t}_{1i}^{systematic})$ = 0.6 µs = 600 ns is used as an example (somewhat arbitrarily). Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±600 ns ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{t}_{1i}^{systematic}) = U(\hat{t}_{1i}^{systematic})/\sqrt{3} = 600\,ns/\sqrt{3}$ = 346.41 ns.

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100 Note that if "dry calibration" is not made at the same transducer distances as used in the USM meter body, diffraction effects may also need to be taken into account when evaluating $u(\hat{t}_{1i}^{systematic})$ [Lunde *et al.*, 1999; 2000a; 2000b]. However, such effects are neglected in the present example.

101 The shift will depend among others on the method used for transit time detection [Lunde *et al.*, 2000a].

2. ***Downstream transit time,*** $u(\hat{t}_{2i}^{systematic})$**:** The difference between $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$ accounts for the uncertainty of the Δt-correction relative to the value at flow calibration conditions (P & T effects, drift, etc.). Here, a tentative value $U(\hat{t}_{2i}^{systematic})$ = 590 ns is used as an example (somewhat arbitrarily), corresponding to an expanded uncertainty of the Δt-correction of 10 ns. Assuming a Type B uncertainty, at a 100 % confidence level and a rectangular probability distribution within the range ±590 ns ($k = \sqrt{3}$, cf. Section B.3), one obtains $u(\hat{t}_{2i}^{systematic}) = U(\hat{t}_{2i}^{systematic})/\sqrt{3} = 590\,ns/\sqrt{3}$ = 340.64 ns.

Table 4.19 gives $E_{time,\Delta}$ for this example, calculated at the same test flow rates as used in Table 4.12. Note that $E_{time,\Delta}$ increases at low flow rates, due to the sensitivity coefficients s_{t1i} and s_{t2i} which increase at low flow rates due to the reduced transit time difference, cf. Eqs. (3.43)-(3.44). At low flow rates $E_{time,\Delta}$ is determined by the uncertainty of the Δt-correction. At higher flow rates $E_{time,\Delta}$ is determined by the uncertainty of the transit times themselves.

Table 4.19. Uncertainty budget for the systematic contributions to the transit time uncertainties of the 12" USM, calculated from Eqs. (3.42)-(3.44) and (3.46). (Corresponds to Fig. 5.16.)

Input uncertainty	**Given expanded uncert.**	**Conf. level & Distribut.**	**Cov. fact., *k***	**Standard uncertainty**
$U(\hat{t}_{1i}^{systematic})$	600 ns	100 % (rect)	$\sqrt{3}$	346.41 ns
$U(\hat{t}_{2i}^{systematic})$	590 ns	100 % (rect)	$\sqrt{3}$	340.64 ns

Test rate no.	**Test flow velocity**	**Test flow rate**	**Rel. comb. standard uncertainty,** $E_{time,\Delta}$	**Rel. exp. uncertainty (95 % c.l.),** $2E_{time,\Delta}$
1	0.4 m/s	q_{min}	0.4206	0.841
2	1.0 m/s	0.10 q_{max}	0.1197	0.239
3	2.5 m/s	0.25 q_{max}	0.0007	0.001
4	4.0 m/s	0.40 q_{max}	0.0308	0.062
5	7.0 m/s	0.70 q_{max}	0.0523	0.105
6	10.0 m/s	q_{max}	0.0609	0.122

4.4.4 Integration method (installation effects)

The relative combined standard uncertainty accounting for installation effects (the integration method), $E_{I,\Delta}$, is given by Eq. (3.47). Only changes of installation conditions from flow calibration to field operation are to be accounted for in $E_{I,\Delta}$. Examples of possible contributions are listed in Section 3.4.3, see also Table 3.8.

In general, for a given type of meter, $E_{I,\Delta}$ should be determined from a (preferably) extensive empirical data set where the type of meter in question has been subjected to testing with respect to various installation conditions (bend configurations, flow conditioners, meter orientation rel. bend, etc., cf. Table 3.8). A great deal of installation effects testing has been carried out over the last decade, together with theoretical / numerical investigations of such effects. USM manufacturers (together with flow calibration laboratories, users and research institutions) today posess a considerable amount of information and knowledge on this subject. USM installation effects is today still a subject of intense testing and research.

However, data for $E_{I,\Delta}$ are today not available from USM manufaturerer data sheets, and $E_{I,\Delta}$ appears to be one of the more difficult uncertainty contributions to specify, especially when it comes to traceability.

Consequenty, only a tentative uncertainty figure can be taken here. As an example, a figure of 0.3 % is used for the effect of changed installation conditions from flow calibration to field operation. Assuming a 95 % confidence level and a normal probability distribution (k = 2, cf. Section B.3), one has $E_{I,\Delta} = 0.3\%/2$ = 0.15 %. This value is further used in Sections 4.4.6 and 4.6.

4.4.5 Miscellaneous USM effects

Uncertainty contributions due to miscellaneous USM effects are discussed briefly in Section 3.4.4. These are meant to account for USM uncertainties not covered by the other uncertainty terms used in the uncertainty model, such as e.g. inaccuracy of the USM functional relationship. In the present calculation example such miscellaneous USM uncertainties are neglected. However, miscellaneous effects can be accounted for in the program *EMU - USM Fiscal Gas Metering Station*, cf. Section 5.10.2.

4.4.6 Summary - Expanded uncertainty of USM in field operation

The relative expanded uncertainty of the USM in field operation E_{USM} is given by Eqs. (3.19)-(3.20), where the involved relative uncertainty terms E_{rept}, $E_{body,\Delta}$, $E_{time,\Delta}$, $E_{I,\Delta}$ and E_{misc} are all evaluated in the sections above.

Sample uncertainty budgets are given in Tables 4.20 and 4.21, for evaluation of E_{USM} according to these equations. Note that in each table only a single test flow

rate is evaluated[102]. Tables 4.20 and 4.21 apply to test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively, cf. Table 4.12.

Table 4.20. Sample uncertainty budget for the USM in field operation (example), calculated according to Eqs. (3.19)-(3.20), for a *single test flow rate*, $0.10q_{max}$. (Corresponds to Figs. 5.18 and 5.26.)

Source	**Input uncertainty**				**Combined uncertainty**	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
USM repeatability in field operation (Table 4.15)	0.2 %	95 % (norm)	2	0.1 %	1	$1.00 \cdot 10^{-6}$
Meter body uncertainty (Table 4.17)	0.26 %	95 % (norm)	2	0.128 %	1	$1.64 \cdot 10^{-6}$
Systematic transit time effects (Table 4.19)	0.239 %	95 % (norm)	2	0.12 %	1	$1.43 \cdot 10^{-6}$
Installation (integration) effects (Section 4.4.4)	0.3 %	95 % (norm)	2	0.15 %	1	$2.25 \cdot 10^{-6}$
Miscellaneous effects (Section 4.4.5)	Neglected	-	-	-	1	0
Sum of relative variances				E_{USM}^2		$6.32 \cdot 10^{-6}$
Relative combined standard uncertainty				E_{USM}		0.251 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{USM}$		**0.50 %**

Table 4.21. Sample uncertainty budget for the USM in field operation (example), calculated according to Eqs. (3.19)-(3.20), for a *single test flow rate*, $0.70q_{max}$. (Corresponds to Figs. 5.18 and 5.26.)

Source	**Input uncertainty**				**Combined uncertainty**	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
USM repeatability in field operation (Table 4.15)	0.2 %	95 % (norm)	2	0.1 %	1	$1.00 \cdot 10^{-6}$
Meter body uncertainty (Table 4.17)	0.26 %	95 % (norm)	2	0.128 %	1	$1.64 \cdot 10^{-6}$
Systematic transit time effects (Table 4.19)	0.105 %	95 % (norm)	2	0.052 %	1	$2.74 \cdot 10^{-7}$
Installation (integration) effects (Section 4.4.4)	0.3 %	95 % (norm)	2	0.15 %	1	$2.25 \cdot 10^{-6}$
Miscellaneous effects (Section 4.4.5)	Neglected	-	-	-	1	0
Sum of relative variances				E_{USM}^2		$5.164 \cdot 10^{-6}$
Relative combined standard uncertainty				E_{USM}		0.227 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{USM}$		**0.45 %**

102 In the program *EMU - USM Fiscal Gas Metering Station*, uncertainty evaluation is made for all test flow rates, cf. Section 5.10.

The calculated relative expanded uncertainties (specified at a 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) are 0.50 % and 0.45 %, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively. These values are further used in Section 4.6.

4.5 Signal communication and flow computer calculations

Effects of uncertainties due to signal communication and flow computer calculations are described briefly in Section 3.5. In the present calculation example such uncertainties are neglected. However, both effects can be accounted for in the program *EMU - USM Fiscal Gas Metering Station*, cf. Section 5.11.

4.6 Summary - Expanded uncertainty of USM fiscal gas metering station

In the following, the relative expanded uncertainty of the USM fiscal gas metering station example is calculated, on basis of the calculations given above. Results are given for each of the four measurands q_v, Q, q_m and q_e.

4.6.1 Volumetric flow rate, line conditions

The relative expanded uncertainty of the volumetric flow rate at line conditions E_{q_v} is given by Eqs. (3.1) and (3.6), where the involved relative uncertainty terms E_{cal}, E_{USM}, E_{comm} and E_{flocom} are evaluated in the sections above.

Sample uncertainty budgets for E_{q_v} are given in Tables 4.22 and 4.23, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively. For the present example, the calculated relative expanded uncertainties (specified at a 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) are 1.00 % and 0.58 %, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively.

Table 4.22. Sample uncertainty budget for the USM fiscal gas metering station, for the volumetric flow rate at line conditions, q_v (example), calculated according to Eqs. (3.1) and (3.6), for a *single test flow rate*, $0.10q_{max}$. (Corresponds to Fig. 5.28.)

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Flow calibration (Table 4.13)	0.87 %	95 % (norm)	2	0.434 %	1	$1.88 \cdot 10^{-5}$
USM field operation (Table 4.20)	0.50	95 % (norm)	2	0.25 %	1	$6.32 \cdot 10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				$E_{q_v}^2$		$2.51 \cdot 10^{-5}$
Relative combined standard uncertainty				E_{q_v}		0.501 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{q_v}$		**1.00 %**

Table 4.23. Sample uncertainty budget for the USM fiscal gas metering station, for the volumetric flow rate at line conditions, q_v (example), calculated according to Eqs. (3.1) and (3.6), for a *single test flow rate*, $0.70q_{max}$.

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Flow calibration (Table 4.14)	0.37 %	95 % (norm)	2	0.187 %	1	$3.35 \cdot 10^{-6}$
USM field operation (Table 4.21)	0.45 %	95 % (norm)	2	0.227 %	1	$5.16 \cdot 10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				$E_{q_v}^2$		$8.514 \cdot 10^{-6}$
Relative combined standard uncertainty				E_{q_v}		0.292 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{q_v}$		**0.58 %**

4.6.2 Volumetric flow rate, standard reference conditions

The relative expanded uncertainty of the volumetric flow rate at standard reference conditions E_Q is given by Eqs. (3.2) and (3.7), where the involved relative uncertainty terms E_P, E_T, $E_{Z/Z0}$, E_{cal}, E_{USM}, E_{comm} and E_{flocom} have been evaluated in the sections above.

Sample uncertainty budgets for E_Q are given in Tables 4.24 and 4.25, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively. For the present example, the calculated relative expanded uncertainties (specified at a 95 % confidence level and a normal

probability distribution, with k = 2, cf. Section B.3) are 1.07 % and 0.70 %, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively.

Table 4.24. Sample uncertainty budget for the USM fiscal gas metering station, for the volumetric flow rate at standard reference conditions, Q (example), calculated according to Eqs. (3.2) and (3.7), for a *single test flow rate*, $0.10q_{max}$. (Corresponds to Fig. 5.29.)

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Pressure measurement, P (Table 4.6)	0.16 %	95 % (norm)	2	0.08 %	1	$6.4\cdot10^{-7}$
Temperature measurement, T (Table 4.8)	0.047 %	95 % (norm)	2	0.024 %	1	$5.52\cdot10^{-8}$
Compressibility, Z/Z_0 (Table 4.9)	0.339 %	95 % (norm)	2	0.170 %	1	$2.878\cdot10^{-6}$
Flow calibration (Table 4.13)	0.87 %	95 % (norm)	2	0.434 %	1	$1.88\cdot10^{-5}$
USM field operation (Table 4.20)	0.50	95 % (norm)	2	0.25 %	1	$6.32\cdot10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				E_Q^2		$2.869\cdot10^{-5}$
Relative combined standard uncertainty				E_Q		0.536 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_Q$		**1.07 %**

Table 4.25. Sample uncertainty budget for the USM fiscal gas metering station, for the volumetric flow rate at standard reference conditions, Q (example), calculated according to Eqs. (3.2) and (3.7), for a *single test flow rate*, $0.70q_{max}$.

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Pressure measurement, P (Table 4.6)	0.16 %	95 % (norm)	2	0.08 %	1	$6.4\cdot10^{-7}$
Temperature measurement, T (Table 4.8)	0.047 %	95 % (norm)	2	0.024 %	1	$5.52\cdot10^{-8}$
Compressibility, Z/Z_0 (Table 4.9)	0.339 %	95 % (norm)	2	0.170 %	1	$2.878\cdot10^{-6}$
Flow calibration (Table 4.14)	0.37 %	95 % (norm)	2	0.183 %	1	$3.35\cdot10^{-6}$
USM field operation (Table 4.21)	0.45 %	95 % (norm)	2	0.227 %	1	$5.16\cdot10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				E_Q^2		$1.208\cdot10^{-5}$
Relative combined standard uncertainty				E_Q		0.348 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_Q$		**0.70 %**

4.6.3 Mass flow rate

The relative expanded uncertainty of the mass flow rate E_m is given by Eqs. (3.3) and (3.8), where the involved relative uncertainty terms E_ρ, E_{cal}, E_{USM}, E_{comm} and E_{flocom} are evaluated in the sections above.

Table 4.26. Sample uncertainty budget for the USM fiscal gas metering station, for the mass flow rate, q_m (example), calculated according to Eqs. (3.3) and (3.8), for a *single test flow rate*, $0.10q_{max}$. (Corresponds to Figs. 5.27 and 5.30.)

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Density measurement, ρ (Table 4.11)	0.19 %	95 % (norm)	2	0.095 %	1	$9.025 \cdot 10^{-7}$
Flow calibration (Table 4.13)	0.87 %	95 % (norm)	2	0.434 %	1	$1.88 \cdot 10^{-5}$
USM field operation (Table 4.20)	0.50	95 % (norm)	2	0.25 %	1	$6.32 \cdot 10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				E_m^2		$2.602 \cdot 10^{-5}$
Relative combined standard uncertainty				E_m		0.510 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_m$		**1.02 %**

Table 4.27. Sample uncertainty budget for the USM fiscal gas metering station, for the mass flow rate, q_m (example), calculated according to Eqs. (3.3) and (3.8), for a *single test flow rate*, $0.70q_{max}$.

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Density measurement, ρ (Table 4.11)	0.19 %	95 % (norm)	2	0.095 %	1	$9.025 \cdot 10^{-7}$
Flow calibration (Table 4.134)	0.37 %	95 % (norm)	2	0.183 %	1	$3.35 \cdot 10^{-6}$
USM field operation (Table 4.21)	0.45 %	95 % (norm)	2	0.227 %	1	$5.16 \cdot 10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				E_m^2		$9.413 \cdot 10^{-6}$
Relative combined standard uncertainty				E_m		0.306 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_m$		**0.61 %**

Sample uncertainty budgets for E_m are given in Tables 4.26 and 4.27, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively. For the present example, the calculated relative expanded uncertainties (specified at a 95 % confidence level and a normal

probability distribution, with k = 2, cf. Section B.3) are 1.03 % and 0.63 %, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively.

4.6.4 Energy flow rate

The relative expanded uncertainty of the energy flow rate E_{q_e} is given by Eqs. (3.4) and (3.9), where the involved relative uncertainty terms E_P, E_T, $E_{Z/Z0}$, E_{H_s}, E_{cal}, E_{USM}, E_{comm} and E_{flocom} are evaluated in the sections above.

Sample uncertainty budgets for E_{q_e} are given in Tables 4.28 and 4.29, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively. For the present example, the calculated relative expanded uncertainties (specified at a 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) are 1.08 % and 0.71 %, for test flow rates $0.10q_{max}$ and $0.70q_{max}$, respectively.

Table 4.28. Sample uncertainty budget for the USM fiscal gas metering station, for the energy flow rate, q_e (example), calculated according to Eqs. (3.4) and (3.9), for a *single test flow rate*, $0.10q_{max}$. (Corresponds to Fig. 5.31.)

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Pressure measurement, P (Table 4.6)	0.16 %	95 % (norm)	2	0.08 %	1	$6.4\cdot10^{-7}$
Temperature measurement, T (Table 4.8)	0.047 %	95 % (norm)	2	0.024 %	1	$5.52\cdot10^{-8}$
Compressibility, Z/Z_0 (Table 4.9)	0.339 %	95 % (norm)	2	0.170 %	1	$2.878\cdot10^{-6}$
Calorific value, H_s (Section 4.2.5)	0.15 %	95 % (norm)	2	0.075 %	1	$5.63\cdot10^{-7}$
Flow calibration (Table 4.13)	0.87 %	95 % (norm)	2	0.434 %	1	$1.88\cdot10^{-5}$
USM field operation (Table 4.20)	0.50	95 % (norm)	2	0.25 %	1	$6.32\cdot10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				$E_{q_e}^2$		$2.926\cdot10^{-5}$
Relative combined standard uncertainty				E_{q_e}		0.541 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k\cdot E_{q_e}$		**1.08 %**

Table 4.29. Sample uncertainty budget for the USM fiscal gas metering station, for the energy flow rate, q_e (example), calculated according to Eqs. (3.4) and (3.9), for a *single test flow rate*, $0.70q_{max}$.

Source	Input uncertainty				Combined uncertainty	
	Relative expand. uncert.	**Conf. level & Distribut.**	**Cov. fact., *k***	**Relative standard uncertainty**	**Rel. sens. coeff.**	**Relative Variance**
Pressure measurement, P (Table 4.6)	0.16 %	95 % (norm)	2	0.08 %	1	$6.4 \cdot 10^{-7}$
Temperature measurement, T (Table 4.8)	0.047 %	95 % (norm)	2	0.024 %	1	$5.52 \cdot 10^{-8}$
Compressibility, Z/Z_0 (Table 4.9)	0.339 %	95 % (norm)	2	0.170 %	1	$2.878 \cdot 10^{-6}$
Calorific value, H_s (Section 4.2.5)	0.15 %	95 % (norm)	2	0.075 %	1	$5.63 \cdot 10^{-7}$
Flow calibration (Table 4.14)	0.37 %	95 % (norm)	2	0.183 %	1	$3.35 \cdot 10^{-6}$
USM field operation (Table 4.21)	0.45 %	95 % (norm)	2	0.227 %	1	$5.16 \cdot 10^{-6}$
Signal commun. and flow computer (Section 4.5)	Neglected	-	-	-	1	0
Sum of relative variances				$E_{q_e}^2$		$1.265 \cdot 10^{-5}$
Relative combined standard uncertainty				E_{q_e}		0.356 %
Relative expanded uncertainty (95 % confidence level, k = 2)				$k \cdot E_{q_e}$		**0.71 %**

4.7 Authors' comments to the uncertainty evaluation example

In the following, some overall comments to the above uncertainty evaluation example are given, with respect to specification of input uncertainties. As described in Section 1.3 (Table 1.5) and in Section 3.6, and also used above, the input uncertainties are conveniently organized in eight groups, where each group corresponds to a worksheet in the program *EMU - USM Fiscal Gas Metering Station*, cf. Chapter 5.

With respect to P, T and USM flow calibration, most of the necessary input uncertainties are normally available from instrument data sheets and calibration certificates, USM manufacturer data sheets and flow calibration results, and the calibration laboratory. For Z/Z_0 and ρ, parts of the necessary specifications of input uncertainties have been available in data sheets or other documents. With respect to USM field operation, only repeatability data have been available, - other significant input uncertainties (at the "detailed level") are not specified in the manufacturers' data sheets (cf. Chapter 6). Where manufacturer specifications have not been available, more tentative input uncertainty figures have been used here.

In the following, each of these eight groups is commented in some more detail, in relation to the present example.

• **Pressure measurement, *P*.** Table 3.1 gives an overview of the input uncertainties to be specified for the pressure measurement at the "detailed level". Except for the atmospheric pressure uncertainty, all input uncertainties used in the calculation example have been available from the manufacturer data sheet [Rosemount, 2000], cf. Section 4.2.1. The stability of the pressure transmitter clearly dominates the uncertainty of the pressure measurement, cf. Table 4.6 and Figs. 5.4, 5.21. The influence of the atmospheric pressure uncertainty, RFI effects, the transmitter uncertainty and ambient temperature effects on the transmitter are relatively smaller.

• **Temperature measurement, *T*.** Table 3.2 gives an overview of the input uncertainties to be specified for the temperature measurement at the "detailed level". Except for the stability of the Pt 100 element (which is taken from [BIPM, 1997]), all input uncertainties used in the calculation example have been available from the manufacturer data sheet [Rosemount, 2000], cf. Section 4.2.2. The stability of the temperature transmitter dominates the uncertainty of the temperature measurement, cf. Table 4.8 and Figs. 5.6, 5.22. Important are also the element and transmitter uncertainy, RFI effects on the temperature transmitter, and the stability of the temperature element. The influence of ambient temperature effects is relatively smaller.

• **Compressibility factor ratio calculation, Z/Z_0.** Table 3.3 gives an overview of the input uncertainties to be specified for the compressibility ratio at the "detailed level". The model uncertainty at line conditions ($E_{Z,mod}$) has been taken from [AGA-8, 1994]. At standard reference conditions, ISO 6976 [ISO, 1995c] has been used for the model uncertainty ($E_{Z0,mod}$). This means that the model uncertainty is smaller at standard reference conditions than at line conditions. The analysis uncertainties are in general more complicated to estimate, cf. Section 3.2.3.2. The present example is based on a simplified Monte Carlo type of simulation with natural variations of the gas composition (at a level which has been observed in practice). At line condition, the analysis uncertainty ($E_{Z,ana}$) is larger than the model uncertainty in this example, while the analysis uncertainty is shown to be negligible at standard reference conditions ($E_{Z0,ana}$), cf. Section 4.2.3 (Table 4.9) and Figs. 5.7, 5.23.

• **Density measurement, ρ.** Table 3.4 gives an overview of the input uncertainties to be specified for the density measurement at the "detailed level". Some of the input uncertainties used in the present example have been available from the manufacturer data sheet [Solartron, 1999], or are calculated from other results (pressure and tem-

perature uncertainties), cf. Section 4.2.4 and Table 4.10. However, a number of relevant input uncertainties related to VOS and installation corrections have not been available in data sheets, such as for the VOS at calibration conditions (c_c), the VOS at densitometer conditions (c_d), the periodic time (τ), the VOS correction constant (K_d), and the pressure deviation between densitometer and line conditions (ΔP_d). Input figures for these are discussed in Section 4.2.4.

In the present example the "densitometer accuracy" $u(\hat{\rho}_u)$ totally dominates the uncertainty of the pressure measurement, cf. Table 4.11 and Figs. 5.9, 5.24. Less important are the repeatability, $u(\hat{\rho}_{rept})$, the uncertainty of the line temperature, $u_c(\hat{T})$, the densitometer temperature, $u(\hat{T}_d)$, the uncertainty of the VOS correction constant, $u(\hat{K}_d)$, and the uncertainty of the temperature correction model, $u(\hat{\rho}_{temp})$. The uncertaintes of the line pressure measurement, $u(\hat{P})$, the periodic time, $u(\hat{\tau})$, and the calibration temperature, $u(\hat{T}_c)$, appear to be negligible. The influences of the pressure difference between densitometer and line conditions, $u(\Delta\hat{P}_d)$, and the uncertaintes of the VOS in the calibration and densitometer gases, $u(\hat{c}_c)$ and $u(\hat{c}_d)$, are also relatively small.

• **Calorific value measurement, H_s.** Table 3.5 gives the input uncertainty to be specified for the calorific value measurement. Only the "overall level" is available at present, - in general an uncertainty evaluation of the expanded uncertainty of the calorific value would be needed. That has not been made here, due to the scope of work for the *Handbook* (cf. Section 2.1). Only a tentative example value has been used, corresponding to the NPD regulation requirements [NPD, 2001], cf. Section 4.2.5 and Fig. 5.10.

• **USM flow calibration.** Table 3.6 gives an overview of the three input uncertainties to be specified for the USM flow calibration. The required input uncertainties are available from flow calibration laboratories and USM manufacturers, cf. Section 4.3. In the present example, and in the low-velocity range, the deviation factor clearly dominates the uncertainty of the USM flow calibration, cf. Table 4.13 and Figs. 5.11, 5.25. Note that this result will depend largely on the actual correction factor K used, i.e. the actual deviation curve, cf. Section 2.2.2 (Fig. 2.1). At higher flow velocities, the uncertainty of the flow calibration laboratory and the USM repeatability dominate, cf. Table 4.14. That is, all three input uncertainties are significant.

• **USM field operation.** Table 3.8 gives an overview of the input uncertainties to be specified for the USM in field operation (deviation relative to flow calibration conditions). These are grouped into four groups: USM repeatability in field operation, meter body uncertainty, uncertainty of systematic transit time effects, and the integration method uncertainty (installation effects). In the present example all four groups contribute significantly to the uncertainty of the USM in field operation, cf. Section 4.4, Tables 4.20, 4.21 and Figs. 5.18, 5.26. In general the latter two groups are the most difficult to specify (only the USM repeatability is available from current USM manufacturer data sheets), and tentative uncertainty figures have been used in the present calculation example, to demonstrate the sensitivity to these uncertainty contributions.

The four groups are commented in some more detail in the following.

Repeatability: The USM repeatability in field operation has been taken as a typical figure from USM manufacturer data sheets (flow rate repeatability), cf. Section 4.4.1, Table 4.15, and Figs. 5.12, 5.26. On lack of other data, the repeatability is taken here to be independent of flow rate (which is probably a simplification). If a flow rate dependent repeatability figure were available, that would be preferred. Alternatively, the repeatability of the measured transit times could be pecified (standard deviations), cf. Section 3.4.2. However, such information is not available from data sheets today, although it should be readily available from USM flow computers, cf. Chapter 6.

Meter body uncertainty: Pressure and temperature correction of the meter body dimensions are not used by all meter manufacturers today, cf. Table 2.6. In case of pressure and temperature deviation from flow calibration conditions, this may lead to significant measurement errors (in excess of the NPD requirements [NPD, 2001]), cf. Section 2.3.4. As correction methods are available, cf. Table 2.6, such correction might preferably be used on a routinely basis.

Evaluation of the meter body uncertainty involves specification of the two relative uncertainty terms $u(\hat{\alpha})/|\hat{\alpha}|$ and $u(\hat{\beta})/|\hat{\beta}|$, i.e. the uncertainties of the linear temperature and pressure expansion coefficients, α and β, respectively, cf. Section 4.4.2. These may often be difficult to specify, e.g. due to lack of data for $u(\hat{\alpha})/|\hat{\alpha}|$, and due to inaccuracy of the model(s) used for β, cf. Section 2.3.4. As these uncertainties have not been directly available, only tentative uncertainty figures have been used in the present example, cf. Table 4.18 and Fig. 5.15. Note that for relatively large ΔP_{cal}

and ΔT_{cal} (several tens of bars and ^{o}C, as in the present example), the actual input uncertainty figures for α and β become important.

Systematic transit time effects: Information on uncorrected systematic transit time effects and associated uncertainties are not available from USM manufacturers today, cf. Chapter 6. It is difficult to specify representative uncertainty figures for these effects at present. Some knowledge is available, however (cf. e.g. [Lunde *et al.*, 1999; 2000a; 2000b]), but yet this knowledge is far from sufficient. More work is definitely required in this area. The uncertainty figure example used here (for 40 ^{o}C and 50 bar deviation from flow calibration conditions) are thus very tentative, but far from unrealistic, cf. Section 4.4.3, Table 4.19 and Figs. 5.16, 5.26. The example shows that expanded uncertainties of systematic transit time effects of 600 and 590 ns (upstream and downstream, respectively) contribute by about 0.12 % to the expanded uncertainty of the flow rate reading at 10 m/s, and 0.84 % at 0.4 m/s (Table 4.19 and Fig. 5.16)[103]. As an uncertainty figure this high-velocity value (0.12 %) may at first glance appear to be small, but when remembering that it represents an uncorrected systematic timing error, the resulting error in flow rate reading may over time still represent a significant economic value [NPD, 2001][104].

Installation (integration) effects: Data on the uncertainty due specifically to installation effects are not available from USM manufacturer data sheets, cf. Chapter 6, despite the considerable effort on investigating such effects made over the last decade. Such uncertainty data might preferably be based on a large number of tests and simulations related to varying installation conditions, cf. Section 3.4.3. For the present calculation example a tentative uncertainty figure has been used, cf. Section 4.4.4, and Figs. 5.17, 5.26.

• **Signal communication and flow computer calculations.** Table 3.7 gives an overview of the input uncertainties to be specified related to signal communication and flow computer calculations. Data on such uncertainties have not been available from USM manufacturer data sheets. In the present example these uncertainties have been neglected, cf. Section 4.5 (but can be accounted for in the program *EMU - USM Fiscal Gas Metering Station*, cf. Fig. 5.27).

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103 The uncertainty at 10 m/s is determined by the magnitude of these expanded uncertainties (about 600 ns), whereas at 0.4 m/s the uncertainty is determined by the *difference* between these expanded uncertainties (10 ns).

104 The example used in Table 4.19 is by no means a “worst case”.

• **Gas metering station.** For the uncertainty evaluation example described in Chapter 4, the dominating contributions to the metering station's expanded uncertainty are due to the deviation factor (at low flow velocities), the flow calibration laboratory, the USM repeatability, the systematic deviations relative to flow calibration, the density measurement, and the compressibility factors, cf. Tables 4.22-4.29, and Figs. 5.27-5.31. The pressure and temperature measurement uncertainties are less important, especially the latter. It should be emphasized, however, that this is an example, and that especially with respect to the USM field operation uncertainties, a number of input uncertainties have only been given tentative example values.

5. PROGRAM "EMU - USM FISCAL GAS METERING STATION"

The present chapter describes the Excel program *EMU - USM Fiscal Gas Metering Station* which has been implemented for performing uncertainty calculations of USM fiscal gas metering stations. The program applies to metering stations equipped as described in Section 2.1, and is based on the uncertainty model for such stations described in Chapter 3. Using the program, uncertainty evaluation can be made for the expanded uncertainty of four measurands (at a 95 % confidence level, using $k = 2$):

- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard reference conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e.

In the following, the various worksheets used in *EMU - USM Fiscal Gas Metering Station* are presented and described, in the order they appear in the program. Example values are those used in the uncertainty evaluation example given in Chapter 4. This evaluation example follows closely the structure of the program, and may thus to some extent serve also as a guideline to using the program, with respect to the specification of input uncertainties.

5.1 General

Overall descriptions of the program *EMU - USM Fiscal Gas Metering Station* are given in Sections 1.2 and 1.3. In the following, some supplementary information is given.

The program is implemented in Microsoft Excel 2000 and is based on worksheets where the input data to the calculations are entered by the user. These “input worksheets” are mainly formed as uncertainty budgets, which are continuously updated as the user enters new input data. Other worksheets provide display of the uncertainty calculation results, and are continuously updated in the same way.

With respect to specification of input parameters and uncertainties, colour codes are used in the program *EMU - USM Fiscal Gas Metering Station*, according to the following scheme:

- Black font: Value that must be edited by the user,
- Blue font[105]: Outputs from the program, or number read from another worksheet (editing prohibited).

In the following subsections the worksheets of the program are shown and explained, and the necessary input parameters are addressed with an indication of where in Chapters 3 and 4 the input values are discussed.

Output data are presented in separate worksheets, graphically (curves and bar-charts), and by listing. An output report worksheet is available, summarizing the main uncertainty calculation results.

The expanded uncertainties calculated by the program may be used in the documentation of the metering station uncertainty, with reference to the present *Handbook* (cf. Appendix B.4). The worksheets are designed so that printouts of these can be used directly as a part of the uncertainty evaluation documentation. They may also conveniently be copied into a text document[106], for documentation and reporting purposes. However, it must be emphasised that the inputs to the program (quantities, uncertainties, confidence levels and probability distributions) must be documented by the user of the program. The user must also document that the calculation procedures and functional relationships implemented in the program (described in Chapter 2) are in conformity with the ones actually applied in the fiscal gas metering station.

With respect to uncertainty calculations using the present *Handbook* and the program *EMU - USM Fiscal Gas Metering Station*, the ”normal” instrument uncertainties (found in instrument data sheets, obtained from manufacturers, calibration laboratories, etc.) are normally to be used. Possible malfunction of an instrument (e.g. loss of an acoustic path in a USM, erronous density, pressure or temperature measurement, etc.), and specific procedures in connection with that, may represent a challenge in

105 These colours refer to the Excel program. Unfortunately, in Figs. 5.1 - 5.33 the colours have not always been preserved correctly when pasting from the Excel program into the present document (“cut and paste special” with “picture” functionality).

106 For instance, by using Microsoft Word 2000, a “cut and paste special” with “picture” functionality may be sufficient for most worksheets. However, for some of the worksheets the full worksheet is (for some reason) not being pasted using the “paste special” with “picture” feature. Only parts of the worksheet is copied. In this case use of the “paste special” with “bitmap” feature may solve the problem.

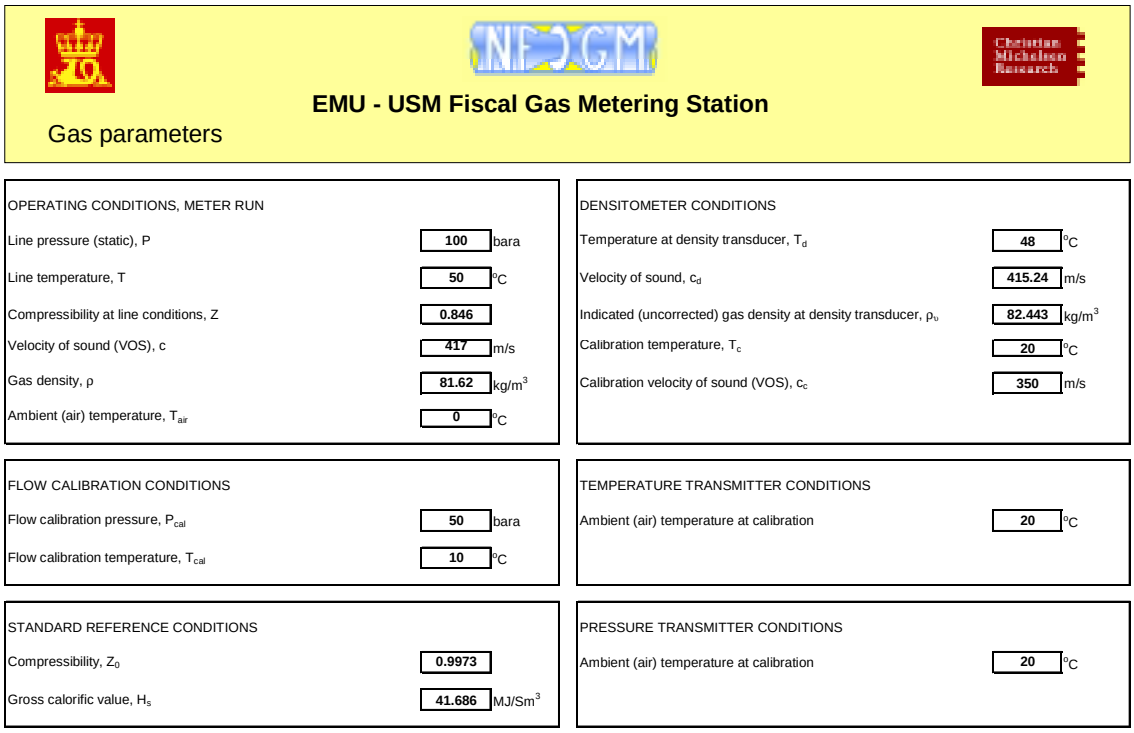
However, if the Word (doc) file is to be converted to a pdf-file, use of the “bitmap” feature results in poor-quality pictures. In this case it is recommended to first convert the Excel worksheet in question into an 8-bit gif-file (e.g. using Corel Photo Paint 7), and then import the gif-file as a picture into the Word document. The resulting quality is not excellent, but still useful. (The latter procedure has been used here, for a number of the figures in Chapter 5.)

this respect. However, if e.g. the instrument manufacturer is able to specify an (increased) uncertainty figure for a malfunctioned instrument, the present *Handbook* and the program may be used to calculate the uncertainty of the metering station also in case of instrument malfunction.

In a practical work situation in the evaluation of a metering station, a convenient way to use the program may be the following. After the desired input parameters and uncertainties have been entered, the Excel file document may be saved e.g. using a modified file name, e.g. "*EMU - USM Fiscal Gas Metering Station - MetStat1.xls*", "*EMU - USM Fiscal Gas Metering Station - MetStat2.xls*", etc. Old evaluations may then conveniently be revisited, used as basis for new evaluations, etc. The file size is about 1.4 MB.

5.2 Gas parameters

In the worksheet denoted "*Gas parameters*" shown in Fig. 5.1, the user enters data for



EMU - USM Fiscal Gas Metering Station

Gas parameters

OPERATING CONDITIONS, METER RUN		
Line pressure (static), P	100	bara
Line temperature, T	50	°C
Compressibility at line conditions, Z	0.846	
Velocity of sound (VOS), c	417	m/s
Gas density, ρ	81.62	kg/m³
Ambient (air) temperature, T_{air}	0	°C

DENSITOMETER CONDITIONS		
Temperature at density transducer, T_d	48	°C
Velocity of sound, c_d	415.24	m/s
Indicated (uncorrected) gas density at density transducer, ρ_u	82.443	kg/m³
Calibration temperature, T_c	20	°C
Calibration velocity of sound (VOS), c_c	350	m/s

FLOW CALIBRATION CONDITIONS		
Flow calibration pressure, P_{cal}	50	bara
Flow calibration temperature, T_{cal}	10	°C

TEMPERATURE TRANSMITTER CONDITIONS		
Ambient (air) temperature at calibration	20	°C

STANDARD REFERENCE CONDITIONS		
Compressibility, Z_0	0.9973	
Gross calorific value, H_s	41.686	MJ/Sm³

PRESSURE TRANSMITTER CONDITIONS		
Ambient (air) temperature at calibration	20	°C

Fig. 5.1. The "*Gas parameters*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.2.)

- The operating gas conditions of the fiscal gas metering station (in the meter run),
- The gas conditions in the densitometer,

- The gas conditions at flow calibration of the USM,
- The ambient temperature at calibration of the pressure and temperature transmitters,
- Some gas conditions at standard reference conditions.

The program uses these data in the calculation of the individual uncertainties of the primary measurements, and in calculation of the combined gas metering station uncertainty. The data used in the input worksheet shown in Fig. 5.1 are the same data as specified in Table 4.2 for the calculation example given in Chapter 4.

5.3 USM setup parameters

With respect to USM technology, the program *EMU - USM Fiscal Gas Metering Station* can be run in two modes:

(A) Completely meter independent, and
(B) Weakly meter dependent.

Mode (A) corresponds to choosing the "overall level" in the "*USM*" worksheet (both for the repeatability and the systematic deviation re. flow calibration), cf. Section 5.10. In this mode the "*USM setup*" worksheet does not need to be specified, since this information is not used in the calculations[107].

Mode (B) corresponds to using the "detailed level" in the "*USM*" worksheet (for the repeatability and/or the systematic deviation re. flow calibration), cf. Section 5.10. In this case some information on the USM is needed, since Mode (B) involves the calculation of certain sensitivity coefficients related to the USM. These depend on ϕ_{i0} y_{i0}, $N_{refl,i}$, w_i and R_0, $i = 1, \ldots, N$.

By "weakly meter independent" is here meant that the number of paths[108] (N) and the number of reflections for each path ($N_{refl,i}$) need to be known. However, actual values for the inclination angles (ϕ_{i0}), lateral chord positions (y_{i0}) and integration weights (w_i) do not need to be known. Only very approximate values for these

107 However, it is useful to give input to the "*USM setup*" worksheet in any case, since then one may conveniently switch between the "overall level" and the "detailed level" in the "*USM*" worksheet.

108 The number of acoustic paths in the USM can be set to any number in the range 1-10.

quantities are needed (for the calculation of the sensitivity coefficients), as also described in Chapter 6, cf. Table 6.3.

The worksheet for setup of the USM parameters is shown in Fig. 5.2. The input parameters are: number of paths, integration method, inclination angles, number of reflections, lateral chord positions, integration weights, and meter body material data (usually steel) (diameter, wall thickness, temperature expansion coefficient, and Young's modulus). The worksheet and the program covers both reflecting-path and non-reflecting-path USMs.

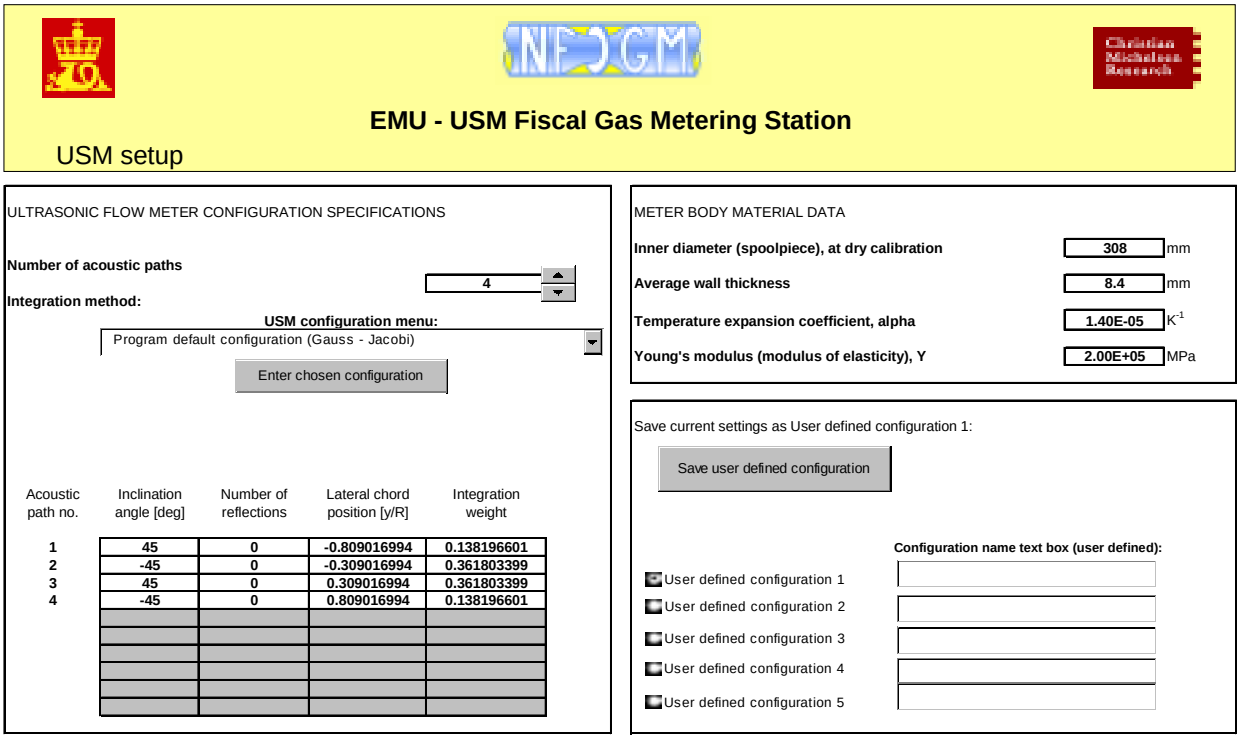


Fig. 5.2. The "*USM setup*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Data partly taken from Table 4.3.)

With respect to the integration method, this concerns the path configuration data. That is, inclination angles, number of reflections, lateral chord positions and integration weights, for each path. One may choose either "program default configuration" (which is the Gauss-Jacobi integration method), or choose among one or several "user default values" (up to 5). That means, the user can set up his own path configuration(s) and store the data for later use. This is done by - for each path - filling in the white boxes in the table to the left of the worksheet (inclination angles, number of reflections, lateral chord positions and integration weights). Then move to the right hand side of the worksheet for storing of the chosen configuration: choose among 1, 2, …, 5 (for example "User default values no. 2"), and give the desired title of the path setup (for example "USM no. 2"). At a later time one may then obtain

this stored configuration by going to the “integration method” box, choose the “User default values no. 2”, and press the “Enter chosen configuration” button.

With respect to meter body data, these are usually steel data, cf. Table 4.3.

5.4 Pressure measurement uncertainty

The worksheet “*P*” for evaluation of the expanded uncertainty of the pressure measurement in the meter run is shown in Figs. 5.3 and 5.4, for the “overall level” and the “detailed level”, respectively. These are described separately below.

5.4.1 Overall level

When the “overall level” is chosen for specification of input uncertainties to calculation of the pressure measurement uncertainty, the user enters only the relative expanded uncertainty of the pressure measurement, and the accompanying confidence level / probability distribution, see Fig. 5.3. Cf. also Table 3.1.

EMU - USM Fiscal Gas Metering Station

Pressure measurement in meter run

Select level of input: Overall input level / Detailed input level

OVERALL INPUT LEVEL

Input variable	Given Uncertainty	Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty	Sensitivity Coefficient	Variance
Pressure measurement	0.16 bar	95 % (normal)	B	0.08 bar	1	0.0064 bar²

Pressure Measurement	Sum of variances, $u_c(P)^2$	0.0064 bar²
	Combined Standard Uncertainty, $u_c(P)$	0.0800 bar
	Expanded Uncertainty (95% confidence level, k=2), k $u_c(P)$	0.1600 bar
	Operating Static Pressure, P	100 bar
	Relative Expanded Uncertainty (95% confidence level, k=2), k E_P	0.1600 %

Fig. 5.3. The “*P*” worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the “overall level” option.

This option is used e.g. when the user does not want to go into the “detailed level” of the pressure measurement, or if the “detailed level” setup does not fit sufficiently

well to the pressure transmitter at hand. The user must himself document the input value used for the relative expanded uncertainty of the pressure measurement (the "given uncertainty"), and its confidence level and probability distribution.

5.4.2 Detailed level

When the "detailed level" is chosen for specification of input uncertainties to calculation of the pressure measurement uncertainty, the user enters the uncertainty figures of the pressure transmitter in question, in addition to the accompanying confidence levels / probability distributions. Cf. Table 3.1 and Section 3.2.1.

EMU - USM Fiscal Gas Metering Station

Pressure measurement in meter run

Select level of input: Overall input level / Detailed input level

DETAILED INPUT LEVEL

Type of instrument:

Maximum calibrated static pressure	120	barg
Minimum calibrated static pressure	50	barg
Calibrated span	70	bar
Upper Range Limit (URL)	138	barg
Ambient temperature deviation	20	°C
Time between calibrations	12	months

Input variable	Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty		Sensitivity Coefficient	Variance	
Transmitter	0.05	%Span	99 % (normal)	A	0.0116667	Bar	1	0.0001361	bar²
Stability	0.1	%URL / 1 year	95 % (normal)	B	0.069	Bar	1	0.004761	bar²
RFI effects	0.1	%Span	99 % (normal)	A	0.0233333	Bar	1	0.0005444	bar²
Ambient temperature effect	(0.006	%URL							
	+ 0.03	%Span)	99 % (normal)	B	0.0069714	Bar	1	4.86E-05	bar²
Atmospheric pressure	0.09	bar	99 % (normal)	A	0.03	Bar	1	0.0009	bar²
		bar	95 % (normal)	B	0	Bar	1	0	bar²

Pressure Measurement			
	Sum of variances, $u_c(P)^2$	0.0063902	bar²
	Combined Standard Uncertainty, $u_c(P)$	0.0799	bar
	Expanded Uncertainty (95% confidence level, k=2), k $u_c(P)$	0.1599	bar
	Operating Static Pressure, P	100	bar
	Relative Expanded Uncertainty (95% confidence level, k=2), k E_P	0.1599	%

Fig. 5.4. The "*P*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the "detailed level" option. (Corresponds to Table 4.6 and Fig. 5.21.)

In Fig. 5.4 the uncertainty data specified for the Rosemount 3051P Reference Class Smart Pressure Transmitter [Rosemount, 2000] have been used, cf. Table 4.1. These are the same as used in Table 4.6, see Section 4.2.1 for details. A blank field denoted "type of instrument" can be filled in to document the actual instrument being evaluated, for reporting purposes.

In addition to the input uncertainty values, the user must specify a few other data, found in instrument data sheets. By selecting the "maximum" and "minimum calibrated static pressure", the program automatically calculates the "calibrated span". The "URL" is entered by the user. The "ambient temperature deviation" is calculated by the program from data given in the "Gas parameters" worksheet. Also the "time between calibrations" has to be specified.

In addition to the "usual" pressure transmitter input uncertainties given in the worksheet, a "blank cell" has been defined, where the user can specify miscellaneous uncertainty contributions to the pressure measurement not covered by the other input cells in the worksheet.

The user must himself document the input uncertainty values used for the pressure measurement (the "given uncertainty"), e.g. on basis of a manufacturer data sheet, a calibration certificate, or other manufacturer information.

5.5 Temperature measurement uncertainty

The worksheet "*T*" for evaluation of the expanded uncertainty of the temperature measurement in the meter run is shown in Figs. 5.5 and 5.6, for the "overall level" and the "detailed level", respectively. These are described separately below.

5.5.1 Overall level

When the "overall level" is chosen for specification of input uncertainties to calculation of the temperature measurement uncertainty, the user specifies only the relative expanded uncertainty of the temperature measurement, and the accompanying confidence level / probability distribution, see Fig. 5.5. Cf. Table 3.2 and Section 3.2.2.

This option is used e.g. when the user does not want to go into the "detailed" level of the temperature measurement, or if the "detailed level" setup does not fit sufficiently well to the temperature element and transmitter at hand. The user must himself

document the input value used for the relative expanded uncertainty of the temperature measurement (the "given uncertainty"), and its confidence level and probability distribution.

EMU - USM Fiscal Gas Metering Station

Temperature measurement in meter run

Select level of input: Overall input level / Detailed input level

OVERALL INPUT LEVEL

Input variable	Given Uncertainty	Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty	Sensitivity Coefficient	Variance
Temperature measurement	0.15 °C	95 % (normal)	B	0.075 °C	1	0.005625 (°C)²

Temperature Measurement		
	Sum of variances, $u_c(T)^2$	0.005625 (°C)²
	Combined Standard Uncertainty, $u_c(T)$	0.0750 °C
	Expanded Uncertainty (95% confidence level, k=2), k $u_c(T)$	0.1500 °C
	Operating temperature, T	50 °C
	Relative Expanded Uncertainty (95% confidence level, k=2), k E_T	0.0464 %

Fig. 5.5. The "*T*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the "overall level" option.

5.5.2 Detailed level

When the "detailed level" is chosen for specification of input uncertainties to calculation of the temperature measurement uncertainty, the user specifies the uncertainty data of the temperature element and transmitter in question, together with the accompanying confidence levels / probability distributions. Cf. Table 3.2. The user must himself document the input uncertainty values for the temperature measurement, e.g. on basis of a manufacturer data sheet, a calibration certificate, or other manufacturer information.

In Fig. 5.6 the uncertainty figures given for the Rosemount 3144 Smart Temperature Transmitter [Rosemount, 2000] used in combination with a Pt 100 temperature element, have been specified, cf. Table 4.1. These are the same as used in Table 4.8, see Section 4.2.2 for details. A blank field denoted "type of instrument" can be filled in to document the instrument evaluated, for reporting purposes.

In addition to the input uncertainty data, the user must specify the "time between calibrations". The "ambient temperature deviation" is calculated by the program from data given in the "Gas parameters" worksheet.

In addition to the "usual" temperature transmitter input uncertainties given in the worksheet, a "blank cell" has been defined, where the user can specify miscellaneous uncertainty contributions to the temperature measurement not covered by the other input cells in the worksheet. The user must himself document the input value used for the "miscellaneous uncertainty" of the temperature measurement, and its confidence level and probability distribution.

EMU - USM Fiscal Gas Metering Station

Temperature measurement in meter run

Select level of input: Overall input level / Detailed input level

DETAILED INPUT LEVEL

Type of instrument:

Ambient temperature deviation 20 °C

Time between calibrations 12 months

Input variable		Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty		Sensitivity Coefficient	Variance	
Temperature element and transmitter		0.1	°C	99 % (normal)	A	0.0333333	°C	1	0.0011111	(°C)²
Stability	Max (	0.1	°C							
		0.1	%MV/24months)	99 % (normal)	B	0.0538583	°C	1	0.0029007	(°C)²
RFI effects		0.1	°C	99 % (normal)	A	0.0333333	°C	1	0.0011111	(°C)²
Ambient temperature effect		0.0015	°C/°C	99 % (normal)	B	0.01	°C	1	0.0001	(°C)²
Stability - temperature element		0.05	°C	95 % (normal)	B	0.025	°C	1	0.000625	(°C)²
			°C	95 % (normal)	B	0	°C	1	0	(°C)²

Temperature Measurement			
	Sum of variances, $u_c(T)^2$	0.0058479	(°C)²
	Combined Standard Uncertainty, $u_c(T)$	0.0765	°C
	Expanded Uncertainty (95% confidence level, k=2), k $u_c(T)$	0.1529	°C
	Operating temperature, T	50	°C
	Relative Expanded Uncertainty (95% confidence level, k=2), k E_T	0.0473	%

Fig. 5.6. The "*T*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the "detailed level" option. (Corresponds to Table 4.8 and Fig. 5.22.)

5.6 Compressibility factor uncertainty

The worksheet "*Z*" for evaluation of the expanded uncertainty of the compressibility factor ratio Z/Z_0 is shown in Fig. 5.7. For each of Z and Z_0, two types of input un-

certainties are to be specified; the model uncertainty and the analysis uncertainty. Cf. Table 3.3 and Sections 3.2.3 and 4.2.3 for details.

For the model uncertainty, there is implemented an option of filling in the AGA-8 (92) model uncertainties for Z and Z_0 [AGA-8, 1994] or the ISO 6976 model uncertainty for Z_0 [ISO, 1995c], on basis of pressure and temperature information given in the "Gas parameters" worksheet. Cf. Section 3.2.3. These are the model uncertainties used in Fig. 5.7. If another equation of state is used for one or both of Z and Z_0, the input model uncertainty figures have to be filled in manually. The user must himself document the uncertainty values used as input to the worksheet, together with its confidence level and probability distribution.

EMU - USM Fiscal Gas Metering Station

Gas compressibility uncertainty

DETAILED INPUT LEVEL

Input variables	Given Relative Uncertainty	Confidence Level (probability distr.)	Type of Uncertainty	Relative Standard Uncertainty	Relative Sensitivity Coefficient	Relative Variance
MODEL UNCERTAINTY[1)]						
Z at line conditions: Fill in AGA-8 uncertainty (option)						
Z at std. conditions: Fill in AGA-8 uncertainty (option)		Z at std. conditions: Fill in ISO 6976 uncertainty (option)				
Gas compressibility factor at line conditions (Z)	0.1 %	95 % (normal)	A	0.05000 %	1	0.00000025
Gas compressibility factor at standard conditions (Z_0)	0.052 %	95 % (normal)	A	0.02600 %	1	6.76E-08
ANALYSIS UNCERTAINTY						
Line conditions	0.16 %	Standard	A	0.16000 %	1	0.00000256
Standard conditions	0 %	Standard	A	0.00000 %		

Gas compressibility calculation		
	Sum of relative variances, $(E_{Z0/Z})^2$	2.8776E-06
	Relative Combined Standard Uncertainty, $E_{Z0/Z}$	0.001696
	Relative Expanded Uncertainty (95% confidence level, k=2), k $E_{Z0/Z}$	0.3393 %

1) Re. model uncertainty: AGA-8 uncertainty used for the model uncertainty of Z (line conditions)
ISO 6976 uncertainty used for the model uncertainty of Z0 (standard reference conditions)

Fig. 5.7. The "*Z*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.9.)

5.7 Density measurement uncertainty

The worksheet "*Density*" for evaluation of the expanded uncertainty of the density measurement in the meter run is shown in Figs. 5.8 and 5.9, for the "overall level" and the "detailed level", respectively. These are described separately below.

5.7.1 Overall level

When the "overall level" is chosen for specification of input uncertainties to calculation of the density measurement uncertainty, the user specifies only the relative expanded uncertainty of the density measurement, and the accompanying confidence level and probability distribution, see Fig. 5.8. Cf. Table 3.4 and Section 3.2.4.

This option is used e.g. when the user does not want to go into the "detailed" level of the density measurement, in case a different method for density measurement is used (e.g. calculation from GC analysis), in case of a different installation of the densitometer (e.g. in-line), or if the "detailed level" setup does not fit sufficiently well to the densitometer at hand. The user must himself document the input value used for the relative expanded uncertainty of the density measurement (the "given uncertainty"), and its confidence level and probability distribution.

EMU - USM Fiscal Gas Metering Station

Density measurement

Select level of input: Overall input level / Detailed input level

OVERALL INPUT LEVEL

Input variable	Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty		Sensitivity Coefficient	Variance	
Density measurement	0.16	kg/m³	95 % (normal)	B	0.08	kg/m³	1	0.0064	(kg/m³)²

Density Measurement			
	Sum of variances, $u_c(\rho)^2$	0.0064	(kg/m³)²
	Combined Standard Uncertainty, $u_c(\rho)$	0.080000	kg/m³
	Expanded Uncertainty (95% confidence level, k=2), k $u_c(\rho)$	0.16	kg/m³
	Operating Density, ρ	81.62	kg/m³
	Relative Expanded Uncertainty (95% confidence level, k=2), k Eρ	0.1960	%

Fig. 5.8. The "*Density*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the "overall level" option.

5.7.2 Detailed level

When the "detailed level" is chosen for specification of input uncertainties to calculation of the density measurement uncertainty, the user specifies the uncertainty figures of the online installed vibrating element densitometer in question, in addition to the accompanying confidence levels / probability distributions. Cf. Table 3.4. The user must himself document the input uncertainty values for the density measure-

ment, e.g. on basis of a manufacturer data sheet, a calibration certificate, or other manufacturer information.

EMU - USM Fiscal Gas Metering Station

Density measurement

Select level of input: Overall input level / Detailed input level

DETAILED INPUT LEVEL

Type of instrument:

K18	-1.36E-05	
K19	8.44E-04	
K_d	21000	µm
c_c	350	m/s
c_d	415.24	m/s
τ	650	µs

Input variable	Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty		Sensitivity Coefficient		Variance	
Densitometer accuracy	0.15	%	95 % (normal)	B	0.0618323	kg/m³	0.9897335		0.0037451	(kg/m³)²
Repeatability	0.04	kg/m³	95 % (normal)	A	0.02	kg/m³	1		0.0004	(kg/m³)²
Calibration temperature, T_{cal}	0.1	°C	95 % (normal)	A	0.05	°C	-0.000274	(kg/m³)/°C	1.884E-10	(kg/m³)²
Line temperature, T	0.1529	°C	95 % (normal)		0.0764718	°C	0.2525762	(kg/m³)/°C	0.0003731	(kg/m³)²
Densitometer temperature, T_d	0.1529	°C	95 % (normal)		0.0764718	°C	0.2538747	(kg/m³)/°C	0.0003769	(kg/m³)²
Line pressure, P	0.1599	bar	95 % (normal)		0.0799385	bar	0.0001632	(kg/m³)/bar	1.702E-10	(kg/m³)²
Press. difference, densitometer to line, ΔP_d	0.02	bar	100 % (rectangular)	B	0.011547	bar	-0.816037	(kg/m³)/bar	8.879E-05	(kg/m³)²
VOS, calibration gas, c_c	1	m/s	100 % (rectangular)	B	0.5773503	m/s	-0.00394	(kg/m³)/(m/s)	5.176E-06	(kg/m³)²
VOS, densitometer gas, c_d	1	m/s	100 % (rectangular)	B	0.5773503	m/s	0.0023655	(kg/m³)/(m/s)	1.865E-06	(kg/m³)²
Periodic time, τ	0.1	µs	100 % (rectangular)	A	0.057735	µs	-0.000611	(kg/m³)/µs	1.243E-09	(kg/m³)²
VOS correction constant, K_d	2100	µm	100 % (rectangular)	B	1212.4356	µm	1.89E-05	(kg/m³)/µm	0.0005252	(kg/m³)²
Temperature correction model	0.048	kg/m³	95 % (normal)	B	0.024	kg/m³	1		0.000576	(kg/m³)²
	0	kg/m³	95 % (normal)	B	0	kg/m³	1		0	(kg/m³)²

Density Measurement			
	Sum of variances, $u_c(\rho)^2$	0.0060921	(kg/m³)²
	Combined Standard Uncertainty, $u_c(\rho)$	0.078052	kg/m³
	Expanded Uncertainty (95% confidence level, k=2), k $u_c(\rho)$	0.1561039	kg/m³
	Operating Density, ρ	81.62	kg/m³
	Relative Expanded Uncertainty (95% confidence level, k=2), k $E\rho$	0.1913	%

Fig. 5.9. The "*Density*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the "detailed level" option. (Corresponds to Tables 4.4 and 4.11, and Fig. 5.24.)

In Fig. 5.9 the uncertainty figures specified for the Solartron Model 7812 Gas Density Transducer [Solartron, 1999] have been used, cf. Table 4.1. These are the same as used in Table 4.11, see Section 4.2.4 for details. A blank field denoted "type of in-

strument" can be filled in to document the actual instrument being evaluated, for reporting purposes.

The input uncertainty of the line temperature (T), the densitometer temperature (T_d), and the line pressure (P), are taken from the "*T*" and "*P*" worksheets. The uncertainty due to pressure difference between line and densitometer conditions (ΔP_d), is calculated by the program, cf. Section 4.2.4.

In addition to the input uncertainty values, the user must specify four gas densitometer constants, K_{18}, K_{19}, K_d and τ, defined in Section 2.4. Cf. Table 4.4 and Section 4.2.4 for details. The "calibration VOS", c_c, and the "densitometer VOS", c_d, are taken directly from the "Gas parameters" worksheet.

In addition to the "usual" densitometer input uncertainties given in the worksheet, a "blank cell" has been defined, where the user can specify miscellaneous uncertainty contributions to the density measurement not covered by the other input cells in the worksheet. The user must himself document the input value used for the "miscellaneous uncertainty" of the densitometer measurement, together with its confidence level and probability distribution.

5.8 Calorific value uncertainty

The worksheet "*Hs*" for evaluation of the expanded uncertainty of the superior (gross) calorific value estimate is shown in Fig. 5.10.

EMU - USM Fiscal Gas Metering Station

Calorific value at standard reference conditions

OVERALL INPUT LEVEL

Input variable	Given Relative Uncertainty	Confidence Level (probability distr.)	Type of uncertainty	Relative Standard Uncertainty	Relative Sensitivity Coefficient	Relative Variance
Gross Calorific Value, H_s	0.15 %	95 % (normal)	B	0.075 %	1	5.625E-07

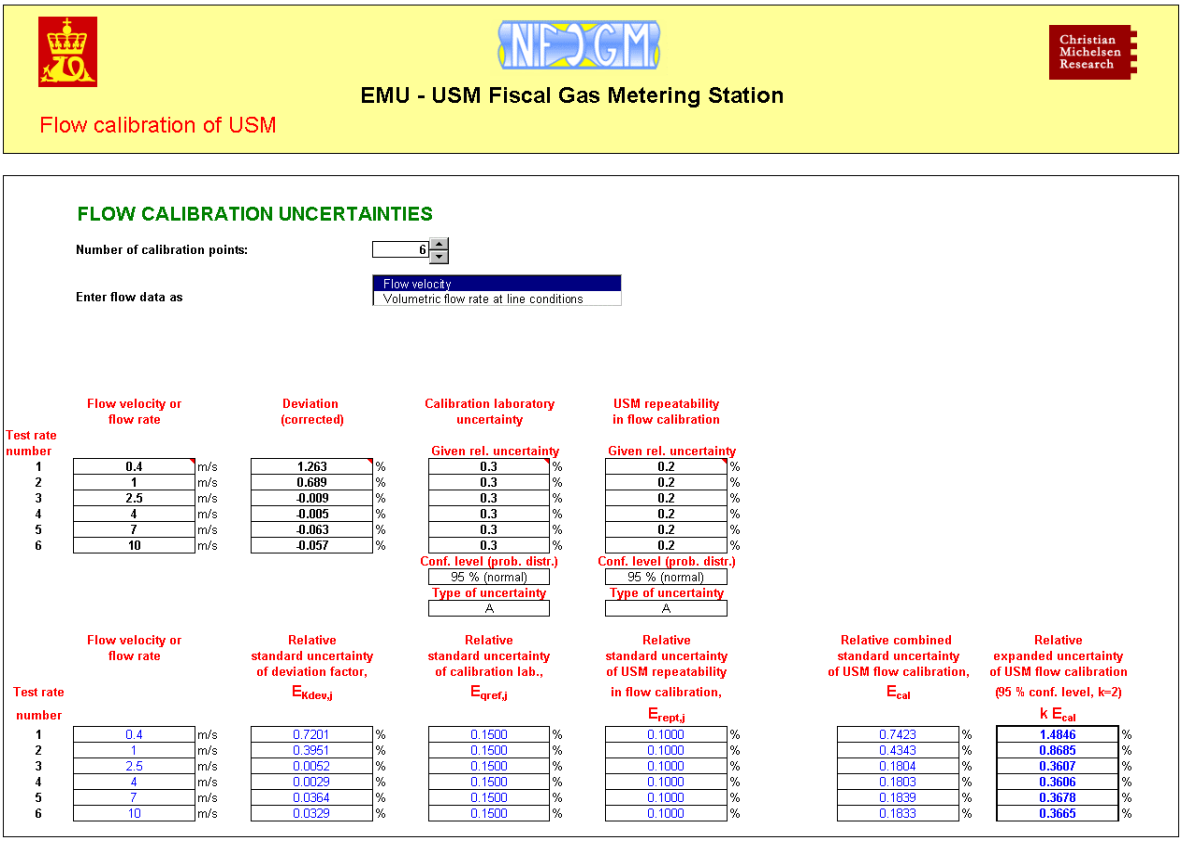
Calorific Value Measurement	Sum of relative variances, E_{Hs}^2	5.625E-07
	Relative Combined Standard Uncertainty, E_{Hs}	0.000750
	Relative Expanded Uncertainty (95% confidence level, k=2), k E_{Hs}	0.1500 %

Fig. 5.10. The "*Hs*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*.

Only the "overall level" is available. That means, the user specifies the relative expanded uncertainty of the superior (gross) calorific value estimate, and the accompanying confidence level and probability distribution, see Fig. 5.10. Cf. Sections 3.2.5 and 4.2.5 for some more details. The user must himself document the input value used for the relative expanded uncertainty of the superior (gross) calorific value estimate (the "given uncertainty"), and its confidence level and probability distribution.

5.9 Flow calibration uncertainty

The worksheet "*Flow cal.*" for evaluation of the expanded uncertainty of the USM flow calibration is shown in Fig. 5.11. First, the number (*M*) of flow calibration points (calibration flow rates) is chosen, in the range 4 - 10. Flow data can then be entered either as (a) flow velocity or (b) volumetric flow rate at line conditions.



EMU - USM Fiscal Gas Metering Station

Flow calibration of USM

FLOW CALIBRATION UNCERTAINTIES

Number of calibration points: 6

Enter flow data as: Flow velocity / Volumetric flow rate at line conditions

Test rate number	Flow velocity or flow rate		Deviation (corrected)		Calibration laboratory uncertainty (Given rel. uncertainty)		USM repeatability in flow calibration (Given rel. uncertainty)	
1	0.4	m/s	1.263	%	0.3	%	0.2	%
2	1	m/s	0.689	%	0.3	%	0.2	%
3	2.5	m/s	-0.009	%	0.3	%	0.2	%
4	4	m/s	-0.005	%	0.3	%	0.2	%
5	7	m/s	-0.063	%	0.3	%	0.2	%
6	10	m/s	-0.057	%	0.3	%	0.2	%
					Conf. level (prob. distr.): 95 % (normal)		Conf. level (prob. distr.): 95 % (normal)	
					Type of uncertainty: A		Type of uncertainty: A	

Test rate number	Flow velocity or flow rate		Relative standard uncertainty of deviation factor, $E_{Kdev,i}$		Relative standard uncertainty of calibration lab., $E_{qref,i}$		Relative standard uncertainty of USM repeatability in flow calibration, $E_{rept,i}$		Relative combined standard uncertainty of USM flow calibration, E_{cal}		Relative expanded uncertainty of USM flow calibration (95 % conf. level, k=2) $k\,E_{cal}$	
1	0.4	m/s	0.7201	%	0.1500	%	0.1000	%	0.7423	%	1.4846	%
2	1	m/s	0.3951	%	0.1500	%	0.1000	%	0.4343	%	0.8685	%
3	2.5	m/s	0.0052	%	0.1500	%	0.1000	%	0.1804	%	0.3607	%
4	4	m/s	0.0029	%	0.1500	%	0.1000	%	0.1803	%	0.3606	%
5	7	m/s	0.0364	%	0.1500	%	0.1000	%	0.1839	%	0.3678	%
6	10	m/s	0.0329	%	0.1500	%	0.1000	%	0.1833	%	0.3665	%

Fig. 5.11. The "*Flow cal.*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.13, for the 1 m/s flow velocity.)

In the first column from the left, the user is to specify the *M* flow velocities (or flow rates) for which flow calibration has been made. In Fig. 5.11 the example discussed in Section 4.3 has been used, with *M* = 6 flow velocities specified, and flow velocities corresponding to the flow rates given in Table 4.12.

In the second column from the left, the user is to specify the "Deviation (corrected)" at the *M* calibration flow rates. That is, the corrected relative deviation $Dev_{C,j}$, j = 1, …, *M*, defined by Eq. (2.10). Note that this is the deviation *after* flow calibration correction using the correction factor *K*, as described in Section 2.2. Cf. Sections 3.3.2, 4.3.2 and Table 4.12 for details.

In the third column from the left, the expanded uncertainty of the flow calibration laboratory is to be specified at the *M* calibration flow rates, together with the accompanying confidence level / probability distribution. This uncertainty contribution may be specified to be flow rate dependent, but in Table 4.12 and in Fig. 5.11 it has been taken to be constant over the flow range (which may be a common approach, although simplified). Cf. Sections 3.3.1 and 4.3.1 for details.

Finally, in the fourth column from the left, the repeatability of the USM in flow calibration is to be specified at the *M* calibration flow rates, together with the accompanying confidence level / probability distribution. This uncertainty contribution may also be specified to be flow rate dependent, but in Fig. 5.11 it has been taken to be constant over the flow range (which may be a common approach, although simplified). Cf. Sections 3.3.3 and 4.3.3 for details.

On basis of these input data, the expanded uncertainty of the USM flow calibration is calculated as shown in Fig. 5.11, in a similar approach as shown in Tables 4.13 and 4.14 (for two of the six flow rates).

5.10 USM field uncertainty

The worksheet "*USM*" for evaluation of the expanded uncertainty of the USM in field operation is basically divided in two main parts:

- The "USM field repeatability", and
- The uncertainty of uncorrected "USM systematic deviations re. flow calibration".

Both of these can be specified at an "overall level" and a "detailed level", cf. Figs. 5.12 -5.18 below. The two parts of the worksheet are described separately below.

5.10.1 USM field repeatability

The "USM field repeatability" can be specified at an "overall level" and a "detailed level", corresponding to specifying (1) the repeatability of the indicated USM flow rate measurement, and (2) the repeatability of the measured transit times, respectively. Both can be given to be flow rate dependent. The two options are described separately below.

5.10.1.1 Overall level

When the "overall level" is chosen for specification of the "USM field repeatability", this corresponds to specification of the *repeatability of the measured flow rate* for the USM in field operation.

The user specifies the relative expanded uncertainty of the flow rate repeatability, for the USM in field operation, at the *M* flow rates chosen in the worksheet "*Flow cal.*", together with the accompanying confidence level / probability distribution. Fig. 5.12 shows this option, for the same example as shown in Table 4.15. Cf. also Table 3.8 and Sections 3.4.2, 4.4.1.

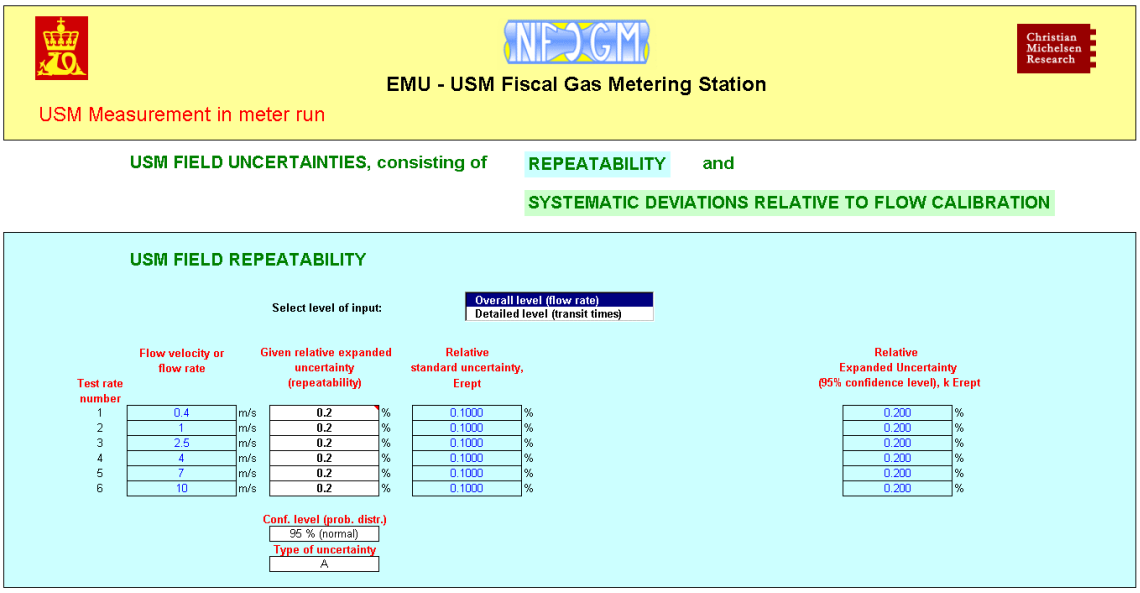


Fig. 5.12. Part of the "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, related to the USM repeatability in field operation (shown for the "overall level" option; specification of flow rate repeatability). (Corresponds to Table 4.15.)

The flow rate repeatability can be specified to be flow rate dependent, although in Fig. 5.12 it is taken to be constant over the flow rate (which in practice may be a common approach, although simplified). For a given flow rate, the standard uncertainty of the flow rate repeatability may simply be taken to be the standard deviation of the flow rate measurements. The user must himself document the input value(s)

used for the relative expanded uncertainty of the flow rate repeatability, together with its confidence level and probability distribution, on basis of the USM manufacturer data sheet or other manufacturer information.

5.10.1.2 Detailed level

When the "detailed level" is chosen for specification of the "USM field repeatability", this corresponds to specification of the *repeatability of the mesured transit times* for the USM in field operation.

The user specifies the expanded uncertainty of random transit time variations (repeatability), for the USM in field operation, at the *M* flow rates chosen in the worksheet "*Flow cal.*", together with the accompanying confidence level / probability distribution. Fig. 5.13 shows this option, for the same example as shown in Table 4.16. Cf. also Table 3.8 and Sections 3.4.2, 4.4.1.

The transit time repeatability can be specified to be flow rate dependent, although in Fig. 5.13 it is taken to be constant over the flow rate (which is a simplified approach, cf. Section 3.4.2). At a given flow rate, the standard uncertainty of the random transit time variations may simply be taken to be the standard deviation of the transit time measurements. The user must himself document the input value(s) used for the uncertainty of the transit time repeatability, together with its confidence level and probability distribution, e.g. on basis of USM manufacturer information.

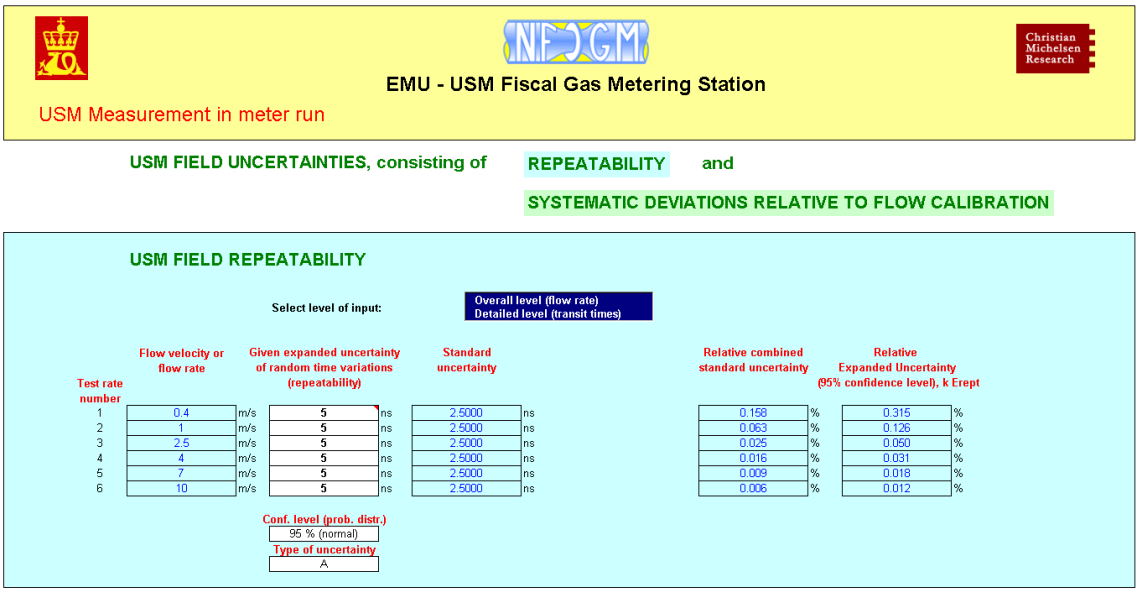


Fig. 5.13. Part of the "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, related to the USM repeatability in field operation (shown for the "detailed level" option; specification of transit time repeatability). (Corresponds to Table 4.16.)

5.10.2 USM systematic deviations re. flow calibration

The uncertainty of uncorrected "USM systematic deviations re. flow calibration" can be specified at an "overall level" and a "detailed level". The two options are described separately below.

5.10.2.1 Overall level

For specification of the "USM systematic deviations re. flow calibration" at the "overall level", the user specifies the relative expanded uncertainty of all uncorrected systematic deviations of the USM relative to flow calibration, see Table 3.8. Fig. 5.14 shows this option, for an example corresponding to the example given in Table 4.20. Cf. also Section 3.4.

EMU - USM Fiscal Gas Metering Station

USM Measurement in meter run

USM FIELD UNCERTAINTIES, consisting of REPEATABILITY and SYSTEMATIC DEVIATIONS RELATIVE TO FLOW CALIBRATION

USM FIELD REPEATABILITY

Select level of input: Overall level (flow rate) / Detailed level (transit times)

Test rate number	Flow velocity or flow rate		Given relative expanded uncertainty (repeatability)		Relative standard uncertainty, Erept		Relative Expanded Uncertainty (95% confidence level), k Erept	
1	0.4	m/s	0.2	%	0.1000	%	0.200	%
2	1	m/s	0.2	%	0.1000	%	0.200	%
3	2.5	m/s	0.2	%	0.1000	%	0.200	%
4	4	m/s	0.2	%	0.1000	%	0.200	%
5	7	m/s	0.2	%	0.1000	%	0.200	%
6	10	m/s	0.2	%	0.1000	%	0.200	%

Conf. level (prob. distr.): 95 % (normal)

Type of uncertainty: A

USM SYSTEMATIC DEVIATIONS RE FLOW CALIBRATION

Select level of input: Overall level / Detailed level

OVERALL INPUT LEVEL

Input variable	Given Relative Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Relative Standard Uncertainty
USM measurement	0.46	%	95 % (normal)	B	0.23

USM FIELD UNCERTAINTIES - SUMMARY

Test rate number	Flow velocity or flow rate		Repeatability Relative combined standard uncertainty, E_{rept}		Relative combined standard uncertainty $E_{USM,A}$		Relative combined standard uncertainty E_{USM}		Relative Expanded Uncertainty (95% confidence level) k E_{USM}	
1	0.4	m/s	0.1000	%	0.2300	%	0.2508	%	0.5016	%
2	1	m/s	0.1000	%	0.2300	%	0.2508	%	0.5016	%
3	2.5	m/s	0.1000	%	0.2300	%	0.2508	%	0.5016	%
4	4	m/s	0.1000	%	0.2300	%	0.2508	%	0.5016	%
5	7	m/s	0.1000	%	0.2300	%	0.2508	%	0.5016	%
6	10	m/s	0.1000	%	0.2300	%	0.2508	%	0.5016	%

Fig. 5.14. The "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, shown for the "overall level" options both for (1) "USM field repeatability" and (2) "USM systematic deviations re. flow calibration".

This option is implemented as a simplified approach, to be used in case the user does not want to go into the "detailed level" of uncertainty input with respect to "USM systematic deviations re. flow calibration". The user must himself document the input value used for the relative expanded uncertainty of the uncorrected systematic deviations of the USM relative to flow calibration, together with the accompanying confidence level and probability distribution, e.g. on basis of a USM manufacturer data sheet, or other information.

5.10.2.2 Detailed level

For specification of the "USM systematic deviations re. flow calibration" at the "detailed level", the user specifies three types of input uncertainties, together with their accompanying confidence level and probability distributions:

- Uncertainty related to systematic USM meter body effects,
- Uncertainty related to uncorrected systematic transit time effects, and
- Uncertainty related to integration method (installation conditions),

cf. Table 3.8 and Sections 3.4, 4.4. These three input types are discussed separately below.

Meter body

With respect to the "USM meter body uncertainty" part of the worksheet, the user specifies whether correction for pressure and temperature effects *is* used by the USM manufacturer or not, and the relative expanded uncertainties of the pressure and temperature expansion coefficients, cf. Table 3.8.

Fig. 5.15 shows this this part of the "*USM*" worksheet, for the same example as given in Table 4.17 (i.e. no *P* and *T* correction used). Cf. Sections 3.4.1 and 4.4.2.

The user must himself document the input uncertainty values used for the pressure and temperature expansion coefficients, together with the associated confidence levels and probability distributions, e.g. on basis of possible USM manufacturer information (cf. Chapter 6), or other information.

USM SYSTEMATIC DEVIATIONS RE FLOW CALIBRATION

Select level of input: Overall level / Detailed level

DETAILED INPUT LEVEL

USM METER BODY UNCERTAINTY

PRESSURE AND TEMPERATURE CORRECTION

Is temperature and pressure correction of the USM used?	No	
Coefficient of linear thermal expansion, α	1.40E-05	1/K
Coefficient of pressure expansion, β	9.17E-11	1/Pa
Temperature correction, K_T	1.00056	
Pressure correction, K_P	1.000458333	

Input variable	Given Relative Uncertainty	Confidence Level (probability distr.)	Type of uncertainty	Relative Standard Uncertainty
Uncertainty in α	20 %	100 % (rectangular)	B	11.55 %
Uncertainty in β	20 %	100 % (rectangular)	B	11.55 %
Relative combined standard uncertainty in temperature correction, E_{KT}				0.0330 %
Relative combined standard uncertainty in pressure correction, E_{KP}				0.0270 %

CONTRIBUTIONS TO METER BODY UNCERTAINTY	Relative combined standard uncertainty	Relative Sensitivity coefficient	Relative combined standard uncertainty
Spoolpiece radius, $E_{rad,\Delta}$	0.0426 %	3.6000	0.1533 %
Lateral chord positions, $E_{chord,\Delta}$	0.0426 %	-0.6000	-0.0256 %
Inclination angles, $E_{angle,\Delta}$			0.0000 %

Meter body uncertainty		
	Relative Combined Standard Uncertainty, $E_{body,\Delta}$	0.1278 %
	Relative Expanded Uncertainty (95% confidence level, k = 2), $k \cdot E_{body,\Delta}$	0.2555 %

Fig. 5.15. Part of the "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, related to the "USM systematic deviations re. flow calibration" (for the "detailed level" option, subsection "USM meter body uncertainty"), for the case of no temperature and pressure correction. (Corresponds to Table 4.17.)

Systematic transit time effects

With respect to the "USM transit time uncertainties (systematic effects)" part of the worksheet, the user specifies the input expanded uncertainty of uncorrected systematic transit time effects on the measured upstream and downstream transit times (deviation from flow calibration to field operation), together with the accompanying confidence levels / probability distributions. Examples of such effects are given in Tables 1.4 and 3.8. The actual uncertainty figure is preferably to be specified by the USM manufacturer, cf. Chapter 6.

Fig. 5.16 shows this part of the "USM" worksheet, for the same example as used in Table 4.19. Cf. Sections 3.4.2 and 4.4.3. The user must himself document the input uncertainty values used for "USM transit time uncertainties (systematic effects)", e.g. on basis of USM manufacturer information (cf. Chapter 6).

USM TRANSIT TIME UNCERTAINTIES (systematic effects)

Input variable	Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Standard Uncertainty	
Upstream time measurement	600	ns	100 % (rectangular)	B	346.4101615	ns
Downstream time measurement	590	ns	100 % (rectangular)	B	340.6366588	ns

Transit Time uncertainties	Test rate number	Flow velocity or flow rate		Relative Combined Standard Uncertainty, $E_{time,A}$		Relative Expanded Uncertainty (95 % conf. level), k $E_{time,A}$	
	1	0.4	m/s	0.4206	%	0.8412	%
	2	1	m/s	0.1197	%	0.2393	%
	3	2.5	m/s	0.0007	%	0.0014	%
	4	4	m/s	0.0308	%	0.0616	%
	5	7	m/s	0.0523	%	0.1046	%
	6	10	m/s	0.0609	%	0.1218	%

Fig. 5.16. Part of the "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, related to the "USM systematic deviations re. flow calibration" (for the "detailed level" option, subsection "USM transit time uncertainties (systematic effects)"). (Corresponds to Table 4.19.)

Integration method (installation effects)

With respect to the "USM integration uncertainty (installation effects)" part of the worksheet, the user specifies the relative expanded uncertainty of uncorrected installation effects (due to possible deviation in conditions from flow calibration to field operation), together with the accompanying confidence level and probability distribution. Examples of such effects are given in Tables 1.4 and 3.8.

The actual uncertainty figure is preferably to be specified by the USM manufacturer, on basis of extensive testing/simulations/experience with different installation conditions for the meter in question (e.g. for a specific type of installation, or more general).

Fig. 5.17 shows this part of the "USM" worksheet, for the same example as given in Section 4.4.4. Cf. also Section 3.4.3. The user must himself document the input uncertainty values used for the "USM integration uncertainty (installation effects)", e.g. on basis of information provided by the USM manufacturer.

USM INTEGRATION UNCERTAINTY (installation effects)

Input variable	Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Relative standard uncertainty, $E_{I,A}$		Relative Expanded Uncertainty (95 % conf. level), k $E_{I,A}$	
Integration uncertainty	0.3	%	95 % (normal)	B	0.15	%	0.3	%

MISCELLANEOUS EFFECTS (other uncertainty contributions)

Input variable	Given Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Relative standard uncertainty, E_{misc}		Relative Expanded Uncertainty (95 % conf. level), k E_{misc}	
	0	%	95 % (normal)	B	0	%	0	%

Fig. 5.17. Part of the "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, related to the "USM systematic deviations re. flow calibration" (for the "detailed level" option, subsection "USM integration uncertainty (installation effects)" and "Miscellaneous effects").

Summary - Expanded uncertainty of USM in field operation

Fig. 5.18 shows the part of the "USM" worksheet which summarizes the uncertainty calculations for the USM field uncertainties. This display is common to the "overall level" and "detailed level" options (cf. Fig. 5.14). The values used here correspond to the example given in Tables 4.20 and 4.21 (for two of the six flow rates), i.e. Figs. 5.12 and 5.15-5.17.

USM FIELD UNCERTAINTIES - SUMMARY

Test rate number	Flow velocity or flow rate		Repeatablility Relative combined standard uncertainty, E_{rept}		Relative combined standard uncertainty $E_{USM,A}$		Relative combined standard uncertainty E_{USM}		Relative Expanded Uncertainty (95% confidence level) $k\ E_{USM}$	
1	0.4	m/s	0.1000	%	0.4645	%	0.4751	%	0.9503	%
2	1	m/s	0.1000	%	0.2305	%	0.2513	%	0.5026	%
3	2.5	m/s	0.1000	%	0.1970	%	0.2210	%	0.4419	%
4	4	m/s	0.1000	%	0.1994	%	0.2231	%	0.4462	%
5	7	m/s	0.1000	%	0.2039	%	0.2271	%	0.4541	%
6	10	m/s	0.1000	%	0.2062	%	0.2292	%	0.4584	%

Fig. 5.18. Part of the "*USM*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, summarizing the USM field uncertainty calculations (example). (Corresponds to Tables 4.20, 4.21 and Fig. 5.26.)

5.11 Signal communication and flow computer calculations

The worksheet "*Computer*" for evaluation of the expanded uncertainty of "Flow computer effects", due to signal communication and flow computer calculations, is shown in Fig. 5.19, for the same example as given in Section 4.5.

EMU - USM Fiscal Gas Metering Station

Flow computer effects

Input variable	Given Relative Uncertainty		Confidence Level (probability distr.)	Type of uncertainty	Relative Standard Uncertainty		Relative Sensitivity Coefficient	Relative Variance
Communication	0	%	95 % (normal)	B	0	%	1	0
Flow computer calculations	0	%	95 % (normal)	B	0	%	1	0

Flow computer effects		
	Sum of relative variances	0
	Relative Combined Standard Uncertainty	0.0000
	Relative Expanded Uncertainty (95% confidence level)	0.0000 %

Fig. 5.19. The "*Computer*" worksheet in the program *EMU - USM Fiscal Gas Metering Station.*

The user specifies the relative expanded uncertainty of signal communication effects and flow computer calculations, together with the accompanying confidence levels /

probability distributions. Cf. also Sections 3.5 and 4.5. The user must himself document the input uncertainty values used for the "Flow computer effects", e.g. on basis of information provided by the USM manufacturer.

5.12 Graphical presentation of uncertainty calculations

Various worksheets are available in the program *EMU - USM Fiscal Gas Metering Station* to plot and display the calculation results, such as curve plots and bar-charts. These worksheets are described in the following.

5.12.1 Uncertainty curve plots

Plotting of uncertainty curves is made using the "*Graph*" worksheet. Editing of plot options is made using the "*Graph menu*" worksheet ("curve plot set-up").

Plotting of the relative expanded uncertainty can be made for the following four "measurands":

- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard ref. conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e.

These can be plotted as a function of (for the set of M flow velocities/rates chosen in the "*Flow cal.*" worksheet):

- Flow velocity,
- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard ref. conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e.

The relative expanded uncertainties above can be plotted together with the following measurands:

- No curve,
- Flow velocity,
- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard ref. conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e.

Axes may be scaled according to user needs (automatic or manual), and various options for curve display (points only, line between points and smooth curve[109]) are available.

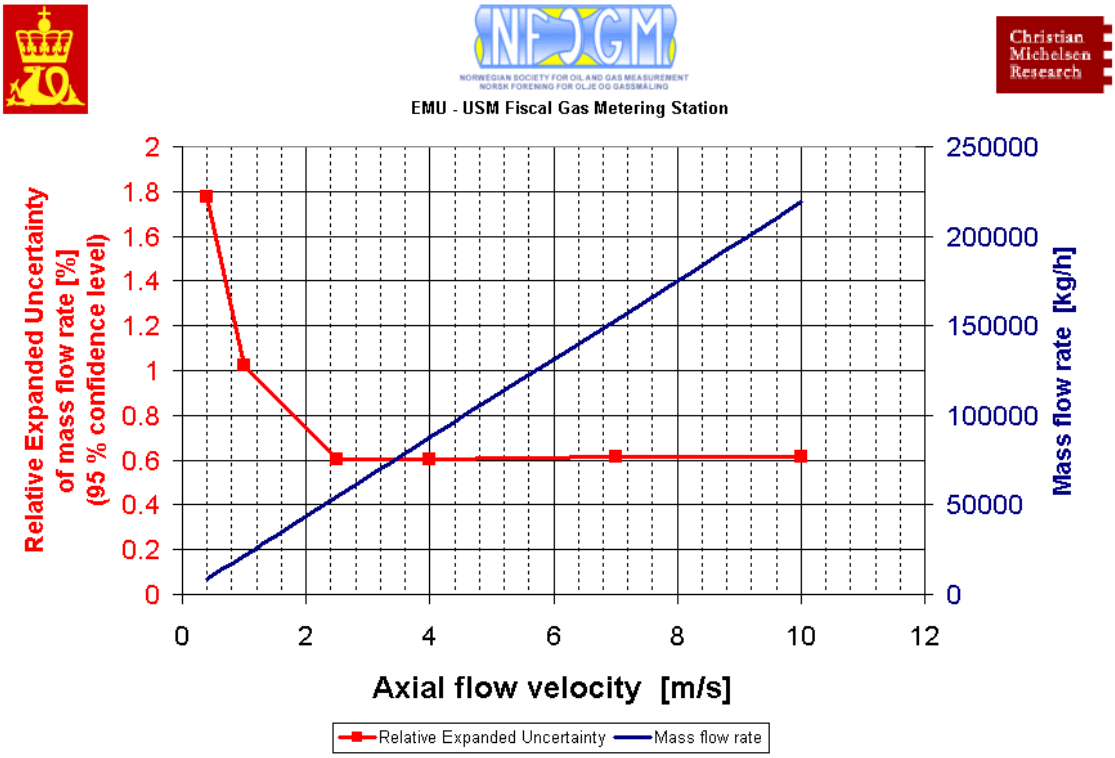


Fig. 5.20. The "*Graph*" worksheet in the program *EMU - USM Fiscal Gas Metering Station* (example).

Fig. 5.20 shows an example where the relative expanded uncertainty of the mass flow rate (at a 95 % confidence level and a normal probability distribution, with k = 2, cf. Section B.3) is plotted together with the mass flow rate itself, as a function of flow velocity. The example used here is the same as used in the text above, and in Section 4.6.3 (cf. Tables 4.26 and 4.27)[110].

5.12.2 Uncertainty bar-charts

Plotting of bar charts is made using the "*NN-chart*" worksheets. Editing of bar chart options is made using the "*Graph menu*" worksheet ("bar-chart set-up" section). Bar

109 For the "smooth curve" display option, the default method implemented in Microsoft Excel 2000 is used.

110 The front page shows the same evaluation example, plotted vs. mass flow rate.

charts are typically used to evaluate the relative contributions of various input uncertainties to the expanded uncertainty of the "measurand" in question.

Such bar-charts are available for the following seven "measurands":

- Pressure measurement ("*P-chart*" worksheet),
- Temperature measurement ("*T-chart*" worksheet),
- Compressibility factor measurement / calculation ("*Z-chart*" worksheet),
- Density measurement ("*D-chart*" worksheet),
- USM flow calibration ("*FC-chart*" worksheet),
- USM field operation ("*USMfield-chart*" worksheet), and
- Gas metering station ("*MetStat-chart*" worksheet).

As for the "Graph" worksheet, axes may be scaled according to user needs (automatic or manual). These bar charts are described separately in the following.

5.12.2.1 Pressure

The pressure-measurement bar chart is given in the "*P-chart*" worksheet. Fig. 5.21 shows an example where the contributions to the expanded uncertainty of the pressure measurement are plotted (blue), together with the expanded uncertainty of the pressure measurement (green). The example used here is the same as the example given in Table 4.6 and Fig. 5.4.

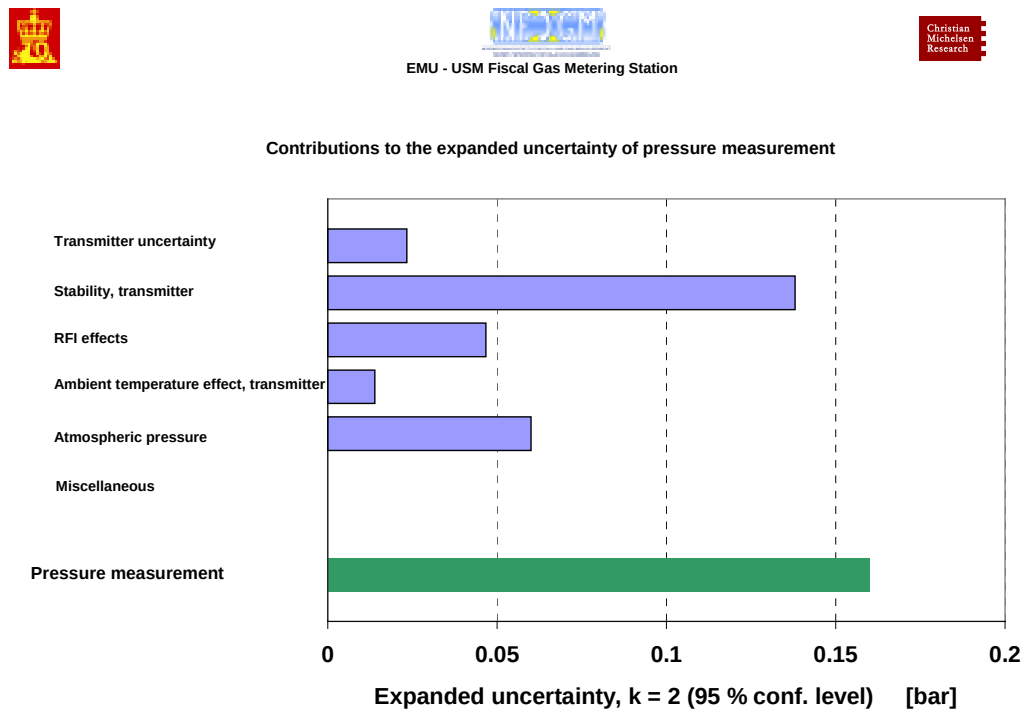


Fig. 5.21. The "*P-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.6 and Fig. 5.4.)

5.12.2.2 Temperature

The temperature-measurement bar chart is given in the "*T-chart*" worksheet. Fig. 5.22 shows an example where the contributions to the expanded uncertainty of the temperature measurement are plotted (blue), together with the expanded uncertainty of the temperature measurement (green). The example used here is the same as the example given in Table 4.8 and Fig. 5.6.

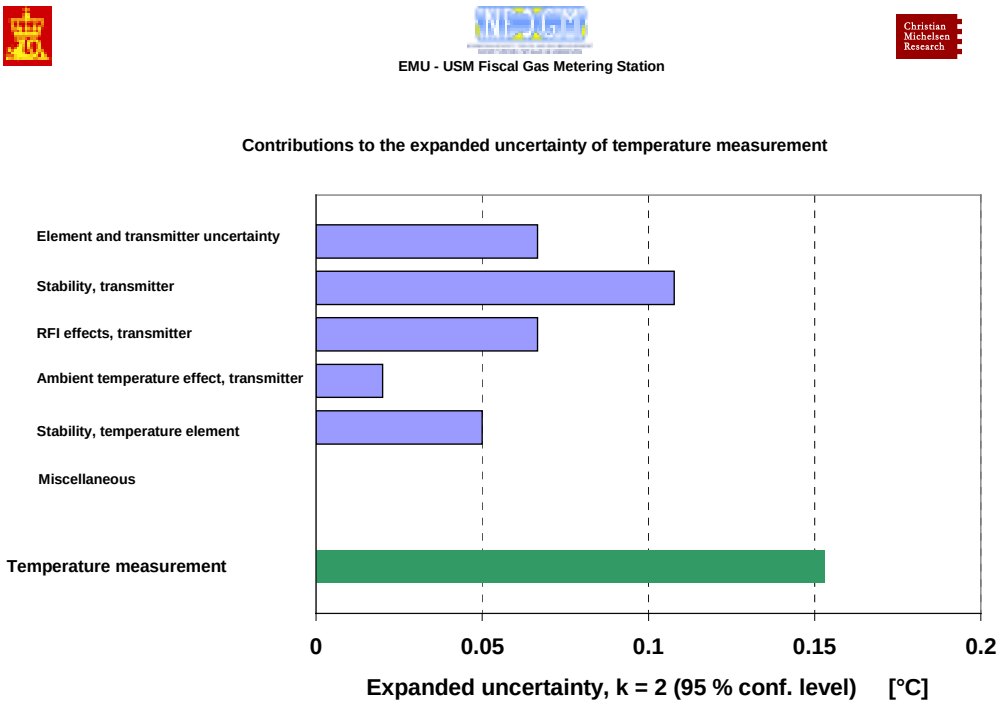


Fig. 5.22. The "*T-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.8 and Fig. 5.6.)

5.12.2.3 Compressibility factors

The compressibility factor bar chart is given in the "*Z-chart*" worksheet. Fig. 5.23 shows an example where the contributions to the expanded uncertainty of the *Z*-factor measurements/calculations are plotted (blue), together with the expanded uncertainty of the *Z*-factor ratio (green). The example used here is the same as the example given in Table 4.9 and Fig. 5.7.

5.12.2.4 Density

The density-measurement bar chart is given in the "*D-chart*" worksheet. Fig. 5.24 shows an example where the contributions to the relative expanded uncertainty of the density measurement are plotted (blue), together with the relative expanded uncertainty of the density measurement (green). The example used here is the same as the example given in Table 4.11 and Fig. 5.9.

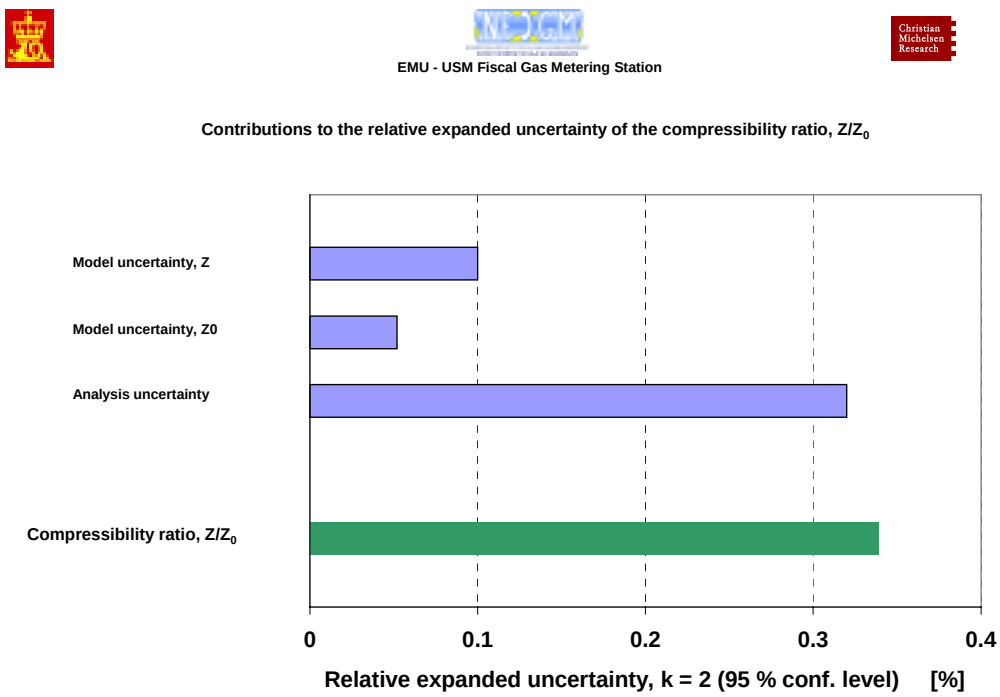


Fig. 5.23. The "*Z-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.9 and Fig. 5.7.)

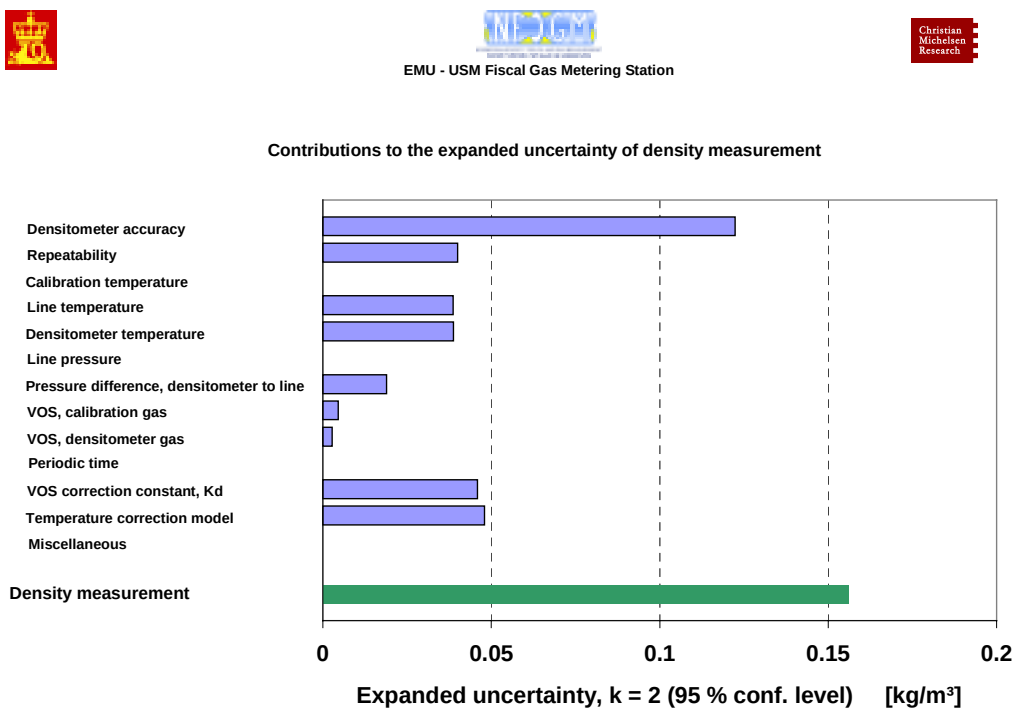


Fig. 5.24. The "*D-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.11 and Fig. 5.9.)

5.12.2.5 USM flow calibration

The USM flow calibration bar chart is given in the "*FC-chart*" worksheet. The bar chart can be shown for one flow velocity (or flow rate) at the time, among the set of

M flow velocities chosen in the "*Flow cal.*" worksheet. The desired flow velocity is set in the "*Graph menu*" worksheet.

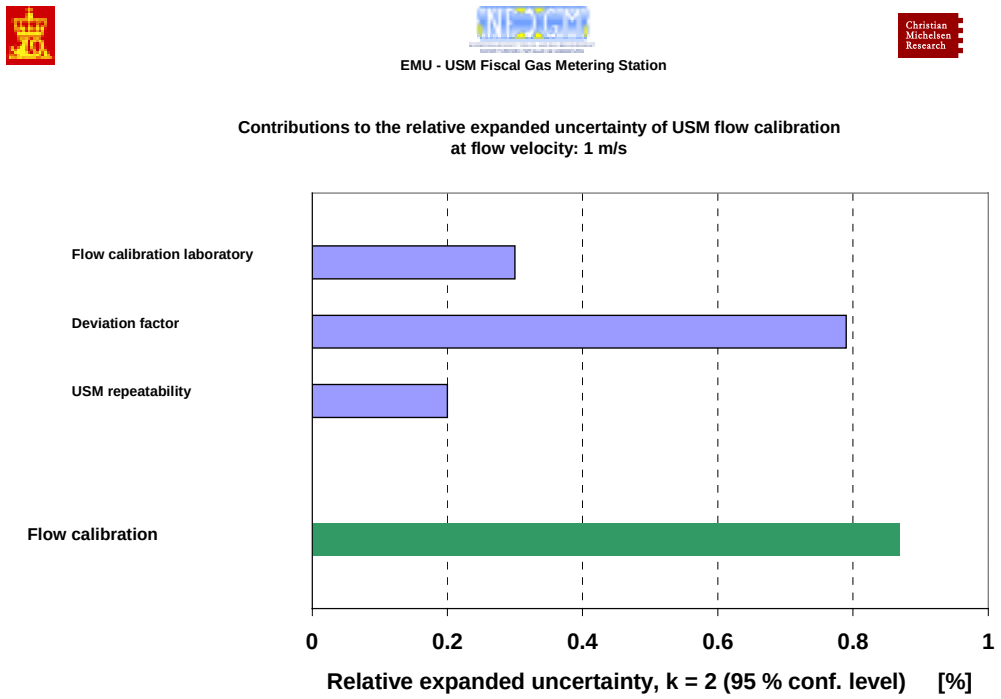


Fig. 5.25. The "*FC-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.13 and Fig. 5.11.)

Fig. 5.25 shows an example where the contributions to the relative expanded uncertainty of the USM flow calibration are plotted (blue), together with the relative expanded uncertainty of the USM flow calibration (green), for a flow velocity of 1 m/s. The example used here is the same as the example given in Table 4.13 and Fig. 5.11.

5.12.2.6 USM field operation

The USM field operation bar chart is given in the "*USMfield-chart*" worksheet. The bar chart can be shown for one flow velocity (or flow rate) at the time, among the set of *M* flow velocities chosen in the "*Flow cal.*" worksheet. The desired flow velocity is set in the "*Graph menu*" worksheet.

Fig. 5.26 shows an example where the contributions to the relative expanded uncertainty of the USM flow calibration are plotted (blue), together with the relative expanded uncertainty of the USM flow calibration (green), for a flow velocity of 1 m/s. The example used here is the same as the example given in Table 4.20 and Figs. 5.13 and 5.15-5.18.

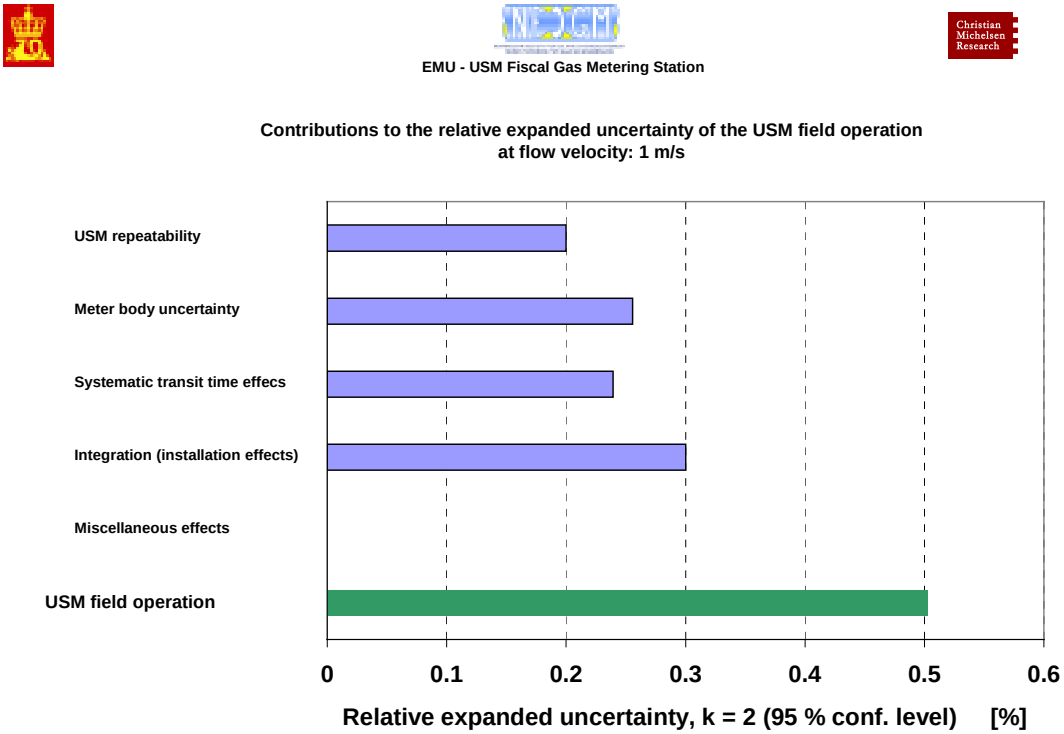


Fig. 5.26. The "*USMfield-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*. (Corresponds to Table 4.20 and Fig. 5.18.)

5.12.2.7 Gas metering station

The bar chart for the overall uncertainty of the USM fiscal gas metering station is given in the "*MetStat-chart*" worksheet.

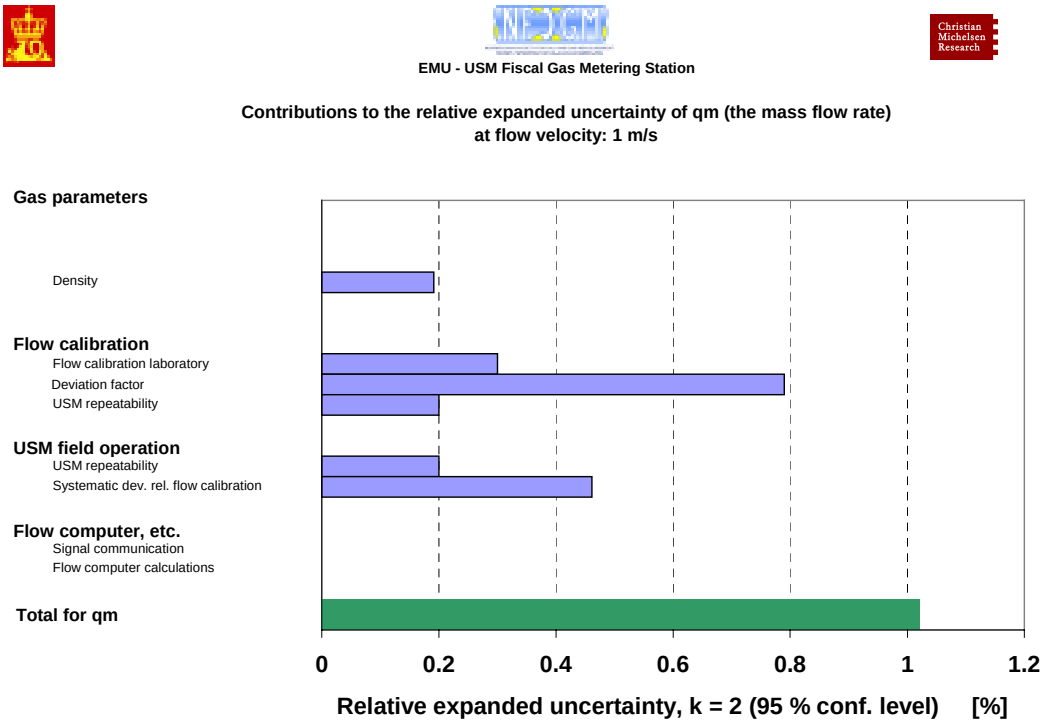


Fig. 5.27. The "*MetStat-chart*" worksheet in the program *EMU - USM Fiscal Gas Metering Station*, for the mass flow rate at 1 m/s flow velocity. (Corresponds to Table 4.26 and Figs. 5.20, 5.31.)

The bar chart can be shown for one flow velocity (or flow rate) at the time, among the set of *M* flow velocities chosen in the "*Flow cal.*" worksheet. The desired flow velocity is set in the "*Graph menu*" worksheet. The desired measurand (type of flow rate to be evaluated) is also set in the "*Graph menu*" worksheet. One may choose among the four measurands in question, q_v, Q, q_m and q_e.

Fig. 5.27 shows an example where the contributions to the relative expanded uncertainty of the mass flow rate are plotted (blue), together with the relative expanded uncertainty of the gas metering station (green), for a flow velocity of 1 m/s. The example used here is the same as the example given in Table 4.26 and Fig. 5.20.

5.13 Summary report - Expanded uncertainty of USM fiscal gas metering station

A "*Report*" worksheet is available in the program *EMU - USM Fiscal Gas Metering Station* to provide a condensed report of the calculated expanded uncertainty of the USM fiscal gas metering station. For documentation purposes, this one-page report can be used alone, or together with printout of other worksheets in the program.

Blank fields are available for filling in program user information and other comments. Also some of the settings of the "*Gas parameter*" and "*USM setup*" worksheets are included for documentation purposes.

A report can be prepared for each of the four flow rate measurands in question (q_v, Q, q_m and q_e), at a given flow velocity (or volumetric flow rate, depending on the type of input used in the "*Flow cal.*" worksheet). The desired flow velocity (or volumetric flow rate) is chosen in the "*Report*" worksheet, among the *M* calibration flow velocities (or volumetric flow rates) specified in the "*Flow cal.*" worksheet.

Figs. 5.28-5.31 show the "*Report*" worksheet, calculated at the flow velocity 1 m/s, for the four flow rate measurands in question: the volumetric flow rate at line conditions (q_v) , the volumetric flow rate at standard reference conditions (Q), the mass flow rate (q_m), and the energy flow rate (q_e), respectively. The examples shown in Figs. 5.28-5.31 are the same as those given in Tables 4.22, 4.24, 4.26 and 4.28, respectively.

EMU - USM Fiscal Gas Metering Station

Uncertainty evaluation report

Calculation performed by: _____

Date: 31-des-2001

OPERATING CONDITIONS, METER RUN			DENSITOMETER CONDITIONS		
Line temperature, T	50	°C	Temperature at density transducer, T_d	48	°C
Line pressure (static), P	100	bara	Velocity of sound, c_d	415.24	m/s
Gas density, ρ	81.62	kg/m³	Indicated (uncorrected) gas density at dens. transd., ρ_u	82.443	kg/m³
Compressibility at line conditions, Z	0.846		Calibration temperature, T_c	20	°C
Velocity of sound (VOS), c	417	m/s	Calibration velocity of sound (VOS), c_c	350	m/s
Ambient (air) temperature, T_{air}	0	°C			
FLOW CALIBRATION CONDITIONS			**STANDARD REFERENCE CONDITIONS**		
Flow calibration temperature, T_{cal}	10	°C	Compressibility, Z_0	0.9973	
Flow calibration pressure, P_{cal}	50	bara	Gross calorific value, H_s	41.686	MJ/Sm³
Inner diameter (spoolpiece), at dry calibration	308	mm			

User comments: _____

Analyse uncertainty in: Volumetric flow rate at line conditions

Choose flow velocity or flow rate: 1 m/s

	Unit	Value	Standard Uncertainty	Rel. Expanded Uncertainty (95 % c. l., k=2)	Contribution to k Eqv
Flow calibration					
Flow calibration laboratory	-	-	-	0.3000 %	**0.3000 %**
Deviation factor	-	-	-	0.7901 %	**0.7901 %**
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
USM field operation					
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
Systematic dev. rel. flow calibration	-	-	-	0.4610 %	**0.4610 %**
Flow computer, etc.					
Signal communication	-	-	-	0.0000 %	**0.0000 %**
Flow computer calculations	-	-	-	0.0000 %	**0.0000 %**
Volumetric flow rate at line conditions, qv	**m³/h**	**268.22**	**1.3457**	**1.0034 %**	

Fig. 5.28. The *"Report"* worksheet in the program *EMU - USM Fiscal Gas Metering Station*, for the volumetric flow rate at line conditions. (Corresponds to Table 4.22.)

EMU - USM Fiscal Gas Metering Station

Uncertainty evaluation report

Calculation performed by: _____

Date: 31-des-2001

OPERATING CONDITIONS, METER RUN			DENSITOMETER CONDITIONS		
Line temperature, T	50	°C	Temperature at density transducer, T_d	48	°C
Line pressure (static), P	100	bara	Velocity of sound, c_d	415.24	m/s
Gas density, ρ	81.62	kg/m³	Indicated (uncorrected) gas density at dens. transd., ρ_u	82.443	kg/m³
Compressibility at line conditions, Z	0.846		Calibration temperature, T_c	20	°C
Velocity of sound (VOS), c	417	m/s	Calibration velocity of sound (VOS), c_c	350	m/s
Ambient (air) temperature, T_{air}	0	°C			
FLOW CALIBRATION CONDITIONS			**STANDARD REFERENCE CONDITIONS**		
Flow calibration temperature, T_{cal}	10	°C	Compressibility, Z_0	0.9973	
Flow calibration pressure, P_{cal}	50	bara	Gross calorific value, H_s	41.686	MJ/Sm³
Inner diameter (spoolpiece), at dry calibration	308	mm			

User comments: _____

Analyse uncertainty in: Volumetric flow rate at standard conditions

Choose flow velocity or flow rate: 1 m/s

	Unit	Value	Standard Uncertainty	Rel. Expanded Uncertainty (95 % c. l., k=2)	Contribution to k EQ
Gas parameters					
Pressure	bar	100	0.0799	0.1599 %	**0.1599 %**
Temperature	°C	50	0.0765	0.0473 %	**0.0473 %**
Compr. factor ratio, Z0/Z, (std/line)	-	1.1788	0.002	0.3393 %	**0.3393 %**
Flow calibration					
Flow calibration laboratory	-	-	-	0.3000 %	**0.3000 %**
Deviation factor	-	-	-	0.7901 %	**0.7901 %**
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
USM field operation					
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
Systematic dev. rel. flow calibration	-	-	-	0.4610 %	**0.4610 %**
Flow computer, etc.					
Signal communication	-	-	-	0.0000 %	**0.0000 %**
Flow computer calculations	-	-	-	0.0000 %	**0.0000 %**
Volumetric flow rate at standard reference conditions, Q	**Sm³/h**	**27826**	**149.19**	**1.0723 %**	

Fig. 5.29. The *"Report"* worksheet in the program *EMU - USM Fiscal Gas Metering Station*, for the volumetric flow rate at standard reference conditions. (Corresponds to Table 4.24.)

EMU - USM Fiscal Gas Metering Station

Uncertainty evaluation report

Calculation performed by: _____

Date: 31-des-2001

OPERATING CONDITIONS, METER RUN			DENSITOMETER CONDITIONS		
Line temperature, T	50	°C	Temperature at density transducer, T_d	48	°C
Line pressure (static), P	100	bara	Velocity of sound, c_d	415.24	m/s
Gas density, ρ	81.62	kg/m³	Indicated (uncorrected) gas density at dens. transd., ρ_u	82.443	kg/m³
Compressibility at line conditions, Z	0.846		Calibration temperature, T_c	20	°C
Velocity of sound (VOS), c	417	m/s	Calibration velocity of sound (VOS), c_c	350	m/s
Ambient (air) temperature, T_{air}	0	°C			
FLOW CALIBRATION CONDITIONS			**STANDARD REFERENCE CONDITIONS**		
Flow calibration temperature, T_{cal}	10	°C	Compressibility, Z_0	0.9973	
Flow calibration pressure, P_{cal}	50	bara	Gross calorific value, H_s	41.686	MJ/Sm³
Inner diameter (spoolpiece), at dry calibration	308	mm			

User comments: _____

Analyse uncertainty in: Mass flow rate

Choose flow velocity or flow rate: 1 m/s

	Unit	Value	Standard Uncertainty	Rel. Expanded Uncertainty (95 % c. l., k=2)	Contribution to k Eqm
Gas parameters					
Density	kg/m³	81.62	0.0781	0.1913 %	**0.1913 %**
Flow calibration					
Flow calibration laboratory	-	-	-	0.3000 %	**0.3000 %**
Deviation factor	-	-	-	0.7901 %	**0.7901 %**
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
USM field operation					
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
Systematic dev. rel. flow calibration	-	-	-	0.4610 %	**0.4610 %**
Flow computer, etc.					
Signal communication	-	-	-	0.0000 %	**0.0000 %**
Flow computer calculations	-	-	-	0.0000 %	**0.0000 %**
Mass flow rate, qm	**kg/h**	**21892**	**111.82**	**1.0215 %**	

Fig. 5.30. The *"Report"* worksheet in the program *EMU - USM Fiscal Gas Metering Station*, for the mass flow rate. (Corresponds to Table 4.26 and Fig. 5.27.)

EMU - USM Fiscal Gas Metering Station

Uncertainty evaluation report

Calculation performed by: _____

Date: 31-des-2001

OPERATING CONDITIONS, METER RUN			DENSITOMETER CONDITIONS		
Line temperature, T	50	°C	Temperature at density transducer, T_d	48	°C
Line pressure (static), P	100	bara	Velocity of sound, c_d	415.24	m/s
Gas density, ρ	81.62	kg/m³	Indicated (uncorrected) gas density at dens. transd., ρ_u	82.443	kg/m³
Compressibility at line conditions, Z	0.846		Calibration temperature, T_c	20	°C
Velocity of sound (VOS), c	417	m/s	Calibration velocity of sound (VOS), c_c	350	m/s
Ambient (air) temperature, T_{air}	0	°C			
FLOW CALIBRATION CONDITIONS			**STANDARD REFERENCE CONDITIONS**		
Flow calibration temperature, T_{cal}	10	°C	Compressibility, Z_0	0.9973	
Flow calibration pressure, P_{cal}	50	bara	Gross calorific value, H_s	41.686	MJ/Sm³
Inner diameter (spoolpiece), at dry calibration	308	mm			

User comments: _____

Analyse uncertainty in: Energy flow rate

Choose flow velocity or flow rate: 1 m/s

	Unit	Value	Standard Uncertainty	Rel. Expanded Uncertainty (95 % c. l., k=2)	Contribution to k Eqe
Gas parameters					
Pressure	bar	100	0.0799	0.1599 %	**0.1599 %**
Temperature	°C	50	0.0765	0.0473 %	**0.0473 %**
Compr. factor ratio, Z0/Z, (std/line)	-	1.1788	0.002	0.3393 %	**0.3393 %**
Calorific value	MJ/Sm³	41.686	0.0313	0.1500 %	**0.1500 %**
Flow calibration					
Flow calibration laboratory	-	-	-	0.3000 %	**0.3000 %**
Deviation factor	-	-	-	0.7901 %	**0.7901 %**
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
USM field operation					
USM repeatability	-	-	-	0.2000 %	**0.2000 %**
Systematic dev. rel. flow calibration	-	-	-	0.4610 %	**0.4610 %**
Flow computer, etc.					
Signal communication	-	-	-	0.0000 %	**0.0000 %**
Flow computer calculations	-	-	-	0.0000 %	**0.0000 %**
Energy flow rate, qe	**MJ/h**	**1E+06**	**6279.5**	**1.0827 %**	

Fig. 5.31. The *"Report"* worksheet in the program *EMU - USM Fiscal Gas Metering Station*, for the energy flow rate. (Corresponds to Table 4.28.)

5.14 Listing of plot data and transit times

Two worksheets are available in the program *EMU - USM Fiscal Gas Metering Station* to provide listing of data involved in the uncertainty evaluation.

The "*Plot data*" worksheet gives a listing of all data used and plotted in the "*Graph*" and "*NN-chart*" worksheets, cf. Fig. 5.32. Such a listing may be useful for reporting purposes, and in case the user needs to present the data in a form not directly available in the program *EMU - USM Fiscal Gas Metering Station*. Note that the contents of the "plot data" sheet will change with the settings used in the "Graph menu" sheet.

For convenience, a "*Transit time*" worksheet has also been included, giving a listing of all transit time data used for the USM calculations, cf. Fig. 5.33. This involves upstream and downstream transit times, and the transit time difference, for the chosen pipe diameter and flow rates involved. The transit time calculations have been made using a uniform axial flow velocity profile, and no transversal flow[111].

Note that the transit time values will change by changing the path configuration setup in the "USM setup" worksheet (i.e., diameter, no. of paths, no. of reflections, inclination angle and lateral chord positions).

5.15 Program information

Two worksheets are available to provide information on the program. These are the "About" and the "Readme" worksheets.

The "About" worksheet, which is displayed at startup of the program *EMU - USM Fiscal Gas Metering Station*, and can be activated at any time, gives general information about the program. The "Readme" worksheet gives regulations and conditions for the distribution of the *Handbook* and the program, etc.

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111 Effects of non-uniform flow profiles and transversal flow ("ray bending") are thus not included here, since in the program *EMU - USM Fiscal Gas Metering Station*, the transit times are used only for calculation of sensitivity coefficients. "Ray bending" effects are negligible in this context.

However, note that "ray bending" effects may influence on the USM reading at high flow velocities [Frøysa *et al.*, 2001]. This effect is included in the uncertainty model and the program through the terms $u(\hat{t}_{1i}^{systematic})$ and $u(\hat{t}_{2i}^{systematic})$, cf. Section 3.4.2.2.

EMU - USM Fiscal Gas Metering Station

Plot data

Plot data for the "Graph" - sheet

First column: Axial flow velocity [m/s]
Second column: Relative Expanded Uncertainty of mass flow rate [%] (95 % confidence level)
Third column: Mass flow rate [kg/h]

0.4	1.773057	8756.9
1	1.021505	21892.25
2.5	0.601644	54730.63
4	0.604728	87569
7	0.614904	153245.8
10	0.617281	218922.5

Plot data for the "MetStat-chart" - sheet
All data: relative expanded uncertainty (k=2) (%)

Gas parameters	
Density	0.191257
Flow calibration	
Flow calibration laboratory	0.3
Deviation factor	0.790145
USM repeatability	0.2
USM field operation	
USM repeatability	0.2
Systematic dev. rel. flow calibration	0.461048
Flow computer, etc.	
Signal communication	0
Flow computer calculations	0
Total for qm	1.021505

Plot data for the "P-chart" - sheet
All data: expanded uncertainty (k=2) (bar)

Transmitter uncertainty	0.023333
Stability, transmitter	0.138
RFI effects	0.046667
Ambient temperature effect, transmitter	0.013943
Atmospheric pressure	0.06
Miscellaneous	0
Total	0.159877

Plot data for the "T-chart" - sheet
All data: expanded uncertainty (k=2) (°C)

Element and transmitter uncertainty	0.066667
Stability, transmitter	0.107717
RFI effects, transmitter	0.066667
Ambient temperature effect, transmitter	0.02
Stability, temperature element	0.05
Miscellaneous	0
Total	0.152944

Plot data for the "Z-chart" - sheet
All data: relative expanded uncertainty (k=2) (%)

Model uncertainty, Z	0.1
Model uncertainty, Z0	0.052
Analysis uncertainty	0.32
Total	0.33927

Plot data for the "D-chart" - sheet
All data: expanded uncertainty (k=2) (kg/m³)

Densitometer accuracy	0.122395
Repeatability	0.04
Calibration temperature	2.74E-05
Line temperature	0.03863
Densitometer temperature	0.038829
Line pressure	2.61E-05
Pressure difference, densitometer to line	0.018846
VOS, calibration gas	0.00455
VOS, densitometer gas	0.002731
Periodic time	7.05E-05
VOS correction constant, Kd	0.045833
Temperature correction model	0.048
Miscellaneous	0
Total	0.156104

Plot data for the "FC-chart" - sheet
All data: relative expanded uncertainty (k=2) (%)

Flow calibration laboratory	0.3
Deviation factor	0.790145
USM repeatability	0.2
Total	0.868521

Plot data for the "USMfield-chart" - sheet
All data: relative expanded uncertainty (k=2) (%)

USM repeatability	0.2
Meter body uncertainty	0.255512
Systematic transit time effecs	0.239332
Integration (installation effects)	0.3
Miscellaneous effects	0
Total	0.502559

Fig. 5.32. The "*Plot data*" worksheet in the program *EMU - USM Fiscal Gas Metering Station* (example).

EMU - USM Fiscal Gas Metering Station

Transit times

Axial flow velocity:	Acoustic path no:	Upstream transit time:	Downstream transit time:	Transit time difference:
0.4 m/s	1	614.3886 µs	613.5557 µs	832.8901 ns
	2	994.1016 µs	992.7539 µs	1347.644 ns
	3	994.1016 µs	992.7539 µs	1347.644 ns
	4	614.3886 µs	613.5557 µs	832.8901 ns

Axial flow velocity:	Acoustic path no:	Upstream transit time:	Downstream transit time:	Transit time difference:
1 m/s	1	615.0155 µs	612.9332 µs	2082.235 ns
	2	995.1159 µs	991.7468 µs	3369.127 ns
	3	995.1159 µs	991.7468 µs	3369.127 ns
	4	615.0155 µs	612.9332 µs	2082.235 ns

Axial flow velocity:	Acoustic path no:	Upstream transit time:	Downstream transit time:	Transit time difference:
2.5 m/s	1	616.5911 µs	611.3854 µs	5205.745 ns
	2	997.6654 µs	989.2423 µs	8423.073 ns
	3	997.6654 µs	989.2423 µs	8423.073 ns
	4	616.5911 µs	611.3854 µs	5205.745 ns

Axial flow velocity:	Acoustic path no:	Upstream transit time:	Downstream transit time:	Transit time difference:
4 m/s	1	618.1789 µs	609.8492 µs	8329.659 ns
	2	1000.234 µs	986.7568 µs	13477.67 ns
	3	1000.234 µs	986.7568 µs	13477.67 ns
	4	618.1789 µs	609.8492 µs	8329.659 ns

Axial flow velocity:	Acoustic path no:	Upstream transit time:	Downstream transit time:	Transit time difference:
7 m/s	1	621.3913 µs	606.8116 µs	14579.67 ns
	2	1005.432 µs	981.8419 µs	23590.4 ns
	3	1005.432 µs	981.8419 µs	23590.4 ns
	4	621.3913 µs	606.8116 µs	14579.67 ns

Axial flow velocity:	Acoustic path no:	Upstream transit time:	Downstream transit time:	Transit time difference:
10 m/s	1	624.6538 µs	603.8195 µs	20834.21 ns
	2	1010.711 µs	977.0005 µs	33710.47 ns
	3	1010.711 µs	977.0005 µs	33710.47 ns
	4	624.6538 µs	603.8195 µs	20834.21 ns

Fig. 5.33. The *"Transit times"* worksheet in the program *EMU - USM Fiscal Gas Metering Station* (example).

6. USM MANUFACTURER SPECIFICATIONS

The present chapter summarizes some input parameters and uncertainty data related to the USM, which should preferably be known to enable an uncertainty evaluation of the USM fiscal gas metering station at a "detailed level" with respect to the USM, and which preferably are to be specified by the USM manufacturer.

Today, information normally available from USM manufacturers includes e.g.:

- USM dimensional data,
- USM path configuration data (to some extent; - integration weights (w_i), lateral chord positions (y_{i0}) and inclination angles (ϕ_{i0}) are not provided by all manufacturers),
- Flow calibration results (e.g. the number of calibration points (M), and the deviation re. reference after applying the correction factor K ($Dev_{C,j}$, cf. Fig. 2.1)),
- Whether pressure and temperature correction of meter body dimensions are used or not,
- USM uncertainty data ("accuracy" and repeatability).

With respect to the USM uncertainty, typical data as specified by USM manufacturers today are given in Table 6.1.

Table 6.1. Typical USM uncertainty data currently specified by USM manufacturers.

	Instromet (2000) (Q.Sonic)	***Daniel (2000) (SeniorSonic)***	***Kongsberg (2000) (MPU 1200)***
"Accuracy"	≤ 0.5 %	Without flow calibration: ≤ 0.5 % of reference. *With* flow calibration: Higher accuracy.	Without flow calibration: ±0.5 % of meas.value. *With* flow calibration: ±0.25 % of meas.value.
Repeatability	≤ 5 mm/s	< 0.2 % of reading in specified velocity range	± 0.2 % of measured value

For evaluation of a USM fiscal gas metering station and its uncertainty, the buyer or user of a USM may occasionally end up with some questions in relation to manufacturer data. Typical "problems" may be:

- The "accuracy" specified in the data sheets is usually not sufficiently defined. Data sheets do not specify what types of uncertainties the "accuracy" term accounts for (e.g. systematic transit time effects, installation effects, etc.). Information on how the "accuracy" varies with pressure, temperature and installation con-

ditions, and deviation from flow calibration to field operation conditions, is generally lacking.

- Confidence level(s) and probability distribution(s) are lacking (both for "accuracy" and repeatability). That means, the user does not know whether the figure specified in the data sheet shall be divided by 1, 2, $\sqrt{3}$ or 3 (or another number) in order to obtain the corresponding standard uncertainty value.
- A single repeatability figure is specified in data sheets. If the repetability is different in flow calibration and in field operation, which may be the case in practice (cf. Sections 3.3.3 and 3.4.2.1), both may be needed. At least it should be specified whether the repeatability figure accounts for both or not.
- The repeatability is often not specified as a function of flow velocity (or flow rate).

For improved evaluation of USM fiscal gas metering stations and their uncertainty, some more specific USM uncertainty data are proposed here, cf. Tables 6.2-6.6. Such data can be used directly as input to the Excel uncertainty evaluation program *EMU - USM Fiscal Gas Metering Station*, at the "detailed level", cf. Chapter 5. Note that for all uncertainties, the confidence level and probability distribution should be specified.

Table 6.2. Proposed "USM meter body" data and uncertainties to be specified by the USM manufacturer, for uncertainty evaluation of the USM fiscal gas metering station. For uncertainties, the confidence level and probability distribution should be specified.

Meter body	Quantity	Symbol	Unit	Ref. Handbook Section
Dimensions	Inner diameter ("dry calibration" value)	$2R_0$	mm	2.3, 5.3
	Average wall thickness	w	mm	2.3, 5.3
Material data	Temperature expansion coefficient	α	K^{-1}	2.3, 5.3
	Young's modulus (or modulus of elasiticity)	Y	MPa	2.3, 5.3
P & T correction	Whether pressure and temperature correction of meter body dimensions is used or not	-	-	2.3.4, 3.4.1, 4.4.2, 5.10.2
Uncertainties	Rel. standard uncertainty of the coefficient of linear temperature expansion for the meter body material, α	$u(\hat{\alpha})/\lvert\hat{\alpha}\rvert$	%	----- " -----
	Rel. standard uncertainty of the coefficient of linear pressure expansion for the meter body material, β	$u(\hat{\beta})/\lvert\hat{\beta}\rvert$	%	----- " -----

Table 6.2 gives the proposed "USM meter body" data and corresponding uncertainties to be specified. Most of these data are normally available from USM manufacturers today, such as R_0, w, α and Y, and whether pressure and temperature correction

of the meter body dimensions are used or not. The two relative uncertainty terms $u(\hat{\alpha})/|\hat{\alpha}|$ and $u(\hat{\beta})/|\hat{\beta}|$ may be more difficult to specify, cf. Section 4.4.2.

Table 6.3. "USM path configuration" data which may preferably be specified by the USM manufacturer, for uncertainty evaluation of the USM fiscal gas metering station.

Quantity	**Symbol**	**Unit**	**Tentative domain**[112]	**Ref. Handbook Section**
No. of acoustic paths	N	-	-	2.3, 5.3
No. of reflections in path no. i, $i = 1, \ldots, N$	$N_{refl,i}$	-	-	----- " -----
Inclination angle of path no. i ("dry calibration" value), $i = 1, \ldots, N$	ϕ_{i0}	^{o}	$\pm 5\ ^{o}$	----- " -----
Relative lateral chord position of path no. i ("dry calibration" value), $i = 1, \ldots, N$	y_{i0}/R_0	-	± 0.1	----- " -----
Integration weight of path no. i, $i = 1, \ldots, N$	w_i	-	± 0.1 $\sum_{i=1}^{N} w_i \approx 1$	----- " -----

Table 6.3 gives the "USM path configuration" data which may preferably be specified by the USM manufacturer. Note that these are needed only if the "detailed level" is used for the USM in field operation. If the "overall level" is used for USM field operation (both with respect to repeatability and systematic deviation relative to flow calibration, cf. Section 5.10), none of the parameters listed in Table 6.3 need to be specified.

Some of the data set up in Table 6.3 are already available from all USM manufacturers today, such as N and $N_{refl,i}$, $i = 1, \ldots, N$. With respect to ϕ_{i0}, y_{i0}/R_0 and w_i, these are available from some USM manufacturers, but may not be available from others. The "ideal" situation with respect to uncertainty evaluation - at least from a user viewpoint - would be that the manufacturer data for these were known. However, manufacturer data may not always be available. In such cases the following compromise approach may be used to run *EMU - USM Fiscal Gas Metering Station*: for each of ϕ_{i0}, y_{i0}/R_0 and w_i the manufacturer may specify a value within a domain around the actual (unavailable) value. Tentative domains have been proposed in Table 6.3: $\pm 5\ ^{o}$ or better for the inclination angles, ϕ_{i0}, ± 0.1 or better for the relative lateral chord positions, y_{i0}/R_0, and ± 0.1 or better for the integration weights, w_i. Note that the sum of the integration weights w_i is to be approximately equal to unity, as also indicated in Table 6.3.

112 The domains for specification of nominal values of ϕ_{i0}, y_{i0}/R_0 and w_i given in Table 6.3 are only tentative, based on only a few limited investigations for a 12" USM. A more systematic analyses with respect to such domains should be carried out.

Table 6.4. Proposed "USM flow calibration" data and uncertainties to be specified by the USM manufacturer, for uncertainty evaluation of the USM fiscal gas metering station. For uncertainties, the confidence level and probability distribution should be specified.

Quantity / Uncertainty	**Symbol**	**Unit**	**Ref. Handbook Section**
No. of flow calibration points	M	-	2.2.2, 2.2.3, 4.3.2, Fig. 2.1
Deviation re. reference (after using the correction factor K), at flow calibration pt. no. j, $j = 1,\ldots, M$ (cf. Fig. 2.1)	$Dev_{C,j}$	-	----- " -----
Repeatability of USM flow rate reading, at flow calibration pt. no. j, $j = 1,\ldots, M$. (i.e., the relative standard deviation of the flow rate reading)	$E_{rept,j}$	%	3.3, 4.3.3, 5.9, Table 3.6

Table 6.4 gives the proposed "USM flow calibration" data and uncertainties to be specified by the USM manufacturer. These data are normally available from the manufacturers.

Table 6.5. Proposed "USM field operation" uncertainty data to be specified by the USM manufacturer, for uncertainty evaluation of the USM fiscal gas metering station. For uncertainties, the confidence level and probability distribution should be specified.

Quantity / Uncertainty	**Symbol**	**Unit**	**Ref. Handbook Section**
Repeatability of USM *flow rate reading* in field operation, at the actual flow rate. (i.e., the relative standard deviation of the flow rate readings)	E_{rept}	%	3.4.2, 4.4.1, 5.10.1, Table 3.8
Repeatability of USM *transit time readings* in field operation, at the actual flow rate, accounting for all paths (i.e., the relative standard deviation of the transit time readings)	$u(\hat{t}_{1i}^{random})$	ns	3.4.2, 4.4.1, 5.10.1, Table 3.8
Standard uncertainty due to systematic effects on the *upstream* transit time of path no. i, $\hat{t}_{1i}$, $i = 1,\ldots, N$, due to *change* of conditions from flow calibration to field operation	$u(\hat{t}_{1i}^{systematic})$	ns	3.4.2, 4.4.3, 5.10.2, Table 3.8
Standard uncertainty due to systematic effects on the *downstream* transit time of path no. i, $\hat{t}_{1i}$, $i = 1,\ldots, N$, due to *change* of conditions from flow calibration to field operation	$u(\hat{t}_{2i}^{systematic})$	ns	3.4.2, 4.4.3, 5.10.2, Table 3.8
Relative standard uncertainty of the USM integration method, due to *change* of installation conditions from flow calibration to field operation.	$E_{I,\Delta}$	%	3.4.3, 4.4.4, 5.10.2, Table 3.8

Table 6.5 gives the proposed "USM field operation" uncertainty data to be specified by the USM manufacturer.

The two first parameters in the table, E_{rept} and $u(\hat{t}_{1i}^{random})$, are related to the USM repeatability in field operation, cf. Section 5.10.1. They represent the repeatability of

the flow rate and the transit times, respectively. E_{rept} is needed if the "overall level" is used for the USM repeatability in field operation (cf. Fig. 5.12), and $u(\hat{t}_{1i}^{random})$ is needed if the "detailed level" is used (cf. Fig. 5.13). Both types of data should be readily available from USM flow computers. Preferably, both parameters should be specified by the USM manufacturer (as indicated in Table 6.5) so that the user of the program could himself choose which one to use. Cf. Sectiuons 3.4.2 and 4.4.1 for a discussion.

The last three parameters included in Table 6.5, $u(\hat{t}_{1i}^{systematic})$, $u(\hat{t}_{2i}^{systematic})$ and $E_{I,\Delta}$, are related to systematic deviations of the USM relative to flow calibration, cf. Section 5.10.2. They represent the systematic effects on the upstream and downstream transit times, and installation effects, respectively. They are needed if the "detailed level" is used for the systematic USM effects in the program *EMU - USM Fiscal Gas Metering Station*. Preferably, all three parameters should be specified by USM manufacturers. Note that if the field conditions and flow calibration conditions are identical (with respect to pressure, temperature, gas composition, flow profiles (axial and transversal)), and there is no drift in the transducers, these three terms would be zero. However, this is not likely to be the situation in practice.

Table 6.6. Proposed "Flow computer" uncertainties to be specified by the USM manufacturer, for uncertainty evaluation of the USM fiscal gas metering station. For uncertainties, the confidence level and probability distribution should be specified.

Uncertainty	**Symbol**	**Unit**	**Ref. Handbook Section**
Relative standard uncertainty of the estimate $\hat{q}_v$ due to signal communication with flow computer	E_{comm}	%	3.5 , 4.5, 5.11, Table 3.7
Relative standard uncertainty of the estimate $\hat{q}_v$ due to flow computer calculations	E_{flocom}	%	----- " -----

Table 6.6 gives the proposed "Flow computer" uncertainties to be specified by the USM manufacturer. These data should be readily available from the manufacturers.

7. CONCLUDING REMARKS

A *Handbook* of uncertainty calculation for fiscal gas metering stations based on a flow calibrated multipath ultrasonic gas flow meters has been worked out, including a Microsoft Excel program *EMU - USM Fiscal Gas Metering Station* for calculation of the expanded uncertainty of such metering stations. The uncertainty of the following four flow rate measurements have been addressed (cf. Chapters 2 and 3):

- Actual volume flow (i.e. the volumetric flow rate at line conditions), q_v,
- Standard volume flow (i.e., the volumetric flow rate at standard reference conditions), Q,
- Mass flow rate, q_m, and
- Energy flow rate, q_e.

The following metering station instrumentation has been addressed (cf. Section 2.1):

- Pressure measurement,
- Temperature measurement,
- Compressibility factor calculation (from GC gas composition measurement),
- Density measurement (vibrating element densitometer),
- Calorific value measurement (calorimeter), and
- Multipath ultrasonic gas flow meter (USM).

The uncertainty evaluation is made in conformity with accepted international standards and recommendations on uncertainty evaluation, such as the *GUM* [ISO, 1995a] and ISO/CD 5168 [ISO, 2000], cf. Appendix B.

The uncertainty model for USM fiscal gas metering stations presented in this *Handbook* is based on present-day “state of the art of knowledge” for stations of this type, and is not expected to be complete with respect to description of effects influencing on such metering stations. In spite of that, the uncertainty model does account for a large number of the important factors that influence on the expanded uncertainty of metering stations of this type. It is expected that the most important uncertainty contributions have been accounted for. Evaluation of the effects of these factors on the uncertainty of the metering station should be possible with the uncertainty model and the program *EMU - USM Fiscal Gas Metering Station* developed here.

It is the intention and hope of the partners presenting this *Handbook* that - after a period of practical use of the *Handbook* and the program - the uncertainty model pre-

sented here will be subject to necessary comments and viewpoints from users and developers of USMs, and others with interest in this field, as a basis for a possible later revision of the *Handbook*. The overall objective of such a process would of course be that - in the end - a useful and accepted method for calculation of the uncertainty of USM fiscal gas metering stations can be agreed on, in the Norwegian metering society as well as internationally.

With respect to possibilities for improvements, the *Handbook* and the program *EMU - USM Fiscal Gas Metering Station* should constitute a useful basis for implementation of upgraded descriptions of the uncertainty model. This may concern e.g.:

(A) The uncertainty of alternative instruments and/or calculation methods,
(B) The USM field uncertainty,
(C) The functionality of the Excel program,

as discussed briefly in the following.

(A) With respect to **possible upgraded uncertainty descriptions of the instruments and/or calculation methods** involved in USM gas metering stations, the following topics may be of relevance:

- For the volumetric flow rate at standard reference conditions (Q), at least 3 different approaches are used by different gas companies, cf. Table 2.1. Only one of these is addressed here (method no. 3 of Table 2.1).

  For the mass flow rate (q_m), 2 different approaches are accepted by the NPD regulations [NPD, 2001], cf. Table 2.2. Only one of these is addressed here (method no. 1 of Table 2.2).

  With respect to measurement of the energy flow rate (q_e), the calorific value uncertainty is only addressed at an “overall level”, without correlation to other measurements involved (gas chromatography). That means, in the present approach the calorific value may implicitely be assumed to be measured using a calorimeter (i.e. method no. 5 of Table 2.3). However, at least 5 different approaches to measure the energy flow rate may be used, cf. Table 2.3.

  In the present *Handbook* and Excel program all methods described in Tables 2.1-2.3 are covered at the “overall level”. Only selected methods are covered at the “detailed level”, as described above. An update of the *Handbook* and

the program *EMU - USM Fiscal Gas Metering Station* on such points, to cover several or all methods indicated in Tables 2.1-2.3 at a "detailed level"[113], might represent a useful extension to cover a broader range of metering methods used in the gas industry, with respect to measurement of Q, q_m and q_e.

- For on-line vibrating element densitometers, several methods are in use for VOS correction, as described in Section 2.4.3. In a possible future revision of the *Handbook* other methods for VOS correction may be implemented than the method used here, as options.

  With respect to the contribution to $u(\hat{\rho}_{misc})$ in Eq. (3.14), a description of the various uncertainty contributions listed in Section 3.2.4 may be included in the uncertainty model, based on the functional relationship for ρ_u, Eq. (2.23).

- Here the instantaneous values of the respective flow rates are addressed. The program can be updated to account for the accumulated flow rates (i.e. including the uncertainty due to the integration of the instantaneous flow rates over time).

(B) The *Handbook* and the program should also constitute a useful basis for implementation of **possible upgraded uncertainty descriptions of the USM field uncertainty**, such as e.g.:

- With respect to pressure expansion of the meter body, a single model for the coefficient of radial pressure expansion β has been implemented in the present version of the program, cf. Eq. (2.19). Several models for β are used in current USMs, and all of these represent simplifications, cf. Table 2.6. Implementation of a choice of various models for β may thus be of interest, especially in connection with high pressure differences between flow calibration and field operation, at which the actual value of β becomes essential.

- Implementation (in the program) of an option with automatic calculation of the possible error made if Eqs. (2.20)-(2.22) are used for pressure correction of the meter body.

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113 Such an upgrade would need to address possible correlation between the compressibility factors, Z and Z_0, and the molar weight, m. Also possible correlation between Z and Z_0 and the superior calorific value, H_s, would need to be addressed.

- Further with respect to pressure and temperature expansion of the meter body, the additional effect of transducer expansion / contraction can be evaluated.

- At present Formulation A is used for input of the geometrical meter body quantities. The program can be extended to optional input of all four formulations A, B, C and D, cf. Table 2.5. As discussed in Section 2.3.3, this does not influence on the uncertainty of the flow calibrated USM or the metering station, but may be more convenient for the user, if the USM manufacturer uses another formulation of the functional relationship than A.

- For the input uncertainties of the compressibility factors, the analysis uncertainties of Z and Z_0 can be evaluated statistically using a Monte Carlo type of method, for various gas compositions of relevance, as described in Section 3.2.3.2.

- Improved flexibility with respect to levels of complexity for entering of input uncertainties can be implemented[114], such as:
  (1) ”Overall level” (completely meter independent, as today's "overall level"),
  (2) ”Detailed level 1” (as today’s ”detailed level”), and
  (3) ”Detailed level 2”: More detailed input uncertainties can be given than in today’s ”detailed level”, e.g. with respect to:
    - input uncertainties entered for individual paths (not only average over all paths)[115],

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114 In the present version of the program *EMU - USM Fiscal Gas Metering Station*, the "detailed level" in the "USM" worksheet is definitely a compromise between what *ideally* should be specified as input uncertainties, and what is available today from USM manufacturers. This is done to avoid a too high "user treshold" with respect to specifying USM input uncertainties.

However, as the USM technology grows more mature, the need for a more detailed level of input uncertainties may also grow. An option of several levels of complexity for input uncertatinties may be convenient, which would provide a possibility of entering the USM input uncertainties in a physically more "correct" way.

115 Entering of input uncertainties for individual paths, and description of the propagation of these uncertainties to the metering stations’s expanded uncertainty, may be very useful in many circumstances, such as e.g.:

- The repeatability may vary between paths.
- In case of transducers exchange, this may be done for only one or two paths. Such exchange may result in changed time delay and Δt-correction (“dry calibration” values).
- In case of erroneous signal period detection (cf. Section 3.4.2.2), this may occur at only one or two paths (either upstream or downstream, or in both directions).
- In situations with a path failure, USM manufacturers may use historical flow profile data to keep the meter “alive”, preferably over a relatively short time period. That means, for the

- decomposition of random and systematic transit time effects into their individual physical contributions (cf. Table 1.4),
- accounting for both correlated and uncorrelated effects between paths (cf. Appendix E).

The user could then choose among these three levels, for a given uncertainty evaluation case.

- The uncertainty model and the program *EMU - USM Fiscal Gas Metering Station* can be extended to account for both
  - meter independent USM technologies (as in today's version), and
  - meter dependent USM technologies.

  That is, to account for e.g. specific path configurations / integration techniques, transit time detection methods, "dry calibration" methods, correction methods, etc. That means, variants of the program can be tailored to optionally describe the uncertainty of a specific meter (or meters), used in a gas metering station. Such extension(s) may be of particular interest for meter manufacturer(s), but also for users of that specific meter type.

(C) With respect to **functionality of the Excel program**, data storage requirements may be an issue. In the present version, storing of an executed uncertainty evaluation is made by saving the complete Excel file (.xls format), which requires about 1.4 MB per file. To save storage space, it would be convenient to enable saving the uncertainty evaluation data in another format (less space demanding, such as an ordinary data file), and reading such stored data files into the Excel program.

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path in question, the lacking upstream and downstream transit times are effectively substituted with "synthetic" transit times. There are thus systematic timing uncertainties associated with such procedures, the consequences of which should preferably be evaluated at an individual path basis.

- Possible transducer deposits such as grease, liquid, etc. may build up differently at the upstream and downstream transducers, and differently for different paths.
- PRV noise have been reported to be detected differently by different paths, and differently by the upstream and downstream transducers within a path.

In the present version of the program such effects are accounted for by input uncertainties which (for the convenience of the user [Ref. Group, 2001]) are *averaged over all paths*, and input uncertainties may thus be difficult to quantify in practice. Upgrading the program with an additional option for specification of input uncertainties at individual paths ("Detailed level 2") would enable a more realististic description. In many cases the specification of input uncertainties for individual paths may also be simpler to understand for the user of the program, as it is closer to the practical metering situation. Thus, the disadvantages of such an option (the specification of a larger number of input transit time uncertainties) should be balanced with the advantages and improved uncertainty evaluation which can be achieved using a "Detailed level 2".

PART B

APPENDICES

APPENDIX A

SOME DEFINITIONS AND ABBREVIATIONS

In the present appendix, some terms and abbreviations related to USM fiscal *gas metering stations* are defined. References to corresponding definitions given elsewhere are included. Note that relevant definitions and terminology related to *uncertainty calculations* are listed in Appendix B, Section B.1.

EMU	"Evaluation of metering uncertainty" [Dahl e*t al.*, 1999].
USM	A multipath ultrasonic flow meter for gas based on measurement of transit times, and calculation of transit time differences. The wording USM refers to the composite of the meter body (“spoolpiece”), the ultrasonic transducers, the control electronics, and the CPU unit / flow computer.
Large USM	A USM with (nominal) diameter ≥ 12” [AGA-9, 1998].
Small USM	A USM with (nominal) diameter < 12” [AGA-9, 1998].
USM functional relationship	The set of mathematical equations describing the USM measurement of e.g. the axial volumetric flow rate at line conditions, q_{USM}.
Spoolpiece	The USM meter body
Meter run	A flow measuring device within a meter bank complete with any associated pipework valves, flow straighteners and auxiliary instrumentation [ISO, 1995b].
Line conditions	Gas conditions at actual pipe flow operational conditions, at the USM installation location, with respect to pressure, temperature, and gas composition.
Standard reference conditions	Reference conditions of pressure, temperature and humidity (state of saturation) equal to: 1 atm. and 15 ^{o}C (1013.25 hPa, 288.15 K), for a dry, real gas [ISO, 2001].

Normal reference conditions	Reference conditions of pressure, temperature and humidity (state of saturation) equal to: 1 atm. and 0 ^{o}C (1013.25 hPa, 273.15 K), for a dry, real gas [ISO, 2001].
Deviation	The difference between the axial volumetric flow rate (or axial flow velocity) measured by the USM under test and the actual axial volumetric flow rate (or axial flow velocity) measured by the reference meter [AGA-9, 1998]. Percentage deviation is given relative to the reference measurement.
Deviation curve[116]	The deviation as a function of axial flow velocity over a given flow velocity range, at a specific installation condition, pressure, temperature and gas composition.
Flow calibration	Measurement of the deviation curve (cf. [AGA-9, 1998], Chapter 5.4).
"Dry calibration"[117]	(or more precisely, “zero flow verification test”, or "zero point control"). Measurement of quantities which are needed for the operation of the USM, such as relevant dimensions, angles, transit time delays through transducers, cables and electronics, and possibly Δt-correction [AGA-9, 1998]. "Dry calibration" measurements are made typically in the factory, at one or several specific conditions of pressure, temperature and gas composition. Various corrections and correction factors are typically

116 By [AGA-9, 1998] the deviation curve is referred to as the “error curve”. Here, the term “error” will be avoided in this context, since error refers to comparison with the (true) value of the flow rate, which is never known. Only the reference measurement of the flow calibration laboratory is compared with, and hence the term “deviation curve” is preferred here.

117 The wording “dry calibration” has come into common use in the USM community today, and is therefore used also here. However, it should be emphasized that this wording may be misleading. The "dry calibration" is not a *calibration* of the meter in the normal meaning of the word, but a procedure to determine, usually in the factory, a set of correction factors to be used in the meter software (including correction of transit times). In [AGA-9, 1998] (Section 5.4), this procedure is more correctly referred to as “zero flow verification test”. By [NPD, 2001] the wording “zero point control” is used.

	established by the "dry calibration", such as for transit times.
Zero flow reading	The maximum allowable flow meter reading when the gas is at rest, i.e. both axial and non-axial flow velocity components are essentially zero [AGA-9, 1998].
Time averaging period	Period of time over which the displayed measured flow velocities and volume flow rates are averaged.
Integration method	(Or flow profile integration method). Numerical technique to calculate the average flow velocity in the pipe from knowledge of the average flow velocity along each acoustic path in the USM.
Gauss-Jacobi quadrature	A specific integration method [Abromowitz and Stegun, 1968].
Ideal flow conditions	Pipe flow situation where no transversal (non-axial) flow velocity components are present, and where the axial flow velocity profile is turbulent and fully developed. That is, flow conditions in an ideal infinite-length straight pipe.
Axial flow velocity	The component of flow velocity along the pipe axis.
Transversal flow velocity	The non-axial components of flow velocity in the pipe.
PRV	Pressure reduction valve.
VOS	Velocity of sound.
GUM	Abbreviation for the ISO document "*Guide to the expression of uncertainty in measurement*" [ISO, 1995a].
VIM	Abbreviation for the ISO document "*International vocabulary of basic and general terms in metrology*" [ISO, 1993].

APPENDIX B

FUNDAMENTALS OF UNCERTAINTY EVALUATION

The NPD regulations [NPD, 2001] and the NORSOK I-104 [NORSOK, 1998a] standard refer to the *GUM* (*Guide to the expression of uncertainty in measurement)* [ISO, 1995a][118] as the "accepted norm" with respect to uncertainty analysis. The uncertainty model and the uncertainty calculations reported here are therefore based primarily on the "*GUM*".

A brief outline of the *GUM* terminology used in evaluating and expressing uncertainty is given in Section B.1. A list of important symbols used in the *Handbook* for expressing uncertainty is given in Section B.2. The *GUM* procedure used here for evaluating and expressing uncertainty is summarized in Section B.3, as a basis for the description of the uncertainty model and the uncertainty calculations. Requirements to documentation of the uncertainty calculations are described in Section B.4.

B.1 Terminology for evaluating and expressing uncertainty

Precise knowledge about the definitions of the terms used in the *Handbook* is important in order to perform the uncertainty calculations with - preferably - a minimum possibility of misunderstandings.

Consequently, the definition of some selected terms regarding uncertainty calculations which are used in the present *Handbook*, or are important for using the *Handbook*, are summarized in Table B.1, with reference to the source documents used, in

118 The *GUM* was prepared by a joint working group consisting of experts nominated by BIPM, IEC, ISO and OIML, on basis of a request from the CIPM. The following seven organizations supported the development, which was published in their name: BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML.

The abbreviations are: CIPM: Comité International des Poids et Mesures, France (International Committee for Weights and Measures); BIPM: Bureau International des Poids et Mesures, Sévres Cedex, France (International Bureau of Weights and Measures); IEC: International Electrotechnical Commission, Genève, Switzerland; IFFC: International Federation of Clinical Chemistry, Nancy, France; ISO: International Organization for Standardization Genève, Switzerland; IUPAC: International Union of Pure and Applied Chemistry, Oxford, UK; IUPAP: International Union of Pure and Applied Physics, Frolunda Sweden; IOML: International Organization of Legal Metrology, Paris, France.

which further details may be given. For definition of terms in which symbols are used, the symbol notation is defined in Section B.2.

For additional definitions of relevance, cf. e.g. the *VIM* [ISO, 1993], and the *GUM*, Appendices E and F [ISO, 1995a][119].

Table B.1. Definitions of some terms regarding uncertainty calculations.

Type of term	Term	Definition	Reference
Quantities and units	Output quantity, y	In most cases a measurand y is not measured directly, but is determined from M other quantities $x_1, x_2, ..., x_M$ through a functional relationship, $y = f(x_1, x_2, ..., x_M)$	*GUM*, §4.1.1 and §4.1.2, p. 9
	Input quantity, x_i,	An *input quantity*, x_i (i = 1, ..., M), is a quantity upon which the *output quantity*, y, depends, through a functional relationship, $y = f(x_1, x_2, ..., x_M)$. The input quantities may themselves be viewed as measurands and may themselves depend on other quantities.	*GUM*, §4.1.2, p. 9
	Value (of a quantity)	Magnitude of a particular quantity generally expressed as a unit measurement multiplied by a number.	*VIM*, §1.18. *GUM*, §B.2.2, p. 31
	True value (of a quantity)	Value consistent with the definition of a given particular quantity. *VIM* notes (selected): 1. This is a value that would be obtained by a perfect measurement. 2. True values are by nature indeterminate. *GUM* comment: 3. The term "true value" is not used, since the terms "value of a measurand" (or of a quantity) and the term "true value of a measurand" (or of a quantity) are viewed as equivalent, with the word "true" to be redundant. *Handbook* comment: 4. Since a true value cannot be determined, in practice a conventional true value is used (cf. the *GUM*, p. 34).	*VIM*, §1.19 *GUM*, §3.1.1, p. 4. *GUM*, §3.2.3, p. 4. *GUM*, §D.3.5, p. 41.
	Conventional true value (of a quantity)	Value attributed to a particular quantity and accepted, sometimes by conventions, as having an uncertainty appropriate for a given purpose. *VIM* note (selected): 1. "Conventional true value" is sometimes called **assigned value**, **best estimate** of the value, **conventional value** or **reference value**.	*VIM*, §1.20 *GUM*, §B.2.4, p. 32
Measurements	Measurand	Particular quantity subject to measurement.	*VIM*, §2.6 *GUM*, §B.2.9
	Influence quantity	Quantity that is not the measurand, but that affects the result of measurement.	*VIM*, §2.7 *GUM*, §B.2.10, pp. 32-33.
Measurement	Result of	Value attributed to a measurand, obtained by measure-	*VIM*, §3.1

119 Note that a number of documents are available in which the basic uncertainty evaluation philosophy of the *GUM* is interpreted and explained in more simple and compact manners, for practical use in metrology. Some documents which may be helpful in this respect are [Taylor and Kuyatt, 1994], [NIS 3003, 1995], [EAL-R2, 1997], [EA-4/02, 1999], [Bell, 1999], [Dahl *et al.*, 1999] and [ISO/CD 5168, 2000].

results	measurement	ment. *VIM* notes: 1. When a result is given, it should be made clear whether it refers to: - the indication, - the uncorrected result, - the corrected result, and whether several values are averaged. 2. A complete statement of the result of a measurement includes information about the uncertainty of measurement.	*GUM*, §B.2.11, p. 33
	Indication (of a measuring instrument)	Value of a quantity provided by a measuring instrument. *VIM* notes (selected): 1. The value read from the display device may be called the **direct indication**; it is multiplied by the instrument constant to give the indication. 2. The quantity may be the measurand, a measurement signal, or another quantity to be used in calculating the value of the measurand.	*VIM*, §3.2
	Uncorrected result	Result of a measurement before correction for systematic error.	*VIM*, §3.3 *GUM*, §B.2.12, p. 33
	Corrected result	Result of a measurement after correction for systematic error.	*VIM*, §3.4 *GUM*, §B.2.13, p. 33
	Correction	Value added algebraically to the uncorrected result of a measurement to compensate for systematic error. *VIM* notes: 1. The correction is equal to the negative of the estimated systematic error. 2. Since the systematic error cannot be known perfectly, the compensation cannot be complete. *Handbook* comment: 3. If a correction is made, the correction must be included in the functional relationship, and the calculation of the combined standard uncertainty must include the standard uncertainty of the applied correction.	*VIM*, §3.15. *GUM*, §B.2.23, p. 34
	Correction factor	Numerical factor by which the uncorrected result of a measurement is multiplied to compensate for systematic error. *VIM* note: 1. Since the systematic error cannot be known perfectly, the compensation cannot be complete. *Handbook* comment: 2. If a correction factor is applied, the correction must be included in the functional relationship, and the calculation of the combined standard uncertainty must include the standard uncertainty of the applied correction factor.	*VIM*, §3.16 *GUM*, §B.2.24, p. 34
	Accuracy of measurement	Closeness of the agreement between the result of a measurement and a true value of the measurand. *VIM* notes: 1. Accuracy is a qualitative concept. 2. The term “precision” should not be used for “accuracy”. *Handbook* comment: 3. Accuracy should not be used quantitatively. The expression of this concept by numbers should be associated with (standard) uncertainty.	*VIM*, §3.5 *GUM*, §B.2.14, p. 33

Repeatability	Closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement. *VIM* notes: 1. These conditions are called repeatability conditions. 2. Repeatbility conditions include: - the same measurement procedure, - the same observer, - the same measuring instrument, used under the same conditions, - the same location, - repetition over a short period of time. 3. Repeatability may be expressed quantitatively in terms of the dispersion characteristics of the results.	*VIM*, §3.6 *GUM*, §B.2.15, p. 33.
Reproducability	Closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement. *VIM* notes: 1. A valid statement of reproducability requires specification of the conditions changed. 2. The changed conditions may include: - principle of measurement, - method of measurement, - observer, - measuring instrument, - reference standard, - location, - conditions of use, - time. 3. Repeatability may be expressed quantitatively in terms of the dispersion characteristics of the results. 4. Results are here usually understood to be corrected results.	*VIM*, §3.7 *GUM*, §B.2.16, p. 33
Experimental standard deviation	A quantity characterizing the dispersion of the results, for a series of measurements of the same measurand.	*VIM*, §3.8 *GUM*, § B.2.17, p. 33
Uncertainty of measurement	Parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.	*VIM*, §3.9. *GUM*, §2.2.4, p. 2-3. *GUM*, §B.2.18, p. 34. *GUM*, Annex D
Error (of measurement)	Result of a measurement minus a true value of the measurand.	*VIM*, §3.10. *GUM*, §B.2.19, p. 34
Deviation	Value minus its reference value.	*VIM*, §3.11
Relative error	Error of a measurement divided by a true value of the measurand.	*VIM*, §3.12. *GUM*, §B.2.20, p. 34.
Random error	Result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions. *VIM* notes: 1. Random error is equal to error minus systematic error. 2. Because only a finite number of measurements can be made, it is possible to determine only an estimate of random error.	*VIM*, §3.13. *GUM*, §B.2.21, p. 34.
Systematic error	Mean that would result from an infinite number of meas-	*VIM*, §3.14

		urements of the same measurand carried out under repeatability conditions minus the true value of the measurand. *VIM* notes: 1. Systematic error is equal to error minus random error. 2. Like true value, systematic error and its causes cannot be completely known. 3. For a measuring instrument, see "bias".	*GUM*, §B.2.22, p. 34.
Characterisation of measuring instruments	Nominal range	Range of indications obtainable with a particular setting of the controls of a measuring instrument. *VIM* note (selected): 1. Nominal range is normally stated in terms of its lower and upper limits.	*VIM*, §5.1
	Span	Modulus of the difference between the two limits of nominal range. *VIM* note: 1. In some fields of knowledge, the difference between the greatest and smallest value is called **range**.	*VIM*, §5.2
	Measuring range, Working range	Set of values of measurands for which the error of a measuring instrument is intended to lie within specified limits	*VIM*, §5.4
	Resolution (of a displaying device)	Smallest difference between indications of a displaying device that can be meaningfully distinguished. *VIM* note (selected): 1. For a digital displaying device, this is the change in the indication when the least significant digit changes by one step.	*VIM*, §5.12
	Drift	Slow change of metrological characteristic of a measuring instrument.	*VIM*, §5.16
	Accuracy of a measuring instrument	Ability of a measuring instrument to give responses close to a true value. *VIM* note: 1. "Accuracy" is a qualitative concept.	*VIM*, §5.18 *GUM*, §B.2.14, p. 33
	Error (of indication) of a measuring instrument	Indication of a measuring instrument minus a a true value of the corresponding input quantity. *VIM* note (selected): 1. This concept applies mainly where the instrument is compared to a reference standard.	*VIM*, §5.20 *GUM*, §B2.2.19, p. 34; Section 3.2
	Datum error (of a measuring instrument)	Error of a measuring instrument at a specified indication of a specified value of the measurand, chosen for checking the instrument.	*VIM*, §5.22
	Zero error (of a measuring instrument)	Datum error for zero value of the measurand.	*VIM*, §5.23
	Bias (of a measuring instrument)	Systematic error of the indication of a measuring instrument. *VIM* note: 1. The bias of a measuring instrument is normally estimated by averaging the error of indication over an appropriate number of repeated measurements.	*VIM*, §5.25 *GUM*, §3.2.3 note, p. 5
	Repeatability (of a measuring instrument)	Ability of a measuring instrument to provide closely similar indications for repeated applications of the same measurand under the same conditions of measurement.	*VIM*, §5.27
Statistical terms and concepts	Random variable	A variable that may take any of the values of a specified set of values, and with which is associated a probability distribution.	*GUM*, §C.2.2, p. 35
	Probability distribution (of a random variable)	A function giving the probability that a random variable takes any given value or belongs to a given set of values.	*GUM*, §C.2.3, p. 35
	Variance	A measure of dispersion, which is the sum of the squared deviations of observations from their average divided by	*GUM*, §C.2.20, p. 36.

	one less than the number of observations. *GUM* note (selected): 1. The sample standard deviation is an unbiased estimator of the population standard deviation.	
Standard deviation	The positive square root of the variance. *GUM* note: 1. The sample standard deviation is a biased estimator of the population standard deviation.	*GUM*, §C.2.12, p. 36; §C.2.21, p. 37; §C.3.3, p. 38.
Normal distribution		*GUM*, §C.2.14, p. 34
Estimation	The operation of assigning, from the observations in a sample, numerical values to the parameters of a distribution chosen as the statistical model of population from which this sample is taken.	*GUM*, §C.2.24, p. 37
Estimate	The value of an estimator obtained as a result of an estimation. *Handbook* comment: 1. Estimated value of a quantity, obtained either by measurement, or by other means (such as by calculations).	*GUM*, §C.2.26, p. 37.
Input estimate, and output estimate	An estimate of the measurand, y, denoted by $\hat{y}$, is obtained from the functional relationship $y = f(x_1, x_2, ..., x_M)$ using input estimates, $\hat{x}_1$, $\hat{x}_2$, ..., $\hat{x}_M$ for the values of the M quantities $x_1, x_2, ..., x_M$. Thus the output estimate, which is the result of the measurement, is given by $\hat{y} = f(\hat{x}_1, \hat{x}_2, ..., \hat{x}_M)$. *Handbook* comment: 1. The symbols used here are those used in this *Handbook*, cf. Section B.2.	*GUM*, §4.1.4, p. 10.
Sensitivity coefficient	Describes how the output estimate y varies with changes in the values of an input estimate, x_i, $i = 1, \ldots, M$.	*GUM*, §5.1.3, p.19; 5.1.4, p. 20
Coverage factor, k:	Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty.	*GUM*, §2.3.6, p. 3. *GUM*, §G.1.3, p. 59.
Level of confidence		*GUM*, Annex G, pp. 59-65.
Standard uncertainty	Uncertainty of the result of a measurement expressed as standard deviation	*GUM*, §2.3.1, p. 3. *GUM*, Chapter 3, pp. 9-18.
Combined standard uncertainty	The standard uncertainty of the result of a measurement, when that result is obtained from the values of a number of other quantities, is termed *combined standard uncertainty*, and denoted u_c. It is the estimated standard deviation associated with the result, and is equal to the positive square root of the combined variance obtained from all variance and covariance components.	*GUM*, §3.3.6, p. 6. *GUM* §4.1.5, p. 10.
Expanded uncertainty	Quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.	*GUM*, §2.3.5, p. 3. *GUM* §6.1.2, p. 23. *GUM* Chapter 5, pp. 23-24.
Systematic effect	The effect of a recognized effect of an influence quantity. Note: By [NIS 81, 1994, p. 4], contributions to uncertainty arising from systematic effects are described as “those that remain constant while the measurement is being made, but	*GUM*, §3.2.3, p. 5

		can change if the measurment conditions, method or equipment is altered".	
	Random effect	The effect of unpredictable or stochastic temporal and spatial variations of influence quantities.	*GUM*, §3.2.2, p. 5
	Linearity	Deviation between a calibration curve for a device and a straight line.	[NPD, 2001]
	Type A evaluation (of uncertainty)	Method of evaluation of uncertainty by the statistical analysis of series of observations.	*GUM*, §2.3.2, p. 3
	Type B evaluation (of uncertainty)	Method of evaluation of uncertainty by means other than the statistical analysis of series of observations, e.g. by engineering/scientific judgement.	*GUM*, §2.3.3, p. 3
Miscellaneous	Functional relationship, *f*	In most cases a measurand *y* is not measured directly, but is determined from *M* other quantities $x_1, x_2, ..., x_M$ through a functional relationship, $y = f(x_1, x_2, ..., x_M)$	*GUM*, §4.1.1 and §4.1.2, p. 9

B.2 Symbols for expressing uncertainty

In general, the following symbols are used in the present *Handbook* for expressing quantities and uncertainties:

$\hat{x}_i$: an *estimated value* (or simply an *estimate*) of an input quantity, x_i,

$\hat{y}$: an *estimated value* (or simply an *estimate*) of an output quantity, y,

$u(\hat{x}_i)$: the *standard uncertainty* of an input estimate, $\hat{x}_i$,

$u_c(\hat{y})$: the *combined standard uncertainty* of an output estimate, $\hat{y}$,

$U(\hat{y})$: the *expanded uncertainty* of an output estimate, $\hat{y}$:

$$U(\hat{y}) = k \cdot u_c(\hat{y}),$$

E_x : the *relative standard uncertainty* of an input estimate, $\hat{x}_i$:

$$E_x = \frac{u(\hat{x}_i)}{|\hat{x}_i|}$$

E_y : the *relative combined standard uncertainty* of an output estimate[120], $\hat{y}$:

$$E_y = \frac{u_c(\hat{y})}{|\hat{y}|}$$

With four exceptions (see Table B.2 and points (1)-(4) below), the symbols used for expression of uncertainty are those used by the *GUM* [ISO, 1995a, §4.1.5 and

120 For simplicity in notatation, and since it should not cause confusion here, the same symbol, E_y, is used for both types of relative (i.e., percentage) standard uncertainties; i.e., relative standard uncertainty, and relative combined standard uncertainty. In each case it will be noted in the text which type of relative uncertainty that is in question.

§6.2.1], see also [Taylor and Kuyatt, 1994], [EAL-R2, 1997], [EA-4/02, 1999], [ISO/CD 5168, 2000].

(1) With respect to the symbols used for *a quantity and the estimate value of the quantity*, the "conventions" of the *GUM* are not followed exactly, mainly for practical reasons[121]. Here, both capital and small letters are used for input quantites, in order to enable use of common and well-established terminology in USM technology (cf. e.g. [ISO, 1997]) and physics in general, involving both capital and small letters as symbols for input/output quantities. To distinguish between a quantity and the estimate value of the quantity, the above defined terminology has thus been chosen (with the symbol "$\hat{x}$" (the "hat notation") to denote the estimate value of the quantity "x").

(2) With respect to *relative uncertainties*, no specific symbol was used in the *GUM*, other than a notation of the type $u_c(\hat{y})/|\hat{y}|$ (for the relative combined standard uncertainty of an output estimate, $\hat{y}$) [ISO, 1995a, §5.1.6, p. 20]. This notation has been used also in [ISO/CD 5168, 2000]. However, for the present document, a simpler symbol than $u_c(\hat{y})/|\hat{y}|$ has been found to be useful, or even necessary, to avoid unnecessary complexity in writing the expressions for the relative expanded uncertainty of the USM fiscal gas metering station. "E_y" is the symbol for relative uncertainty used by e.g. [ISO, 1997]; [ISO 5168:1978], and has been adopted here[122,123].

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121 In the *GUM* [ISO, 1995, Section 3.1, pp. 9-10], a quantity and an estimate value for the quantity are denoted by capital and small letters, respectively (such as "X" and "x", respectively) (cf. Note 3 to §4.1.1). (Cf. also [NIS 3003, 1995, pp. 16-17], [ISO/CD 5168, 2000, p. 7]).

This notation is considered to be impractical for the present Handbook. For example, in physics, engineering and elsewhere the temperature is uniformly denoted by T, while in the USM community a transit time is commonly denoted by t (cf. e.g. [ISO, 1997]). This is one of several examples where this notation is considered to be impractical.

Moreover, also in the *GUM*, the "*GUM* conventions" are not used consequently. For example, in the illustration examples [ISO, 1995, Annex H, cf. p. 68], the same symbol has been used for a quantity and its estimate, for simplicity in notation.

122 By [EAL-R2, 1997], the notation $w(\hat{x}) = u(\hat{x})/|\hat{x}|$ has been used for the relative standard uncertainty of an estimate $\hat{x}$ (cf. their Eq. (3.11)). [Taylor and Kuyatt, 1994] has proposed to denote relative uncertainties by using a subscript "r" for the word "relative", i.e., $u_r(\hat{x}) \equiv u(\hat{x})/|\hat{x}|$, $u_{c,r}(\hat{y}) \equiv u_c(\hat{y})/|\hat{y}|$ and $U_r \equiv U/|\hat{y}|$ for the relative standard uncertainty, the relative combined standard uncertainty, and the relative expanded uncertainty, respectively (cf. their §D1.4).

123 The "E_y" - notation for relative uncertainties was used also in [Lunde *et al.*, 1997; 2000a].

(3) With respect to the symbol "$U(\hat{y})$, the use of simply "U" has been recommended by the *GUM*. In the present document that would lead to ambiguity, since the expanded uncertainties of four output estimates are considered in the USM uncertainty model: $\hat{q}_v$, $\hat{Q}$, $\hat{q}_m$ and $\hat{q}_e$, cf. Chapters 2 and 3. Hence, the symbols $U(\hat{q}_v)$, $U(\hat{Q})$, $U(\hat{q}_m)$ and $U(\hat{q}_e)$ are used for these, to avoid confusion.

(4) With respect to the symbols used for *dimensional (absolute)* and *dimensionless (relative) sensitivity coefficients*, the *GUM* has recommended use of the symbols c_i and c_i^*, respectively. These symbols are used also by [ISO/CD 5168, 2000]. However, to avoid confusion with the well established notation c used in acoustics for the sound velocity (VOS), the symbols s_i and s_i^* are used here for the dimensional (absolute) and dimensionless (relative) sensitivity coefficients of the output estimate $\hat{y}_i$ to the input estimate $\hat{x}_i$.

In Table B.2, the symbol notation used in the *Handbook* is summarized and compared with the symbol notation recommended by the *GUM*.

Table B.2. Symbol notation used in the *Handbook* in relation to that recommended by the *GUM*.

Term	***GUM symbol***	***Handbook symbol***	***Deviation ?***
Input quantity & estimate value of the input quantity Output quantity & estimate value of the output quantity	Capital and small letters, respectively (" X_i " and " x_i ") Capital and small letters, respectively ("Y" and "y")	" $\hat{x}_i$ " denotes the estimate value of the input quantity " x_i " " $\hat{y}$ " denotes the estimate value of the output quantity "y"	Yes
Standard uncertainty of an input estimate	$u(x_i)$	$u(\hat{x}_i)$	No
Combined standard uncertainty of an output estimate	$u_c(y)$	$u_c(\hat{y})$	No
Relative standard uncertainty of an input estimate Relative combined standard uncertainty of an output estimate	$\frac{u(x_i)}{\lvert x_i \rvert}$ $\frac{u_c(y)}{\lvert y \rvert}$	$E_{x_i} = \frac{u(\hat{x}_i)}{\lvert \hat{x}_i \rvert}$ $E_y = \frac{u_c(\hat{y})}{\lvert \hat{y} \rvert}$	Yes
Expanded uncertainty	U	$U(\hat{y})$	Yes / No
Relative expanded uncertainty	$\frac{U}{\lvert y \rvert}$	$\frac{U(\hat{y})}{\lvert \hat{y} \rvert}$	No
Dimensional (absolute) sensitivity coefficients Dimensionless (relative) sensitivity coefficients	c_i c_i^*	s_i s_i^*	Yes

B.3 Procedure for evaluating and expressing uncertainty

The procedure used here for evaluating and expressing uncertainty is the procedure recommended by the *GUM* [124] [ISO, 1995a, Chapter 7], given as[125]:

1. The (mathematical) *functional relationship* is expressed between the measurand, y, and the input quantities, x_i, on which y depends: $y = f(x_1, x_2, ..., x_M)$, where M is the number of input quantities (in accordance with the *GUM*, Chapter 7, §1)[126]. The function, f, should preferably contain every quantity, including all corrections and correction factors, that can contribute significantly to the uncertainty of the measurement result.

2. $\hat{x}_i$, the *estimated value* of the input quantity, x_i, is determined, either on the basis of a statistical analysis of a series of observations, or by other means (in accordance with the *GUM*, Chapter 7, §2)[127].

3. The *standard uncertainty* $u(\hat{x}_i)$ of each input estimate $\hat{x}_i$ is evaluated; either as Type A evaluation of standard uncertainty (for an input estimate obtained from a statistical analysis of observations), or as Type B evaluation of standard uncertainty (for an input estimate obtained by other means), in accordance with the *GUM*, Chapter 7, §3 (cf. also the *GUM*, Chapter 3; [EAL-R2, 1997], [EA-4/02, 1999]).

If the uncertainty of the input estimate $\hat{x}_i$ is given as an expanded uncertainty, $U(\hat{x}_i)$, this expanded uncertainty may be converted to a standard uncertainty by dividing with the coverage factor, k:

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124 Other documents of interst in this context are e.g. [Taylor and Kuyatt, 1994], [NIS 3003, 1995], [EAL-R2, 1997], [EA-4/02, 1999], [Bell, 1999] and [ISO/CD 5168, 2000], which are all based on (and are claimed to be consistent with) the *GUM*. However, the *GUM* is considered as the authoritative text.

125 The *GUM* procedure is here given in our formulation. The substance is meant to be the same, but the wording may be different in some cases. In case of possible inconsistency or doubt, the text given in Chapter 7 of the *GUM* is authoritative.

126 In the general overview given in Appendix B, the symbols "y" and "x_i" are used for the output and input quantities, respectively. In Chapters 2-4, other symbols are used, which are more in agreement with the general literature on USMs (see also Section B.2).

127 With respect to the estimated value of a quantity x_i (input or output), the "hat" symbol, $\hat{x}_i$, is used here, to distinguish between these. Cf. Section B.2.

$$u(\hat{x}_i) = \frac{U(\hat{x}_i)}{k} \tag{B.1}$$

For example, if $U(\hat{x}_i)$ is given at a 95 % confidence level, and a normal probability distribution is used, k = 2. If the confidence level is 99 %, and a normal probability distribution is used, k = 3. If the confidence level is 100 %, and a rectangular probability distribution is used, converting to standard uncertainty is done by using $k = \sqrt{3}$. When an expanded uncertainty is given as a specific number of standard deviations, the standard uncertainty is achieved by dividing the given expanded uncertainty with the specific number of standard deviations.

4. *Covariances* are evaluated associated with input estimates that are *correlated*, in accordance with the *GUM*, Chapter 7, §4 (cf. also the *GUM*, Section 4.2). For two input estimates $\hat{x}_i$ and $\hat{x}_j$, the covariance is given as

$$u(\hat{x}_i, \hat{x}_j) = u(\hat{x}_i)u(\hat{x}_j)r(\hat{x}_i, \hat{x}_j) \qquad (i \neq j), \tag{B.2}$$

where the degree of correlation is characterised by $r(\hat{x}_i, \hat{x}_j)$, the correlation coefficient between $\hat{x}_i$ and $\hat{x}_j$ (where $i \neq j$ and $|r| \leq 1$). The value of $r(\hat{x}_i, \hat{x}_j)$ may be determined by engineering judgement or based on simulations or experiments. The value is a number between -1 and 1, where $r(\hat{x}_i, \hat{x}_j)$ = 0 represents uncorrelated quantities, and $|r(\hat{x}_i, \hat{x}_j)|$ = 1 represents fully correlated quantities

5. The *result of the measurement* is to be calculated in accordance with the *GUM*, Chapter 7, §5. That is, the estimate, $\hat{y}$, of the measurand, *y*, is to be calculated from the functional relationship, *f*, using for the input quanties the estimates $\hat{x}_i$ obtained in Step 2.

6. The *combined standard uncertainty*, $u_c(\hat{y})$, of the measurement result (output estimate), $\hat{y}$, is evaluated from the standard uncertainties and the covariances associated with the input estimates, in accordance with the *GUM*, Chapter 7, §6 (cf. also the *GUM*, Chapter 4).

$u_c(\hat{y})$ is given as the positive square root of the combined variance $u_c^2(\hat{y})$,

$$u_c^2(\hat{y}) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(\hat{x}_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(\hat{x}_i, \hat{x}_j), \tag{B.3}$$

where N is the number of input esitmates $\hat{x}_i$, i = 1, …, N, and the partial derivatives are the sensitivity coefficients, s_i, i.e.

$$s_i \equiv \frac{\partial f}{\partial x_i}. \tag{B.4}$$

7. The *expanded uncertainty U* is determined by multiplying the combined standard uncertainty, $u_c(\hat{y})$, by a coverage factor, *k*, to obtain

$$U = k \cdot u_c(\hat{y}) \tag{B.5}$$

on basis of the level of confidence required for the uncertainty interval $\hat{y} \pm U$, in accordance with the *GUM*, Chapter 7, §7 (cf. also the *GUM*, Chapter 5 and Annex G).

For example, if *U* is to be given at a 95 % confidence level, and a normal probability distribution is assumed, k = 2. If the confidence level is 99 %, and a normal probability distribution is used, *k* = 3. If the confidence level is 100 %, and a rectangular probability distribution is supposed, *k* = $\sqrt{3}$.

In the program *EMU - USM Fiscal Gas Metering Station*, the coverage factor *k* is set to *k* = 2, cf. Section 1.2, corresponding to a level of confidence of 95.45 % in case of a normal probability distribution of the output estimate, $\hat{y}$ [128]. In the uncertainty caclulations of Chapters 5 and 6, *k* = 2 has also been used

8. The result of the measurement (the output estimate), $\hat{y}$, is to be *reported*, together with its expanded uncertainty, *U*, and the method by which *U* has been obtained, in accordance with the *GUM*, Chapter 7, §7 (cf. also the *GUM*, Chapter 6).

This includes documentation of the value of each input estimate, $\hat{x}_i$, the individual uncertainties $u(\hat{x}_i)$ which contribute to the resulting uncertainty, and the evaluation method used to obtain the reported uncertainties of the output estimate as summarised in steps 1 to 7.

Tables 4.6, 4.8, 4.9, 4.11, 4.13, 4.14, 4.17-4.29 in Chapter 4, as well as Figs. 5.3-5.19 and 5.28-5.31 in Chapter 5, show typical uncertainty budgets used for documentation of the calculated expanded uncertainties.

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128 Note that a coverage factor of *k* = 2 produces an interval corresponding to a level of confidence of 95.45 % while that of *k* = 1.96 corresponds to a level of confidence of 95 %. The calculation of intervals having specified levels of confidence is at best only approximate. The *GUM* justifiably emphasises that for most cases it does not make sense to try to distinguish between e.g. intervals having levels of confidence of say 94, 95 or 96 %, cf. Annex G of the *GUM*. In practice, it is therefore recommended to use *k* = 2 which is assumed to produce an interval having a level of confidence of approximately 95 %. This is also in accordance with NPD regulations [NPD, 2001].

The above procedure (given by steps 1-8), recommended by the *GUM*, serves as a basis for the USM uncertainty model described in Chapter 3, the uncertainty calculations reported in Chapters 4, and the program *EMU - USM Fiscal Gas Metering Station* described in Chapter 5.

In the NPD regulations [NPD, 2001] the uncertainties are specified as relative expanded uncertainties, at a 95 % confidence level (assuming a normal probability distribution), with $k = 2$.

In the formulas which are implemented in the program, the input standard uncertainties, combined standard uncertainties, and the expanded uncertainties, are in many cases expressed as *relative* uncertainties, defined as

$$\frac{u(\hat{x}_i)}{|\hat{x}_i|}, \qquad \frac{u_c(\hat{y})}{|\hat{y}|}, \qquad \frac{U}{|\hat{y}|}, \tag{B.6}$$

respectively.

B.4 Documentation of uncertainty evaluation

According to the *GUM* [ISO, 1995a, Chapter 6], all the information necessary for a re-evaluation of the measurement should be available to others who may need it.

In Chapter 6.1.4 of the *GUM* it is stated that one should:

(1) describe clearly the methods used to calculate the measurement result and its uncertainty from the experimental observations and input data,
(2) list all uncertainty components and document fully how they were evaluated,
(3) present the data analysis in such a way that each of its important steps can be readily followed and the calculation of the reported result can be independently repeated if necessary,
(4) give all corrections and constants used in the analysis and their sources.

The present *Handbook* together with the program *EMU - USM Fiscal Gas Metering Station* should fill essential parts of the documentation requirements (1)-(4) above. A printout of the worksheets used for uncertainty evaluation of the measurand in question, together with the "*Report*" worksheet and the mathematical expressions given in Chapters 2 and 3, may be used in a documentation of the uncertainty evaluation of the metering station.

In addition, the user of the program must himself document the uncertainties used as input to the program[129]. Such documentation may be calibration certificates, data sheets, manufacturer information or other specifications of the metering station.

For uncertainty calculations on fiscal metering stations this requires that every quantity input to the calculations should be fully documented with its value (if needed), and its uncertainty, together with the confidence level and probability distribution. Furthermore, it must be documented that the functional relationships used in the uncertainty calculation programs following the *Handbook* are equal to the ones actually implemented in the metering station. An uncertainty evaluation report should be generated, containing the uncertainty evaluations and copies of (or at least reference to) the documentation described above.

129 In many cases this is a difficult point, especially with respect to some of the USM input uncertainties, such as the integration uncertainty (installation conditions), and the uncertainty of uncorrected systematic transit time effects.

APPENDIX C

SELECTED REGULATIONS FOR USM FISCAL GAS METERING STATIONS

The present *Handbook* relates to USM fiscal gas metering stations designed and operated according to NPD regulations [NPD, 2001]. For fiscal metering of gas using ultrasonic meters, the regulations refer to the NORSOK I-104 national standard and the AGA Report No. 9 as recognised standards ("accepted norm").

As a basis for the uncertainty calculations of the present *Handbook*, a selection of NPD regulations [NPD, 2001] and NORSOK requirements [NORSOK, 1998a] which apply to USM metering stations for sales metering of gas, are summarized in the following (with citations from the respective documents).

Only selected regulations are included here. That is, regulations which are related to uncertainty evaluation and to fiscal gas metering stations based on USMs. The selection is not necessarily complete. For the full regulations it is referred to the above referenced documents.

C.1 NPD regulations (selection)

The following selection of regulations of relevance for USM fiscal gas metering stations are taken from [NPD, 2001].

- ***Section 8, Allowable measuring uncertainty.*** Measurement system, Gas metering for sale and allocation purposes: ±1.0 % of mass, at 95 % confidence level (expanded uncertainty with coverage factor k = 2).

  It shall be possible to document the total uncertainty of the measurement system. An uncertainty analysis shall be prepared for the measurement system within a 95 % confidence level. In the present regulations a confidence level equal to ±2 σ, i.e. a coverage factor k = 2, is used. This gives a confidence level slightly higher than 95 %.

  In respect of the measurement system's *individual components* the following maximum limits apply:

Component	Circuit uncertainty limits	Linearity limits (band)	Uncertainty limits component	Repeatability limits (band)
Ultrasonic flow meter gas (sales - allocation)	1 pulse of 100 000, 0.001 %, at pulse transmission of signal	1.0 % in working range (20:1). Deviation from reference in calibration shall be less than ±1.50 % in working range (20:1) before use of calibration factor	± 0.70 % in working range (20:1) after zero point control	0.50 % in working range (20:1) after zero point control
Pressure measuring	± 0.30 % of measured value in working range	NA	± 0.10 % of measured value in working range	NA
Temperature measuring oil, gas	± 0.30 ^{o}C	NA	± 0.20 ^{o}C	NA
Density measuring gas	± 0.25 % of measured value	NA	± 0.20 % of measured value	NA
Calorific value gas	NA	NA	± 0.15 % of calorific value	NA
Uncertainty computer part for oil and gas	NA	NA	± 0.001 %	NA

- ***Re. Section 8, Allowable measuring uncertainty.*** The basic principles for uncertainty analysis are stated in the ISO "Guide to the expression of uncertainty in measurement" (the Guide).

- ***Section 10, Reference conditions.*** Standard reference conditions for pressure and temperature shall in measuring oil and gas be 101.325 kPa and 15 ^{o}C.

- ***Section 11, Determination of energy content, etc.*** Gas composition from continuous flow proportional gas chromatography or from automatic flow proportional sampling shall be used for determination of energy content.

  With regard to sales gas metering stations two independent systems shall be installed.

- ***Re. Section 11, Determination of energy content, etc.*** Recognized standard for determination of energy content will be ISO 6976 or equivalent. Reference temperature for energy calculation should be 25 ^{o}C / 15 ^{o}C (^{o}C reference temperature of combustion / ^{o}C volume). When continuous gas chromatography is used, recognized standard will be NORSOK I-104.

- ***Section 13, Requirements to the metering system in general.*** The metering system shall be planned and built according to the requirements of the present regulations and in accordance with recognized standards for metering systems.

  On sales metering stations the number of parallel meter runs shall be such that the maximum flow of hydrocarbons can be measured with one meter run out of service, whilst the rest of the meter runs operate within their specified operating range.

  If necessary, flow straighteners shall be installed.

  In areas where inspection and calibration takes place, there shall be adequate protection against the outside climate and vibration.

  The metering tube and associated equipment shall be installed upstream and downstream for a distance sufficient to prevent temperature changes affecting the instruments that provide input signals for the fiscal calculations.

- ***Re. Section 13, Requirements to the metering system in general.*** If, on an allocation metering station with ultrasonic metering a concept based on only one metering tube is selected, there should

be possibilities to check the meter during operation and to have the necessary spare equipment ready for installation in the metering tube.

In gas metering the maximum flow velocity during ultrasonic metering should not exceed 80 percent of the maximum flow rate specified by the supplier.

- ***Section 14, The mechanical part of the metering system.*** It shall be documented that surrounding equipment will not affect the measured signals.

- ***Re. Section 14, The mechanical part of the metering system.*** Re. design of the metering system for hydrocarbons in gas phase, recognized standards are NORSOK I-104, ISO 5167-1, AGA Report no. 9 and ISO 9951.

- ***Section 15, The instrument part of the metering system.*** Pressure, temperature, density and composition analysis shall be measured in such way that representative measurements are achieved as input signals for the fiscal calculations (cf. Section 8).

- ***Re. Section 15, The instrument part of the metering system.*** Recognized standards are NORSOK I-104 and I-105.

  The signals from the sensors and transducers should be transmitted so that measurement uncertainty is minimized. Transmission should pass through as few signal converters as possible. Signal cables and other parts of the instruments loops should be designed and installed so that they will not be affected by electromagnetic interference.

  When density meters are used at the outlet of the metering station, they should be installed at least 8D after upstream disturbance.

  When gas metering takes place, density may be determined by continuous gas chromatography, if such determination can be done within the uncertainty requirements applicable to density measurement. If only one gas chromatograph is used, a comparison function against for example one densitometer should be carried out. This will provide independent control of the density value and that density is still measured when GC is out of operation.

  Measured density should be monitored.

- ***Re. Section 16, The computer part of the metering system.*** Recognized standards are NORSOK I-104 and I-105.

  In ultrasonic measuring the computer part should contain control functions for continuous monitoring of the quality of the measurements. It should be possible to verify time measurements.

- ***Re. Section 19, General.*** When equipment is taken into use, the calibration data furnished by the supplier may be used, if they are having adequate traceability and quality. If such is not the case, the equipment should be recalibrated by a competent laboratory. By competent laboratory is meant a laboratory which has been accredited as mentioned in recognized standard EN 45000/ISO 17025, or in some other way has documented competence and ensures traceability to international or national standards.

- ***Section 20, Calibration of mechanical part.*** The mechanical parts critical to measurement uncertainty shall be measured or subjected to flow calibration in order to document calibration curve.

  The assembled fluid metering system shall be flow tested at the place of manufacture and flow meter calibration shall be carried out.

- ***Re. Section 20, Calibration of mechanical part.*** The checks referred to will for example be measuring of critical mechanical parameters by means of traceable equipment.

The linearity and repeatability of the flow meters should be tested in the highest and lowest part of the operating range, and at three points naturally distributed between the minimum and the maximum values.

- ***Section 21, Calibration of instrument part.*** The instrument loops shall be calibrated and the calibration results shall be accessible.

- ***Section 23, Maintenence.*** The equipment which is an integral part of the metering system, and which is of significant importance to the measuring uncertainty, shall be calibrated using traceable equipment before start of operation, and subsequently be maintained to that standard.

- ***Section 25, Operating requirements for flow meters.*** In case of ultrasonic flow measurement of gas the condition parameters shall be verified.

  During orifice plate gas measuring or or ultrasonic gas measuring the meter tupes shall be checked if there is indication of change in internal surface.

- ***Re. Section 25, Operating requirements for flow meters.*** Ultrasonic flow meters for gas should be checked after pressurization and before they are put into operation to verify sound velocity and zero point for each individual sound path. Deviation limits for the various parameters shall be determined before start-up.

  Recalibration should be carried out if the meter has a poor maintenance history.

  Sound velocity and the velocity of each individual sound path should be followed up continuously in order to monitor the meter.

- ***Section 26, Operation requirements for instrument part.*** The calibration methods shall be such that systematic measurement errors are avoided or are compensated for.

  Gas densitometers shall be verified against calculated density or other relevant method.

- ***Section 28, Documentation prior to start-up of the metering system.*** Prior to start-up of the metering system, the operator shall have the following documents available: ……., (g) uncertainty analysis.

- ***Section 29, Documentation relating to the metering system in operation.*** Correction shall be made for documented measurement errors. Correction shall be carried out if the deviation is larger than 0.02 % of the total volume. If measurement errors have a lower percentage value, correction shall nevertheless be carried out when the total value of the error is considered to be significant.

  If there is doubt as to the time at which a measurement error arose, correction shall apply for half of the maximum possible time span since it could bave occurred.

- ***Section 30, Information.*** The Norwegian Petroleum Directorate shall be informed about : …, (b) measurement errors, (c) when fiscal measurement data have been corrected based upon calculations, …, etc.

- ***Section 31, Calibration documents.*** Description of procedure during calibration and inspection, as well as an overview of results where measurement deviation before and after calibration is shown, shall be documented. This shall be available for verification at the place of operation.

C.2 NORSOK I-104 requirements (selection)

The NPD regulations for fiscal metering of gas [NPD, 2001] refer to NORSOK I-104 [NORSOK, 1998a] as an accepted norm. The following selection of functional and technical requrements are taken from NORSOK I-104.

- ***§4.1, General.*** Fiscal measurement systems for hydrocarbon gas include all systems for:
  - Sales and allocation meassurement of gas,
  - Measurement of fuel and flare gas,
  - Sampling,
  - Gas chromatograph.

- ***§4.2, Uncertainty.*** Uncertainty limits for sales and allocation measurement (expanded uncertainty with a coverage factor k = 2): ±1.0 % of standard volume (other units may be requested (project specific), e.g. mass, energy, etc.). (Class A.)

  The uncertainty figure shall be calculated for each component and accumulated for the total system in accordance with the following reference document: Guide to the Expression of Uncertainty in Measurement.

- ***§4.4, Calibration.*** All instruments and field varibles used for fiscal calculations or comparison with fiscal figures shall be traceably calculated by an accredited laboratory to international/national standards.

  All geometrical dimensions used in fiscal calculations shall be traceably measured and certified to international/national standards.

- ***§5.1.3, Equipment / schematic.*** The measurement system shall consist of:
  - A mechanical part, including the flow meter,
  - An instrument part,
  - A computer part performing calculations for quantity, reporting and control functions.

- ***§5.1.7, Maintenance requirements:***

  §5.1.7.2, Calibration. If it is impossible to calibrate the meter at the relevant process conditions, the meter shall at least be calibrated for the specific flow velocity range.

- ***§5.1.8, Layout requirements.*** Ultrasonic flow meters shall not be installed in the vicinity of pressure reduction systems (valves, etc.) which may affect the signals.

- ***§5.2.2, Mechanical part:***

  §5.2.2.1, Sizing. The measurement system shall be designed to measure any expected flow rate with the meters operating within 80 % of their standard range (not extended).

  §5.2.2.4, Flow meter designs / ultrasonic meters. The number of paths for ultrasonic meters shall be determined by required uncertainty limits.

  All geometric dimensions of the ultrasonic flow meter that affect the measurement result shall be measured and certified using traceable equipment, at know temperatures. The material constants shall be available for corrections.

  In order for the ultrasonic meter to be accepted and considered to be of good enough quality the maximum deviation from the reference during flow calibration shall be less than ±1.50 %. The linearity shall be better than ±1.0 % (band) and the repeatablity shall be better than ±0.5 % (band).

These requirements are applicable after application of zero flow point calibration but before application of any correction factors, for flow velocities above 5 % of the maximum measuring range.

For the meter run, the minimum straight upstream length shall be 10 ID. The minimum straight downstream length shall be 3 ID. Flow conditioner of a recognized standard shall be installed, unless it is verified that the ultrasonic meter is not influenced by the layout of the piping upstream or downstream, in such a way that the overall uncertainty requirements are exceeded.

§5.2.2.7, Thermal insulation. The ultrasonic flow meter with associated meter tube should be thermally insulated upstream and downstream including temperature measurement point, in order to reduce temperature gradients.

- ***§5.2.3, Instrument part:***

§5.2.3.1, Location of sensors: Pressure and temperature shall be measured in each of the meter runs. Density shall be measured by at least two densitometers in the metering station. The density measurement device shall be installed so that representative measurements are achieved. Pressure and temperature measurement shall be measured as close as possible to the density measurement.

§5.2.3.5, Temperature loop. For fiscal measurement applications the smart temperature transmitter and Pt 100 element should be two separate devices where the temperature transmitter shall be installed in an instrument enclosure connected to the Pt 100 element via a 4-wire system. Alternatively, the Pt 100 element and temperature transmitter may be installed as one unit where the temperature transmitter is head mounted onto the Pt 100 element (4- or 3-wire system).
The Pt 100 element should as minimum be in accordance to EN 60751 tolerance A.

The temperature transmitter and Pt 100 element shall be calibrated as one system where the Pt 100 element's curve-fitted variables shall be downloaded to the temperature transmitter before final accredited calibration. The total uncertainty for the temperature loop shall be better than ±0.15 ^{o}C.

§5.2.3.7, Direct density measurement. Continuous measurement of density is required. The density shall be measured by the vibrating element technique. Density calculation and calibration shall be in accordance with company practice. The density shall be corrected to the conditions at the fiscal measurement point. If density is of the by-pass type temperature compensation shall be applied.

The uncertainty (expanded uncertainty with a coverage factor k = 2) of the complete density circuit, including drift between calibrations, shall not exceed ±0.30 % of measured value.

§5.2.3.9, Ultrasonic flow meter. For the ultrasonic flow meter, critical parameters relating to electronics and transducers shall be determined. It shall be possible to verify the quality of the electric signal, which represents the acoustic pulse, by automatic monitoring procedures in the instrument or by connecting external test equipment.

The transducers shall be marked by serial number or similar to identify their location in the meter body, etc. A dedicated certificate stating critical parameters shall be attached.

- ***§5.2.4, Computer part:***

§5.2.4.5, Calculations. The computer shall calculate flow rates and accumulated quantities for
(a) Actual volume flow,
(b) Standard volume flow,
(c) Mass flow, and
(d) Energy flow (application specific).
All calculations shall be performed to full computer accuracy. (No additional truncation or rounding.)

The interval between each cycle for computation of instantaneous flow shall be less than 10 seconds.

APPENDIX D

PRESSURE AND TEMPERATURE CORRECTION OF THE USM METER BODY

The present appendix gives the theoretical basis of the model for pressure and temperature correction of the USM meter body given by Eqs. (2.12)-(2.17).

Another objective of the present appendix is to show that Eqs. (2.12)-(2.17) are practically equivalent to Eqs. (2.20)-(2.22), for USMs where all inclination angles are ±45°, and to evaluate the error of Eqs. (2.20)-(2.22) for inclination angles of practical interest for current USMs, 40° - 60°.

Having defined the notation and most of the quantities involved in Chapter 2, take as starting point the temperature and pressure expansion of the inner radius of the meter body given by Eqs. (2.13) and (2.16)-(2.18), i.e. (cf. e.g. [Lunde et al., 1997, 2001], [AGA-9, 1998])

$$R \approx K_T K_P R_0 \ , \qquad \text{where} \quad K_T \equiv 1 + \alpha \Delta T_{dry} \ , \qquad K_P \equiv 1 + \beta \Delta P_{dry} \ . \tag{D.1}$$

In the following, the influence of temperature and pressure on the other geometrical quantities of the meter body are examined, and the consequences for Eq. (2.12) addressed.

Temperature change

First, consider a **temperature change** only, ΔT_{dry}. The consequences for the geometrical quantities are illustrated in Fig. D.1. From the figure one has

$$y_{i0} = R_0 \cos\theta_{i0} \tag{D.2}$$

$$D_{i0} = 2\sqrt{R_0^2 - y_{i0}^2} \tag{D.3}$$

$$L_{i0} = \frac{D_{i0}}{\sin\phi_{i0}} \tag{D.4}$$

$$x_{i0} = \frac{D_{i0}}{\tan\phi_{i0}} \tag{D.5}$$

where D_{i0} is the chord length of path no. *i*, and subscript "*0*" is used to denote the relevant geometrical quantity at dry calibration conditons.

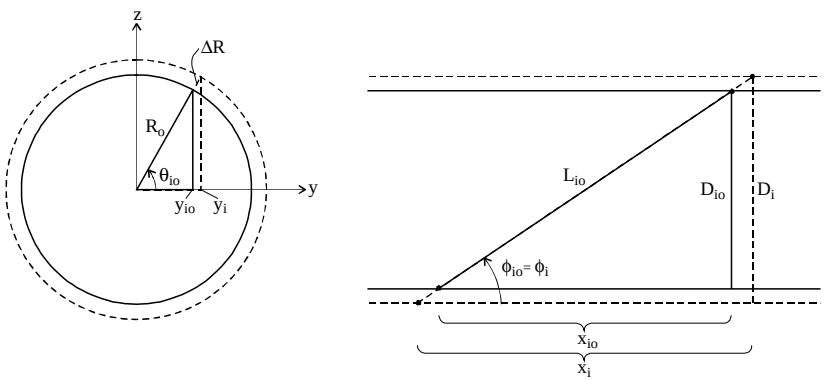

Fig. D.1 Geometry for acoustic path no. *i* in the USM, showing changes with temperature, *T* (schematically).

Now, for a temperature change ΔT_{dry} only, it is assumed here - as a simplification, depending on the type of pipe support for the meter body - that the meter body will expand equally in the axial and radial directions[130]. In this case the angles remain unchanged, cf. Fig. D.1, i.e.

$$\theta_i = \theta_{i0} \quad \text{and} \quad \phi_i = \phi_{i0} \quad . \tag{D.6}$$

Consequently, one obtains

$$y_i = R\cos\theta_i \approx K_T R_0 \cos\theta_{i0} = K_T y_{i0} \ , \tag{D.7}$$

$$D_i = 2\sqrt{R^2 - y_i^2} \approx 2\sqrt{K_T^2 R_0^2 - K_T^2 y_{i0}^2} = K_T 2\sqrt{R_0^2 - y_{i0}^2} = K_T D_{i0} \ , \tag{D.8}$$

$$L_i = \frac{D_i}{\sin\phi_i} \approx \frac{K_T D_{i0}}{\sin\phi_{i0}} = K_T L_{i0} \ , \tag{D.9}$$

$$x_i = \frac{D_i}{\tan\phi_i} \approx \frac{K_T D_{i0}}{\tan\phi_{i0}} = K_T x_{i0} \ . \tag{D.10}$$

130 This assumption may be a simplification, depending on the type of pipe support for the meter body. For instance, for nearly "clamped" axial boundary conditions (such as a meter body mounted in an infinitely long pipe), the meter body is not likely to expand much in the axial direction, and the assumption may be poor. At the other extreme, for "free" axial boundary conditions (a finite pipe section with free ends), the meter body is free to expand in the axial direction, and the assumption should be a reasonable one. Between these two extremes is expected to be the case of a meter body mounted in a finite pipe section (between bends).

Pressure change

Next, consider a **pressure change**, ΔP_{dry}. The consequences for the geometrical quantities are illustrated in Fig. D.2. For a uniform internal pressure change, the angle θ_i remains unchanged, i.e.

$$\theta_i = \theta_{i0} \,. \tag{D.11}$$

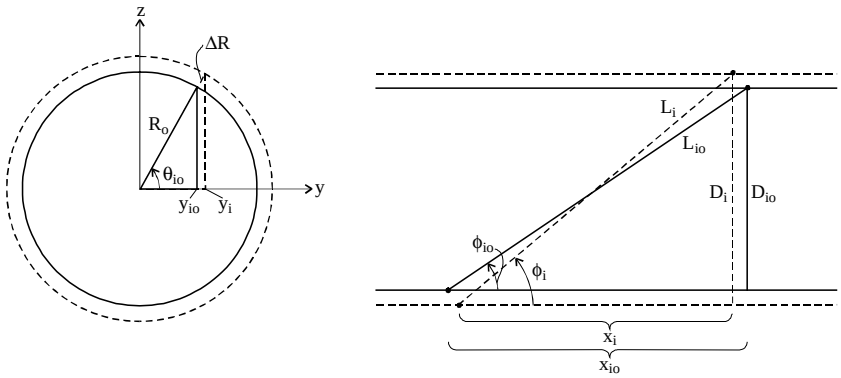

Fig. D.2 Geometry for acoustic path no. *i* in the USM, showing changes with pressure, *P* (schematically). Illustrated here - without loss of generality - for a cylindrical pipe section model (ends free), with axial contraction.

Consequently, one obtains

$$y_i = R\cos\theta_i \approx K_P R_0 \cos\theta_{i0} = K_P y_{i0} \,, \tag{D.12}$$

$$D_i = 2\sqrt{R^2 - y_i^2} \approx 2\sqrt{K_P^2 R_0^2 - K_P^2 y_{i0}^2} = K_P 2\sqrt{R_0^2 - y_{i0}^2} = K_P D_{i0} \,. \tag{D.13}$$

With respect to displacement in the axial direction, x_i, different models for the meter body pressure expansion / contraction are in use (cf. Table 2.6), and these give different results for the axial displacement. However, the different models can be treated here conveniently in one single description. Let

$$x_i = K_P^* x_{i0} \,, \qquad \text{where} \quad K_P^* \equiv 1 + \beta^* \Delta P_{dry} \,, \tag{D.14}$$

is the correction factor for the for the *axial* displacement of the USM meter body, and β^* is the coefficient of linear pressure expansion for the *axial* displacement[131].

131 For the cylindrical pipe section model (with free ends), one has $\beta^* = R_0/Yw$ and $\beta^* = -R_0\sigma/Yw$ [Roark, 2001, p. 592], so that $\beta^*/\beta = -\sigma$. For the cylindrical tank model (with ends capped), one has $\beta^* = (R_0/Yw)(1-\sigma/2)$ and $\beta^* = (R_0/Yw)(0.5-\sigma)$ [Roark, 2001, p. 593], so that $\beta^*/\beta = (1-2\sigma)/(2-\sigma)$.

One needs to express x_i in terms of K_P instead of K_P^*. For this purpose, the ratio of the two correction factors is written as

$$C \equiv \frac{K_P^*}{K_P} = \frac{1+\beta^* \Delta P_{dry}}{1+\beta \Delta P_{dry}} \approx 1-\left(\beta-\beta^*\right)\Delta P_{dry} = 1-\left(1-\frac{\beta^*}{\beta}\right)(K_P-1), \qquad \text{(D.15)}$$

where C is a number very close to unity. Consequently, Eq. (D.14) can be written as

$$x_i = K_P C x_{i0} \ , \qquad \text{(D.16)}$$

leading to

$$\begin{aligned} L_i &= \sqrt{x_i^2 + D_i^2} \approx \sqrt{K_P^2 C^2 x_{i0}^2 + K_P^2 D_{i0}^2} = \sqrt{K_P^2\left[1-\left(\beta-\beta^*\right)\Delta P_{dry}\right]^2 x_{i0}^2 + K_P^2 D_{i0}^2} \\ &\approx \sqrt{K_P^2\left[1-2\left(\beta-\beta^*\right)\Delta P_{dry}\right]x_{i0}^2 + K_P^2 D_{i0}^2} = K_P L_{i0}\sqrt{1-2\frac{x_{i0}^2}{L_{i0}^2}\left(\beta-\beta^*\right)\Delta P_{dry}} \qquad \text{(D.17)} \\ &= K_P L_{i0}\sqrt{1-2\cos^2\phi_{i0}\left(\beta-\beta^*\right)\Delta P_{dry}} \end{aligned}$$

and

$$\tan\phi_i = \frac{D_i}{x_i} \approx \frac{K_P D_{i0}}{K_P C x_{i0}} = \frac{\tan\phi_{i0}}{C} \ . \qquad \text{(D.18)}$$

<u>Combined temperature and pressure change</u>

Hence, for a combined temperature and pressure change, one has

$$y_i \approx K_T K_P y_{i0} \ , \qquad \text{(D.19)}$$

$$D_i \approx K_T K_P D_{i0} \ , \qquad \text{(D.20)}$$

$$x_i \approx K_T K_P C x_{i0} \qquad \text{(D.21)}$$

$$\phi_i \approx \tan^{-1}\left(\frac{\tan\phi_{i0}}{C}\right) \ , \qquad \text{(D.22)}$$

For steel (σ = 0.3), the ends-free model gives $\beta^*/\beta = -0.3$, and the ends-capped model gives $\beta^*/\beta = 0.235$. The two models thus have different sign, where the former model gives axial contracton, and the latter model gives axial expansion.

For the infinitely long pipe model (ends clamped, with no axial displacement), one has $\beta^* = 0$ so that $\beta^*/\beta = 0$.

The parameters involved are defined in Section 2.3.4 (cf. Table 2.6).

$$L_i \approx K_T K_P L_{i0} \sqrt{1 - 2\cos^2 \phi_{i0} \left(\beta - \beta^*\right) \Delta P_{dry}} \ . \tag{D.23}$$

Eqs. (D.19)-(D.23) should constitute a relatively general model for the effect of pressure and temperature on the USM meter body, accounting for any model being used for the radial and axial pressure expansion / contraction of the meter body (i.e. for β and β^*, respectively). These expressions form the basis of Section 2.3.4.

<u>Meters with inclination angles equal to ±45°</u>

From the above analysis, the validity of the commonly used expression for q_{USM} given by Eqs. (2.20)-(2.22) may be evaluated, as addressed in the following.

From Eqs. (D.1), (D.21) and (D.23) one obtains

$$\begin{aligned} \frac{R^2 L_i^2}{x_i} &\approx \frac{\left(K_T K_P R_0\right)^2 \left(K_T K_P L_{i0} \sqrt{1 - 2\cos^2 \phi_{i0} \left(\beta - \beta^*\right) \Delta P_{dry}}\right)^2}{K_T K_P C x_{i0}} \\ &= \frac{{R_0}^2 L_{i0}^2}{x_{i0}} K_T^3 K_P^3 \frac{1 - 2\cos^2 \phi_{i0} \left(\beta - \beta^*\right) \Delta P_{dry}}{1 - B(K_P - 1)} \\ &= \frac{{R_0}^2 L_{i0}^2}{x_{i0}} K_T^3 K_P^3 \left[1 + (1 - 2\cos^2 \phi_{i0}) \left(\beta - \beta^*\right) \Delta P_{dry}\right] \end{aligned} \tag{D.24}$$

For USMs for which all inclination angles are ±45°, i.e. ϕ_{i0} = ±45°, i = *1*, …, *N*, one has $1 - 2\cos^2 \phi_{i0} = 0$, and Eq. (D.24) reduces to

$$\frac{R^2 L_i^2}{x_i} \approx \frac{{R_0}^2 L_{i0}^2}{x_{i0}} K_T^3 K_P^3 \ , \qquad i = 1, \ldots, N, \tag{D.25}$$

so that insertion into Formulation C given in Table 2.5 leads to

$$q_{USM} = \pi R^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i} + 1) L_i^2 (t_{1i} - t_{2i})}{2 x_i t_{1i} t_{2i}} \approx K_T^3 K_P^3 q_{USM,0} \tag{D.26}$$

where

$$q_{USM,0} \equiv \pi R_0^2 \sum_{i=1}^{N} w_i \frac{(N_{refl,i} + 1) L_{i0}^2 (t_{1i} - t_{2i})}{2 x_{i0} t_{1i} t_{2i}} \tag{D.27}$$

is Formulation C with the “dry calibration” values for the geometrical quantities inserted.

It has thus been shown that for meters with inclination angles equal to ±45°, Formulation C of Table 2.5, in combination with Eqs. (2.13)-(2.17), lead exactly to Eq. (D.26), which is the same as Eqs. (2.20)-(2.22). That means, the relationship has been shown for formulation C.

Since formulations A, B, C and D, given by Table 2.5, are equivalent (can easily be derived from one another), it follows that Eqs. (2.20)-(2.22) apply to all the four formulations.

Meters with inclination angles different from 45°

Current USMs use inclination angles in the range 40 to 60°, approximately. To evaluate whether Eq. (D.26) is a good approximation also for such inclination angles, the relative error made by using Eq. (D.26) is investigated. From Eq. (D.24) this relative error is equal to

$$(1-2\cos^2\phi_{i0})(\beta-\beta^*)\Delta P_{dry}, \qquad \text{(D.28)}$$

which for the three meter body expansion models becomes

$$\begin{aligned}
&(1-2\cos^2\phi_{i0})\beta(1+\sigma)\Delta P_{dry}, && \text{for the cylindrical pipe section model (ends free),}\\
&(1-2\cos^2\phi_{i0})\beta\Delta P_{dry}, && \text{for the infinite cylindr. pipe model (ends clamped),}\\
&(1-2\cos^2\phi_{i0})\beta\frac{1+\sigma}{2-\sigma}\Delta P_{dry}, && \text{for the cylindrical tank model (ends capped),}
\end{aligned} \qquad \text{(D.29)}$$

respectively, where β is given in Table 2.6 for two of the models considered here (ends free and ends capped).

For example, consider the ends-free model, steel (σ = 0.3), and a "worst case" example with ϕ_{i0} = 60° for path no. *i*. The relative error made by using Eq. (D.26) is then $\frac{1}{2}(1+\sigma)\beta\Delta P_{dry} = \frac{1}{2}\cdot 1.3\cdot\beta\Delta P_{dry}$, which is typically of the order of $6\cdot 10^{-6}\cdot\Delta P_{dry}$. (Here, ΔP_{dry} is given in bar, and β is given in Table 2.6.) For the 12" USM data given in Table 4.3, pressure deviations of e.g. ΔP_{dry} = 10 and 100 bar yield relative errors in flow rate of about $6\cdot 10^{-5}$ = 0.006 % and $6\cdot 10^{-4}$ = 0.06 %, respectively.

Consequently, for moderate pressure deviations ΔP_{dry} (a few tens of bar), such errors can be neglected, and Eq. (D.26) may be used also for inclination angles in the range 40° to 60°. However, for large pressure deviations ΔP_{dry} (100 bars or more), such er-

rors may *not* be negligible[132], and Eq. (D.26) represents an approximation which is not so good for inclination angles approaching 60°.

Hence, for such pressure deviations and inclination angles, Eq. (2.20) represents only an approximation (with respect to separation of K_P). It should be noted, however, that K_T can be separated out in all cases.

132 The NPD regulations [NPD, 2001] state that "Correction shall be made for documented measurement errors. Correction shall be carried out if the deviation is larger than 0.02 % of the total volume." (cf. section C.1).

APPENDIX E

THEORETICAL BASIS OF UNCERTAINTY MODEL

The present appendix gives the theoretical basis and mathematical derivation of the uncertainty model for the USM fiscal gas metering station which is summarized in Chapter 3.

As described in Section 1.4, the present appendix is included essentially for documentation of the theoretical basis of the uncertainty model. For practical use of the Handbook and the computer program *EMU - USM Fiscal Gas Metering Station*, it is not necessary to read Appendix E. Chapter 3 is intended to be self-contained in that respect.

For convenience and compactness the detailed derivation of the uncertainty model is given here for the actual USM volumetric flow rate measurement only, q_v, given by Eqs. (2.1) and (2.9) for the functional relationship, and Eqs. (3.1) and (3.6) for the relative expanded uncertainty, respectively.

The uncertainty models for the volumetric flow rate at standard conditions, Q, the mass flow rate, q_m, and the energy flow rate, q_e, are then obtained easily from the uncertainty model derived for the volumetric flow rate measurement, q_v, simply by including the uncertainties of P, T, Z/Z_0 (for Q), ρ (for q_m), and P, T, Z/Z_0 and H_s (for q_e), as given by Eqs. (3.7)-(3.9), respectively (cf. the footnote accompanying Eqs. (2.1)-(2.4)).

The overall uncertainty model for the fiscal gas metering station is derived first, cf. Section E.1. The uncertainty model for the un-flow-corrected USM is then given in Section E.2. In Section E.3 the results of Section E.2 are combinded with the uncertainty model for the gas metering station, so that the model accounts for the USM flow calibration, the USM field measurement, the flow calibration laboratory, etc.

On basis of the detailed derivations given here, Chapter 3 summarizes the expressions which are implemented and used in the program *EMU – USM Fiscal Gas Metering Station*.

E.1 Basic uncertainty model for the USM fiscal gas metering station

For the volumetric flow rate measurement, the functional relationship is given by Eq. (2.1), which is equivalent to Eq. (2.9). The various quantities involved are defined in Section 2.2.

Since $q_{USM,j}$ and q_{USM} are measurement values obtained using *the same meter* (at the flow laboratory and in the field, respectively), $q_{USM,j}$ and q_{USM} are partially correlated. That is, some contributions to $q_{USM,j}$ and q_{USM} are correlated, while others are uncorrelated. This partial correlation has to be accounted for in the uncertainty model.

There are several ways to account for the partial correlation between $q_{USM,j}$ and q_{USM} [133]. The method used here (for the purpose of deriving the uncertainty model) is to decompose $q_{USM,j}$ and q_{USM} into their correlated and uncorrelated parts, i.e.:

$$q_{USM,j} = q^{U}_{USM,j} + q^{C}_{USM,j}\,, \tag{E.1}$$

$$q_{USM} = q^{U}_{USM} + q^{C}_{USM}\,, \tag{E.2}$$

respectively[134].

Here, $q^{C}_{USM,j}$ and q^{C}_{USM} are those parts of the estimates $q_{USM,j}$ and q_{USM} which are assumed to be mutually *correlated*. They are associated with standard uncertainties

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133 Three possible approaches to account for the partial correlation between $q_{USM,j}$ and q_{USM} are addressed in Appendix F. This includes the approach used here, and its relationship to and equivalence with the approach recommended by the *GUM*, as given by Eqs. (B.2)-(B.3).

134 This method of decomposition into correlated and uncorrelated parts (of partially correlated quantities), such as used in Eqs. (E.1)-(E.2), has not been found to be mentionened in other documents on uncertainty evaluation, such as e.g. [ISO, 1995], [EAL-R2, 1997], [ISO/CD 5168, 2000], [NIS, 1995], [Taylor and Kuyatt, 1994]. However, the method was applied succesfully in [Lunde *et al.*, 1997], and a variant of this approach (although for a different type of problem) has been used in [EA-4/02, 1999, Appendix G, p. 23 (D5)].

This method has its main advantage when partial correlations are involved, so that the correlation coefficient, *r*, involved in the ”covariance method” recommended by the *GUM* (cf. Eqs. (B.2)-(B.3)), is different from 0 and ±1 ($|r| < 1$). The evaluation of *r* may then in practice be difficult. However, by using the ”decomposition method”, the inclusion of the correlation coefficient *r* in the evaluation of the standard uncertainty of the measurand is avoided. In fact, the ”decomposition method” used here and ”covariance method” recommended by the *GUM* are equivalent, as shown in Appendix F.

which are due to systematic effects. $q^U_{USM,j}$ and q^U_{USM} are those parts of the estimates $q_{USM,j}$ and q_{USM} which are assumed to be mutually *uncorrelated.*

Formally[135], thus, by inserting Eqs. (E.1)-(E.2), Eq. (2.9) can be written as

$$q_v = 3600 \cdot K_{dev,j} q_{ref,j} \frac{q^U_{USM} + q^C_{USM}}{q^U_{USM,j} + q^C_{USM,j}} \quad . \tag{E.3}$$

From Eq. (E.3), the combined variance of the estimate $\hat{q}_v$ is given as

$$\begin{aligned} u_c^2(\hat{q}_v) = & \left[\frac{\partial \hat{q}_v}{\partial \hat{K}_{dev,j}} u_c(\hat{K}_{dev,j})\right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{ref,j}} u_c(\hat{q}_{ref,j})\right]^2 \\ & + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}^U_{USM}} u_c(\hat{q}^U_{USM})\right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}^U_{USM,j}} u_c(\hat{q}^U_{USM,j})\right]^2 \\ & + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}^C_{USM}} u_c(\hat{q}^C_{USM}) + \frac{\partial \hat{q}_v}{\partial \hat{q}^C_{USM,j}} u_c(\hat{q}^C_{USM,j})\right]^2 \\ & + \left[u(\hat{q}_{comm})\right]^2 + \left[u(\hat{q}_{flocom})\right]^2 \end{aligned} \tag{E.4}$$

where

$u_c(\hat{q}_v) \equiv$ combined standard uncertainty of the output volumetric flow rate estimate, $\hat{q}_v$,

$u_c(\hat{K}_{dev,j}) \equiv$ combined standard uncertainty of the deviation factor estimate, $\hat{K}_{dev,j}$,

$u_c(\hat{q}_{ref,j}) \equiv$ combined standard uncertainty of the reference measurement, $\hat{q}_{ref,j}$, at test flow rate no. *j* (representing the uncertainty of the flow calibration laboratory),

$u_c(\hat{q}^U_{USM}) \equiv$ combined standard uncertainty of the estimate $\hat{q}^U_{USM}$ (representing the USM uncertainty contributions in field operation, which are *uncorrelated* with flow calibration),

135 Note that the decomposition of each of the flow rate measurements $q_{USM,j}$ and q_{USM} into two terms, a correlated and an uncorrelated term, as made in Eqs. (E.1)-(E.3), is used ***only*** for the purpose of developing the uncertainty model. This type of description, with such a decomposition of the flow rate measurements, is of course never used in any gas metering station or USM algorithm.

$u_c(\hat{q}^U_{USM,j}) \equiv$ combined standard uncertainty of the estimate $\hat{q}^U_{USM,j}$, at test flow rate no. *j* (representing the USM uncertainty contributions in flow calibration, which are *uncorrelated* with field operation),

$u_c(\hat{q}^C_{USM}) \equiv$ combined standard uncertainty of the estimate $\hat{q}^C_{USM}$ (representing the USM uncertainty contributions in field operation, which are *correlated* with flow calibration),

$u_c(\hat{q}^C_{USM,j}) \equiv$ combined standard uncertainty of the estimate $\hat{q}^C_{USM,j}$, at test flow rate no. *j* (representing the USM uncertainty contributions in flow calibration, which are *correlated* with field operation),

$u(\hat{q}_{comm}) \equiv$ standard uncertainty of the estimate $\hat{q}_v$, due to signal communication between USM field electronics and flow computer (e.g. use of analog frequency or digital signal output), in flow calibration and field operation (assembled in on term),

$u(\hat{q}_{flocom}) \equiv$ standard uncertainty of the estimate $\hat{q}_v$, due to flow computer calculations, in flow calibration and field operation (assembled in on term).

As explained in Section B.2, the symbol "$\hat{x}$" (the "hat notation") has been used to denote the estimated value of a quantity "*x*", in order to distinguish between a quantity and the estimated value of the quantity.

The contributions appearing in the third line of Eq. (E.4) represent the contributions to the combined variance from the correlated input estimates $\hat{q}^C_{USM,j}$ and $\hat{q}^C_{USM}$. The other terms represent the contributions from the uncorrelated input estimates. To obtain Eq. (E.4), it has been assumed that the deviation factor estimate $\hat{K}_{dev,j}$ is uncorrelated with the other quantities appearing in Eq. (E.3). This assumption may be questioned, as $\hat{K}_{dev,j}$ may be correlated with possible systematic contributions to $\hat{q}_{ref,j}$, but is not addressed further here.

Now, since it follows from Eq. (E.3) that

$$\begin{aligned}
&\frac{\partial \hat{q}_v}{\partial \hat{K}_{dev,j}} = \frac{\hat{q}_v}{\hat{K}_{dev,j}}, && \frac{\partial \hat{q}_v}{\partial \hat{q}_{ref,j}} = \frac{\hat{q}_v}{\hat{q}_{ref,j}}, \\
&\frac{\partial \hat{q}_v}{\partial \hat{q}^U_{USM}} = \frac{\hat{q}_v}{\hat{q}_{USM}}, && \frac{\partial \hat{q}_v}{\partial \hat{q}^C_{USM}} = \frac{\hat{q}_v}{\hat{q}_{USM}}, \\
&\frac{\partial \hat{q}_v}{\partial \hat{q}^U_{USM,j}} = -\frac{\hat{q}_v}{\hat{q}_{USM,j}}, && \frac{\partial \hat{q}_v}{\partial \hat{q}^C_{USM,j}} = -\frac{\hat{q}_v}{\hat{q}_{USM,j}},
\end{aligned} \tag{E.5}$$

Eq. (E.4) becomes

$$\left[\frac{u_c(\hat{q}_v)}{\hat{q}_v}\right]^2 = \left[\frac{u_c(\hat{K}_{dev,j})}{\tilde{K}_{dev,j}}\right]^2 + \left[\frac{u_c(\hat{q}_{ref,j})}{\hat{q}_{ref,j}}\right]^2 + \left[\frac{u_c(\hat{q}^U_{USM})}{\hat{q}_{USM}}\right]^2 + \left[\frac{u_c(\hat{q}^U_{USM,j})}{\hat{q}_{USM,j}}\right]^2 + \left[\frac{u_c(\hat{q}^C_{USM})}{\hat{q}_{USM}} - \frac{u_c(\hat{q}^C_{USM,j})}{\hat{q}_{USM,j}}\right]^2 + \left[\frac{u(\hat{q}_{comm})}{\hat{q}_v}\right]^2 + \left[\frac{u(\hat{q}_{flocom})}{\hat{q}_v}\right]^2 \tag{E.6}$$

To simplify the notation, Eq. (E.6) can be written more conveniently as

$$\boxed{E^2_{q_v} = E^2_{K_{dev,j}} + E^2_{q_{ref,j}} + E^2_{q^U_{USM}} + E^2_{q^U_{USM,j}} + \left[E_{q^C_{USM}} - E_{q^C_{USM,j}}\right]^2 + E^2_{comm} + E^2_{flocom}} \tag{E.7}$$

where the definitions

$$\begin{aligned}
E_{q_v} &\equiv \frac{u_c(\hat{q}_v)}{|\hat{q}_m|}, \\
E_{K_{dev,j}} &\equiv \frac{u(\hat{K}_{dev,j})}{|\hat{K}_{dev,j}|}, & E_{q_{ref,j}} &\equiv \frac{u(\hat{q}_{ref,j})}{|\hat{q}_{ref,j}|}, \\
E_{q^U_{USM}} &\equiv \frac{u_c(\hat{q}^U_{USM})}{|\hat{q}_{USM}|}, & E_{q^U_{USM,j}} &\equiv \frac{u_c(\hat{q}^U_{USM,j})}{|\hat{q}_{USM,j}|}, \\
E_{q^C_{USM}} &\equiv \frac{u_c(\hat{q}^C_{USM})}{|\hat{q}_{USM}|}, & E_{q^C_{USM,j}} &\equiv \frac{u_c(\hat{q}^C_{USM,j})}{|\hat{q}_{USM,j}|}, \\
E_{comm} &\equiv \frac{u(\hat{q}_{comm})}{|\hat{q}_v|}, & E_{flocom} &\equiv \frac{u(\hat{q}_{flocom})}{|\hat{q}_v|},
\end{aligned} \tag{E.8}$$

have been used, and

$E_{q_v} \equiv$ relative combined standard uncertainty of the output volumetric flow rate estimate, $\hat{q}_v$,

$E_{K_{dev,j}} \equiv$ relative standard uncertainty of the deviation factor estimate, $\hat{K}_{dev,j}$.

$E_{q_{ref,j}} \equiv$ relative standard uncertainty of the reference measurement, $\hat{q}_{ref,j}$ (representing the uncertainty of the flow calibration laboratory),

$E_{q^U_{USM}}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}^U_{USM}$ (the USM uncertainty contributions in field operation, which are *uncorrelated* with flow calibration),

$E_{q^U_{USM,j}}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}^U_{USM,j}$, at test flow rate no. *j* (the USM uncertainty contributions in flow calibration, which are *uncorrelated* with field operation),

$E_{q^C_{USM}}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}^C_{USM}$ (the USM uncertainty contributions in field operation, which are *correlated* with flow calibration),

$E_{q^C_{USM,j}}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}^C_{USM,j}$, at test flow rate no. *j* (the USM uncertainty contributions in flow calibration, which are *correlated* with field operation),

E_{comm} ≡ relative standard uncertainty of the estimate $\hat{q}_v$, due to signal communication between USM field electronics and flow computer (e.g. use of analog frequency or digital signal communication), in flow calibration and field operation (assembled in on term).

E_{flocom} ≡ relative standard uncertainty of the estimate $\hat{q}_v$, due to flow computer calculations, in flow calibration and field operation (assembled in on term).

In the calculation program *EMU - USM Fiscal Gas Metering Station*, $E_{K_{dev,j}}$, $E_{q_{ref,j}}$, E_{comm} and E_{flocom} are input relative uncertainties. The terms related to the USM measurement are analysed in more detail in the following.

E.2 Uncertainty model for the un-flow-corrected USM

Before proceeding with the full uncertainty model for the gas metering station, given by Eq. (E.7), the relative uncertainty contributions $E_{q^U_{USM}}$, $E_{q^U_{USM,j}}$ and $E_{q^C_{USM}}$ - $E_{q^C_{USM,j}}$ need to be associated with particular physical effects and uncertainty contributions in question for the USM. To evaluate this, a model for the uncertainty of the USM is needed.

For this purpose it serves to be useful to *first* use an expression for the uncertainty of a un-flow-corrected USM, in which “all” (or at least most of) the uncertainty contributions are described (Section E.2), and *next* see what uncertainty terms which are

cancelled when applying this model to a flow-corrected USM (cf. Section E.3), so that only deviations relative to flow calibration conditions are accounted for in the model, as well as possible correlation between flow calibration and field operation.

The uncertainty model of the USM is to be meter independent, in the sense that the analysis does not rest on specific meter dependent technologies which possibly might exclude application to some meters. This means that the analysis has to be kept on a relatively general level, so that e.g. meter dependent integration techniques, transit time detection methods, "dry calibration" methods, etc., are not to be accounted for[136].

E.2.1 Basic USM uncertainty model

From [Lunde *et al.*, 1997 (Eqs. (5.3) and (5.18)] and [Lunde *et al.*, 2000a (Eq. (7.2)], it is known that the relative combined standard uncertainty of the *un-flow-corrected* USM volumetric flow rate reading, $\hat{q}_{USM}$, can be modelled as[137, 138, 139]

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136 However, it should be noted that the uncertainty model and the program *EMU - USM Fiscal Gas Metering Station* can be extended to account for meter dependent technologies, such as specific integration techniques, transit time detection methods, "dry calibration" methods, etc. That means, variants of the program can be tailored to described the uncertainty of a specific meter (or meters), used in a gas metering station. Such extension(s) may be of particular interest for meter manufacturer(s), but also for users of that specific meter. Cf. Chapter 6.

137 The derivation of Eq. (E.9) has not been included here since that would further increase the present appendix, and since the detailed derivation is available in [Lunde *et al.*, 1997].

138 One modification has been made here relative to the expression given in [Lunde *et al.*, 1997] (Eqs. (5.3) and (5.18) of that report). In the last term of Eq. (E.9) the expression $\sum_{i=1}^{N}\left[s_{t1i}^{*}E_{t1i,C}^{(n)}+s_{t2i}^{*}E_{t2i,C}^{(n)}\right]^{2}$ has ben replaced by $\left(\sum_{i=1}^{N}\left[s_{t1i}^{*}E_{t1i,C}^{(n)}+s_{t2i}^{*}E_{t2i,C}^{(n)}\right]\right)^{2}$. That means, not only is the correlation between upstream and downstream propagation in path no. *i* accounted for, but also the correlation between the different paths.

It should be noted that this approach represents a simplification. Ideally, terms of both types mentioned above should be accounted for in the uncertainty model, since some effects will be correlated between paths (electronics/transducer delay and Δt-correction (P & T effects, drift), etc.), while others may be uncorrelated between paths (transducer deposits, etc.). The model can easily be extended to account for both correlated and uncorrelated effects between paths, by accountinfg for both types of uncertainty terms. That has not been done here, to avoid a too complex user interface (difficulty in specifying USM input uncertainties). However, after some time, as the USM technology grows more mature, it may be more relevant to include both effects in possible future updates of the *Handbook* and the program *EMU - USM Fiscal Gas Metering Station*. Cf. Chapter 6.

139 In the *GARUSO* model, an expression corresponding to Eq. (E.9) is used (with the modification described in the footnote above). In addition, expressions are given for the various uncertainty terms appearing in Eq. (E.9). In the present *Handbook*, these more detailed expressions are not used, in order to keep the program *EMU - USM Fiscal Gas Metering Station* on a more generic level, to avoid a too high "user treshold" (difficulty in specifying USM input uncertainties).

$$
\begin{aligned}
E_{q_{USM}}^2 \approx\ & E_I^2 + (s_R^* E_R)^2 + \sum_{i=1}^{N}\left[(s_{yi}^* E_{yi})^2 + (s_{\phi i}^* E_{\phi i})^2\right] \\
& + \frac{1}{N_{ave}}\sum_{i=1}^{N}\left[(s_{t1i}^* E_{t1i,U}^{(n,U)})^2 + (s_{t2i}^* E_{t2i,U}^{(n,U)})^2\right] + \sum_{i=1}^{N}\left[(s_{t1i}^* E_{t1i,U}^{(n,C)})^2 + (s_{t2i}^* E_{t2i,U}^{(n,C)})^2\right] \qquad \text{(E.9)} \\
& + \left(\sum_{i=1}^{N}\left[s_{t1i}^* E_{t1i,C}^{(n)} + s_{t2i}^* E_{t2i,C}^{(n)}\right]\right)^2,
\end{aligned}
$$

where the definitions

$$
\begin{aligned}
E_I &\equiv \frac{u(\hat{q}_{USM,I})}{|\hat{q}_{USM}|}, & E_R &\equiv \frac{u_c(\hat{R})}{\hat{R}}, \\
E_{yi} &\equiv \frac{u_c(\hat{y}_i)}{|\hat{y}_i|}, & E_{\phi i} &\equiv \frac{u_c(\hat{\phi}_i)}{|\hat{\phi}_i|}, \\
E_{t1i,U}^{(n,U)} &\equiv \frac{u_c(\hat{t}_{1i,U}^{(n,U)})}{|\hat{t}_{1i}|}, & E_{t2i,U}^{(n,U)} &\equiv \frac{u_c(\hat{t}_{2i,U}^{(n,U)})}{|\hat{t}_{2i}|}, \qquad \text{(E.10)} \\
E_{t1i,U}^{(n,C)} &\equiv \frac{u_c(\hat{t}_{1i,U}^{(n,C)})}{|\hat{t}_{1i}|}, & E_{t2i,U}^{(n,C)} &\equiv \frac{u_c(\hat{t}_{2i,U}^{(n,C)})}{|\hat{t}_{2i}|}, \\
E_{t1i,C}^{(n)} &\equiv \frac{u_c(\hat{t}_{1i,C}^{(n)})}{|\hat{t}_{1i}|}, & E_{t2i,C}^{(n)} &\equiv \frac{u_c(\hat{t}_{2i,C}^{(n)})}{|\hat{t}_{2i}|},
\end{aligned}
$$

have been used for the respective relative standard uncertainties. Here, N_{ave} is the number of time averagings used for a transit time measurement, and

$u(\hat{q}_{USM,I}) \equiv$ standard uncertainty of the estimate $\hat{q}_{USM}$, due to approximating the integral expression for q_{USM} by a numerical integration (i.e. a finite sum over the number of acoustic paths).

$u_c(\hat{R}) \equiv$ combined standard uncertainty of the pipe radius estimate, $\hat{R}$.

$u_c(\hat{y}_i) \equiv$ combined standard uncertainty of the lateral path position estimate, $\hat{y}_i$.

For example, in *GARUSO* the integration uncertainty E_I is calculated from more basic input (e.g. flow velocity profiles (analytical or based on CFD modelling), USM integration method, etc.). In the program *EMU - USM Fiscal Gas Metering Station*, E_I is to be specified and documented by the USM manufacturer. Similar situations apply to e.g. the transit time uncertainties of Eq. (E.9).

It may be noted, however, that such more detailed expressions can be implemented in the program as well, if that should is of interest (e.g. for USM manufacturers), cf. Chapter 6.

$u_c(\hat{\phi}_i) \equiv$ combined standard uncertainty of the inclination angle estimate, $\hat{\phi}_i$.

$u_c(\hat{t}_{1i,U}^{(n,U)}) \equiv$ combined standard uncertainty of $\hat{t}_{1i,U}^{(n,U)}$ (the part of the upstream transit time estimate $\hat{t}_{1i}^{(n)}$ which is *uncorrelated* with respect to *downstream* propagation, and *uncorrelated* with respect to the N_{ave} *upstream* "shots"[140] of path no. *i*)[141].

$u_c(\hat{t}_{2i,U}^{(n,U)}) \equiv$ combined standard uncertainty of $\hat{t}_{2i,U}^{(n,U)}$ (the part of the downstream transit time estimate $\hat{t}_{2i}^{(n)}$ which is *uncorrelated* with respect to *upstream* propagation, and *uncorrelated* with respect to the N_{ave} *downstream* "shots" of path no. *i*).

$u_c(\hat{t}_{1i,U}^{(n,C)}) \equiv$ combined standard uncertainty of $\hat{t}_{1i,U}^{(n,C)}$ (the part of the upstream transit time estimate $\hat{t}_{1i}^{(n)}$ which is *uncorrelated* with respect to *downstream* propagation, and *correlated* with respect to the N_{ave} *upstream* "shots" of path no. *i*).

$u_c(\hat{t}_{2i,U}^{(n,C)}) \equiv$ combined standard uncertainty of $\hat{t}_{2i,U}^{(n,C)}$ (the part of the downstream transit time estimate $\hat{t}_{2i}^{(n)}$ which is *uncorrelated* with respect to *upstream* propagation, and *correlated* with respect to the N_{ave} *downstream* "shots" of path no. *i*).

$u_c(\hat{t}_{1i,C}^{(n)}) \equiv$ combined standard uncertainty of $\hat{t}_{1i,C}^{(n)}$ (the part of the upstream transit time estimate $\hat{t}_{1i}^{(n)}$ which is *correlated* with respect to *downstream* propagation, and *correlated* with respect to the N_{ave} *upstream* "shots" of path no. *i*).

$u_c(\hat{t}_{2i,C}^{(n)}) \equiv$ combined standard uncertainty of $\hat{t}_{2i,C}^{(n)}$ (the part of the downstream transit time estimate $\hat{t}_{2i}^{(n)}$ which is *correlated* with respect to *upstream* propagation, and *correlated* with respect to the N_{ave} *downstream* "shots" of path no. *i*).

Here, E_I is the relative standard "integration uncertainty", i.e. the contribution to the relative combined standard uncertainty of the estimate $\hat{q}_{USM}$, due to use of the *finite-sum integration* formula instead of the integral. E_I accounts for effects of installa-

140 For simplicity, a "shot" refers here to a single pulse in a sequence of *N* pulses being averaged. The superscript "*n*" refers to "shot" no. *n* in the sequence *n* = 1, …, *N*.

141 With respect to transit times and their uncertainties, ***subscripts*** "U" and "C" refer to uncorrelated and correlated estimates, respectively, *with respect to upstream and downstream propagation* (for "shot" no. *n*, at path no. *i*).

Superscripts "U" and "C" refer to uncorrelated and correlated estimates, respectively, *with respect to different "shots"* (for a given propagation direction at path no. *i*).

tion conditions influencing on flow profiles, such as pipe bend configurations, flow conditioners, pipe roughness, etc.

E_R is the relative standard uncertainty of the meter body inner radius estimate, $\hat{R}$. E_R accounts for the instrument (measurement) uncertainty when measuring $\hat{R}$ at "dry calibration" conditions, pressure / temperature effects on $\hat{R}$ (including possible pressure / temperature corrections), and out-of-roundness.

E_{yi} is the relative standard uncertainty of the lateral chord position estimate, $\hat{y}_i$, for path no. *i*. E_{yi} accounts for the instrument (measurement) uncertainty when measuring $\hat{y}_i$ at "dry calibration", and pressure / temperature effects on $\hat{y}_i$ (including possible pressure / temperature corrections).

$E_{\phi i}$ is the relative standard uncertainty of the inclination angle estimate $\hat{\phi}_i$, for path no. *i*. $E_{\phi i}$ accounts for the instrument (measurement) uncertainty when measuring $\hat{\phi}_i$ at "dry calibration", and temperature effects on $\hat{\phi}_i$ (including possible temperature corrections).

$E_{t1i,U}^{(n,U)}$ and $E_{t2i,U}^{(n,U)}$ are the relative combined standard uncertainties of those contributions to the upstream and downstream transit time estimates $\hat{t}_{1i}^{(n)}$ and $\hat{t}_{2i}^{(n)}$, respectively, which are **uncorrelated** with respect to upstream and downstream propagation, and also **uncorrelated** with respect to the N_{ave} "shots" (for a given propagation direction of path no. *i*). They represent random time fluctuation effects[142], such as incoherent noise and turbulence effects (random velocity fluctuations, and random temperature fluctuations).

$E_{t1i,U}^{(n,C)}$ and $E_{t2i,U}^{(n,C)}$ are the relative combined standard uncertainties of those contributions to the upstream and downstream transit time estimates $\hat{t}_{1i}^{(n)}$ and $\hat{t}_{2i}^{(n)}$, respectively, which are **uncorrelated** with respect to upstream and downstream propagation, but **correlated** with respect to the N_{ave} "shots" (for a given propagation direction of path no. *i*). They represent e.g. effects of the clock resolution of the time detection system in the meter, and coherent noise.

$E_{t1i,C}^{(n)}$ and $E_{t2i,C}^{(n)}$ are the relative combined standard uncertainties of those contributions to the upstream and downstream transit time estimates $\hat{t}_{1i}^{(n)}$ and $\hat{t}_{2i}^{(n)}$, respec-

142 By random effects are here meant effects which are ***not*** equal or correlated from "shot" to "shot", and which therefore will be affected significantly by averaging of measurements over time.

tively, which are **correlated**, both with respect to upstream and downstream propagation, and with respect to the N_{ave} "shots" (for a given propagation direction of path no. *i*). They represent systematic effects due to cable/electronics/transducer /diffraction time delay correction (including finite beam effects), possible cavity delay correction, possible transducer deposits, possible beam reflection at the pipe wall (for USMs using reflecting paths) and sound refraction (profile effects on transit times).

$s_R^*, s_{yi}^*, s_{\phi i}^*, s_{t1i}^*, s_{t2i}^*$ are the *relative (non-dimensional) sensitivity coefficients* for the sensitivity of the estimate $\hat{Q}$ to the input estimates $\hat{R}, \hat{y}_i, \hat{\phi}_i, \hat{t}_{1i}, \hat{t}_{2i}$, respectively, given as

$$s_R^* = \frac{1}{\left|\hat{Q}\right|}\sum_{i=1}^{N}\hat{Q}_i\left(2+\frac{1}{1-\left(\hat{y}_i/\hat{R}\right)^2}\right), \quad s_{yi}^* = -sgn(\hat{y}_i)\frac{\hat{Q}_i}{\left|\hat{Q}\right|}\frac{\left(\hat{y}_i/\hat{R}\right)^2}{1-\left(\hat{y}_i/\hat{R}\right)^2}, \tag{E.11a}$$

$$s_{\phi i}^* = -\frac{\hat{Q}_i}{\left|\hat{Q}\right|}\frac{2\left|\hat{\phi}_i\right|}{tan\,2\hat{\phi}_i}, \quad s_{t1i}^* = \frac{\hat{Q}_i}{\left|\hat{Q}\right|}\frac{\hat{t}_{2i}}{\hat{t}_{1i}-\hat{t}_{2i}}, \quad s_{t2i}^* = -\frac{\hat{Q}_i}{\left|\hat{Q}\right|}\frac{\hat{t}_{1i}}{\hat{t}_{1i}-\hat{t}_{2i}}, \tag{E.11b}$$

respectively, where for convenience in notation, the definititions

$$\hat{Q} \equiv \sum_{i=1}^{N}\hat{Q}_i, \qquad \hat{Q}_i \equiv 7200\pi\hat{R}^2\frac{\hat{P}T_0\hat{Z}_0}{P_0\hat{T}\hat{Z}}w_i\frac{(N_{refl}+1)\sqrt{\hat{R}^2-\hat{y}_i^2}(\hat{t}_{1i}-\hat{t}_{2i})}{\hat{t}_{1i}\hat{t}_{2i}\left|sin\,2\hat{\phi}_i\right|}, \tag{E.12}$$

have been used.

The last (sum) term appearing in Eq. (E.9) involves the upstream and downstream relative sensitivity coefficients of path no. i, s_{t1i}^* and s_{t2i}^*. From Eqs. (E.11) these are about equal in magnitude but have opposite signs. Thus, for equal drift in the upstream and downstream transit times - that means, equal relative combined standard uncertainties $E_{t1i,C}^{(n)}$ and $E_{t2i,C}^{(n)}$, the resulting relative combined variance due to such drift will be comparably small and nearly negligible relative to the other (uncorrelated) terms. The partial cancelling effect obtained in USMs through the transit time difference $\hat{t}_{1i}$ - $\hat{t}_{2i}$, is thus accounted for in the uncertainty model through this last (sum) term in Eq. (E.9).

E.2.2 Simplified USM uncertainty model

In order to avoid a too high "user treshold" of the program *EMU - USM Fiscal Gas Metering Station*, with respect to specification of USM input uncertainties, the USM uncertainty model given by Eq. (E.9) can be further simplified with respect to transit time uncertainties, without much loss of generality. Note that this is a simplification relative to the *GARUSO* model [Lunde *et al.*, 1997].

E.2.2.1 Uncorrelated transit time contributions

In the following, it is assumed that $E_{t1i,U}^{(n,U)}$ and $E_{t2i,U}^{(n,U)}$ are about equal, so that they can be replaced by a single relative uncertainty term, here denoted as $E_{t1i,U}^{U}$. That means,

$$E_{t2i,U}^{(n,U)} \approx E_{t1i,U}^{(n,U)} \equiv E_{t1i,U}^{U} \quad . \tag{E.13}$$

This assumption is motivated by the fact that $E_{t1i,U}^{(n,U)}$ and $E_{t2i,U}^{(n,U)}$ are associated with random transit time fluctuation effects on two acoustic signals that are propagating in opposite directions almost simultanously in time (upstream and downstream "shot" no. *n*), see Section E.2.1.

Similarly, it is assumed that $E_{t1i,U}^{(n,C)}$ and $E_{t2i,U}^{(n,C)}$ are about equal, so that they can be replaced by a single relative uncertainty term, to be denoted by $E_{t1i,U}^{C}$, i.e.

$$E_{t2i,U}^{(n,C)} \approx E_{t1i,U}^{(n,C)} \equiv E_{t1i,U}^{C} \quad . \tag{E.14}$$

This assumption is motivated by the fact that $E_{t1i,U}^{(n,C)}$ and $E_{t2i,U}^{(n,C)}$ are associated with random and systematic transit time effects on two acoustic signals that are propagating in opposite directions almost simultanously in time (upstream and downstream "shot" no. *n*), see Section E.2.1.

To further simplify the notation, and without loss of generality, it is then natural to define

$$E_{t1i,U} \equiv \frac{1}{N_{ave}} (E_{t1i,U}^{U})^2 + (E_{t1i,U}^{C})^2 \quad . \tag{E.15}$$

This single relative uncertainty term $E_{t1i,U}$ thus accounts for the effects of turbulence (random velocity fluctuations, and random temperature fluctuations), noise (incoherent and coherent), finite clock resolution, electronics stability (possible random effects), possible random effects in signal detection/processing (e.g. erronous signal

period identification), and power supply variations, on the upstream and downstream transit times of path no. *i* (cf. Section E.2.1).

E.2.2.2 Correlated transit time contributions

For the correlated uncertainties, it is in the following assumed that $E_{t1i,C}^{(n)}$ are about equal for all N_{ave} upstream "shots" of path no. *i*, and simply denoted $E_{t1i,C}$, i.e.

$$E_{t1i,C}^{(n)} \equiv E_{t1i,C} \quad . \tag{E.16}$$

Similarly, it is assumed that $E_{t2i,C}^{(n)}$ are about equal for all N_{ave} downstream "shots" of path no. *i*, and simply denoted $E_{t2i,C}$, i.e.

$$E_{t2i,C}^{(n)} \equiv E_{t2i,C} \quad . \tag{E.17}$$

These assumptions are motivated by the fact that each of $E_{t1i,C}^{(n)}$ and $E_{t2i,C}^{(n)}$ is associated with systematic transit time effects on N_{ave} acoustic signals that are propagating in the same direction almost simultanously in time, see Section E.2.1.

Such systematic effects are e.g. cable/electronics/transducer/diffraction time delay correction, possible cavity delay correction, possible transducer deposits, possible beam reflection at the pipe wall (for USMs using reflecting paths) and sound refraction (profile effects on transit times), cf. Section E.2.1.

E.2.2.3 Simplified USM uncertainty model

Insertion of Eqs. (E.13)-(E.17) into Eq. (E.9), and using the fact that $s_{t1i}^{*} \approx s_{t2i}^{*}$, yields a simplified expression for the relative standard uncertainty of the un-flow-corrected USM,

$$\begin{aligned} E_{q_{USM}}^{2} \approx\ & E_{I}^{2} + (s_{R}^{*}E_{R})^{2} + \sum_{i=1}^{N}\left[(s_{yi}^{*}E_{yi})^{2} + (s_{\phi i}^{*}E_{\phi i})^{2} + 2(s_{t1i}^{*}E_{t1i,U})^{2}\right] \\ & + \left(\sum_{i=1}^{N}\left[s_{t1i}^{*}E_{t1i,C} + s_{t2i}^{*}E_{t2i,C}\right]\right)^{2} \end{aligned} \tag{E.18}$$

As a basis for the identification of uncertainty contributions to be made in Section E.3, a description of the various relative uncertainties appearing in Eq. (E.18) is summarized in Table E.1.

Table E.1. Relative uncertainties involved in the uncertainty model of the *un-flow-corrected* USM.

Uncertainty term	**Description**
E_R	Relative standard uncertainty of the meter body inner radius estimate, $\hat{R}$, due to e.g.: • instrument uncertainty when measuring *R* at "dry calibration" conditions, • *P & T* effects on $\hat{R}$ (including possible *P & T* corrections), • out-of-roundness.
E_{yi}	Relative standard uncertainty of the lateral chord position estimate, $\hat{y}_i$, due to e.g.: • instrument uncertainty when measuring $\hat{y}_i$ at "dry calibration" conditions, • *P & T* effects on $\hat{y}_i$ (including possible *P & T* corrections),
$E_{\phi i}$	Relative standard uncertainty of the inclination angle estimate, $\hat{\phi}_i$, due to e.g.: • instrument uncertainty when measuring ϕ_i at "dry calibration" conditions, • *P* effects on $\hat{\phi}_i$ (including possible *P* correction),
$E_{t1i,U}$	Relative standard uncertainty of those contributions to the transit time estimates $\hat{t}_{1i}$ and $\hat{t}_{2i}$ which are *uncorrelated* with respect to upstream and downstream propagation, such as: • turbulence (transit time fluctuations due to random velocity and temperature fluctuations), • incoherent noise (RFI, pressure control valve (PRV) noise, pipe vibrations, etc.), • coherent noise (acoustic and electromagnetic cross-talk, acoustic reverberation in pipe, etc.) • finite clock resolution, • electronics stability (possible random effects), • possible random effects in signal detection/processing (e.g. erronous signal period identification), • power supply variations.
$E_{t1i,C}$ $(E_{t2i,C})$	Relative standard uncertainty of those contributions to the upstream transit time estimate $\hat{t}_{1i}$ (downstream transit time estimate $\hat{t}_{2i}$) which are *correlated* with respect to upstream and downstream propagation, such as: • cable/electronics/transducer/diffraction time delay, including finite-beam effects (line P and T effects, ambient temperature effects, drift, effects of possible transducer exchange), • *Δt*-correction (line P and T effects, ambient temperature effects, drift, reciprocity, effects of possible transducer exchange), • possible systematic effects in signal detection/processing (e.g. erronous signal period identification) • possible cavity delay correction, • possible deposits at transducer front (lubricants, liquid, wax, grease), • sound refraction (flow profile effects on transit times), • possible beam reflection at the meter body wall.
E_I	Relative standard "integration uncertainty", due to effects of installation conditions influencing on flow velocity profiles, such as: • pipe bend configurations upstream of USM, • in-flow profile to upstream pipe bends, • meter orientation relative to pipe bends, • initial wall roughness, • changed wall roughness over time (corrosion, wear, pitting, etc.), • possible wall deposits / contamination (lubricants, liquid, grease, etc.).

E.3 Combining the uncertainty models of the gas metering station and the USM

In the following, the uncertainty model for the *un-flow-corrected* USM, Eq. (E.18), is to be used in combination with the uncertainty model for the gas metering station, Eq. (E.7), in which flow calibration of the USM *is* accounted for. That means, one is to associate each of the relative uncertainty contributions $E_{q^U_{USM}}$, $E_{q^U_{USM,j}}$ and $E_{q^C_{USM}}$ - $E_{q^C_{USM,j}}$ with specific physical effects and uncertainty contributions in the USM. For this purpose, Eq. (E.18) and Table E.1 is used. However, Eq. (E.18) needs some modification to account for flow calibration effects.

E.3.1 Consequences of flow calibration on USM uncertainty contributions

The method used here to associate the various uncertainty contributions of Table E.1 with $E_{q^U_{USM}}$, $E_{q^U_{USM,j}}$ and $E_{q^C_{USM}}$ - $E_{q^C_{USM,j}}$, is to evaluate qualitatively each of the uncertainty contributions of Table E.1 with respect to

(1) which effects that are (practically) eliminated by flow calibration, and
(2) for those effects that are *not* eliminated by flow calibration, whether they are correlated or uncorrelated with respect to field operation re. flow calibration.

Table E.2 summarizes the results of this evaluation.

E.3.2 Modified uncertainty model for flow calibrated USM

In Eqs. (E.9) and (E18), the uncertainty terms related to $\hat{R}$, $\hat{y}_i$ and $\hat{\phi}_i$, $i = 1, \ldots, N$, have been assumed to be uncorrelated. This is a simplification, as commented in [Lunde et al., 1997]. In practice, they will be partially correlated. For example, instrument (measurement) uncertainties are typically uncorrelated, as well as the out-of-roundness uncertainty of $\hat{R}$. However, *P* and *T* effects are correlated, cf. Table E.1.

By flow calibration, the uncorrelated contributions to E_R, E_{y_i} and E_{ϕ_i} are practically eliminated (instrument uncertainties, out-of-roundness), cf. Table E.2. The remaining uncertainty terms are those related to *P* and *T* effects, which are correlated. For application to a *flow calibrated* USM, thus, the uncertainty model given by Eq. (E.18) needs to be modified, to accont for such contributions. That means, in this case the term

$$(s_R^* E_R)^2 + \sum_{i=1}^{N} \left[(s_{yi}^* E_{yi})^2 + (s_{\phi i}^* E_{\phi i})^2 \right]$$

appearing in Eq. (E.18) is not valid and needs to be modified (replaced by one of the terms in Eq. (E.24), see below).

Table E.2. Evaluation of uncertainty contributions in the uncertainty model for the *un-flow-corrected* USM, Eq. (E.18), with respect to effects of flow calibration.

Uncertainty term in USM model	**Contribution (examples)**	**Eliminated by flow calibration?**	**Correlated or uncorrelated effect ? (flow calibration re. field)**
E_R	• instrument uncertainty (at "dry calibration"), • *P & T* effects (including possible *P & T* corrections) • out-of-roundness	Eliminated Eliminated	 Correlated
E_{yi}	• instrument uncertainty (at "dry calibration") • *P & T* effects (including possible *P & T* corrections)	Eliminated	 Correlated
$E_{\phi i}$	• instrument uncertainty ("dry calibration") • *P* effect (including possible *P* correction)	Eliminated	 Correlated
$E_{t1i,U}$	• turbulence (random velocity and temperature fluct.) • incoherent noise • coherent noise • finite clock resolution • electronics stability (possible random effects) • possible random effects in signal detection/processing • power supply variations		Uncorrelated Uncorrelated Uncorrelated Uncorrelated Uncorrelated Uncorrelated Uncorrelated
$E_{t1i,C}$	• cable/electronics/transducer/diffraction time delay • Δt-correction • possible cavity delay correction • possible systematic effects in signal detection • possible deposits at transducer front • sound refraction (flow profile effects on transit times) • possible beam reflection at the pipe wall	Eliminated	Correlated Correlated Correlated Correlated Correlated Correlated
$E_{t2i,C}$	• cable/electronics/transducer/diffraction time delay • Δt-correction • possible cavity delay correction • possible systematic effects in signal detection • possible deposits at transducer front • sound refraction (flow profile effects on transit times) • possible beam reflection at the pipe wall	Eliminated	Correlated Correlated Correlated Correlated Correlated Correlated
E_I	• pipe bend configurations upstream of USM • in-flow profile to upstream bends • meter orientation relative to pipe bends • initial wall roughness (corrosion, wear, pitting, etc.) • changed wall roughness over time • possible wall deposits / contamination • possible use of flow conditioners	Eliminated Eliminated	Correlated Correlated Correlated Correlated Correlated

For this purpose, one needs to revisit the basic USM uncertainty model derivation given in [Lunde *et al.*, 1997]. However, it is necessary here to consider only the terms related to $\hat{R}$, $\hat{y}_i$ and $\hat{\phi}_i$, $i = 1, \ldots, N$, and account for correlations between

those. With respect to these parameters, Eq. (5.5) in [Lunde *et al.*, 1997] is then re-written as (using Eq. (B.3))

$$
\begin{aligned}
u_c^2(\hat{Q}) = & \left(\frac{\partial \hat{Q}}{\partial \hat{R}} u_c(\hat{R})\right)^2 + \sum_{i=1}^{N}\left(\frac{\partial \hat{Q}}{\partial \hat{y}_i} u_c(\hat{y}_i)\right)^2 + \sum_{i=1}^{N}\left(\frac{\partial \hat{Q}}{\partial \hat{\phi}_i} u_c(\hat{\phi}_i)\right)^2 \\
& + \sum_{i=1}^{N} 2r(\hat{R},\hat{y}_i)\frac{\partial \hat{Q}}{\partial \hat{R}}\frac{\partial \hat{Q}}{\partial \hat{y}_i} u_c(\hat{R})u_c(\hat{y}_i) + \sum_{i=1}^{N} 2r(\hat{R},\hat{\phi}_i)\frac{\partial \hat{Q}}{\partial \hat{R}}\frac{\partial \hat{Q}}{\partial \hat{\phi}_i} u_c(\hat{R})u_c(\hat{\phi}_i) \\
& + \sum_{i=1}^{N}\sum_{j=1}^{N} 2r(\hat{y}_i,\hat{\phi}_j)\frac{\partial \hat{Q}}{\partial \hat{y}_i}\frac{\partial \hat{Q}}{\partial \hat{\phi}_j} u_c(\hat{y}_i)u_c(\hat{\phi}_j) + \sum_{i=1}^{N}\sum_{j=1}^{N} 2r(\hat{y}_i,\hat{y}_j)\frac{\partial \hat{Q}}{\partial \hat{y}_i}\frac{\partial \hat{Q}}{\partial \hat{y}_j} u_c(\hat{y}_i)u_c(\hat{y}_j) \\
& + \sum_{i=1}^{N}\sum_{j=1}^{N} 2r(\hat{\phi}_i,\hat{\phi}_j)\frac{\partial \hat{Q}}{\partial \hat{\phi}_i}\frac{\partial \hat{Q}}{\partial \hat{\phi}_j} u_c(\hat{\phi}_i)u_c(\hat{\phi}_j) \quad + \quad other\ terms
\end{aligned}
\tag{E.19}
$$

where $r(\hat{a},\hat{b})$ is the correlation coefficient of the estimates $\hat{a}$ and $\hat{b}$. Now, let

$$r(\hat{a},\hat{b}) = sign(\hat{a}) \cdot sign(\hat{b}), \tag{E. 20}$$

where $sign(\hat{a})$ = 1 for $\hat{a} \geq 0$, and -1 for $\hat{a} < 0$. This is valid for the correlation between $\hat{R}$ and $\hat{y}_i$, and between $\hat{y}_i$ and $\hat{y}_j$, *i, j = 1, …, N*. For the correlation between $\hat{R}$ and $\hat{\phi}_i$, between $\hat{y}_i$ and $\hat{\phi}_i$, and between $\hat{\phi}_i$ and $\hat{\phi}_j$, *i, j = 1, …, N*, however, Eq. (E.20) represents an approximation, since $\hat{\phi}_i$ is not temperature dependent whereas $\hat{R}$ and $\hat{y}_i$ are (cf. Eqs. (D.1), (D.19) and (D.22)). However, the approximation may be reasonably valid since the $\hat{\phi}_i$-dependence of the meter body uncertainty is very weak (cf. Table 4.17 and Fig. 5.15).

Eqs. (E.19) and (E.20) lead to

$$
\begin{aligned}
u_c^2(\hat{Q}) = & \left(\frac{\partial \hat{Q}}{\partial \hat{R}} u_c(\hat{R})\right)^2 + \sum_{i=1}^{N}\left(\frac{\partial \hat{Q}}{\partial \hat{y}_i} u_c(\hat{y}_i)\right)^2 + \sum_{i=1}^{N}\left(\frac{\partial \hat{Q}}{\partial \hat{\phi}_i} u_c(\hat{\phi}_i)\right)^2 \\
& + \sum_{i=1}^{N} 2sign(\hat{R})sign(\hat{y}_i)\frac{\partial \hat{Q}}{\partial \hat{R}}\frac{\partial \hat{Q}}{\partial \hat{y}_i} u_c(\hat{R})u_c(\hat{y}_i) + \sum_{i=1}^{N} 2sign(\hat{R})sign(\phi_i)\frac{\partial \hat{Q}}{\partial \hat{R}}\frac{\partial \hat{Q}}{\partial \hat{\phi}_i} u_c(\hat{R})u_c(\hat{\phi}_i) \\
& + \sum_{i=1}^{N}\sum_{j=1}^{N} 2sign(\hat{y}_i)sign(\hat{\phi}_j)\frac{\partial \hat{Q}}{\partial \hat{y}_i}\frac{\partial \hat{Q}}{\partial \hat{\phi}_j} u_c(\hat{y}_i)u_c(\hat{\phi}_j) + \sum_{i=1}^{N}\sum_{j=1}^{N} 2sign(\hat{y}_i)sign(\hat{y}_i)\frac{\partial \hat{Q}}{\partial \hat{y}_i}\frac{\partial \hat{Q}}{\partial \hat{y}_j} u_c(\hat{y}_i)u_c(\hat{y}_j) \\
& + \sum_{i=1}^{N}\sum_{j=1}^{N} 2sign(\hat{\phi}_i)sign(\hat{\phi}_i)\frac{\partial \hat{Q}}{\partial \hat{\phi}_i}\frac{\partial \hat{Q}}{\partial \hat{\phi}_j} u_c(\hat{\phi}_i)u_c(\hat{\phi}_j) \quad + \quad other\ terms
\end{aligned}
\tag{E.21}
$$

which can be written as (since $sign(\hat{R}) = 1$)

$$u_c^2(\hat{Q}) = \left(\frac{\partial \hat{Q}}{\partial \hat{R}} u_c(\hat{R}) + \sum_{i=1}^{N} sign(\hat{y}_i) \frac{\partial \hat{Q}}{\partial \hat{y}_i} u_c(\hat{y}_i) + \sum_{i=1}^{N} sign(\hat{\phi}_i) \frac{\partial \hat{Q}}{\partial \hat{\phi}_i} u_c(\hat{\phi}_i) \right)^2 + other\ terms\,. \qquad \text{(E.22)}$$

Consequently, for a flow calibrated USM, Eq. (E.9) is to be replaced by

$$\begin{aligned} E_{q_{USM}}^2 \approx\ & E_I^2 + \left(s_R^* E_R + \sum_{i=1}^{N} \left[sign(\hat{y}_i) s_{yi}^* E_{yi} + sign(\hat{\phi}_i) s_{\phi i}^* E_{\phi i} \right] \right)^2 \\ & + \frac{1}{N_{ave}} \sum_{i=1}^{N} \left[(s_{t1i}^* E_{t1i,U}^{(n,U)})^2 + (s_{t2i}^* E_{t2i,U}^{(n,U)})^2 \right] + \sum_{i=1}^{N} \left[(s_{t1i}^* E_{t1i,U}^{(n,C)})^2 + (s_{t2i}^* E_{t2i,U}^{(n,C)})^2 \right] \qquad \text{(E.23)} \\ & + \left(\sum_{i=1}^{N} \left[s_{t1i}^* E_{t1i,C}^{(n)} + s_{t2i}^* E_{t2i,C}^{(n)} \right] \right)^2 , \end{aligned}$$

and it follows that Eq. (E.18) is to be replaced by

$$\boxed{\begin{aligned} E_{q_{USM}}^2 \approx\ & E_I^2 + \left(s_R^* E_R + \sum_{i=1}^{N} \left[sign(\hat{y}_i) s_{yi}^* E_{yi} + sign(\hat{\phi}_i) s_{\phi i}^* E_{\phi i} \right] \right)^2 \\ & + 2 \sum_{i=1}^{N} (s_{t1i}^* E_{t1i,U})^2 + \left(\sum_{i=1}^{N} \left[s_{t1i}^* E_{t1i,C} + s_{t2i}^* E_{t2i,C} \right] \right)^2 \end{aligned}} \,. \qquad \text{(E.24)}$$

Eq. (E.24) is the expression used in the following for the uncertainty of the flow calibrated USM.

E.3.3 Identification of USM uncertainty terms

E.3.3.1 Uncorrelated contributions, for flow calibration re. field operation (repeatability)

In Eq. (E.7), the term $E_{q_{USM}^U}$ represents the USM uncertainty contributions in field operation, which are *uncorrelated* with flow calibration, cf. Section E.1. Thus, on basis of the fourth column in Table E.2 and Eq. (E.24), the identification

$$E_{q_{USM}^U}^2 = 2 \sum_{i=1}^{N} (s_{t1i}^* E_{t1i,U})^2 \qquad \text{(E.25)}$$

has been made here. Moreover, from of the second column in Table E.2, $E_{t1i,U}$ is to be associated with the repeatability of the transit time measurements in field operation, at the flow rate in question. Consequently, $E_{q_{USM}^U}$ represents the repeatability of the USM measurement in field operation, at the flow rate in question. Hence, it is convenient to define

$$E_{rept} \equiv E_{q^U_{USM}} \quad , \tag{E.26}$$

where

E_{rept} ≡ repeatability (relative combined standard uncertainty, i.e. standard deviation) of the USM measurement in field operation (volumetric flow rate), at the flow rate in question (due to random transit time effects).

Similarly, in Eq. (E.7) the term $E_{q^U_{USM,j}}$ represents the USM uncertainty contributions in flow calibration (at test flow rate no. *j*), which are *uncorrelated* with field operation, cf. Section E.1. On basis of the fourth column in Table E.2 and Eq. (E.24), the identification

$$E^2_{q^U_{USM,j}} = 2\sum_{i=1}^{N}(s^*_{t1i}E_{t1i,U,j})^2 \tag{E.27}$$

has been made here. From the second column in Table E.2, $E_{t1i,U,j}$ is to be associated with the repeatability of the transit time measurements in flow calibration, at test flow rate no. *j*. Consequently, $E_{q^U_{USM,j}}$ is to represent the repeatability of the USM measurement in flow calibration, at test flow rate no. *j*. Hence, it is convenient to define

$$E_{rept,j} \equiv E_{q^U_{USM,j}} \quad , \tag{E.28}$$

where

$E_{rept,j}$ ≡ repeatability (relative combined standard uncertainty, i.e. standard deviation) of the flow calibration measurement (volumetric flow rate), at test flow rate no. *j*, *j* = 1, …, *M* (due to random transit time effects on the *N* acoustic paths of the USM, and the repeatability of the flow laboratory reference measurement).

E.3.3.2 Correlated contributions, for flow calibration re. field operation

In Eq. (E.7), the terms $E_{q^C_{USM}}$ and $E_{q^C_{USM,j}}$ represent those USM uncertainty contributions in field operation, which are *correlated* with flow calibration, and *vice versa*, cf. Section E.1. Thus, on basis of the third and fourth columns in Table E.2, the identification

$$\left[E_{q^C_{USM}} - E_{q^C_{USM,j}}\right]^2 \approx E^2_{I,\Delta} + \left(s^*_R E_{R,\Delta} + \sum_{i=1}^{N} \left[sign(\hat{y}_i) s^*_{yi} E_{yi,\Delta} + sign(\hat{\phi}_i) s^*_{\phi i} E_{\phi i,\Delta} \right] \right)^2 + \left(\sum_{i=1}^{N} \left[s^*_{t1i} E^{\Delta}_{t1i,C} + s^*_{t2i} E^{\Delta}_{t2i,C} \right] \right)^2 \quad \text{(E.29)}$$

has been made here, where

$E_{I,\Delta}$ ≡ relative standard uncertainty of the USM integration method due to change of installation conditions from flow calibration to field operation.

$E_{R,\Delta}$ ≡ relative combined standard uncertainty of the meter body inner radius, $\hat{R}$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{yi,\Delta}$ ≡ relative combined standard uncertainty of the lateral chord position of acoustic path no. *i*, $\hat{y}_i$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{\phi i,\Delta}$ ≡ relative combined standard uncertainty of the inclination angle of acoustic path no. *i*, $\hat{y}_i$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E^{\Delta}_{t1i,C}$ ≡ relative standard uncertainty of uncorrected systematic transit time effects on upstream propagation of acoustic path no. *i*, $\hat{t}_{1i}$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

$E^{\Delta}_{t2i,C}$ ≡ relative standard uncertainty of uncorrected systematic transit time effects on downstream propagation of acoustic path no. *i*, $\hat{t}_{2i}$, due to possible deviation in pressure and/or temperature between flow calibration and field operation.

Note that the sub/superscript "Δ" used in Eq. (E.29) denotes that *only deviations relative to the conditions at the flow calibration* are to be accounted for in the expressions involving this sub/superscript. That means, systematic effects which are (practically) eliminated at flow calibration (cf. Table E.2), are *not* to be accounted for in this expression.

Now, for convenience in notation, define

$$E_{rad,\Delta} \equiv s^*_R E_{R,\Delta} \ , \quad \text{(E.30)}$$

$$E_{chord,\Delta} \equiv \sum_{i=1}^{N} sign(\hat{y}_i) s_{yi}^{*} E_{yi,\Delta} \quad , \tag{E.31}$$

$$E_{angle,\Delta} \equiv \sum_{i=1}^{N} sign(\hat{\phi}_i) s_{\phi i}^{*} E_{\phi i,\Delta} \quad , \tag{E.32}$$

$$E_{time,\Delta} \equiv \sum_{i=1}^{N} \left(s_{t1i}^{*} E_{t1i,C}^{\Delta} + s_{t2i}^{*} E_{t2i,C}^{\Delta} \right) , \tag{E.33}$$

where

$E_{rad,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to uncertainty of the meter body inner radius, $\hat{R}$, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{chord,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to uncertainty of the lateral chord positions of the *N* acoustic paths, $\hat{y}_i$, *i* = 1, …, *N*, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{angle,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to uncertainty of the inclination angles of the *N* acoustic paths, ϕ_i, *i* = 1, …, *N*, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

$E_{time,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to systematic effects on the transit times of the *N* acoustic paths, $\dot{t}_{1i}$ and $\dot{t}_{2i}$, *i* = 1, …, *N*, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

E.3.4 Uncertainty model of the USM fiscal gas metering station

From Eqs. (E.7) and (E.26)-(E.33), the relative combined standard uncertainty of the USM volumetric flow rate measurement becomes

$$\begin{aligned} E_{q_v}^2 &= E_{K_{dev,j}}^2 + E_{q_{ref,j}}^2 + E_{rept,j}^2 + E_{rept}^2 \\ &\quad + E_{I,\Delta}^2 + \left(E_{rad,\Delta} + E_{chord,\Delta} + E_{angle,\Delta}\right)^2 + E_{time,\Delta}^2 + E_{comm}^2 + E_{flocom}^2 \end{aligned} \tag{E.34}$$

Since most conveniently, input to the program *EMU - USM Fiscal Gas Metering Station* may be given at different levels, it may be useful to rewrite Eq. (E.34) as

$$\boxed{E_{q_v}^2 = E_{cal}^2 + E_{USM}^2 + E_{comm}^2 + E_{flocom}^2} \quad , \tag{E.35}$$

where

$$\boxed{E_{cal}^2 \equiv E_{K_{dev,j}}^2 + E_{q_{ref,j}}^2 + E_{rept,j}^2} \quad , \tag{E.36}$$

$$\boxed{E_{USM}^2 \equiv E_{rept}^2 + E_{USM,\Delta}^2} \quad , \tag{E.37}$$

$$\boxed{E_{USM,\Delta}^2 \equiv E_{body,\Delta}^2 + E_{time,\Delta}^2 + E_{I,\Delta}^2} \quad , \tag{E.38}$$

$$\boxed{E_{body,\Delta} \equiv E_{rad,\Delta} + E_{chord,\Delta} + E_{angle,\Delta}} \quad . \tag{E.39}$$

Here,

E_{cal} ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, related to flow calibration of the USM.

E_{USM} ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, related to field operation of the USM.

$E_{USM,\Delta}$ ≡ relative standard uncertainty of the estimate $\hat{q}_v$, related to field operation of the USM.

$E_{body,\Delta}$ ≡ relative combined standard uncertainty of the estimate $\hat{q}_v$, due to change of the USM meter body from flow calibration to field operation. That is, uncertainty of the meter body inner radius, $\hat{R}$, the lateral chord positions of the *N* acoustic paths, $\hat{y}_i$, and the inclination angles of the *N* acoustic paths, ϕ_i, *i* = 1, …, *N*, caused by possible deviation in pressure and/or temperature between flow calibration and field operation.

Eqs. (E.35)-(E.39) are the same as Eqs. (3.6), (3.16), (3.19), (3.20) and (3.22).

The corresponding relative expanded uncertainty of the volumetric flow rate measurement is given from Eq. (E.35) to be

$$\boxed{\frac{U(\hat{q}_v)}{|\hat{q}_v|} = k \cdot \frac{u_c(\hat{q}_v)}{|\hat{q}_v|} = k \cdot E_{q_v}} \quad , \tag{E.40}$$

where $U(\hat{q}_v)$ is the expanded uncertainty of the estimate $\hat{q}_v$. For specification of the expanded uncertainty at a 95 % confidence level, a coverage factor k = 2 is recommended [ISO, 1995; EAL-R2, 1997].

APPENDIX F

ALTERNATIVE APPROACHES FOR EVALUATION OF PARTIALLY CORRELATED QUANTITIES

As mentioned in Section E.1, there are several ways to account for the partial correlation between $q_{USM,j}$ and q_{USM}. Three possible approaches to account for such partial correlation are addressed briefly here. This includes (1) the ”covariance method” recommended by the *GUM* [ISO, 1995] (Section F.1), (2) the”decomposition method” used in Appendix E (Section F.2), and (3) a ”variance method”, which is often useful, but which for the present application (with a rather complex functional relationship) has been more complex to use than the method chosen (Section F.3).

In Section F.2 the ”decomposition approach” used in Appendix E is shown to be equivalent with the ”covariance method” recommended by the *GUM*, for evaluation of partially correlated quantities. The reason for choosing the ”decomposition method” for the analysis of Appendix E is addressed.

F.1 The “covariance method”

The “classical” way to account for partially correlation between quantities is the “covariance method” recommended by the *GUM* [ISO, 1995], given by Eqs. (B.2)-(B.3). From the functional relationship given by Eq. (2.9), the combined variance of the estimate $\hat{q}_v$ is then given as

$$\begin{aligned} u_c^2(\hat{q}_v) = & \left[\frac{\partial \hat{q}_v}{\partial \hat{K}_{dev,j}} u_c(\hat{K}_{dev,j})\right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{ref,j}} u_c(\hat{q}_{ref,j})\right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}} u_c(\hat{q}_{USM})\right]^2 \\ & + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}} u_c(\hat{q}_{USM,j})\right]^2 + 2r(\hat{q}_{USM}, \hat{q}_{USM,j}) \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}} \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}} u_c(\hat{q}_{USM}) u_c(\hat{q}_{USM,j}) \\ & + \left[u(\hat{q}_{comm})\right]^2 + \left[u(\hat{q}_{flocom})\right]^2 \end{aligned} \qquad \text{(F.1)}$$

where $r(\hat{q}_{USM}, \hat{q}_{USM,j})$ is the correlation coefficient for the correlation between the estimates $\hat{q}_{USM}$ and $\hat{q}_{USM,j}$ ($|r| < 1$), and

$u_c(\hat{q}_{USM}) \equiv$ combined standard uncertainty of the estimate $\hat{q}_{USM}$ (representing the USM uncertainty in field operation),

$u_c(\hat{q}_{USM,j}) \equiv$ combined standard uncertainty of the estimate $\hat{q}_{USM,j}$ (representing the USM uncertainty in flow calibration),

The other terms are defined in Appendix E.

Use of the "covariance method", Eq. (F.1), involves evaluation of the correlation coefficient $r(\hat{q}_{USM}, \hat{q}_{USM,j})$, which may be difficult in practice, as r is different from 0 and ±1. For this reason an alternative (and equivalent) method has been used, as described in Section F.2.

F.2 The "decomposition method"

The "decomposition method" which has been used in the present *Handbook* is described in detail in Appendix E. Here some more general aspects and properties of the method are addressed.

The main advantage of this method is that no correlation coefficient $r(\hat{q}_{USM}, \hat{q}_{USM,j})$ is involved, so that evaluation of a numerical value for this coefficient is avoided. Instead, first the *physical contributions* to $\hat{q}_{USM}$ and $\hat{q}_{USM,j}$ are identified, i.e. the physical effects which are influencing them (cf. Table E.1). Next, each contribution is evaluated, i.e. whether it is correlated or uncorrelated between flow calibration and field operation, based on skilled judgement and experience (cf. Table E.2). Identification of terms is then made, as described in Section E.3.

However, since the "covariance method" is the method recommended by the *GUM*, the "decomposition method" is to be shown to be equivalent to the "covariance method", to justify its use. Hence, Eq. (E.4) is to be related to Eq. (F.1). For this purpose, Eq. (E.4) can be rewritten as

$$
\begin{aligned}
u_c^2(\hat{q}_v) = & \left[\frac{\partial \hat{q}_v}{\partial \hat{K}_{dev,j}} u_c(\hat{K}_{dev,j})\right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{ref,j}} u_c(\hat{q}_{ref,j})\right]^2 \\
& + \left(\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}}\right)^2 \left[u_c^2(\hat{q}_{USM}^U) + u_c^2(\hat{q}_{USM}^C)\right] + \left(\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}}\right)^2 \left[u_c^2(\hat{q}_{USM,j}^U) + u_c^2(\hat{q}_{USM,j}^C)\right] \\
& + 2\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}} \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}} u_c(\hat{q}_{USM}^C) u_c(\hat{q}_{USM,j}^C) + \left[u(\hat{q}_{comm})\right]^2 + \left[u(\hat{q}_{flocom})\right]^2
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\frac{\partial \hat{q}_v}{\partial \hat{K}_{dev,j}} u_c(\hat{K}_{dev,j}) \right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{ref,j}} u_c(\hat{q}_{ref,j}) \right]^2 \\
&+ \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}} u_c(\hat{q}_{USM}) \right]^2 + \left[\frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}} u_c(\hat{q}_{USM,j}) \right]^2 \\
&+ 2 \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}} \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}} u_c(\hat{q}^C_{USM}) u_c(\hat{q}^C_{USM,j}) + \left[u(\hat{q}_{comm}) \right]^2 + \left[u(\hat{q}_{flocom}) \right]^2
\end{aligned}
\tag{F.2}
$$

where the identities

$$
\frac{\partial \hat{q}_v}{\partial \hat{q}^U_{USM}} = \frac{\partial \hat{q}_v}{\partial \hat{q}^C_{USM}} = \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM}}, \qquad \frac{\partial \hat{q}_v}{\partial \hat{q}^U_{USM,j}} = \frac{\partial \hat{q}_v}{\partial \hat{q}^C_{USM,j}} = \frac{\partial \hat{q}_v}{\partial \hat{q}_{USM,j}} \tag{F.3}
$$

and

$$
u_c^2(\hat{q}_{USM}) = u_c^2(\hat{q}^U_{USM}) + u_c^2(\hat{q}^C_{USM}), \quad u_c^2(\hat{q}_{USM,j}) = u_c^2(\hat{q}^U_{USM,j}) + u_c^2(\hat{q}^C_{USM,j}) \tag{F.4}
$$

have been used, cf. Eq. (E.3) and Eqs. (E.1)-(E.2), respectively.

Eq. (F.2) (obtained by the “decomposition method”) becomes identical to Eq. (F.1) (obtained by the “covariance method”) by defining

$$
r(\hat{q}_{USM}, \hat{q}_{USM,j}) = \frac{u_c(\hat{q}^C_{USM}) u_c(\hat{q}^C_{USM,j})}{u_c(\hat{q}_{USM}) u_c(\hat{q}_{USM,j})}, \tag{F.5}
$$

or, equivalently,

$$
\begin{aligned}
r(\hat{q}_{USM}, \hat{q}_{USM,j}) &= \frac{u_c(\hat{q}^C_{USM}) u_c(\hat{q}^C_{USM,j})}{\sqrt{u_c^2(\hat{q}^C_{USM}) + u_c^2(\hat{q}^U_{USM})} \sqrt{u_c^2(\hat{q}^C_{USM,j}) + u_c^2(\hat{q}^U_{USM,j})}} \\
&= \frac{1}{\sqrt{1 + \left(\frac{u_c(\hat{q}^U_{USM})}{u_c(\hat{q}^C_{USM})} \right)^2} \sqrt{1 + \left(\frac{u_c(\hat{q}^U_{USM,j})}{u_c(\hat{q}^C_{USM,j})} \right)^2}}
\end{aligned}
\tag{F.6}
$$

for the correlation coefficient between the estimates $\hat{q}_{USM}$ and $\hat{q}_{USM,j}$. Its value ranges from 0 to +1 depending on the numerical values of the two ratios appearing in the last expression of Eq. (F.6).

The “decomposition method” given by Eq. (E.4) is thus equivalent to the “covariance method” given by Eq. (F.1).

In the "decomposition method", *r* is usually not evaluated as a number (i.e. Eqs. (F.5)-(F.6) are not used). On the other hand, it could be said that *r* by this method is evaluated more indirectly, by the identification procedure described above and used in Appendix E.

In fact, it may be noted that by this method, *r* can be evaluated quantitatively. When the identification described above has been done, numbers for $u_c(\hat{q}^U_{USM})$, $u_c(\hat{q}^C_{USM})$, $u_c(\hat{q}^U_{USM,j})$ and $u_c(\hat{q}^C_{USM,j})$ are in principle available, and *r can* be calculated from Eq. (F.6).

F.3 The "variance method"

Another method, which is here referred to as the "variance method", is perhaps the most "intuitive" of the methods discussed here for evaluation of the uncertainty of partially correlated quantities. In some cases (for a relatively simple functional relationship) this approach is very useful, and leads to (or should lead to!) the same results as the methods described in Sections F.1 and F.2. This method is briefly addressed here for completeness.

In the "variance method" approach, the *full* functional relationship for q_v (including the USM measurements q_{USM} and $q_{USM,j}$) would be written out as a *single expression*, in terms of *the basic uncorrelated input quantities*. $u_c(\hat{q}_v)$ would then be evaluated as the square root of the sum of input variances. In this method, the covariance term (the last term in Eq. (F.1)) and the correlation coefficient $r(\hat{q}_{USM}, \hat{q}_{USM,j})$ would thus be avoided.

However, in the present application, involving the combination of Eqs. (2.9), (2.12) and (2.13)-(2.18), the *full* functional relationship for q_v is considered to be too complex to be practically useful. In this case the "variance method" has been considered to be considerably more complex to use than the method chosen.

APPENDIX G

UNCERTAINTY MODEL FOR THE GAS DENSITOMETER

The present appendix gives the theoretical basis for Eq. (3.14), describing the uncertainty model for the gas density measurement, including temperature, VOS and bypass installation corrections.

For the gas density measurement, the functional relationship is given by Eq. (2.28). The various quantities involved are defined in Section 2.4.

From Eqs. (B.3) and (2.28), the combined standard uncertainty of the gas density measurement is given as

$$\begin{aligned} u_c^2(\hat{\rho}) = & \left[\frac{\partial\hat{\rho}}{\partial\hat{\rho}_u}u(\hat{\rho}_u)\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{T}}u_c(\hat{T})\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{T}_d}u(\hat{T}_d)\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{T}_c}u(\hat{T}_c)\right]^2 \\ & + \left[\frac{\partial\hat{\rho}}{\partial\hat{K}_{18}}u(\hat{K}_{18})\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{K}_{19}}u(\hat{K}_{19})\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{K}_d}u(\hat{K}_d)\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{\tau}}u(\hat{\tau})\right]^2 \\ & + \left[\frac{\partial\hat{\rho}}{\partial\hat{c}_c}u(\hat{c}_c)\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{c}_d}u(\hat{c}_d)\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\Delta\hat{P}_d}u(\Delta\hat{P}_d)\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{P}}u_c(\hat{P})\right]^2 \\ & + \left[\frac{\partial\hat{\rho}}{\partial(\hat{Z}_d/\hat{Z})}u_c(\hat{Z}_d/\hat{Z})\right]^2 + u^2(\hat{\rho}_{rept}) + u^2(\hat{\rho}_{misc}) \end{aligned} \qquad \text{(G.1)}$$

where

$u(\hat{\rho}_u) \equiv$ standard uncertainty of the indicated (uncorrected) density estimate, $\hat{\rho}_u$, including the calibration laboratory uncertainty, the reading error during calibration, and hysteresis,

$u_c(\hat{T}) \equiv$ combined standard uncertainty of the line gas temperature estimate, $\hat{T}$,

$u(\hat{T}_d) \equiv$ combined standard uncertainty of the gas temperature estimate in the densitometer, $\hat{T}_d$,

$u(\hat{T}_c) \equiv$ combined standard uncertainty of the estimate of the densitometer calibration temperature, $\hat{T}_c$,

$u(\hat{K}_{18}) \equiv$ standard uncertainty of the estimate of the temperature correction calibration coefficient, $\hat{K}_{18}$,

$u(\hat{K}_{19}) \equiv$ standard uncertainty of the estimate of the temperature correction calibration coefficient, $\hat{K}_{19}$,

$u(\hat{K}_d) \equiv$ standard uncertainty of the estimate of the densitometer transducer constant, $\hat{K}_d$,

$u(\hat{\tau}) \equiv$ standard uncertainty of the periodic time estimate, $\hat{\tau}$,

$u(\hat{c}_c) \equiv$ standard uncertainty of the calibration gas VOS estimate, $\hat{c}_c$,

$u(\hat{c}_d) \equiv$ standard uncertainty of the densitometer gas VOS estimate, $\hat{c}_d$,

$u_c(\hat{P}) \equiv$ combined standard uncertainty of the line gas pressure estimate, $\hat{P}$,

$u(\Delta\hat{P}_d) \equiv$ standard uncertainty of assuming that $P_d = P$, due to possible deviation of gas pressure from densitometer to line conditions, $\Delta\hat{P}_d$,

$u_c(\hat{Z}_d/\hat{Z}) \equiv$ combined standard uncertainty of the gas compressibility factor ratio between densitometer and line conditions, $\hat{Z}_d/\hat{Z}$,

$u(\hat{\rho}_{rept}) \equiv$ standard uncertainty of the repeatability of the indicated (uncorrected) density estimate, $\hat{\rho}_u$,

$u(\hat{\rho}_{misc}) \equiv$ standard uncertainty of the indicated (uncorrected) density estimate, $\hat{\rho}_u$, accounting for miscellaneous uncertainty contributions, such as due to:

- stability (drift, shift between calibrations),
- reading error during measurement (for digital display instruments),
- possible deposits on the vibrating element,
- possible corrosion of the vibrating element,
- possible liquid condensation on the vibrating element,
- mechanical (structural) vibrations on the gas line,
- variations in power supply,
- self-induced heat,
- flow in the bypass density line,
- possible gas viscosity effects,
- neglecting possible pressure dependency in the regression curve, Eq. (2.23),
- model uncertainty of the VOS correction model, Eq. (2.25).

As formulated by Eq. (G.1), the estimates $\hat{T}$, $\hat{T}_d$ and $\hat{T}_c$ are assumed to be uncorrelated, as well as the estimates $\hat{P}$ and $\Delta\hat{P}_d$.

From the functional relationship of the gas densitometer, Eq. (2.28), one obtains

$$\frac{\partial\hat{\rho}}{\partial\hat{\rho}_u} = \frac{\hat{\rho}}{\hat{\rho}_u}\left[\frac{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right] \equiv \frac{\hat{\rho}}{\hat{\rho}_u}s^*_{\rho_u} \equiv s_{\rho_u} \tag{G.2}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{T}} = -\frac{\hat{\rho}}{\hat{T}} \equiv \frac{\hat{\rho}}{\hat{T}}s^*_{\rho,T} \equiv s_{\rho,T} \tag{G.3}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{T}_d} = \frac{\hat{\rho}}{\hat{T}_d}\left[1+\frac{\hat{T}_d\left[\hat{\rho}_u\hat{K}_{18}+\hat{K}_{19}\right]}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right] \equiv \frac{\hat{\rho}}{\hat{T}_d}s^*_{\rho,T_d} \equiv s_{\rho,T_d} \tag{G.4}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{T}_c} = -\frac{\hat{\rho}}{\hat{T}_c}\left[\frac{\hat{T}_c\left[\hat{\rho}_u\hat{K}_{18}+\hat{K}_{19}\right]}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right] \equiv \frac{\hat{\rho}}{\hat{T}_c}s^*_{\rho,T_c} \equiv s_{\rho,T_c} \tag{G.5}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{K}_{18}} = \frac{\hat{\rho}}{\hat{K}_{18}}\left[\frac{\hat{K}_{18}\hat{\rho}_u(\hat{T}_d-\hat{T}_c)}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right] \equiv \frac{\hat{\rho}}{\hat{K}_{18}}s^*_{\rho,K_{18}} \equiv s_{\rho,K_{18}} \tag{G.6}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{K}_{19}} = \frac{\hat{\rho}}{\hat{K}_{19}}\left[\frac{\hat{K}_{19}\left(\hat{T}_d-\hat{T}_c\right)}{\hat{\rho}_u\left[1+\hat{K}_{18}(\hat{T}_d-\hat{T}_c)\right]+\hat{K}_{19}(\hat{T}_d-\hat{T}_c)}\right] \equiv \frac{\hat{\rho}}{\hat{K}_{19}}s^*_{\rho,K_{19}} \equiv s_{\rho,K_{19}} \tag{G.7}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{K}_d} = \frac{\hat{\rho}}{\hat{K}_d}\left[\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_c)^2}-\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_d)^2}\right] \equiv \frac{\hat{\rho}}{\hat{K}_d}s^*_{\rho,K_3} \equiv s_{\rho,K_3} \tag{G.8}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{\tau}} = -\frac{\hat{\rho}}{\hat{\tau}}\left[\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_c)^2}-\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_d)^2}\right] \equiv \frac{\hat{\rho}}{\hat{\tau}}s^*_{\rho,\tau} \equiv s_{\rho,\tau} \tag{G.9}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{c}_c} = -\frac{\hat{\rho}}{\hat{c}_c}\left[\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_c)^2}\right] \equiv \frac{\hat{\rho}}{\hat{c}_c}s^*_{\rho,c_c} \equiv s_{\rho,c_c} \tag{G.10}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{c}_d} = \frac{\hat{\rho}}{\hat{c}_d}\left[\frac{2\hat{K}_d^2}{\hat{K}_d^2+(\hat{\tau}\hat{c}_d)^2}\right] \equiv \frac{\hat{\rho}}{\hat{c}_d}s^*_{\rho,c_d} \equiv s_{\rho,c_d} \tag{G.11}$$

$$\frac{\partial\hat{\rho}}{\partial\Delta\hat{P}_d} = -\frac{\hat{\rho}}{\Delta\hat{P}_d}\left[\frac{\Delta\hat{P}_d}{\hat{P}+\Delta\hat{P}_d}\right] \equiv \frac{\hat{\rho}}{\Delta\hat{P}_d} s^*_{\rho,\Delta P_d} \equiv s_{\rho,\Delta P_d} \tag{G.12}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{P}} = \frac{\hat{\rho}}{\hat{P}}\left[\frac{\Delta\hat{P}_d}{\hat{P}+\Delta\hat{P}_d}\right] \equiv \frac{\hat{\rho}}{\hat{P}} s^*_{\rho,P} \equiv s_{\rho,P} \tag{G.13}$$

$$\frac{\partial\hat{\rho}}{\partial(\hat{Z}_d/\hat{Z})} = \frac{\hat{\rho}}{\hat{Z}_d/\hat{Z}} \equiv \frac{\hat{\rho}}{\hat{Z}_d/\hat{Z}} s^*_{\rho,\hat{Z}_d/\hat{Z}} \equiv s_{\rho,\hat{Z}_d/\hat{Z}} \tag{G.14}$$

Here, $s^*_{\rho,X}$ and $s_{\rho,X}$ are the relative (non-dimensional) and absolute (dimensional) sensitivity coefficients of the estimate $\hat{\rho}$ with respect to the quantity "X".

Now, define

$$u^2(\hat{\rho}_{temp}) = \left[\frac{\partial\hat{\rho}}{\partial\hat{K}_{18}} u(\hat{K}_{18})\right]^2 + \left[\frac{\partial\hat{\rho}}{\partial\hat{K}_{19}} u(\hat{K}_{19})\right]^2 \tag{G.15}$$

where

$u(\hat{\rho}_{temp}) \equiv$ standard uncertainty of the temperature correction factor for the density estimate, $\hat{\rho}$ (represents the *model uncertainty* of the temperature correction model used, Eq. (2.24)).

By dividing Eq. (G.1) through with $\hat{\rho}$, and using Eqs. (G.2)-(G.15), one obtains

$$\begin{aligned} E_\rho^2 = {} & (s^*_{\rho_u})^2 E^2_{\rho_u} + E^2_{\rho,rept} + (s^*_{\rho,T})^2 E_T^2 + (s^*_{\rho,T_d})^2 E^2_{T_d} + (s^*_{\rho,T_c})^2 E^2_{T_c} \\ & + (s^*_{\rho,K_d})^2 E^2_{K_d} + (s^*_{\rho,\tau})^2 E_\tau^2 + (s^*_{\rho,c_c})^2 E^2_{c_c} + (s^*_{\rho,c_d})^2 E^2_{c_d} \\ & + (s^*_{\rho,\Delta P_d})^2 E^2_{\Delta P_d} + (s^*_{\rho,P})^2 E_P^2 + (s^*_{\rho,Z_d/Z})^2 E^2_{Z_d/Z} + E^2_{\rho,temp} + E^2_{\rho,misc} \end{aligned} \tag{G.16}$$

where

$E_{\rho_u} \equiv$ relative standard uncertainty of the indicated (uncorrected) density estimate, $\hat{\rho}_u$, including the calibration laboratory uncertainty, the reading error during calibration, and hysteresis,

$E_{\rho,rept} \equiv$ relative standard uncertainty of the repeatability of the indicated (uncorrected) density estimate, $\hat{\rho}_u$,

$E_T \equiv$ relative combined standard uncertainty of the line gas temperature estimate, $\hat{T}$,

$E_{T_d} \equiv$ relative combined standard uncertainty of the gas temperature estimate in the densitometer, $\hat{T}_d$,

$E_{T_c} \equiv$ relative combined standard uncertainty of the estimate of the densitometer calibration temperature, $\hat{T}_c$,

$E_{K_d} \equiv$ relative standard uncertainty of the estimate of the densitometer transducer constant, $\hat{K}_d$,

$E_{\tau} \equiv$ relative standard uncertainty of the periodic time estimate, $\hat{\tau}$,

$E_{c_c} \equiv$ relative standard uncertainty of the calibration gas VOS estimate, $\hat{c}_c$,

$E_{c_d} \equiv$ relative standard uncertainty of the densitometer gas VOS estimate, $\hat{c}_d$,

$E_P \equiv$ relative combined standard uncertainty of the line gas pressure estimate, $\hat{P}$,

$E_{\Delta P_d} \equiv$ relative standard uncertainty of assuming that $P_d = P$, due to possible deviation of gas pressure from densitometer to line conditions, $\Delta\hat{P}_d$,

$E_{Z_d/Z} \equiv$ relative combined standard uncertainty of the gas compressibility ratio between densitometer and line conditions, $\hat{Z}_d / \hat{Z}$,

$E_{\rho,temp} \equiv$ relative standard uncertainty of the temperature correction factor for the density estimate, $\hat{\rho}$ (represents the *model uncertainty* of the temperature correction model used, Eq. (2.24)).

$E_{\rho,misc} \equiv$ relative standard uncertainty of the indicated (uncorrected) density estimate, $\hat{\rho}_u$, accounting for uncertainties due to:

- stability (drift, shift between calibrations),
- reading error during measurement (for digital display instruments),
- possible deposits on the vibrating element,
- possible corrosion of the vibrating element,
- possible liquid condensation on the vibrating element,
- mechanical (structural) vibrations on the gas line,
- variations in power supply,
- self-induced heat,
- flow in the bypass density line,
- possible gas viscosity effects,

- neglecting possible pressure dependency in the regression curve, Eq. (2.23),
- model uncertainty of the VOS correction model, Eq. (2.25),

defined as

$$
\begin{aligned}
&E_{\rho_u} \equiv \frac{u_c(\hat{\rho}_u)}{|\hat{\rho}_u|}, && E_T \equiv \frac{u_c(\hat{T})}{|\hat{T}|}, && E_{T_d} \equiv \frac{u_c(\hat{T}_d)}{|\hat{T}_d|}, \\
&E_{T_c} \equiv \frac{u_c(\hat{T}_c)}{|\hat{T}_c|}, && E_{K_d} \equiv \frac{u(\hat{K}_d)}{|\hat{K}_d|}, && E_\tau \equiv \frac{u(\hat{\tau})}{|\hat{\tau}|}, \\
&E_{c_c} \equiv \frac{u(\hat{c}_c)}{|\hat{c}_c|}, && E_{c_d} \equiv \frac{u(\hat{c}_d)}{|\hat{c}_d|}, && \\
&E_{\Delta P_d} \equiv \frac{u(\Delta\hat{P}_d)}{|\Delta\hat{P}_d|}, && E_P \equiv \frac{u_c(\hat{P})}{|\hat{P}|}, && E_{\hat{Z}_d/\hat{Z}} \equiv \frac{u_c(\hat{Z}_d/\hat{Z})}{|\hat{Z}_d/\hat{Z}|}, \\
&E_{\rho,rept} \equiv \frac{u(\hat{\rho}_{rept})}{|\hat{\rho}|}, && E_{\rho,temp} \equiv \frac{u(\hat{\rho}_{temp})}{|\hat{\rho}|}, && E_{\rho,misc} \equiv \frac{u(\hat{\rho}_{misc})}{|\hat{\rho}|},
\end{aligned}
\tag{G.17}
$$

respectively.

For each of the estimates $\hat{Z}_d$ and $\hat{Z}$, two kinds of uncertainties are accounted for here [Tambo and Søgaard, 1997]: the *model uncertainty* (i.e. the uncertainty of the model used for calculation of $\hat{Z}_d$ and $\hat{Z}$), and the *analysis uncertainty* (due to the inaccurate determination of the gas composition in the line). The model uncertainties are here assumed to be mutually correlated[143], and so are the analysis uncertainties. That means,

$$
E^2_{Z_d/Z} = (E_{Zd,mod} - E_{Z,mod})^2 + (E_{Zd,ana} - E_{Z,ana})^2 \quad , \tag{G.18}
$$

where

$E_{Zd,\mathrm{mod}} \equiv$ relative standard uncertainty of the estimate $\hat{Z}_d$ due to *model uncertainty* (the uncertainty of the equation of state itself, and the uncertainty of the "basic data" underlying the equation of state),

143 In the derivation of Eq. (G.18), the model uncertainties of the *Z*-factor estimates $\hat{Z}_d$ and $\hat{Z}$ have been assumed to be correlated. The argumentation is as follows: $\hat{Z}_d$ and $\hat{Z}$ relate to nearly equal pressures and temperatures. Since the equation of state is empirical, it may be correct to assume that the error of the equation is systematic in the pressure and temperaure range in question.

$E_{Z,mod} \equiv$ relative standard uncertainty of the estimate $\hat{Z}$ due to *model uncertainty* (the uncertainty of the equation of state itself, and the uncertainty of the "basic data" underlying the equation of state),

$E_{Zd,ana} \equiv$ relative standard uncertainty of the estimate $\hat{Z}_d$ due to *analysis uncertainty* (measurement uncertainty of the gas chromatograph used to determine the gas composition, and variation in gas composition),

$E_{Z,ana} \equiv$ relative standard uncertainty of the estimate $\hat{Z}$ due to *analysis uncertainty* (measurement uncertainty of the gas chromatograph used to determine the line gas composition, and variation in gas composition).

From Eq. (G.18) $E_{Z_d/Z}$ will in practice be negligible, since the same equation of state is normally used for calculation of $\hat{Z}_d$ and $\hat{Z}$. Consequently, Eq. (G.16) can be approximated by

$$\begin{aligned} E_\rho^2 = {} & (s^*_{\rho_u})^2 E^2_{\rho_u} + E^2_{\rho,rept} + (s^*_{\rho,T})^2 E^2_T + (s^*_{\rho,T_d})^2 E^2_{T_d} + (s^*_{\rho,T_c})^2 E^2_{T_c} \\ & + (s^*_{\rho,K_d})^2 E^2_{K_d} + (s^*_{\rho,\tau})^2 E^2_\tau + (s^*_{\rho,c_c})^2 E^2_{c_c} + (s^*_{\rho,c_d})^2 E^2_{c_d} \\ & + (s^*_{\rho,\Delta P_d})^2 E^2_{\Delta P_d} + (s^*_{\rho,P})^2 E^2_P + E^2_{\rho,temp} + E^2_{\rho,misc} \end{aligned} \tag{G.19}$$

By multiplication with $\hat{\rho}^2$, the corresponding expanded uncertainty of the density measurement is given as

$$\begin{aligned} u_c^2(\hat{\rho}) = {} & s^2_{\rho_u} u^2(\hat{\rho}_u) + u^2(\hat{\rho}_{rept}) + s^2_{\rho,T} u_c^2(\hat{T}) + s^2_{\rho,T_d} u^2(\hat{T}_d) + s^2_{\rho,T_c} u^2(\hat{T}_c) \\ & + s^2_{\rho,K_d} u^2(\hat{K}_d) + s^2_{\rho,\tau} u^2(\hat{\tau}) + s^2_{\rho,c_c} u^2(\hat{c}_c) + s^2_{\rho,c_d} u^2(\hat{c}_d) \\ & + s^2_{\rho,\Delta P_d} u^2(\Delta\hat{P}_d) + s^2_{\rho,P} u_c^2(\hat{P}) + u^2(\hat{\rho}_{temp}) + u^2(\hat{\rho}_{misc}) \end{aligned} \tag{G.20}$$

which is the expression used in Chapter 3, Eq. (3.14), and implemented in the program *EMU - USM Fiscal Gas Metering Station*.

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