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7 STEPS TO CALCULATE

Measurement Uncertainty

By Rick Hogan

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The Measurement Uncertainty Process

1. **Specify the Process and Equation**
2. **Identify and Characterize the Uncertainty Sources**
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Introduction

Calculating measurement uncertainty is not easy. In fact, I speak with people every day who are having problems with estimating uncertainty. Therefore, I decided to put together this guide disclosing my exclusive seven step process to calculating measurement uncertainty.

In this guide, you will learn how to calculate measurement uncertainty in seven easy steps. Also, you will learn what information you need to calculate uncertainty, how to identify contributors to uncertainty, and how to evaluate your calculations to prevent overestimating or underestimating uncertainty. Furthermore, I will share with you some of my exclusive tips to help you calculate uncertainty like a pro.

Now, this guide is not a complete "how to" manual. Nor, will it answer all of your questions. Instead, it should be used as a quick reference guide to simplify the uncertainty estimation process into seven steps and learn some of my personnel secrets used when I calculate uncertainty.

So, read this guide and use my advice to help you calculate uncertainty. If you have questions, make sure to **contact me**. Additionally, feel free to use this guide to help you write an uncertainty procedure for your laboratory.

1. Specify the Process and Equation

Before you dive in and begin calculating uncertainty, it is best to have a plan. The first part of your plan should be to identify the measurement process or system that you wish to evaluate.

To help you out, start by answering the following questions;

-What are you measuring?

-How will you measure it?

-What equipment will you use?

-Who will perform the measurements?

-Where will the measurements be performed?

-What factors may affect the measurement results?

After answering these questions, use your answers to identify what measurement process you are evaluating. Once you have identified what you are evaluating, you move on to the step.

If you are performing indirect measurements that require you to calculate your measurement results, you should specify the equation used. This is because each variable in the equation will have its own uncertainty, which will directly affect the uncertainty associated with the calculated result.

To help you out, think of using dead weight testers or calibrating torque transducers and standard resistors. Each of these measurement processes require you to use an equation to calculate a result for comparison purposes. To estimate uncertainty, you will want to break down the equation and evaluate the uncertainty of each variable.

2. Identify and Characterize the Uncertainty Sources

Now that you know your measurement process, you need to identify the factors that influence uncertainty in measurement results. This process is not always easy and can get frustrating. So, stay calm, be patient, and keep researching. You may be surprised by how many influences can affect your measurement results.

Before you begin, I recommend that you find a book or guide on the measurement process you are evaluating. Physics, Chemistry, and Engineering textbooks can come in handy for understanding background and detailed information about your measurement process. If new textbooks are too expensive, you should be able to buy reasonably priced used books on websites like **Ebay**, **Amazon**, **Chegg**, or **Half.com**.

Other resources that you may want to consider are **ASTM** and **ISO** methods. However, if you like free resources (like I do), you may want to search National Metrology Institute websites, such as **NIST**, **NPL**, and **BIPM**. They may have downloadable guides related to your specific measurement processes.

If you are evaluating an equation, then the process is a little different. You will want to identify each variable in the equation and think about what influences each variable.

For example, if you are evaluating the calibration of a torque transducer, you will first write out the equation.

$$M = r \cdot F \cdot sin\theta$$

Where,
M = Moment of Force (i.e. Torque)
r = radius from the center of the fulcrum
F = Force applied
θ = Theta or the angle of applied force

When you evaluate the equation further, you begin to account for other factors that influence the equation. In this example, we begin to consider the radius of the torque arm and cable, the mass of the weights and pan, and the local gravity. If needed, we can evaluate the equation even further to account for more influences and increase the complexity of our uncertainty analysis.

$$M = \left(r_{arm} + \frac{1}{2} r_{cable}\right) \cdot \left(m_{pan} \cdot g_L + m_{wgt} \cdot g_L\right) \cdot sin\theta$$

Where,
M = Moment of Force (i.e. Torque)
r_{arm} = radius of the torque wheel/arm
r_{cable} = radius of the cable
m_{pan} = mass of the weight pan
m_{wgt} = mass of the applied weights
g_L = local gravity
θ = the angle of applied force

Now that you have identified the equation and the variables, you can start to research what factors may cause changes or variations to each variable. Using my example, think about how temperature variations can cause thermal linear expansion or contraction of the radius, or how friction or cable strain can vary the magnitude of applied force.

Evaluating equations can sometimes be easy, but it can become quite difficult depending on the complexity of the equation. Knowing the **rules for the propagation of uncertainty** can come in handy in step 5.

3. Quantify the Magnitude of Uncertainty Components

Before calculating measurement uncertainty, you must first determine the magnitude of each contributing factor. To accomplish this, you may need to perform some data reduction and analysis.

To get started, you may want to gather the following items. They will be helpful when analyzing data.

-Calibration Reports

-Repeatability and Reproducibility (R&R) Studies

-Experiment Results

-Manufacture Manuals and Specifications

-Technical Documents and Guides

-Published papers, studies, journal articles, etc.

Using the items listed above, you should be able determine how much uncertainty is contributed from each source. If you need help, you can **contact me** for additional guidance or **hire me** to analyze the data for you.

One trend that I have observed over last five years is accreditation bodies and assessors want to see Type A data in your uncertainty analysis. This is data collected from Repeatability and Reproducibility studies. If you have not included this data, I recommend that you make it a priority. If you need help, you read my article "**How to Perform A Repeatability Test**."

Now, I know that you are thinking, "How do I determine the magnitude of these weird influences I found in a textbook, journal article, or that my assessor told me to consider?" Most likely, you are not NIST or a research laboratory capable of testing the physical influences that you have read or been told about. So, in these cases, simulation experiments can come in handy.

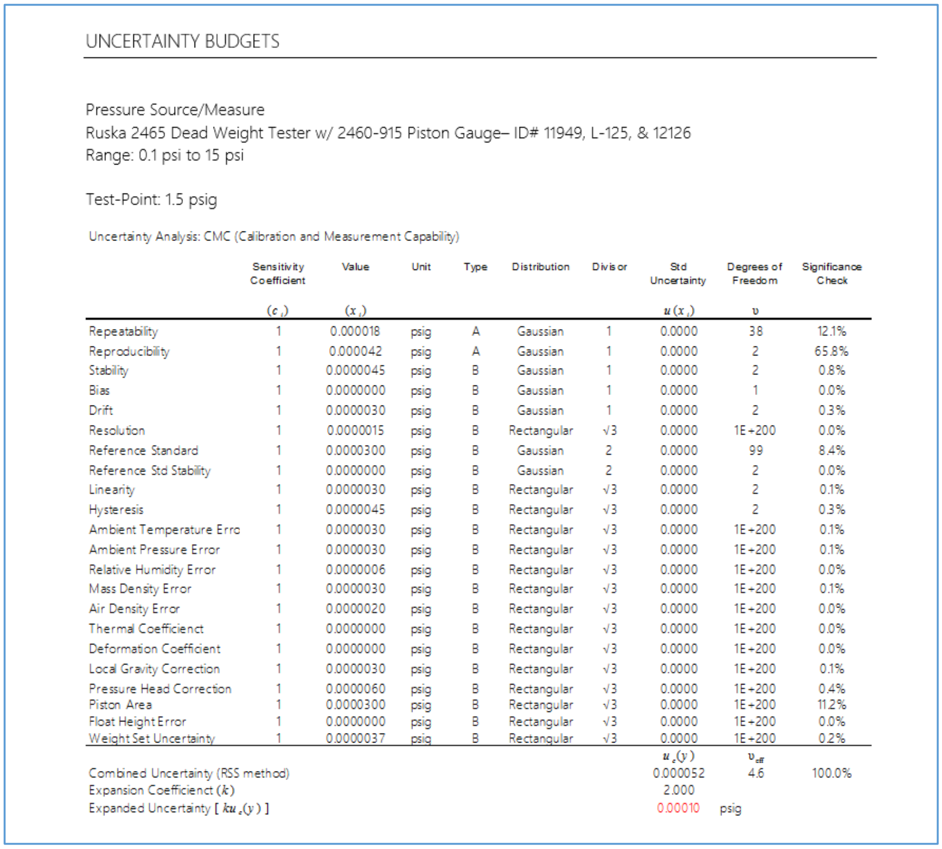
Simulation experiments are simply the mathematical estimation of outcomes using equations and varying the input variables to see how it changes the output variable.

While it may sound difficult, simulation can be pretty easy to do yourself using an excel spreadsheet. Using our example in step 2, create a calculator in your excel spreadsheet. Next, simulate an experiment by varying your inputs (preferably one at a time) and monitoring the variation of the output.

From the example in step 2, think about how a change in temperature can influence the linear expansion of the torque arm. If you know approximately how much the temperature in your laboratory varies, you can calculate the approximate linear expansion of your torque arm and determine the magnitude of change in the calculated value of torque.

This is how simulation works, and I recommend that you give a try. If my explanation does not make much sense to you, send me an email telling me you need more explanation. I will be glad to write an article about it.

Once you have analyzed your data and determined the magnitude of each uncertainty source, you will want to use an **uncertainty budget** help you calculate your total expanded uncertainty. Uncertainty budgets are a great tool for organizing your data and demonstrating how you calculated your estimation of measurement uncertainty. I recommend that you use one.



UNCERTAINTY BUDGETS

Pressure Source/Measure
Ruska 2465 Dead Weight Tester w/ 2460-915 Piston Gauge– ID# 11949, L-125, & 12126
Range: 0.1 psi to 15 psi

Test-Point: 1.5 psig

Uncertainty Analysis: CMC (Calibration and Measurement Capability)

	Sensitivity Coefficient	Value	Unit	Type	Distribution	Divisor	Std Uncertainty	Degrees of Freedom	Significance Check
	(c_i)	(x_i)					$u(x_i)$	υ	
Repeatability	1	0.000018	psig	A	Gaussian	1	0.0000	38	12.1%
Reproducibility	1	0.000042	psig	A	Gaussian	1	0.0000	2	65.8%
Stability	1	0.0000045	psig	B	Gaussian	1	0.0000	2	0.8%
Bias	1	0.0000000	psig	B	Gaussian	1	0.0000	1	0.0%
Drift	1	0.0000030	psig	B	Gaussian	1	0.0000	2	0.3%
Resolution	1	0.0000015	psig	B	Rectangular	√3	0.0000	1E+200	0.0%
Reference Standard	1	0.0000300	psig	B	Gaussian	2	0.0000	99	8.4%
Reference Std Stability	1	0.0000000	psig	B	Gaussian	2	0.0000	2	0.0%
Linearity	1	0.0000030	psig	B	Rectangular	√3	0.0000	2	0.1%
Hysteresis	1	0.0000045	psig	B	Rectangular	√3	0.0000	2	0.3%
Ambient Temperature Erro	1	0.0000030	psig	B	Rectangular	√3	0.0000	1E+200	0.1%
Ambient Pressure Error	1	0.0000030	psig	B	Rectangular	√3	0.0000	1E+200	0.1%
Relative Humidity Error	1	0.0000006	psig	B	Rectangular	√3	0.0000	1E+200	0.0%
Mass Density Error	1	0.0000030	psig	B	Rectangular	√3	0.0000	1E+200	0.1%
Air Density Error	1	0.0000020	psig	B	Rectangular	√3	0.0000	1E+200	0.0%
Thermal Coefficienct	1	0.0000000	psig	B	Rectangular	√3	0.0000	1E+200	0.0%
Deformation Coefficient	1	0.0000000	psig	B	Rectangular	√3	0.0000	1E+200	0.0%
Local Gravity Correction	1	0.0000030	psig	B	Rectangular	√3	0.0000	1E+200	0.1%
Pressure Head Correction	1	0.0000060	psig	B	Rectangular	√3	0.0000	1E+200	0.4%
Piston Area	1	0.0000300	psig	B	Rectangular	√3	0.0000	1E+200	11.2%
Float Height Error	1	0.0000000	psig	B	Rectangular	√3	0.0000	1E+200	0.0%
Weight Set Uncertainty	1	0.0000037	psig	B	Rectangular	√3	0.0000	1E+200	0.2%
							$u_c(y)$	υ_{eff}	
Combined Uncertainty (RSS method)							0.000052	4.6	100.0%
Expansion Coefficienct (k)							2.000		
Expanded Uncertainty [$ku_c(y)$]							0.00010	psig	

4. Convert Uncertainty Components to Standard Deviation Equivalents

The hardest part of the uncertainty analysis is complete, now you can focus on the easier process of calculating measurement uncertainty. To begin, you need to reduce each uncertainty contributor to a standard deviation equivalent.

This process starts with identifying a probability distribution that characterizes or resembles the dispersion of your data set. You should choose a probability distribution for each uncertainty contributor. If you are not sure which probability distribution best describes your data, most people choose the Rectangular distribution by default.

Some of the most common probability distributions used are;

-Normal or Gaussian

-Rectangular or Uniform

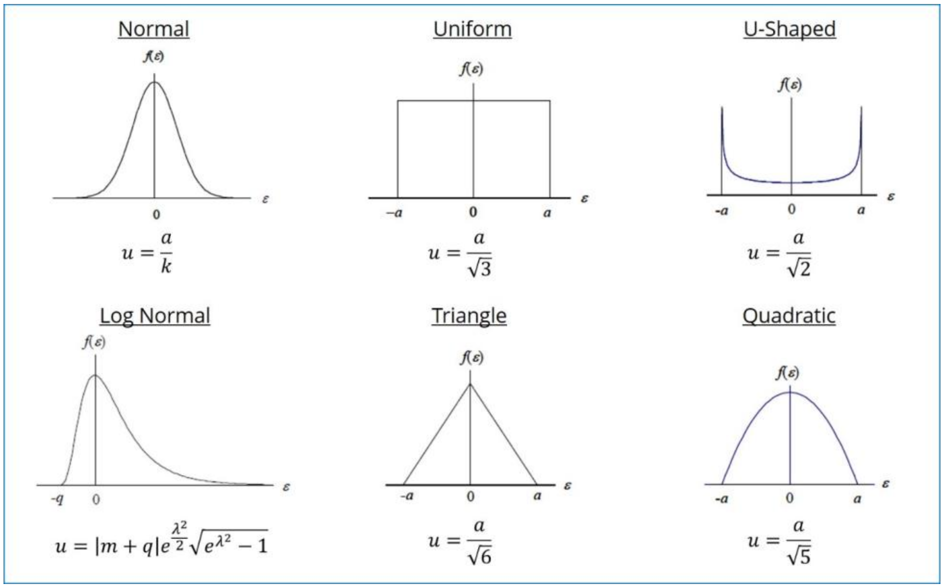
-Triangular

-Log-Normal

-Quadratic

-U-shaped

Use the following chart to help you select the appropriate probability distribution.



Once you select a probability distribution, you can identify the equation to reduce each uncertainty contributor to a standard deviation. This will reduce each source to a 1-sigma equivalent, so you can properly combine them in the next step. Make sure to perform this task for each uncertainty contributor that you quantified in step 3.

5. Calculate the Combined Standard Uncertainty

After you have reduced your uncertainty sources to standard deviation equivalents, it is time to combine them. To accomplish this, you will use the root sum of squares method or RSS. This will mathematically combine your uncertainty sources in quadrature. So, keep reading to learn **how to combine uncertainty**.

To begin, simply square the value of each uncertainty source. Next, add them all together to calculate the sum (i.e. the sum of squares). Then, calculate the square-root of the summed value (i.e. the root sum of squares). The result will be your Combined Uncertainty.

If you are a more visual learner, like me, take a look at the process below to see if it makes more sense.

$$CU = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$$

or

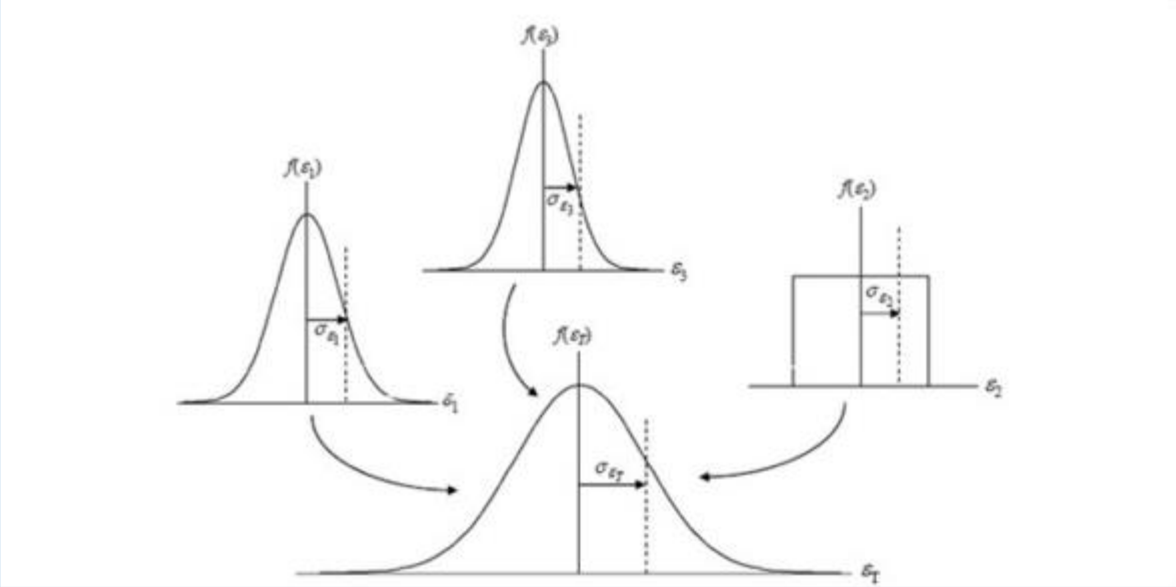
$$CU = (u_1^2 + u_2^2 + \cdots + u_n^2)^{\frac{1}{2}}$$

Both equations yield the same result. So, use the equation that is best for you. If you are using an excel spreadsheet calculator, you may find this function beneficial.

```
=SQRT(SUMSQ(cell1,cell2,…)
```

When you combine uncertainty sources, you are also combining the probability distributions. According to the Central Limit Theorem, the sum of the set of independent random variables (i.e. uncertainty sources) will approach a normal distribution regardless of the individual variable's distribution. Simply, the probability distribution associated with your combined uncertainty will now be normal.

If you need visual guidance, use the image below to see if it helps you understand.



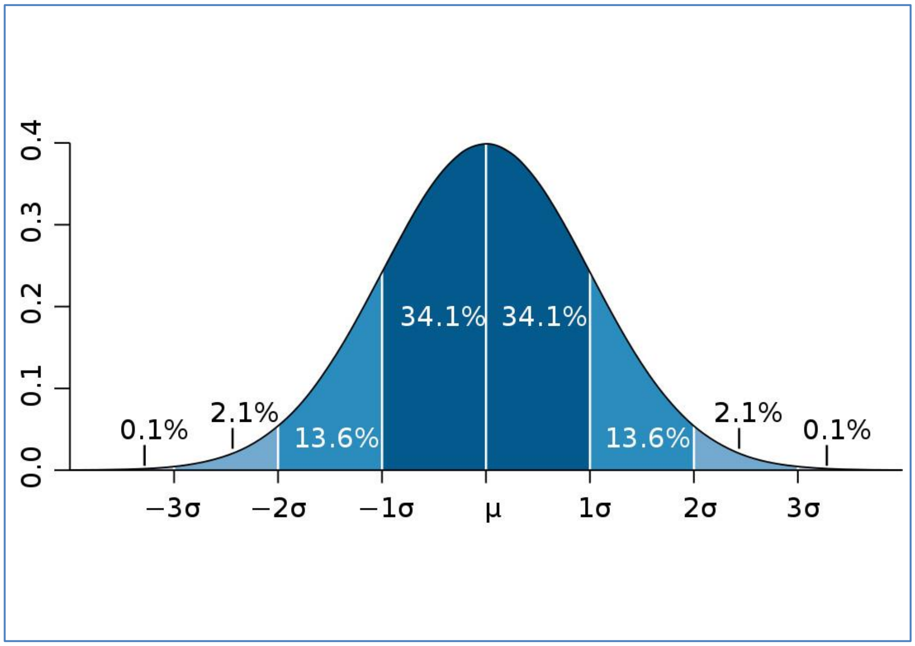
6. Calculate the Expanded Uncertainty

You are now entering the home stretch, so stay with me. I am going to show you how to calculate expanded uncertainty. In this step, you are going to expand your combined uncertainty to an acceptable level of confidence.

To satisfy ISO/IEC 17025:2005 requirements, you must expand uncertainty to approximately 95%. Most people use an expansion factor (k) of 2 to achieve a confidence interval of 95.45%. However, you can also use an expansion factor of 1.96 for a confidence interval of exactly 95.00%.

The choice is yours. Just make sure to select an expansion factor that you will consistently use in each of your uncertainty analyses. Also, it helps to know why you chose your expansion factor so you can justify it to assessors (if they ask).

Use the image below to see the breakdown of the normal probability distribution when expand your estimate to 2-sigma or 95.45% confidence.



To calculate your expanded uncertainty, simply multiply the expansion factor (k) and the combined uncertainty (CU). The result will be your expanded uncertainty (EU).

$$EU = k \cdot CU$$

That's it! You have just calculated your expanded uncertainty and completed the estimation of uncertainty in measurement. However, you are not done yet. I recommend that you verify your calculations. In the next section, I will tell you how to evaluate uncertainty calculations for appropriateness.

7. Evaluate Uncertainty for Appropriateness

Once you have calculated the expanded uncertainty, it is best to evaluate your uncertainty estimate for appropriateness. Essentially, you want to make sure that your measurement uncertainty estimate adequately represents your measurement process and is not overestimated or underestimated.

A good place to start your evaluation is to check and verify that your uncertainty estimate is not larger than the Reference Standard Uncertainty, or the uncertainty that was reported in your equipment's calibration certificates.

Next, try looking at other laboratories scopes of accreditation to verify your uncertainty calculations are reasonably comparable. I personally recommend that you review at least 3 to 5 other scopes. Make sure your estimates are relatively similar to other laboratories. If not, you may have overestimated or underestimated uncertainty.

Then, check the **BIPM Key Comparison Database** to make sure that you are not reporting uncertainty that is less than any national metrology institutes. I know this may sound silly, but I have observed other laboratories making this mistake. So, do yourself a favor and double-check your uncertainty calculations.

Finally, perform a Repeatability and Reproducibility study in your laboratory. Verify that your results are not larger than your uncertainty estimation. If so, you may have understated your uncertainty estimates.

Evaluating your uncertainty estimates are critical. You do not want to go through all the work of calculating measurement uncertainty only find mistakes during an assessment. It is better to perform the hard work upfront than deal with all the paperwork and headaches that result from being cited a deficiency.

Conclusion

In this guide, I have laid out seven steps to help you calculate measurement uncertainty. While this is not a complete how-to guide, I have given you plenty of information to help you perform uncertainty estimation yourself. So, start estimating uncertainty and tell me what works for you and what you struggle with.

If you have additional questions or suggestions that will help improve this guide, **contact me** and share your comments. I will glad to hear your feedback.

References

Castrup, H. (2007). *Distributions for Uncertainty Analysis.* Bakersfield, CA: Integrated Sciences Group.

Miller, V. (2002). *NISTIR 6919: Recommended Guide for Determining and Reporting Uncertainties for Balances and Scales.* Gaithersburg: National Institute of Standards and Technology.

United Kingdom Accreditation Service. (2012). *M3003: The Expression of Uncertainty and Confidence in Measurement.* Middlesex: United Kingdom Accreditation Service.

Help & Feedback

All good things have to come to an end, including this uncertainty guide. Don't worry. If you need additional help, I am only an email or phone call away from helping you overcome your challenges. Enjoy this uncertainty guide, share it with your friends, and be sure give me feedback.

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