The Complete Reference Guide to

PROBABILITY DISTRIBUTIONS AND DIVISORS FOR ESTIMATING Measurement Uncertainty

By Rick Hogan

Probability Distributions and Divisors For Estimating Measurement Uncertainty

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Introduction

Probability distributions are a part of measurement uncertainty analysis that people continually struggle with. Today, my goal is to help you learn more about probability distributions without having to grab a statistics textbook. Although there are hundreds of probability distributions that you could use, I am going to focus on the 6 that you **need to know**.

If you constantly struggle with probability distributions, keep reading. I am going to explain what are probability distributions, why they are important, and how they can help you when estimating measurement uncertainty.

What is a probability distribution

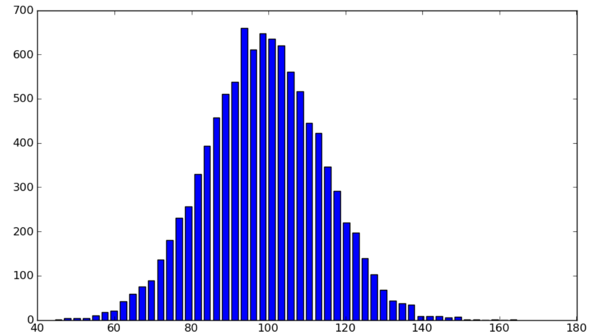
Simply explained, probability distributions are a function, table, or equation that shows the relationship between the outcome of an event and its frequency of occurrence.

Probability distributions are helpful because they can be used as a graphical representation of your measurement functions and how they behave. When you know how your measurement function have performed in the past, you can more appropriately analyze it and predict future outcomes.

Before jumping head-first into the different types of probability distribution, let's first learn a little more about probability distributions. In the next few paragraphs, I am going to explain some characteristics that you should know.

Histogram

A histogram is a graphical representation used to understand how numerical data is distributed. Take a look below at the histogram of a Gaussian distribution.

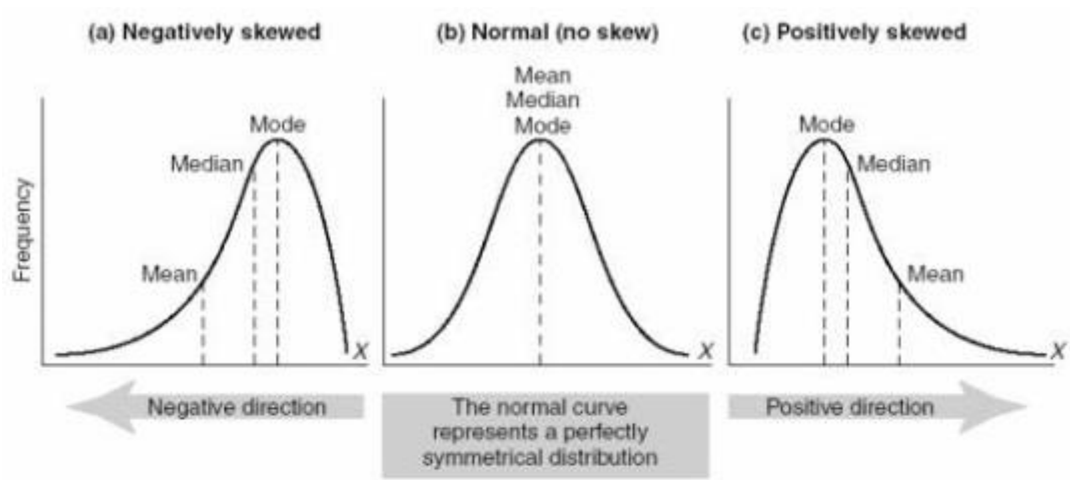


Look at the histogram and view how the majority of the data collected is grouped at the center. This is called central tendency.

Now look at height of each bar in the histogram. The height of the bars indicate how frequent the outcome it represents occurs. The taller the bar, the more frequent the occurrence.

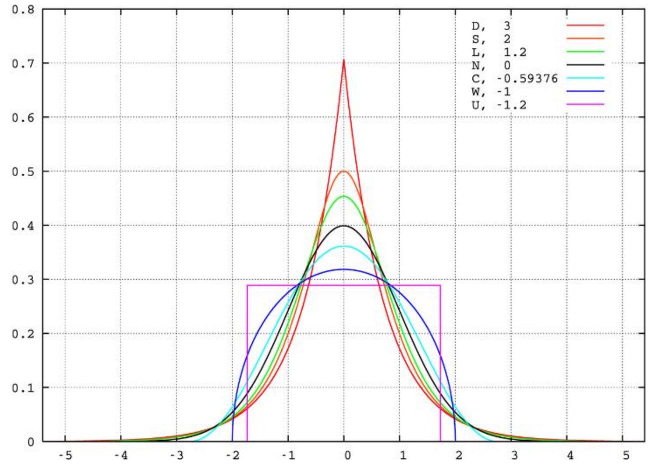
Skewness

Skewness is a measure of the probability distributions symmetry. Look at the chart below to visually understand how probability distributions can skew to the left or the right.



Kurtosis

Kurtosis is a measure of the tailedness and peakedness relative to a normal distribution. As you can see from the image below, distributions with wider tails have smaller peaks while distributions with greater peaks have narrower tails. Do you see the relationship?



Why is it important

I know it seems like I am making you read more information that you want to know, but it is important to know these details so you can select the appropriate probability distribution that characterizes your data.

If you are uncertain how your data is distributed, create histogram and compare it to the following probability distributions.

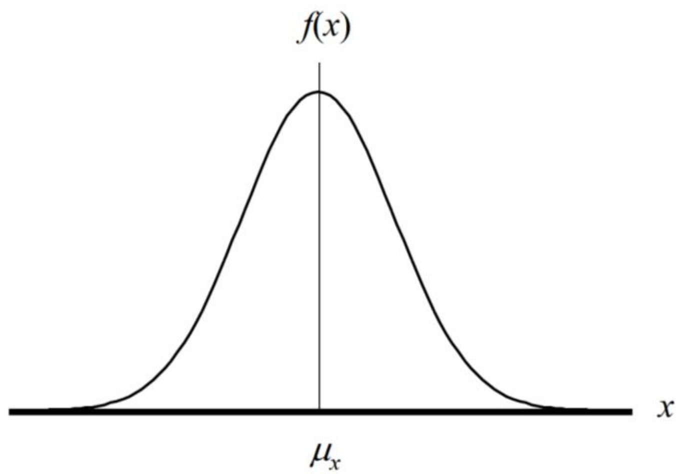
Most commonly used

The most commonly used probability distributions for estimating measurement uncertainty are;

- Normal
- Rectangular
- U-Shaped
- Triangle
- Log-Normal
- Rayleigh

Below you will find a list of the most common probability distributions used in uncertainty analysis. After reading this article, you should be able identify which probability distributions you should use and how to reduce your uncertainty contributors to standard deviation equivalents.

Gaussian (a.k.a. Normal) Distribution



The Normal distribution is a function that represents the distribution of many random variables as a symmetrical bell-shaped graph where the peak is centered about the mean and is symmetrically distributed in accordance with the standard deviation.

The normal distribution is the most commonly used probability distribution for evaluating Type A data. If you do not know what Type A data is, it is the data that you collect from experimental testing, such as repeatability, reproducibility, and stability testing.

To get a better understanding, imagine you are going to collect 100 measurement samples and create a histogram graph with your results. The histogram for your data should resemble a shape close to a normal distribution.

The more data that you collect, the closer your histogram will begin to resemble a normal distribution.

Now, I do not expect you to collect 100 samples every time you perform repeatability and reproducibility test. Instead, I recommend that you begin by collecting 20 to 30 samples for each test. This should give you a good baseline to begin with, and allow you to characterize your data with a normal distribution.

To reduce normally distributed data to a standard deviation equivalent, use the following equation. The variable a will be the value of your uncertainty contributor and k is the value of your expansion factor.

$$u_i = \frac{U_i}{k}$$

For example, if you collect 20 samples for a repeatability experiment and calculate the standard deviation, the value of k is 1. If you are wondering, it is equal to 1 because your standard deviation is already at the 1-sigma level (i.e. 68.27% confidence).

So, if your calculated standard deviation is 1 ppm, then;

$$u_i = \frac{U_i}{k} = \frac{1}{1} = 1.000$$

When using Microsoft Excel to calculate measurement uncertainty, use the following equation:

=[Cell1]/1

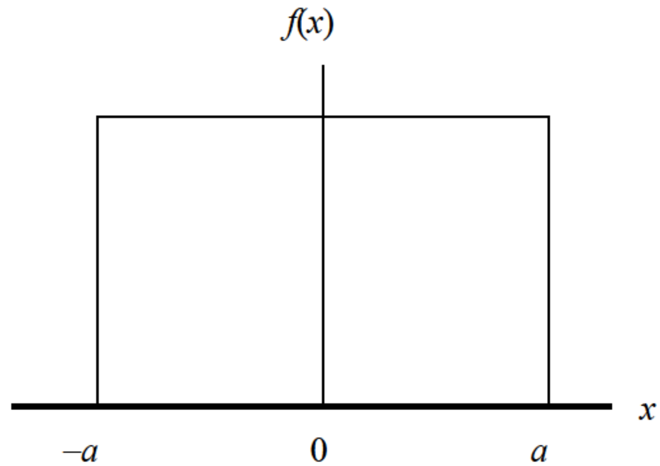
For the next example, imagine you are evaluating the measurement uncertainty from your calibration report. Most likely, it is reported to 95% confidence where k equals 2 (I am sure that you have read this somewhere before). If your reported uncertainty is 1 ppm, then;

$$u_i = \frac{U_i}{k} = \frac{1}{2} = 0.500$$

When using Microsoft Excel to calculate measurement uncertainty, use the following equation:

=[Cell1]/2

Rectangular (a.k.a. Uniform) Distribution



The Rectangular Distribution is a function that represents a continuous uniform distribution and constant probability. In a rectangular distribution, all outcomes are equally likely to occur.

The rectangular distribution is the most commonly used probability distribution in uncertainty analysis. If you are wondering why, it is because it covers the majority of uncertainty factors where the evaluator says, "I am not sure how the data is distributed."

When you are not confident how your data is distributed, it is best evaluate it conservatively. In this situation, the rectangular distribution is a great default option which is why most ISO/IEC 17025 assessors recommend it. So, make sure to pay attention, you will be using this probability distribution a lot.

To reduce your uncertainty contributors to standard deviation equivalents, you will want to divide your values by the square-root or 3.

$$u_i = \frac{U_i}{\sqrt{3}}$$

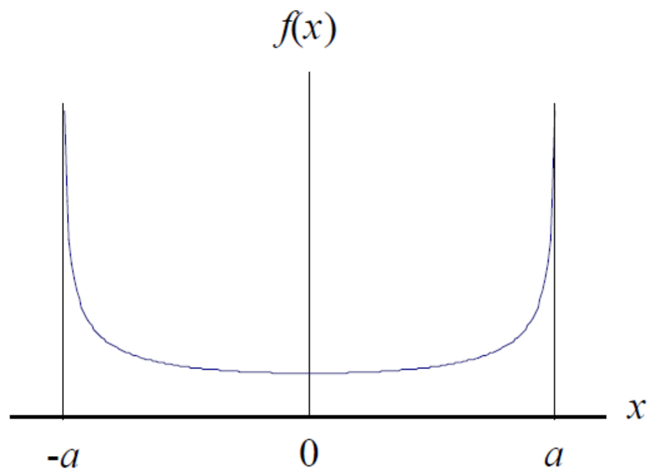
For example, if you performing measurement uncertainty analysis and evaluating the contribution of a factor that has an influence of 1 part-per-million and you propose that the data is uniformly distributed, then;

$$u_i = \frac{U_i}{\sqrt{3}} = \frac{1}{\sqrt{3}} = 0.577350 \approx 0.577$$

When using Microsoft Excel to calculate measurement uncertainty, use the following equation:

=[Cell1]/SQRT(3)

U-shaped Distribution



The U-shaped Distribution is a function that represents outcomes that are most likely to occur at the extremes of the range. The distribution forms the shape of the letter 'U,' but does not necessarily have to be symmetrical.

The u-shaped distribution is helpful where events frequently occur at the extremes of the range. Consider the thermostat that controls the temperature of your laboratory. If you are not using a PID controller, your thermostat controller only attempts to control temperature by activating at the extremes.

For example, imagine that your laboratory thermostat is set at 20°C and controls temperature to 1°C. Most likely, your thermostat does not activate your HVAC system until the laboratory temperature reaches either 19°C or 21°C. This means that your laboratory is not normally at 20°C. Instead, your laboratory temperatures are floating around the limits of the thermostat's thresholds before activating or deactivating.

For this reason, it is best to characterize your laboratory temperature data using a u-shaped distribution.

To reduce your uncertainty contributors to standard deviation equivalents, you will want to divide your values by the square-root or 2.

$$u_i = \frac{U_i}{\sqrt{2}}$$

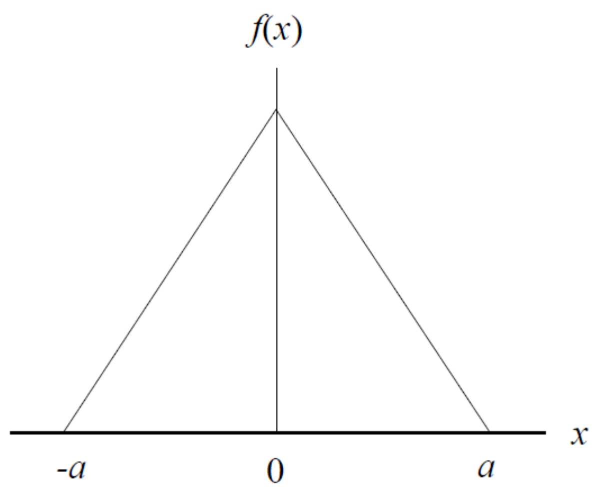
So, if you performing measurement uncertainty analysis and evaluating the contribution of a factor that has an influence of 1 part-per-million and you propose that the data for this factor is u-shaped distributed, then;

$$u_i = \frac{U_i}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707107 \approx 0.707$$

When using Microsoft Excel to calculate measurement uncertainty, use the following equation:

=[Cell1]/SQRT(2)

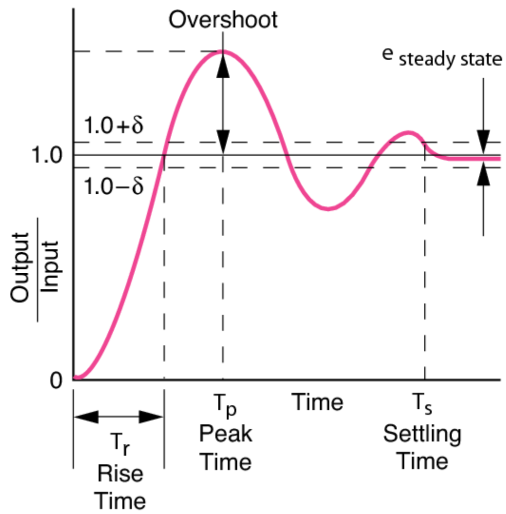
Triangle Distribution



The Triangle Distribution is a function that represents a known minimum, maximum, and estimated central value. It is commonly referred to as the “lack of knowledge” distribution because it is typically used where a relationship between variables is known, but data is scare.

Additionally, the triangle distribution is commonly used where the data collection is difficult or expensive.

For a real world example, image your laboratory is temperature controlled using a PID thermostat controller. The PID thermostat controller is constantly trying to achieve the target temperature set-point. For this reason, the temperatures in your laboratory are constantly floating around 20°C and rarely reach the temperature thresholds (i.e. limits) of a typical thermostat controller.



What this means is that most of your laboratory's temperature data is centered around your set temperature. Therefore, it is best characterized by a triangular distribution because we know the limits and the estimated mean but we are unsure how the data is distributed between these points.

To reduce your uncertainty contributors to standard deviation equivalents, you will want to divide your values by the square-root or 6.

$$u_i = \frac{U_i}{\sqrt{6}}$$

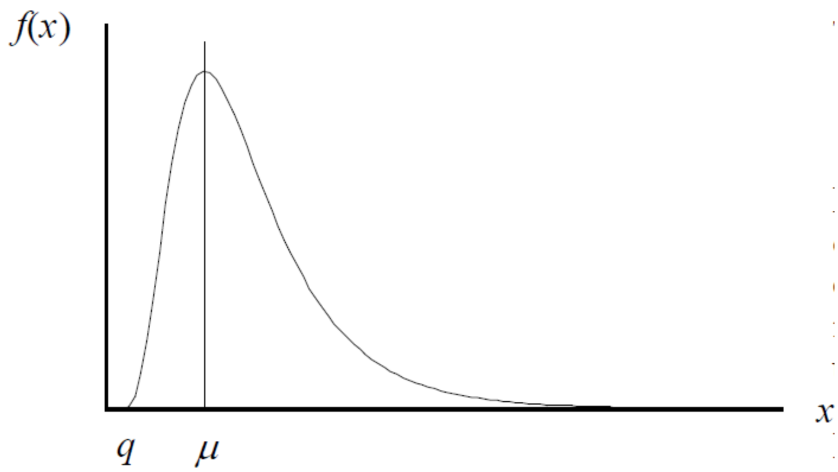
So, if you performing measurement uncertainty analysis and evaluating the contribution of a factor that has an influence of 1 part-per-million and you propose that the data for this factor is triangle distributed, you would reduce it's value by using the equation above.

$$u_i = \frac{U_i}{\sqrt{6}} = \frac{1}{\sqrt{6}} = 0.408248 \approx 0.408$$

If you using Microsoft Excel to calculate measurement uncertainty, use the following equation:

=[Cell1]/SQRT(6)

Log-Normal Distribution



The log-normal distribution is a function of a natural logarithm that is normally distributed.

The log-normal distribution is a distribution is commonly encountered but rarely used. Most of the time it is the result of lack of knowledge or failure to develop a histogram for your data.

For example, if you are performing measurements that are finite, such as length, height, weight, etc., you are most likely going to end up with data that resembles a log-normal distribution. It is most common in dimensional and mechanical metrology.

To give a better understanding, think of calibrating a gage block. Before you begin calibration, you know the target length. If you perform repeated measurements at the single point on the gage block, the majority of your measurement results will be centered around the actual length of the gage block. Some measurement results will be larger than the actual value of the gage block, and much fewer measurement results will be less than the actual value of the gage block.

The reason this happens is your measurement results are limited by the length of the gage block. Realistically, you cannot measure less than the length of the block; so, your measurement results are finite or limited.

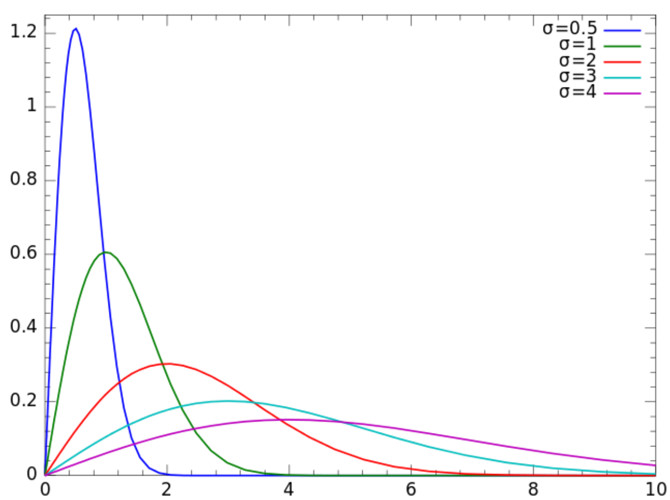
Make sure to consider the log-normal distribution next time you are performing measurements that are finite. It may prevent you from encountering measurement errors and mis-calculated uncertainties.

To reduce your uncertainty contributors to standard deviation equivalents, you will want use the following equation.

$$u_i = (m - q)e^{\frac{\lambda^2}{2}}\sqrt{e^{\lambda^2} - 1}$$

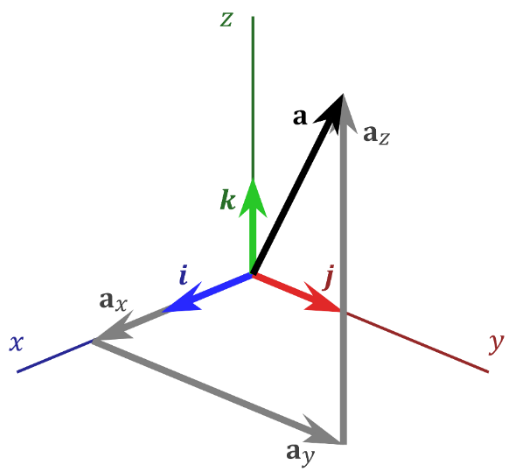
Where,
m = median
q = limit

Rayleigh Distribution



Rayleigh distributions are used when the magnitude of a vector is associated with it's directional components (e.g. x and y), which can also be real and imaginary components.

When directional components are orthogonal and normally distributed, the resulting vector will be Rayleigh distributed.



Rayleigh distributions are commonly used in electrical metrology for RF and Microwave functions. Additionally, they are commonly used in mechanical metrology where vectors are involved.

For example, when wind velocity is analyzed by its 2 dimensional vector components, x and y, the resulting vector is Rayleigh distributed. For this to happen, x and y must be orthogonal and normally distributed.

Reducing uncertainty components to standard deviation equivalents is tricky with the Rayleigh distributions. You will need to know the standard deviation of each directional component to calculate the measurement uncertainty of the vector component. Afterward, you can use the equation below to reduce your uncertainty component to a standard deviation equivalent.

$$u = \frac{U_i}{\sqrt{2 \cdot In(20)}}$$

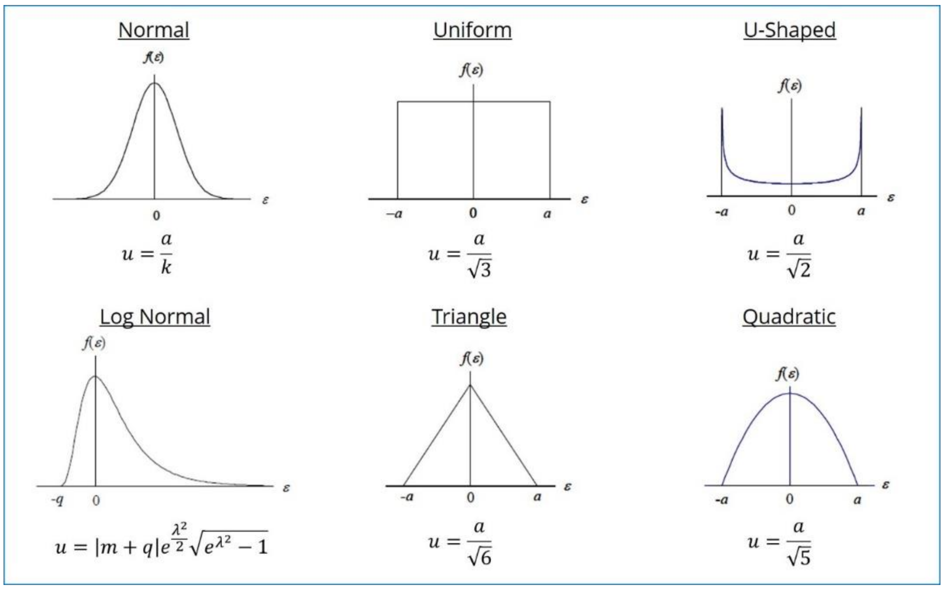
For a better explanation, click the link below to read this paper by Michael Dobbert from Agilent (now Keysight).

Revisiting Mismatch Uncertainty with the Rayleigh Distribution

Conclusion

Probability distributions are an important part of understanding the behavior of functions, analyzing data, and predicting future outcomes. This is why they are a critical component of uncertainty analysis. If you are estimating measurement uncertainty without considering probability distributions, you are going to make mistakes. So make sure to use this guide as a reference when calculating uncertainty.

Additionally, it never hurts to use this chart.



I hope that you have found this article helpful for your uncertainty analysis. **Leave me a comment and tell me the probability distributions you use in your uncertainty analysis.**

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If I have left anything out of this guide or if you can think of any additional tips that would improve this list, please leave a comment or **contact me** to share your advice.

Help & Feedback

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