1 ABSTRACT

Although there are, in many countries, uncertainty ranges acceptable by law and by contracts signed in fiscal metering of natural gas, in many cases is not defined the methodology for calculation of this uncertainty. These data, in addition to verify the certainty of the measurement, can be used in management decisions in equipment and process applications.

This paper applied the methodology shown in GUM (JCGM, 2008), mapping the uncertainties involved in the metrological process. In the analysis were verified several parameters that are normally discarded such as the differences between the calibration and process conditions, errors due the heat transfer between the pipe and the thermowell, variations in atmospheric pressure, and chromatographic uncertainty. In total, fifty four sources of uncertainty were verified. In addition to these data, were considered the uncertainties that affect the amount billed, producing a monetary representation of uncertainty values.

As a case study, was used a custody transfer station in Maranhao / Brazil, with ultrasonic meters in a flow rate of 2,200,000 Sm³/day, at 32 bar.

The results showed the relevance of factors that generate uncertainty in the final measurement, generating reliable data to guide management decisions, showing, for example, if is economically feasible to purchase more expensive and precise equipment, comparing them with the uncertainty values obtained with these equipment.

2 OBJECTIVE

This paper aims to demonstrate the natural gas measurement uncertainty calculations applied at Companhia Distribuidora de Gás Natural do Maranhão - Gasmar.

The uncertainty calculation in accordance with this document aims to:
- Ensure the measurement reliability of consumed volumes;
- Ensure contractual and regulatory compliance with respect to the criterion of uncertainty maximum;
- Add value to the product and service provided in volume measurement;
- Ensure traceability and transparency in uncertainty calculation;
- Provide auditable material to give basis for an arbitration in volumes disputes;
- Ensure that invoiced volumes are fair and not unduly benefit any of the parties;
- Give basis for management decisions.

3 INTRODUCTION

The Joint Resolution ANP/INMETRO No. 1, from June 10, 2013, Item 6.4.7 - b, sets the maximum uncertainty of flow measurement for custody transfer in Brazil at 1.5%. Several other standards in other countries establish maximum uncertainty parameters. However, the methodology for calculation of this uncertainty is not defined. This document defines a methodology for calculation of uncertainty and error in the fiscal measurement based on ISO GUM (JCGM, 2008).

The methodology consists of the individual analysis of each source of uncertainty, combining uncertainties that interfere in each variable separately and calculating the contribution of each variable on
the final value. After the calculation of the uncertainty of the measured volume, the uncertainty is then analyzed over the invoiced amount corresponding to the volume, which includes other sources of uncertainty, such as tariff calculation.

The methodology was tested and applied in a custody transfer measurement system in Maranhao / Brazil using a 10” measurement system, pressure approximately at 32 bar, 40 °C gas temperature, average flow of 2,200,000 Sm³/day and ultrasonic meter with four pairs of transducers. The straight segment, as well as the entire system, was designed in accordance with AGA Report No. 9.

In addition to the confirmation of the legal and contractual compliance and measurement transparency and reliability, these data give basis for management decisions. It can be seen, for example, if the application of an in line chromatograph at the point of delivery reduces the financial risk of this application in order to enable the investment cost, just by comparing the difference in billing uncertainty with and without the chromatograph with the cost of purchasing and installation of the equipment.

4 CALCULATION METHODOLOGY

4.1 Concept and volume uncertainty method

The method demonstrated below is defined in ISO GUM (JCM 2008), applied to the fiscal measurement of natural gas, in order to quantify the uncertainty arising from this measurement. There may be variations depending on the measurement system design (equipment, dimensions, etc.) and processes adopted.

4.1.1 Calculation Method

The calculation of the uncertainty is performed from the partial uncertainties of all variables that act on the system. To analyze which variables impact the system the definition of the mathematical model is required. Typical mathematical models in the gas measurement are detailed in Item 4.2.

After defining the mathematical model, the variables that comprise it and the uncertainties of each variable are analyzed.

4.1.1.1 Partial Uncertainties

Initially, the individual analysis of each variable that impacts the final result is performed. A given variable can result in several sources of uncertainty. For example, the variable temperature may have uncertainty due to repeatability, response time, drift, resolution, etc.

All the variable uncertainties must be identified. The quantification of individual uncertainties is detailed in Item 4.3.

In order all uncertainties are on the same level of confidence, every uncertainty must be divided by their respective coverage factor.

4.1.1.2 Combined uncertainty of the variables

After the definition and quantification of the partial uncertainties, the combined uncertainty is calculated for each variable, algebraically equal to geometric sum of the partial uncertainties (Equation 1).
\[ u_{cj} = \frac{k_j}{2} \sqrt[n_j]{\sum_{i=1}^{n_j} u_{ji}^2} \]  

(1)

Where:
- \( u_{cj} \): Combined uncertainty of variable \( j \) (same unit of variable \( j \));
- \( k_j \): Coverage factor of uncertainty \( u_{cj} \) calculated according to Item 4.1.1.2.1 for a confidence level of 95.45\% (dimensionless);
- \( n_j \): Number of uncertainties identified from variable \( j \) (dimensionless);
- \( u_{ji} \): Partial uncertainty \( i \) of variable \( j \) (same unit of variable \( j \)).

### 4.1.1.2.1 Effective degrees of freedom

Some quantified uncertainties may have been obtained by empirical measurement, in this case, the number of measurements can affect the results obtained. It is applied then the t-Student method to correct the measured value to a theoretical condition in which endless measurements would be performed. This is the case, for example, of the repeatability, which is obtained by repeated measurement of the same value, checking the variations that occur. The more measurements are made, the more reliable the result.

Algebraically, this method is reflected in the coverage factor shown in Equation 1, which is obtained from Table 1 or by the formula shown in Figure 1 (MS Excel - BR) using the effective degrees of freedom, calculated according to Equation 2.

\[ \nu_{eff} = \frac{u_c^4}{\sum_{i=1}^{N} \frac{u_i^4}{\nu_i}} \]  

(2)

Where:
- \( \nu_{eff} \): Effective degrees of freedom (dimensionless);
- \( \nu_i \): Uncertainty degrees of freedom \( u \) (dimensionless);
- \( u_c \): Combined uncertainty of the variable analyzed (same unit of the variable analyzed);
- \( u_i \): Uncertainty \( i \) of the variable analyzed (same unit of the variable analyzed);

![Figure 1. Calculation of the coverage factor in Excel](image)
Table 1. Coverage Factor in function of Effective Degree of Freedom and Level of Confidence

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### 4.1.2 Sensitivity Coefficient

For the calculation of the global uncertainty, combined to the same analysis variable (in this case the volume converted to reference conditions), it is necessary to calculate the Sensitivity Coefficient, which represents the impact the uncertainty of a variable causes in the uncertainty of the result. When the variables are independent, the sensitivity coefficient is calculated according to Equation 3. When the sensitivity coefficient of a variable is multiplied by the uncertainty of this variable, it results in the Contribution of this uncertainty. The geometric sum of the contributions of each variable generates the overall uncertainty (Taylor series considering that the system is linear), which multiplied by a coverage factor results in the Expanded Uncertainty (Equation 4).
\[ c_j = \frac{\partial f}{\partial x_j} \]  

\[ U = k \sqrt{\sum_{j=1}^{N} (u_{c_j} \cdot c_j)^2} \]  

Where:
- \( f \): Mathematical model to obtain the results from the variables analyzed;
- \( x_j \): Variable j;
- \( c_j \): Sensitivity Coefficient of variable j;
- \( U \): Expanded overall uncertainty;
- \( k \): Coverage Factor considering infinite degrees of freedom;
- \( N \): Number of variables of the mathematical model.

Follow below the sensitivity coefficient of each variable of the mathematical model defined in Item 4.2.

\[ \frac{\partial V_R}{\partial N} = \frac{V_R}{N} \quad \frac{\partial V_R}{\partial P_F} = \frac{V_R}{P_F + P_{ATM}} \quad \frac{\partial V_R}{\partial P_{ATM}} = \frac{V_R}{P_F + P_{ATM}} \quad \frac{\partial V_R}{\partial T_F} = -\frac{V_R}{T_F} \]

\[ \frac{\partial V_R}{\partial Z_R} = \frac{V_R}{Z_R} \quad \frac{\partial V_R}{\partial Z_F} = -\frac{V_R}{Z_F} \quad \frac{\partial V_R}{\partial PCS_F} = \frac{V_R}{PCS_F} \]  

**4.2 Definition of the mathematical model**

The volume conversion for the base conditions is performed from the Clapeyron equation and it is shown in Equation 6, considering as negligible the systematic error of the flowmeter (pulse compensation on the meter).

\[ V_R = V_F \cdot \left(\frac{P_F + P_{atm}}{P_R} \right) \cdot \left(\frac{T_R}{T_F} \right) \cdot \left(\frac{Z_R}{Z_F} \right) \]  

Where:
- \( V_R \): Volume at base conditions (m³);
- \( V_F \): Volume at flow conditions (m³);
- \( P_F \): Pressure at flow conditions (Pa);
- \( P_{atm} \): Local atmospheric pressure (Pa);
- \( P_R \): Pressure at base conditions (Pa);
- \( T_R \): Temperature at base conditions (K);
- \( T_F \): Temperature at flow conditions (K);
- \( Z_R \): Compressibility at base conditions (dimensionless);
- \( Z_F \): Compressibility at flow conditions (dimensionless).

To offset the calorific value of the gas, a calorific value factor is included as per Equation 7.
\[ V_R = V_F \cdot \frac{(P_F + P_{atm})}{P_R} \cdot \frac{T_R}{T_F} \cdot \frac{Z_R}{Z_F} \cdot \frac{PCS_F}{PCS_R} \quad (7) \]

Where:
- \( PCS_R \) : Reference Superior Calorific Value (J/m³);
- \( PCS_F \) : Superior Calorific Value of the Gas at flow conditions.

Assuming a constant factor \( K \), the volume in flow conditions is detailed depending on the amount of pulses, according to Equation 8, which also details the calorific value calculated as the method shown in ISO 6976.

\[ V_R = \frac{N}{K} \cdot \frac{(P_F + P_{atm})}{P_R} \cdot \frac{T_R}{T_F} \cdot \frac{Z_R}{Z_F} \cdot \frac{\sum_{j=1}^{n} x_j \cdot \bar{H}^o_j}{PCS_R} \quad (8) \]

Where:
- \( N \) : Accumulated amount of pulses (pulses);
- \( K \) : Pulses correspondence factor per cubic meter (pulse/m³);
- \( x_j \) : Mole fraction of the jth component (dimensionless);
- \( \bar{H}^o_j \) : Ideal calorific value in volumetric base at the base conditions (MJ/m³);
- \( n \) : Quantity of natural gas constituent components (dimensionless).

Considering the expansion of the flowmeter body due to pressure and thermal expansion, and detailing the calculation of the atmospheric pressure based on local altitude, Equation 9 is formed as follow.

\[ V_R = V_{raw} \cdot \text{ExpCorr}_P \cdot \text{ExpCorr}_T \cdot \frac{P_F + P_0 \cdot \left(1 + \frac{a \cdot h}{T_0} \right)^{-M \cdot g}}{P_R} \cdot \frac{T_R}{T_F} \cdot \frac{Z_R}{Z_F} \cdot \frac{\sum_{j=1}^{n} x_j \cdot \bar{H}^o_j}{PCS_R} \quad (9) \]

Where:
- \( V_{raw} \) : Raw volume measured by flowmeter (m³);
- \( \text{ExpCorr}_P \) : Correction factor of the flowmeter expansion due to pressure (dimensionless);
- \( \text{ExpCorr}_T \) : Correction factor of the flowmeter expansion due to thermal expansion (dimensionless);
- \( P_0 \) : Atmospheric pressure at reference location (Pa);
- \( a \) : Rate of temperature variation in relation to altitude (K/m);
- \( h \) : Altitude in relation to atmospheric base point (m);
- \( T_0 \) : Temperature at atmospheric base point (K);
- \( M \) : Air mole mass (kg/m³);
- \( g \) : Local gravity (m/s²);
- \( R \) : Universal gas constant (J/mol.K).

Details of the correction factors of the flowmeter expansion due to pressure and temperature is shown in Equation 10.
\[
ExpCorr_P \cdot ExpCorr_T = 1 + \left[ 3 \cdot \left( \frac{D_{out}^2 \cdot (1 + \nu)}{E \cdot (D_{out}^2 - D_{in}^2)} \right) \cdot \left( P_F + P_{atm} - P_R \right) \right]
\] (10)

Where:
- \( D_{out} \): Outer diameter of the piping or flowmeter (m);
- \( D_{in} \): Inner diameter of piping or flowmeter (m);
- \( \nu \): Poisson Coefficient (dimensionless);
- \( E \): Young Module (MPa).

To simplify the analysis and calculation process of the uncertainty of the parameters analyzed, it is used the basic model defined in Equation 7, details shown in Equations 8, 9 and 10 are calculated separately.

4.3 Uncertainty Sources

To check the uncertainties in the reference volume, the individual variables are analyzed, which are shown below.

4.3.1 Uncertainties related to the quantity of accumulated pulses (N)

4.3.1.1 Reading Resolutions

The transmission of flow by pulses has the range uncertainty of a pulse due to the resolution. The higher the frequency of pulses per cubic meter is, the lower the impact of this uncertainty on the volume.

4.3.1.2 Loss of Pulse

Accumulated pulses differences between the generated by the flow meter and the received by flow computer accumulator may eventually occur. These differences may happen for various reasons, such as the interferences in the inductive environment, contact with corrosion, and incompatible impedance. The estimation of loss of pulse is performed based on the analysis of the history between the flow meter accumulator and the flow computer accumulator.

4.3.1.3 Uncertainty Inherited From the Calibration Laboratory

The analyzed measurement system has a calibration curve correction in the flow meter. The values used in the curve correction are subject to the calibration laboratory uncertainties and should be taken into consideration in the analysis. It is obtained with the multiplication of the uncertainty informed in the calibration certificate of the flowmeter (comprising all the uncertainties of the laboratory, such as the ones inherited from patterns, repeatability, etc.) by the calibrated flow range and the K factor.

4.3.1.4 Drift of Flowmeter

Every measurement instrument has a tendency to increase the uncertainty with the use of the equipment. This can be observed in the As Found calibration report. In order to estimate how much the uncertainty increases during the inter calibration period, a linear regression is made considering the last adjustment period and the geometric sum of the repeatability and the systematic error. The resulting equation is applied to estimate the increase of uncertainty for the analyzed period considering the last calibration performed in the flowmeter.
4.3.1.5 Expansion by Pressure

The uncertainty related to the increase in distance between the ultrasonic transducers due to the expansion by pressure. This is obtained with the individual analysis of the uncertainties that compose the equation of expansion by pressure, as seen in Equation 11.

\[
Exp\text{Corr}_p = 1 + 3 \cdot \frac{[D_{\text{out}}^2 \cdot (1 + v)] + [D_{\text{in}}^2 \cdot (1 - 2 \cdot v)]}{E \cdot (D_{\text{out}}^2 - D_{\text{in}}^2)} \cdot (P_F + P_{\text{atm}} - P_R)
\]  

(11)

In this case study, the uncertainties that came from the measurements of the flowmeter’s inner and outer diameter, the Poisson coefficient, and the Young Module, were not taken into account, considering only the uncertainties that came from the pressure and atmospheric pressure variations, which the flowmeter is submitted. The uncertainty of the pressure effect expansion is obtained with Equation 12, where \( k \) stands for the coverage factor obtained by comparing the values fixed in the GUM (JCGM, 2008) to the effective degrees of freedom (calculated with Equation 13).

\[
u_{N-\text{pre}-pf} = k \cdot \sqrt{\frac{S_P}{n_P}}^2 + u_P^2
\]  

(12)

\[
u_{\text{eff puxos-pressao}} = \frac{\left(\frac{S_P}{n_P}^4 + u_P^4\right)}{n_P - 1 + \nu_{\text{eff pressao}}^4}
\]  

(13)

Where:

\( u_{N-\text{pre}-pf} \): Uncertainty of the gas pressure for the uncertainty calculation due to the expansion by pressure effect, including the values variation (MPa);

\( u_P \): Combined uncertainty of the gas pressure measurement (MPa);

\( n_P \): Quantity of pressure measurements collected in the period (dimensionless);

\( S_P \): Standard deviation of the pressure measurements collected in the period (MPa);

\( \nu_{\text{eff pressao}} \): Effective degrees of freedom of the combined uncertainty in the pressure measurement (dimensionless).

The atmospheric pressure uncertainty is obtained as shown in Item 4.3.7. The sensitivity coefficient is equal to the partial derivative of the pressure effect correction in relation to the measured pressure and in relation to the atmospheric pressure, as shown in Equation 14.

\[
\frac{\partial \text{ExpCorr}_p}{\partial P_F} = \frac{\partial \text{ExpCorr}_p}{\partial P_{\text{atm}}} = 3 \cdot \frac{[D_{\text{out}}^2 \cdot (1 + v)] + [D_{\text{in}}^2 \cdot (1 - 2 \cdot v)]}{E \cdot (D_{\text{out}}^2 - D_{\text{in}}^2)} \cdot K
\]  

(14)

Where:

\( K \): Correspondence Factor of pulses per cubic meter (pulse/m³).

The uncertainty in the pulses emitted by the flowmeter due to the pressure effect expansion is calculated as shown in Equation 15.
\[
\begin{align*}
\mu_{N-pre} &= \sqrt{\left(\frac{\partial \text{ExpCorr}_{P}}{\partial P_{F}} \cdot u_{N-pre-pf} \right)^2 + \left(\frac{\partial \text{ExpCorr}_{P}}{\partial \text{atm}} \cdot u_{\text{atm}} \cdot k_{\text{atm}}\right)^2} \\
&= \frac{\sqrt{\left(\partial \text{ExpCorr}_{P} \cdot u_{N-pre-pf} \right)^2 + \left(\partial \text{ExpCorr}_{P} \cdot u_{\text{atm}} \cdot k_{\text{atm}}\right)^2}}{2}
\end{align*}
\]

Where:
- \( u_{N-pre} \): Uncertainty of the number of pulses due to the pressure effect expansion (pulses);
- \( u_{\text{atm}} \): Uncertainty of the atmospheric pressure (MPa).

**4.3.1.6 Expansion by Temperature Effect**

The distance between the ultrasonic transducers can also be changed due to the thermal expansion of the flowmeter body. The uncertainty from this variation is obtained similarly to what was demonstrated for the pressure expansion, based on Equation 16. The uncertainty is calculated in accordance with Equation 17 (disregarding the uncertainty of the linear expansion coefficient), with the coverage coefficient proportional to the effective degrees of freedom calculated in the Equation 18.

\[
\begin{align*}
\text{ExpCorr}_T &= 1 + 3 \cdot \alpha \cdot (T_F - T_R) \\
\mu_{N-tem-tf} &= \frac{k \cdot \sqrt{s_T^2 + u_{T_F}^2}}{\sqrt{n_T} + \mu_{T_F}^4} \\
\frac{\nu_{\text{eff-pulsos-tem}}}{\nu_{\text{eff-temperatura}}} &= \frac{(s_T^2 + u_{T_F}^2)^4}{n_T - 1} + \frac{u_{T_F}^4}{\nu_{\text{eff-temperatura}}}
\end{align*}
\]

Where:
- \( \alpha \): Coefficient of thermal expansion of body material of flowmeter.
- \( s_T \): Sampling standard deviation of the temperature variation during the period (K);
- \( n_T \): Number of temperature samples (dimensionless);
- \( k \): Coverage factor (dimensionless);
- \( u_{T_F} \): Combined uncertainty of the temperature measurement (K);
- \( \nu_{\text{eff-temperatura}} \): Effective degrees of freedom of the partial uncertainties that compose the uncertainty of temperature measurement (dimensionless);
- \( u_{N-tem-tf} \): Gas temperature uncertainty for the uncertainty calculation due to the expansion by temperature effect (K).

The sensitivity coefficient is equal to the partial derivative of correction by temperature effect in relation to the gas temperature, resulting in what is demonstrated in Equation 19, which also demonstrates the uncertainty calculation in the pulses due to the flowmeter thermal expansion.
\[
\left( \frac{\partial \text{ExpCorr}_F}{\partial T_F} = 3 \cdot \alpha \cdot K \right) \rightarrow u_{N-\text{tem}} = u_{N-\text{tem}-tf} \cdot \frac{\partial \text{ExpCorr}_F}{\partial T_F}
\]

(19)

Where:

\( K \): Correspondence Factor of pulses per cubic meter (pulse/m³);

\( u_{N-\text{tem}} \): Uncertainty of the number of pulses due to the expansion by temperature effect (pulses).

### 4.3.2 Uncertainties Related to the Gas Pressure’s Measurement (PF)

#### 4.3.2.1 Repeatability

Uncertainty related to the capacity of the instrument to show the same measurement in the same point repeatedly. It is estimated during the calibration of the pressure transmitter and is numerically equal to the greatest calculated value of the division between the standard deviation and the square root of the number of measurements, for each measurement point during the calibration, and multiplied by a coverage factor that equals two, as per demonstrated in Equation 20.

\[
u_{P_F-\text{rep}} = \frac{s}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \frac{\sum_{j=1}^{n} x_j}{n})^2}{n - 1}}
\]

(20)

Where:

\( u_{P_F-\text{rep}} \): Uncertainty related to the repeatability of the instrument (Pa);

\( s \): Standard Deviation of Measurements (Pa);

\( n \): Number of Measurements (dimensionless);

\( x_i, x_j \): Pressure Measurement (Pa).

#### 4.3.2.2 Hysteresis

The hysteresis is related to the capacity of the instrument to show the same measurement at the same point in ascending and descending measurements. It is estimated during the calibration of the pressure transmitter and is numerically equal to the greatest calculated value found in the module of the average difference between the ascending and descending calibration measurements, as per Equation 21.

\[
u_{P_F-\text{his}} = \left| \frac{n}{2} \frac{\sum_{\alpha=1}^{n} x_{\alpha}}{n} - \frac{n}{2} \frac{\sum_{d=1}^{n} x_d}{n} \right|
\]

(21)

Where:

\( u_{P_F-\text{his}} \): Uncertainty due to hysteresis of the pressure meter instrument (Pa);

\( n \): Number of measurements (dimensionless);

\( x_{\alpha} \): Ascending pressure measurement (Pa);

\( x_d \): Descending pressure measurement (Pa).
4.3.2.3 Standard Instrument Resolution

The resolution of the standard instrument used in the calibration is also a source of uncertainty and is obtained in the certificate of calibration of the standard instrument.

4.3.2.4 Uncertainty inherited from calibration standard instrument

Like the resolution, the uncertainty inherited from the standard instrument is also considered and obtained from the certificate of calibration of the standard instrument multiplying by the calibrated amplitude of the standard instrument.

4.3.2.5 Instrument Resolution

The instrument resolution represents the truncation of the measured value. In the analyzed system, the signal transmission of the instrument is made via the 4-20mA communication and the resolution is limited by the D/A converter, which is obtained from the transmitter manual, multiplying the value by the signal transmission amplitude.

4.3.2.6 Uncertainty of the 4-20mA current meter

The uncertainty related to the 4-20mA current meter instrument used in the calibration. It is obtained in the certificate of calibration of the current meter. It is algebraically obtained multiplying the uncertainty in the certificate of calibration of the current meter by the amplitude of the signal transmission (in pressure unit) and divided by the amplitude signal transmission in current (with the 4-20mA, the amplitude would be 16mA), as shown in Equation 22.

\[ u_{P_{\text{mcc}}} = \frac{u_{\text{corrente}\_mcc} \cdot a_p}{a_c} \]  

(22)

Where:
- \( u_{P_{\text{mcc}}} \): Uncertainty inherited from 4-20mA signal meter used in calibration (Pa);
- \( u_{\text{corrente}\_mcc} \): Uncertainty of 4-20mA signal meter used in calibration (mA);
- \( a_p \): Transmitter measurement amplitude (Pa);
- \( a_c \): Signal transmitter amplitude (for transmission of 4-20mA ac=16mA) (mA).

4.3.2.7 Signal Generator Uncertainty in Loop Test

In the analyzed system, the instruments are removed from installation site for calibration and the communication mesh is verified separately. The values obtained are compensated in the calibration values. A signal generator is used for the test of the communication mesh. The signal generator uncertainty multiplied by the signal transmission amplitude (in pressure unit) divided by the signal transmission amplitude in current units is the value considered as uncertainty due to the use of this equipment (similar to what was demonstrated in Equation 18).

4.3.2.8 Pressure Variations

The flow computer has a calculation cycle that performs the data reading of the instruments and calculates the volume at base conditions. If there are pressure variations in time shorter than the flow computer sampling, these variations will not be recorded. The shorter the sampling time is, the lower the uncertainty due to the pressure in the sampling intervals. The uncertainty value is the standard deviation of the variations in time of the flow computer cycle divided by the square root of the quantity of analyzed variations.
4.3.2.9 Environment Temperature Effect

The electronic components of the transmitters go through minor changes due to the temperature. These changes can be adjusted in the calibration, but some differences may happen when the temperature in the calibration is different from the temperature in the process. The closest the process temperature is to the temperature in the calibration, the lower this uncertainty will be. Each manufacturer has a calculation method to achieve this uncertainty, which can be found in the manual of the instrument. For the Rosemount transmitter 3051, the uncertainty due to the effect of the environment temperature is calculated according to Equation 23.

\[ u_{P-\eta} = \left( \frac{25 \cdot LSF + 125 \cdot a}{28} \right) \cdot 10^{-5} \cdot |T_{amb-pro} - T_{amb-cal}| \]  

(23)

Where:
- \( u_{P-\eta} \): Uncertainty due to the environment temperature variation (Pa);
- \( LSF \): Upper range limit of transmitter (bar);
- \( a \): Calibration amplitude of transmitter (bar);
- \( T_{amb-pro} \): Average environment temperature at the installation site (°C);
- \( T_{amb-cal} \): Average environment temperature during the calibration (°C).

4.3.2.10 Pressure Meter Drift

Similar to what was demonstrated in Item 4.3.1.4, the pressure meter is subject to the drift effect, and this uncertainty is calculated by the linear regression considering the last adjustment period and the geometric sum of the repeatability, the hysteresis, and the systematic error. The resulting equation is used to estimate the increase in uncertainty for the analyzed period considering the last calibration performed in the instrument.

4.3.2.11 Assembling position of Transmitter

The assembling position (horizontal, vertical, or inclined) can interfere the pressure measurement. This uncertainty can be eliminated in the calibration adjustment. In the analyzed measurement system, the calibration is performed in the same vertical inclination of the installation position. However, once the instrument is moved during the calibration, there might be a slight difference between the calibration measurement and the installed instrument measurement. The manufacturer informs this uncertainty estimative.

4.3.2.12 Electric Variation

Voltage variations of the instruments might propagate in signal transmission and result in interferences in measurement. The uncertainty value depends on the instrument voltage and is informed by the manufacturer.

4.3.3 Uncertainties related to flow temperature (TF)

4.3.3.1 Repeatability

Such as the calculation for the pressure transmitter (Equation 20), the repeatability is also taken into consideration for the temperature measurement.
4.3.3.2 Uncertainty inherited from the standard instrument

The uncertainty of the standard instrument used in the calibration of temperature instrument constitutes the variable uncertainty calculation and must be in absolute values of temperature.

4.3.3.3 Resolution of Calibration Standard Instrument

The standard temperature instrument used in the calibration should be considered (instrument of measurement or indication of temperature bath).

4.3.3.4 Axial Homogeneity of the temperature bath

There might be differences in the measurement of the calibration bath depending of the vertical insertion position of the sensor. These differences create errors in the calibration process. The estimate of the calibration bath homogeneity is achieved with tests in various conditions, and is informed by the manual of manufacturer equipment.

4.3.3.5 Radial Homogeneity of temperature bath

Similar to the axial homogeneity, there is also an uncertainty due to the insertion horizontal position of the sensor. This value can also be found in the manual of the equipment.

4.3.3.6 Uncertainty of the 4-20mA meter

Similar to the calculation of the pressure instrument, the 4-20mA signal meter equipment is considered in the temperature uncertainty.

4.3.3.7 Precision of the Signal Generator in Loop Test

Similar to the calculation of pressure meter instrument, when the Loop Test is used to compensate the error in the communication mesh, the uncertainty of the signal generator is considered.

4.3.3.8 Thermal Transfer of the Thermowell

When a thermowell is used in the temperature measurement, there might be thermal transfers between the thermowell and the piping, considering that normally the device is welded, threaded, or flanged in the piping, and the piping is subject to environmental conditions. These transfers are greater when the gas temperature and the pipe temperature are not in thermal equilibrium, or when there is consumption intermittence.

A study has been conducted to verify the impact of the environmental conditions in the measurement of temperature in the analyzed system. Several computer simulations of the fluid dynamics (CFD system) were performed, using the exact dimensions of the meter system with the materials installed, and using the hourly measurements of temperature in various parts of the system and environment temperature for 30 days. The data were statistically analyzed providing indications as shown in Figure 2. Due to the complexity of the study, this will not be detailed in this paper.
4.3.3.9  Response Time of the Temperature Sensor

The temperature sensor has a period that is necessary to identify the temperature variation and to stabilize the signal in the real value. The sensors are tested by the manufacturer under determined conditions to verify the necessary time for the sensor to reach 50% of the step disturbance value. The uncertainty estimation regarding the response time is achieved by verifying the greatest variation registered by the meter in a period corresponding to the thermal response time of the instrument. This value corresponds to half of the disturbance theoretical maximum value registered by the meter (considered as a Type B uncertainty). Figure 3 shows the method used.

\[
\begin{align*}
T \text{ (°C)} & = T_{\text{Re}} \\
\text{t (s)} & = t_{\text{Re}} \\
\text{u}_{\text{temp-res}} & = \text{Uncertainty in temperature}
\end{align*}
\]

\[
\begin{align*}
\text{Perturbação tipo degrau} & \quad \text{Resposta do sensor à perturbação} \\
\text{t}_{\text{Re}} & \quad \text{Tempo necessário para 50% da variação} \\
T_{\text{Re}} & \quad \text{Indicação de 50% da variação} \\
\text{u}_{\text{temp-res}} & \quad \text{Incerteza no tempo de resposta}
\end{align*}
\]

Figure 3. Response Time Estimation of the Meter

4.3.3.10  Digital-to-Analog Conversion

Inside the temperature transmitter, there is a digital-to-analog converter that comes with a rounding proportional to the quantity of bits of the converter. This rounding contributes to the temperature’s uncertainty and is obtained directly from the transmitter’s datasheet, multiplying the value by the amplitude set up in the transmitter.
4.3.3.11 Drift of temperature meter

Similar to what has been demonstrated in Item 4.3.2.10, the temperature meter is subject to the drift effect, and this uncertainty is calculated by the linear regression considering the last adjustment period, the geometric sum of the repeatability, and the systematic error. The resulting equation is used to estimate the increase in uncertainty for the analyzed period considering the last calibration made in the instrument.

4.3.3.12 Effect of ambient temperature on the temperature transmitter

Similar to Item 4.3.2.9, the temperature transmitter also shows differences in measurement proportional to the ambient temperature’s difference in the calibration and at flow conditions. The calculation method depends on the meter’s model used and can be found in the manufacturer’s manual. The Rosemount 3144P transmitter has an uncertainty of 0.0015°C for each 1°C of difference between the ambient temperature in the calibration and the ambient temperature in the installation place (by using the average ambient temperature).

4.3.3.13 Ambient Temperature Effect on Digital Analog Converter

The difference between the ambient temperature in the calibration and in the installation place also produces an uncertainty in the D/A converter. For the Rosemount 3144P transmitter this uncertainty can be found in the manufacturer’s manual as being numerically equal to 0.001% of the amplitude set up in the transmitter for each 1°C of difference between the ambient temperature in the calibration and in the installation place.

4.3.3.14 Electrical Variation

Similar to the pressure’s meter, the electrical variation in the voltage of the temperature’s transmitter produces an uncertainty in the measurement that can be obtained in the manufacturer’s manual.

4.3.4 Compressibility at Flow Conditions

4.3.4.1 Calculation Methodology

The method used to calculate the compressibility is an important factor to be taken into account. Each calculation method holds an uncertainty related to the constants used and to the method itself, which is normally empirical. The compressibility calculated according to the AGA Report nº 8 Detailed Method (2003), shows the uncertainties exposed in Figure 4 according to pressure and temperature.
4.3.4.2 Uncertainty Inherited from the Measurement of Pressure and Temperature

In the mathematical model shown in Item 4.2, the compressibility was inserted as an independent variable to simplify the analysis. In order to compensate this fact, the insertion of the impact that the pressure and temperature uncertainties cause in the compressibility’s uncertainty is necessary. The compressibility’s calculation method is explained in Equation 24. The uncertainty inherited from the gas pressure measurements, the atmospheric pressure, and the gas temperature is calculated according to Equation 25 (the parameters and constants are calculated according to AGA 8 Detailed Method).

\[
Z = 1 + \frac{D \cdot B}{K^3} - D \cdot \sum_{n=13}^{18} C_n^* \cdot T^{-u_n} + \sum_{n=13}^{58} C_n^* \cdot T^{-u_n} \cdot (b_n - c_n \cdot k_n \cdot D^{k_n}) \cdot D^{b_n} \cdot \exp(-c_n \cdot D^{k_n}) \quad (24)
\]

Where:
- \(Z\) : Compressibility
- \(B\) : Second virial coefficient;
- \(K\) : Size parameter;
- \(D\) : Gas reduced density;
- \(C_n^*\) : Coefficient as a result of the composition;
- \(T\) : Gas temperature;
- \(u_n, b_n, c_n, k_n\) : Constants contained in AGA 8.
\[ u_{ZF-\text{pt}} = \frac{\sqrt{\left( \frac{\partial Z}{\partial P_F} \cdot u_{P_F} \cdot k_{P_F} \right)^2 + \left( \frac{\partial Z}{\partial P_{\text{atm}}} \cdot u_{P_{\text{atm}}} \cdot k_{P_{\text{atm}}} \right)^2 + \left( \frac{\partial Z}{\partial T_F} \cdot u_{T_F} \cdot k_{T_F} \right)^2}}{2} \] (25)

Where:

\[ \frac{\partial Z}{\partial T_F} = T_F \cdot \frac{d B}{dT_F} - d \cdot B + K^3 \cdot d \cdot \sum_{n=13}^{18} C_n^* \cdot T_F^{-u_n} \cdot (u_n + 1) - \alpha + \sum_{n=13}^{18} \left[ c_n^* \cdot (K^3 \cdot d)^{b_n} \cdot \exp(-c_n \cdot K^3 \cdot d)^{k_n} \cdot T_F^{-u_n} \cdot (c_n \cdot K^3 \cdot d)^{k_n} \cdot u_n - b_n \cdot u_n \right] \]

\[ \frac{\partial Z}{\partial P_F} = \frac{\partial Z}{\partial P_{\text{atm}}} = \frac{B \cdot d - \beta + \alpha}{P + \frac{P}{Z_F} \cdot B \cdot d - \frac{P}{Z_F} \cdot \beta + \frac{P}{Z_F} \cdot \alpha} \]

\[ \alpha = \sum_{n=13}^{18} \left[ \frac{\exp(-c_n \cdot K^3 \cdot d)^{k_n} \cdot C_n^* \cdot (K^3 \cdot d)^{b_n}}{T_F^{-u_n}} \cdot \left( b_n^2 - b_n \cdot c_n \cdot (K^3 \cdot d)^{k_n} \cdot k_n - c_n \cdot k_n \cdot (K^3 \cdot d)^{k_n} \cdot (b_n + k_n) + c_n^2 \cdot k_n^2 \cdot (K^3 \cdot d)^{k_n} \cdot K^3 \right) \right] \]

\[ \beta = K^3 \cdot d \cdot \sum_{n=13}^{18} (C_n^* \cdot T_F^{-u_n}) \]

\[ \frac{\partial B}{\partial T_F} = - \sum_{n=1}^{18} u_n \cdot a_n \cdot T_F^{-u_n} \cdot (u_n + 1) \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot x_j \cdot E_{ij} \cdot (K_i^* \cdot K_j^*) \cdot B_{nij}^* \]

\[ k_{P_F}, k_{P_{\text{atm}}}, k_{T_F} : \text{Coverage factor, calculated with Distribution of Student;} \]
\[ d : \text{Molar density;} \]
\[ B : \text{Second virial coefficient;} \]
\[ K : \text{Size parameter;} \]
\[ C_n^* : \text{Coefficient as a result of the composition;} \]
\[ u_n, b_n, c_n, k_n, a_n : \text{Constants;} \]
\[ P : \text{Pressure (equal to } P_{\text{atm}} \text{ or } P_F, \text{depending on sensibility coefficient calculated);} \]
\[ x_i, x_j : \text{Molar fraction of components i and j;} \]
\[ E_{ij} : \text{Binary energy parameter of second virial coefficient;} \]
\[ K_i^*, K_j^* : \text{Parameter of component size;} \]
\[ B_{nij}^* : \text{Coefficient of binary characterization.} \]
4.3.4.3 Uncertainty Inherited from the Gas Composition’s Measurement

The gas composition, normally verified with the chromatography, holds a mixed uncertainty of various factors, such as repeatability, uncertainty inherited from the standard gas used in the calibration, mix normalization, method of analyses, etc. Regardless of the method for the compressibility calculation, the gas composition will be necessary in the calculation, hence, the uncertainty in the measurement of the fractions of the gas components will be passed to the compressibility.

To obtain the composition’s uncertainty in the compressibility, the method of extremes with triangular distribution was adopted. By using the certificate of chromatograph’s calibration, the uncertainty of the mole fraction of each component was verified, and by considering the fractions in extreme points of uncertainty, both minimum and maximum compressibility values were verified in the points of maximum uncertainty with constant pressure and temperature (equal to flow pressure, atmospheric pressure, and average flow temperatures). The difference between these data is regarded as a type B uncertainty.

4.3.4.4 Composition Variations

Even with the online equipment, the chromatographic analysis occurs by batch. Any change in the gas composition in the interval between the analyses will not be taken into account. When a change in the composition is verified between one analysis and another, it is not possible to deduce in which moment between the analyses this event took place. Considering the chromatography used for the compressibility calculation is the last valid one, there is an uncertainty concerning the possibility of a change in composition variation since the last analysis. Algebraically, this uncertainty is achieved with the compressibility’s calculation at constant pressure and temperature (utilizing the average gas pressure, atmospheric pressure, and gas temperature) analyzing the greatest variation in the period inter samplings. This value represents the uncertainty with rectangular distribution.

4.3.5 Compressibility at reference conditions

4.3.5.1 Calculation Methodology

Like in the compressibility calculation at flow condition, the uncertainty related to the calculation method should be taken into account.

4.3.5.2 Uncertainty Inherited from the Gas Composition’s Measurement

Similar to what has been demonstrated in Item 4.3.4.3, when it comes to the compressibility at flow conditions, the uncertainty in compressibility at base conditions inherited from the chromatography is reached with the analysis of the uncertainty’s extremes, however, considering the constant pressure and temperature equal to the base conditions.

4.3.5.3 Composition variations

Just like in the compressibility at flow conditions, the variations in the composition in time shorter than the one in the chromatographic sampling bring about an uncertainty in the compressibility at reference condition. In order to reach this uncertainty, the maximum variations of compressibility at constant pressure and temperature equal to the base conditions must be calculated.

4.3.6 Superior Calorific value of the Gas
4.3.6.1 Uncertainty Inherited from the Chromatograph

The gas calorific value is contained in the volume conversion’s equation for the compensation of the quantity of energy that the gas can provide in the combustion. The gas calorific value is calculated based on the average individual calorific value of each fraction that constitutes the gas, and as such, the chromatograph’s uncertainty used to obtain the molar fractions will impact upon the calorific value uncertainty, which is calculated in Equation 26.

\[
u_{PCS\text{-}cro} = \sqrt{\sum_{j=1}^{N} u_{x_j} \cdot \left( H_{i}^o - \sum_{i=1}^{N} H_{i}^o \cdot x_{i} \right)^2}
\]

(26)

Where:
- \( u_{PCS\text{-}cro} \): Uncertainty of calorific value due to molar fractions uncertainty (MJ/m³);
- \( u_{x_j} \): Uncertainty of j (adm) component molar fraction;
- \( H_{i}^o, H_{j}^o \): Ideal calorific power of i and j components in volumetric base, obtained in ISO 6976 (MJ/m³);
- \( x_{i} \): Molar fraction of i (MJ/m³) component;
- \( N \): Quantity of components in the gas (adm).

4.3.6.2 Calculation Method

The calorific value’s calculation method used in the case study is demonstrated in the ISO 6976 and holds an uncertainty inherent to the method of 0.015% of the calculated calorific value.

4.3.6.3 Base Values Uncertainty

The calculation method used in the case study is based on the pondered average of the individual calorific value of the components. The values used as base were measured empirically and hold an uncertainty inherited from these measurements. The base values demonstrated in ISO 6976 cause an uncertainty of 0.05% in the calculated calorific value.

4.3.6.4 Daily Average

When the daily arithmetical average of the calorific value, which is applied for the conversion of the daily volume, is accomplished instead of an online application, this causes an uncertainty that is algebraically equal to the calorific value’s standard deviation of the analyses throughout the day, divided by the square root of the quantity of analyses. Evidently, the greater the variation of calorific value throughout the day is, the greater uncertainty this will be.

4.3.6.5 Composition Variation

Equal to what has been demonstrated for the compressibility, the chromatograph’s sampling period may hide possible variations in the gas composition. In conservative way, in order to estimate the uncertainty due to this sampling period, the greatest variation in the composition between one analysis and another is verified, and then considering this variation as an uncertainty of rectangular distribution.

4.3.6.6 Rounding

In some cases, the rounding of the calorific value occurs before the application of the
calculation. This rounding causes an uncertainty similar to the uncertainty due to the resolution.

4.3.7 Atmospheric Pressure

4.3.7.1 Atmospheric Variations

In the case of manometric transducers, the atmospheric pressure used in the calculation is constant. Nonetheless, variations effectively occur mainly because the variations of the density due to the humidity, composition, and temperature in the atmospheric layer, and from the atmospheric tides that can change the air layer’s height. In order to estimate these variations, periodic measurements of the local atmospheric pressure are made. The sampling standard deviation divided by the square root of the quantity of measurements provides the uncertainty value. It is important that the sampling collection take place in a representative period (at least a year), so that the data be collected in various weather conditions. The application of the Chauvenet method for elimination of spurious errors can be necessary depending on the data source involved..

4.3.7.2 Barometric Calculation

In the case study, altitude measurements were made, and with this value the barometric equation was applied, as shown in Equation 27. The uncertainties inherent to the method were disregarded, applying only the uncertainties acquired from the variables used. In the case study, the uncertainties inherent to the measured values of reduced gravity at the reference point and the air’s molar mass were also disregarded. The calculation of the atmospheric pressure uncertainty inherited from the variables that form the barometric equation is demonstrated in Equation 28. The values were reached with the instrumentation in atmospheric balloons, such as the ones provided by the INPE/CEPETEC’s metrological station.

\[
P_{atm} = P_0 \cdot \left(1 + \frac{a \cdot h}{T_0} \right)^{-\frac{M \cdot g}{R}}
\]

Where:
- \(P_{atm}\) : Local atmospheric pressure (mbar);
- \(P_0\) : Atmospheric pressure in the reference place (mbar);
- \(a\) : Temperature variation rate in relation to altitude (K/m);
- \(h\) : Geopotential altitude in relation to the reference point (m);
- \(T_0\) : Temperature at the reference point (K);
- \(M\) : Air molar mass (Kg/m³);
- \(g\) : Local gravity (m/s²);
- \(R\) : Constant universal of gases (J/mol.K).
\[ u_{atm-cba} = \frac{\sqrt{\left(\frac{\partial P_{atm}}{\partial P_0} \cdot u_{P_0} \cdot k_{P_0}\right)^2 + \left(\frac{\partial P_{atm}}{\partial a} \cdot u_a \cdot k_a\right)^2 + \left(\frac{\partial P_{atm}}{\partial h} \cdot u_h \cdot k_h\right)^2 + \left(\frac{\partial P_{atm}}{\partial T_0} \cdot u_{T_0} \cdot k_{T_0}\right)^2}}{2} \] (28)

Where:

\[ \frac{\partial P_{atm}}{\partial P_0} = \frac{P_{atm}}{P_0} \]
\[ \frac{\partial P_{atm}}{\partial a} = - \frac{P_0 \cdot M \cdot g \cdot h}{T_0 \cdot a \cdot R} \cdot \left(1 + \frac{a \cdot h}{T_0}\right)^{-\frac{M \cdot g}{a \cdot R}} \]
\[ \frac{\partial P_{atm}}{\partial h} = - \frac{M \cdot g \cdot P_0}{R \cdot T_0} \cdot \left(1 + \frac{a \cdot h}{T_0}\right)^{-\frac{M \cdot g}{a \cdot R}} \]

\( u_{P_0} \): Uncertainty of atmospheric pressure estimated in the reference place (mbar);
\( u_a \): Uncertainty of estimate of temperature variation rate in relation to altitude (K/m);
\( u_h \): Uncertainty of altitude measurement in relation to the reference point (m);
\( u_{T_0} \): Uncertainty of temperature estimate in the reference point (K);
\( k_{P_0}, k_a, k_h, k_{T_0} \): Coverage factor, calculated through the Student (adm) distribution.

### 4.3.8 Volume at reference conditions

#### 4.3.8.1 Rounding of volume

The uncertainty from volume rounding should also be considered.

### 4.4 Methodology for the calculation of billing uncertainty

For the application of uncertainty values in management decisions, it is necessary to verify how much this uncertainty impacts on the billing. This is done by applying the same methodology demonstrated in Item 4.1. The mathematical model for the billing will depend upon the contractual conditions. In this case study, the mathematical model for the billing in a specific type of contract is shown in Equation 29 (the model may vary according to the contract).

\[ R_B = \frac{V_R \cdot t_R \cdot f \cdot 0.0094}{1 - (PIS + COFINS + ISS)} \] (29)

Where:
\( R_B \): Gross revenue (R$);
\( t_R \): Gas tariff in BTU (R$/BTU);
\( f \): Conversion factor from kcal to BTU (kcal/BTU);
\( PIS + COFINS + ISS \): Gross revenue (adm).

The billing uncertainty is obtained with the calculation shown in Equation 30, disregarding the uncertainties from the taxation.
\[ U_{RB} = 2 \cdot \sqrt{\left( \frac{\partial R_B}{\partial t_R} \cdot \frac{u_{tr}}{\sqrt{12}} \right)^2 + \left( \frac{\partial R_B}{\partial V_R} \cdot \frac{U_{VR}}{2} \right)^2} \]  

(30)

Where:

\[ \frac{\partial R_B}{\partial t_R} = \frac{R_B}{t_R} \quad \parallel \quad \frac{\partial R_B}{\partial V_R} = \frac{R_B}{V_R} \]

\[ u_{tr} \]: Uncertainty due to rounding of tariff (similar to the resolution of an instrument) (R$);  
\[ U_{RB} \]: Gross revenue expanded uncertainty with level of trust of 95.45% (R$).

5 PRESENTATION OF RESULTS

It is recommended that all the calculation memory is recorded evidencing the background of the values utilized and the results presented according to the following model, with the information in the same unit and with the same quantity of decimal places:

\[ Y = (y + E) \pm U \quad \text{un} \quad k = k', \text{level of confidence}=IC \]

Where:

- \( Y \): Variable analyzed (Volume in the flow conditions or Gross Revenue);
- \( y \): Value of variable;
- \( E \): Systematic Error, including the signal;
- \( U \): Expanded uncertainty;
- \( \text{un} \): Unit;
- \( k' \) = Coverage factor;
- \( IC \) = level of trust.

For example:

\[ V_R = (2.124.356,5 \pm 86,2 \pm 261,0) \text{ m}^3 : k=2, \text{level of trust}=95,45%; \]
\[ R_B = (658.352,32 +91,51 \pm 154,01) \text{ R$} : k=1,96, \text{level of trust}=95%. \]

6 CASE STUDY

This methodology was applied in the Companhia Maranhense de Gás – Gasmar, in measurement systems with ultrasonic meters average flow of 2.2 million m³ at reference conditions (101.325 kPa, 20°C), average manometric pressure of 32 bar, gas temperature of approximately 40°C, in 10" measurement drains. Considering the processes applied by the company, by applying the proposed method, an expanded uncertainty of ±0.46% measurement and a billing uncertainty of ±1.35% were reached. Figure 5 demonstrates the contributions of each variable in the measurement uncertainty value composition. Through the analysis of this image it is possible to verify which variables cause more impact to trigger a reduction in the uncertainty.
Figure 5. Contribution of each variable to measurement uncertainty

7  APPLICABILITY AND BENEFITS

The analysis of the measurement uncertainty together with the financial impact assessment provide data that can be utilized as subsidy to beacon the management decisions, such as the process mode changes, so that the risk inherent to uncertainty be compatible with the application cost of process change. This is the case, for example, of applying the risk based maintenance of components inherent to the measurement (in addition to the continuity related risks), where the risk is defined by the uncertainty.

For the purpose of analysis, please see below examples of some cases where the methodology can be used in management decisions:

i. Precision instruments generally have a higher cost because they use differentiating technologies and systems. The acquisition of more precise and more expensive instruments should precede a more detailed analysis concerning the impact that the variables may have on the uncertainty, for instance, a simulation of the uncertainty using a pressure instrument with a greater accuracy and a simulation of the uncertainty using an instrument with a lower accuracy. The difference in invoicing between the application of both instruments should be compared to the cost difference of the equipment investment, aiming at the best payback.
ii. The instruments calibration in the field provides a lower uncertainty in comparison to a calibration in the laboratory. A higher cost is generated, though. With the analysis of the measurement’s system uncertainty, it is possible to verify the financial risk of holding a calibration with a higher uncertainty and compare it with the cost. The best cost relation by risk is the ideal operation.

iii. The monthly cost of the chromatographic analyses should not override the cost concerning the uncertainty that the chromatography causes in the measurement. The best periodicity of chromatographic analysis can be defined from that.

iv. The periodicity of instruments calibration demands a more improved analysis than the legal requirement’s compliance only. The measurement may hold an uncertainty that represents a financial risk big enough to demand the adoption of a shorter calibration period. This analysis is based on the As Found, verifying the instrument uncertainty before the calibration, and on the As Left, verifying how much the uncertainty decreases executing the adjustments. The uncertainty also guides the acceptance criteria in the calibrations, analyzing which impact the uncertainty in the calibration would cause in the system as a whole, adopting a criterion that would not cause a considerable impact on the volume.

v. In financial decisions, where the maximum possible difference between buying and selling is controlled, based on independent measurements, providing subsidies for the dimensioning of values that the company should keep in cash reserves to absorb the possible differences. Moreover, it is possible to use this analysis in the decisions related to the tariff plan, encompassing the financial risks inherent to the uncertainties regarding measurements in buying and selling in the gas unit price.

The uncertainty of measurement generates intangible values related to the company’s image, such as the increase in measurement’s reliability for both internal and external clients, thus enabling the traceability for audits and arbitrations. The measurement becomes more transparent and clear, showing the compliance to the contractual and legal requirements.

The possibility of analyses using the measurement system’s uncertainty is not limited to the examples presented and can be used in various applications providing distinct results.

8 CONCLUSION

The guided analysis of uncertainty of gas flow measurement and billing can provide inputs for management decisions, improving process efficiency, however, the use of those values for this purpose must have great accuracy. The methodology presented in this paper outlines many variables that are normally disregarded without any study performed to estimate its impact providing the required accuracy.

Due to the volume of data generated, the results obtained in the case study have not been explained, since the values can differ depending on the systems and processes applied. The impact analysis of each uncertainty should be made for each measurement system, so that the relevant uncertainties can be defined.
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