Generalized Wet Gas Venturi Meter Correlations: Assessment and Improvement

Martin Bjørner\textsuperscript{a}, Philip Fosbøl\textsuperscript{b}, Mads Lisberg\textsuperscript{a} and Henrik Lisberg\textsuperscript{a}

\textsuperscript{a}Emco Controls A/S, Høgevej 6, DK-3400 Hillerød, Denmark.
\textsuperscript{b}Center for Energy Resources Engineering (CERE), Department of Chemical and Biochemical Engineering, Technical University of Denmark, Building 229, DK-2800 Kgs. Lyngby, Denmark.

Abstract

The venturi meter is perhaps the most suitable device for gas flow measurements in wet gas streams. As the presence of liquid in the venturi meter increases the differential pressure, a correction term is required to determine the actual gas flow. In this work we assess the associated error of six common wet gas correlations, against experimental venturi meter data from the literature. The results indicate that the Reader-Harris Graham correlation has the smallest deviations from experimental data. At low β-ratios ($\beta = d/D$) the de Leeuw correlation achieves similar deviations, but the errors become considerable at high β-ratios. The de Leeuw correlation is improved for application with arbitrary β-values. A correction term is suggested, to reduce the observed bias at low Lockhart-Martinelli values. Finally, it is shown that the function for $n$ in the Reader-Harris Graham correlation can be approximated by a de Leeuw type correlation with a β-dependency.

1 Introduction

The term ‘wet gas’, implies the presence of a relatively small amount of liquid in a gas stream. Wet gas flow metering is becoming increasingly important to the natural gas production industry and the potential applications for wet gas flow metering have been growing for more than 30 years [1]–[4]. In natural gas reservoirs, which initially produced dry gas, the operating conditions may change as the reservoir approach the end of its production life, with the gas becoming ‘wet’. Furthermore so-called marginal assets, reservoirs which would not be financially viable by themselves but which exploit already existing platforms, are often wet, and become increasingly wet as they age [1], [4].

Although dedicated wet gas meters have been developed, it is common to employ classical ‘dry gas’ differential pressure (DP) meters for wet gas flow measurements. The perhaps most suitable DP meter for wet gas flow measurements is the venturi meter, as it is simple, robust and cost-effective. Unlike the orifice plate meter, which can act as a trap for debris and liquid, the venturi meter allows both liquid and gas to flow unhindered and the flow conditions of a wet gas is thus unlikely to be altered significantly by the throat of the venturi [3]–[5]. Furthermore, the venturi meter is known to give reliable wet gas flow readings and exhibits low pressure loss.

The presence of liquid in the gas stream causes an increase in the measured differential pressure, and resultingy a higher apparent gas mass flow rate, than would have been predicted had the gas...
flowed alone. To calculate the actual gas mass flow rate a correlation which corrects for the liquid induced over-reading needs to be applied. The flow rate of the liquid phase must be determined by an external source such as tracer dilution experiments, separator data, pressure loss measurements or multiple single-phase meters in series.

During the last half-century several wet gas correlations have been suggested to account for the over-reading of specific DP meters. Originally most correlations were developed for orifice meters. However, due to a lack of alternatives, it has often been assumed that the orifice plate meter correlations could be used for other DP meters without any modifications to the models or their parameters [1], [3]. This implies that the specific over-reading response is approximately the same for various kinds of DP meters. It is now recognized, however, that venturi meters have higher over-readings than orifice meters. This suggests that the models, or at least their parameters, are meter specific.

In this work we have chosen to evaluate six wet gas correlations, which are available in the open literature: The homogeneous model [3], the Murdock [5], the Chisholm [6], [7], the Steven [2], the de Leeuw [8] and the Reader-Harris Graham (RHG) correlation [9]. The latter three have been specifically developed for venturi meters. The homogeneous model can be theoretically derived as the upper limit for a generic DP meter. The remaining two are popular orifice plate meter correlations. The correlations are compared against both theoretical limiting values as well as published experimental venturi meter over-reading data.

To improve the general applicability of the de Leeuw correlation, the correlation is re-parameterized based on the available experimental over-reading data. A modification to the de Leeuw correlation, which should reduce the bias observed at low liquid loads, is furthermore suggested and assessed. Finally, by making a few reasonable simplifications we show how the RHG correlation can be reduced to a de Leeuw type correlation with fewer adjustable parameters than the RHG, without sacrificing its accuracy.

2 Definitions of Wet Gas Flow Parameters
A wet gas flow can be defined as a subset of two-phase gas/liquid flow, where the liquid content is below a certain threshold [3], [4]. Different parameters have been employed in the past to describe and define the allowable amount of liquid flow, see the ASME technical report [3] for more details. The most common upper threshold for a gas/liquid flow to be termed as ‘wet’ is that the Lockhart-Martinelli (LM) parameter has a value less than or equal to 0.3 (or sometimes 0.35).

\[^{1}\text{Note that in this definition of a two-phase flow the liquid is considered a single phase (or state) regardless of its composition and the number of distinct liquid phases. Counterintuitively, this means that multiphase gas/liquid/liquid systems are considered a subset to two-phase flow [3], [4].}\]
The LM parameter is a dimensionless number used to express the relative amount of liquid in the gas:

\[ X_{LM} = \sqrt{\frac{\text{Inertia of liquid flowing alone}}{\text{Inertia of gas flowing alone}}} = \frac{Fr_l}{Fr_g} = \frac{m_i}{m_g} \frac{\rho_g}{\rho_l} = \frac{Q_l}{Q_g} \frac{\rho_l}{\rho_g} \]  

(1)

where subscripts \( g \) and \( l \) denotes the gas and liquid phases respectively, \( m \) is the mass flow rate, \( \rho \) is the density, \( Q \) is the volumetric flow rate and \( Fr \) is the densiometric Froude number. If the flow has multiple liquid phases then \( m_l \) in eq. (1) is the sum of the different liquid mass flow rates and the density is that which is obtained if the liquid phases are assumed to be a single homogenously mixed phase. The gas and liquid densiometric Froude numbers are common terms when describing wet gas flow conditions. They are defined as:

\[ Fr_g = \sqrt{\frac{\text{superficial gas inertia force}}{\text{liquid gravity force}}} = \frac{U_{sg}}{gD} \sqrt{\frac{\rho_g}{\rho_l - \rho_g}} \]  

(2)

\[ Fr_l = \sqrt{\frac{\text{superficial liquid inertia force}}{\text{liquid gravity force}}} = \frac{U_{sl}}{gD} \sqrt{\frac{\rho_l}{\rho_l - \rho_g}} \]  

(3)

Where \( g \) is the gravitational constant, \( D \) the internal diameter of the pipe and \( U_{sl} \) and \( U_{sg} \) are the superficial liquid and gas velocity respectively:

\[ U_{sl} = \frac{m_l}{\rho_l A} \]  

(4)

\[ U_{sg} = \frac{m_g}{\rho_g A} \]  

(5)

The output of many wet gas meters depends on pressure. Due to the (approximate) incompressibility of the liquid phase it has proven to be convenient to incorporate this effect through the dimensionless gas to liquid density ratio:

\[ DR = \frac{\rho_g}{\rho_l} \]  

(6)

3 Wet Gas Metering Principle

If the compressibility of a dry gas is accounted for through the expansibility of the gas, \( \varepsilon \), then the mass flow equation for a generic single-phase differential pressure meter can be derived from a combination of mass continuity laws and the Bernoulli equation for incompressible fluids [10]–[12].

\[ m_g = \frac{C_d \varepsilon A_t}{\sqrt{1 - \beta^4}} \sqrt{2 \rho_g \Delta P_g} = C_d \varepsilon A_t E \sqrt{2 \rho_g \Delta P_g} \]  

(7)

Where \( C_d \) is the discharge coefficient, \( A_t \) is the cross-sectional area of the throat, \( \rho_g \) is the density of the gas, \( \Delta P_g \) is the differential pressure, \( \beta = d/D \), where \( d \) is the internal diameter of the throat.
and $D$ is the internal diameter of the pipe. Finally, $E = 1/\sqrt{1 - \beta^4}$ is the velocity of approach factor.

Due to the presence, or blockage, of liquid the measured differential pressure of a wet gas, $\Delta P_{tp}$, is higher than it would have been had the gas flowed alone. If unaccounted for in eq. (7) the increased differential pressure will result in an erroneous over-estimation of the gas mass flow rate [1], [13]. This uncorrected flow is referred to as the apparent gas mass flow, and is calculated as:

$$m_{g apparent} = C_{d,tp} \epsilon_{tp} A_t E \sqrt{2 \rho_g \Delta P_{tp}}$$

(8)

Where $C_{d,tp}$ and $\epsilon_{g,tp}$ are the discharge coefficient and expansibility under wet gas conditions. The liquid induced over-reading, is defined as the ratio between the apparent gas mass flow rate, eq. (8), to the actual gas mass flow rate, eq. (9)

$$OR = \frac{m_{g apparent}}{m_g} = \frac{C_{d,tp} \epsilon_{tp} \sqrt{\Delta P_{tp}}}{C_g \epsilon \sqrt{\Delta P_g}} \approx \frac{\Delta P_{tp}}{\Delta P_g}$$

(9)

where it has been assumed in eq. (9) that $C_{d,tp} \epsilon_{tp} \approx C_d \epsilon$. Estimation of the over-reading is the basis for almost all DP dry meter wet gas correlations.

4 Differential Pressure Meter Wet Gas Correlations

Today several wet gas venturi meter correlations exist. However, several well-known correlations for orifice meters are still sometimes used for venturi meters, although there is a growing agreement the correlation should match the type of meter it is applied to. In this work, we have selected to evaluate six general over-reading correlations; the homogeneous model [3], the Murdock [5], the Chisholm [6], [7], the Steven [2] the de Leeuw [8] and the Reader-Harris Graham (RHG) correlation. There are several other relevant correlations, for instance those suggested by Smith and Leang [14], [15], Lin [16], Lide et al. [17], He et al. [18] and He and Bai [19], which didn't make it into this work.

4.1 The Homogeneous Model

The homogeneous model is by far the oldest correction factor for wet two-phase flow dating back before the 1950s. It is assumed in its derivation that the flow can be treated as a pseudo-single-phase where the density can be determined from a homogeneous density expression [3], [20]. In terms of the over-reading the model can be expressed as

$$OR = \sqrt{1 + CX_{LM} + X_{LM}^2}$$

(10)

where
\[ C = \left( \frac{\rho_g}{\rho_l} \right)^n + \left( \frac{\rho_l}{\rho_g} \right)^n \] (11)

and where \( n = 0.5 \). The homogeneous model should be a good over-reading approximation for flow rates close to the \textit{annular mist} regime. Unlike the Murdock correlation (see next section) the homogeneous correlation takes the effect of pressure into account (through the DR ratio) independently of the LM parameter.

Eq. (10) and (11) are also known as the ‘general’ over-reading equation, since its functional form can be derived theoretically, with the value of the exponent, \( n \), differing depending on the assumptions. Most modern wet gas correlation employ the general over-reading equation as their fundamental base equation, and determine the value of \( n \) from an empirical expression fitted to experimental data.

4.2 \textit{The Murdock Correlation}

Murdock derived the first wet gas meter correlation based on a range of two-phase flow data through and orifice plate meter under diverse conditions [5]. The model is derived under the assumption that the flow is \textit{stratified}, although the data used for the fit was not necessarily from stratified flow. The Murdock correlation is a simple linear function in the LM parameter given by eq. (12)

\[ OR = 1 + MX_{LM} \approx 1 + 1.26X_{LM} \] (12)

Where \( M \) is the Murdock gradient. The Murdock gradient may be modified for other meter types, Philips Petroleum for instance, has claimed that a gradient of 1.5 is suitable for wet gas flow in venturi meters (sometimes called the modified Murdock correlation) [13].

4.3 \textit{The Chisholm Model}

The Chisholm model for orifice plate meters [6], [7] is derived under the assumption of separated flow (not necessarily stratified). The shear force at the gas/liquid boundary is directly considered. This results in a model with the functional form of the general over-reading equation, where \( n \) is equal to 0.25 instead of 0.5. Chisholm notes in his work that the agreement to the investigated orifice plate data is unsatisfactory [7]. Despite this the model has for a time been popular. Both the homogeneous model as well as the Chisholm model can be derived from theory. The transition between stratified flow and annular mist flow is a gradual process, one would expect that the value of \( n \) would lie somewhere in between the value of the two models for typical production conditions.
4.4 The de Leeuw Correlation

The de Leeuw correlation was the first published correlation for venturi meters [8]. It also uses eq. (10) and (11) as its base equations, but rather than a fixed value for \( n \) its value depends on the \( Fr_g \) number and is calculated from eq. (13) or (14):

\[
    n = 0.41 \quad \text{for} \quad 0.5 \leq Fr_g \leq 1.5
\]

\[
    n = 0.606\left(1 - e^{-0.746Fr_g}\right) \quad \text{for} \quad Fr_g \geq 1.5
\]

According to the Shell Expro two-phase flow pattern map [8] the two expressions for \( n \) correspond to two different flow regimes. When \( Fr_g \leq 1.5 \) the flow is stratified, but as the Froude number increases the flow gradually changes towards an annular mist, and \( n \) begins to increase. The value of \( n=0.41 \) at low \( Fr_g \) numbers is larger than Chisholm’s value of 0.25, and for an annular mist the exponent approaches 0.606 rather than the theoretical limit of 0.5.

Although this correlation is developed for a venturi meter and over a range of flows that are widely employed to wet gas in the industry, a pronounced disadvantage of the de Leeuw correlation is, that the correlation is meant only for a 4” venturi with \( \beta=0.4 \). This means that the effect of different \( \beta \)-ratios isn’t even accounted for implicitly in the adjustable parameters. de Leeuw notes that the error from this is expected to be minimal, however, later studies by Stewart [21] does suggest that the over-reading depends on the beta value of the venturi.

4.5 The Steven Correlation

In 2002 Stevens [2] found the accuracy of the available correlations to be insufficient at low liquid loads. To improve the fit to the experimental over-reading data Stevens used an entirely empirical function, arguing that the Chisholm exponent in the de Leeuw correlation was empirical in any case. His correlation is

\[
    OR = \frac{1 + AX_{LM} + BFr_g}{1 + CX_{LM} + DFr_g}
\]

Where the expressions for A, B, C and D are shown in Table 1. The correlation is expected to perform well if conditions are close to the conditions for the fit. However, as the correlation is entirely empirical one would expect high deviations for conditions differing from those the correlation has been adjusted to.

4.6 The Reader-Harris and Graham Correlation

In 2010 Reader-Harris and Graham (RHG) suggested a new correlation for venturi meters [9]. It too is (primarily) based on the general over-reading equations. The exponent, \( n \), is given by the empirical function
\[ n = \max \left[ 0.583 - 0.18\beta^2 - 0.578e^{-0.8Fr_g/H}, 0.392 - 0.18\beta^2 \right] \] (16)

\( H \) is a parameter which depends on the type of liquid. It was characterized in [9] by:

\[ H = 1 \text{(Hydrocarbon) or } H = 1.35 \text{(water) or } H = 0.79 \text{(water in steam flow)} \] (17)

Eq. (17) is not directly applicable to 3-phase gas/water/hydrocarbon flow as it does not consider mixtures with varying water cuts. Graham et al. [22] and Collins et al. [23] have independently suggested a simple modifications of eq. (17), where the \( H \) parameter is a linear function of the water cut.

\[ H = 1 + 0.35 \frac{Q_{mw}}{Q_{ml}} \] (18)

With this modification the performance of the correlation seems to be promising for 3-phase flow. Finally, the correlation also provides an empirical wet gas discharge coefficient given by:

\[ C_d = 1 - 0.0463e^{-0.05Fr_g/\beta^{2.5} \min \left( 1, \frac{X_{LM}}{0.016} \right)} \] (19)

This wet gas discharge coefficient has been the matter of some debate. In this work, as in [23], \( C_d \) is considered as an additional part of the wet gas correction term.

The RHG correlation has previously been the subject of some criticism, amongst partly because of its many parameters [20] (at least 8, but depends on parameterization). The other correlations, except for Stevens correlation, have between zero and two adjustable parameters. Increasing the number of parameters to a least fourfold that of other correlations does inevitable increase the risk of over-fitting, and thus the error in predictions.

4.7 Discussion
Considerable effort has been put into determining the parameters of importance for wet gas flow metering. As is obvious from the correlations and models presented in the previous sections most researchers agree that the most important wet gas parameters are; \( X_{LM}, DR \) and \( Fr_g \). How many of these parameters are considered in a correlation primarily depends on how old it is. The effect of the \( Fr_g \) number, for instance, was not fully appreciated until de Leeuw suggested his correlation.

Steward has later demonstrated that the over-reading decreases with increasing \( \beta \)-ratios [3], [21], an effect which has been confirmed by Reader-Harris et al. using CFD analysis [24]. It is also known, although less understood, that the physical properties of the liquid has a considerable effect on the over-reading [3], [4], [9]. The orientation of the meter is known to have a pronounced effect on the over-reading, due to alterations in the flow regime. There is almost no available data for inclined or vertical wet gas flow, and all correlations in this paper, are for horizontal pipes. Steven has also suggested that there might be diameter effects on venturi meter over-readings [1], [4].
The consequence of the $\beta$-ratio dependency may be, that the de Leeuw and the Stevens correlation, which were developed from data for a single $\beta$-ratio, may give a systematic bias in the over-reading, when the correlation is employed for other $\beta$-ratios. Only the RHG correlation attempts to account for the effect of different $\beta$-ratios, as well as the type of liquid, in the functional form of the correlation.

Table 1 summarizes the different wet gas models and correlations investigated in this work. The table includes the correlation data range, the meter type which the correlation was developed for and the type of fluids in the correlation data. The range of the correlations is limited compared to the field conditions encountered in industry and extrapolations are common. In this work all correlations will be employed to the full over-reading database, which means that most correlations will be extrapolated beyond the wet gas parameters for which they have been correlated. Adverse effects are thus likely.

Recently a number of independent researchers have studied several of the most popular correlations for the over-reading under diverse conditions [2], [13], [18], [23], [25]. With the exception of the more recent work of Collins et al. [23] the reoccurring theme from these investigations, was that the de Leeuw correlation typically performed well, whereas Stevens correlation performed well at low values of the $X_{LM}$ parameter. The simple homogenous model performed surprisingly well, especially when the flow could be considered close to the annular mist region. Collins et al. [23] also considered the more recent RHG correlation, and found this correlation to be the most accurate publicly available correlation.

5 Theoretical Boundary Conditions and the Exponent, $n$

The value of the over-reading is limited by certain physically determined boundary conditions, which can be exploited in the design of a wet gas correlation:

- As the liquid content goes towards zero, the over-reading must approach 1, i.e. $OR \to 1$ as $m_l \to 0$, ultimately yielding the mass flow rate of the pure gas.
- At elevated pressures the gas and liquid density may approach the same value, which is called the dense phase condition. As this condition is approached $OR_{dense} \to 1 + X_{LM}$.
- Finally, it can be shown that, for a given liquid content, the smallest value which the over-reading can take is the same expression as for the dense phase condition.

Not all wet gas correlations tend towards these boundary conditions. However, correlations which does approach the boundary conditions has a better chance of being successfully extrapolated outside the range of the data on which it has been based.
Table 1 - Summary of the discussed wet gas correlations including the data range which they were correlated to.

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Applicable range</th>
<th>Primary meter and components.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous [3]</td>
<td>( OR = \sqrt{1 + CX_{LM} + X_{LM}^2} ), ( C = \left( \frac{\rho_d}{\rho_l} \right)^n \left( \frac{\rho_l}{\rho_g} \right)^n ), ( n = 0.5 )</td>
<td>N/A</td>
<td>Theoretical. Applies to all meter types</td>
</tr>
<tr>
<td>Murdock [5]</td>
<td>( OR = 1 + 1.26X_{LM} )</td>
<td>( 0.1 \leq P (MPa) \leq 6.3 ) ( ), ( 0.26 \leq \beta \leq 0.5 ) ( ), ( 0.041 \leq X_{LM} \leq 0.25 ) ( ), ( D = 2.5^n ) ( ), ( 1.3 \cdot 10^4 \leq Re_g \leq 1.27 \cdot 10^6 )</td>
<td>Orifice plate. Wet steam, air/water, Gas/salt water, Gas/liquid hydrocarbons</td>
</tr>
<tr>
<td>Chisholm [6], [7]</td>
<td>( OR = \sqrt{1 + CX_{LM} + X_{LM}^2} ), ( C = \left( \frac{\rho_d}{\rho_l} \right)^n \left( \frac{\rho_l}{\rho_g} \right)^n ), ( n = 0.25 )</td>
<td>( 1 \leq P (MPa) \leq 7 ) ( ), ( 0.186 \leq \beta \leq 0.49 ) ( ), ( 0.5 \leq X_{LM} \leq 5 ) ( D = 2^n )</td>
<td>Orifice plate Steam/water</td>
</tr>
<tr>
<td>de Leeuw [8]</td>
<td>( OR = \sqrt{1 + CX_{LM} + X_{LM}^2} ), ( C = \left( \frac{\rho_d}{\rho_l} \right)^n \left( \frac{\rho_l}{\rho_g} \right)^n ), ( n = 0.41 ) ( ), ( 0.5 \leq F_{rg} \leq 1.5 ), and ( n = 0.606(1 - e^{-0.746F_{rg}}) ) for ( F_{rg} \geq 1.5 )</td>
<td>( 1.5 \leq P (MPa) \leq 9.8 ) ( ), ( \beta = 0.4 ) ( ), ( 0.5 \leq F_{rg} \leq 4.8 ) ( ), ( 0 \leq X_{LM} \leq 0.34 ) ( D = 4^n )</td>
<td>Venturi meter Natural gas/water Nitrogen/diesel oil</td>
</tr>
<tr>
<td>Steven [2]</td>
<td>( OR = \frac{1 + AX_{LM} + BF_{rg}}{1 + CX_{LM} + DF_{rg}} ), ( )</td>
<td>( 2 \leq P (MPa) \leq 6 ) ( ), ( \beta = 0.55 ) ( ), ( 0.4 \leq F_{rg} \leq 4 ) ( ), ( 0 \leq X_{LM} \leq 0.3 ) ( D = 6^n )</td>
<td>Venturi meter. Kerosene/nitrogen Decane/nitrogen Stoddard/nitrogen</td>
</tr>
<tr>
<td>Reader-Harris and Graham</td>
<td>( OR = \sqrt{1 + CX_{LM} + X_{LM}^2} ), ( C = \left( \frac{\rho_d}{\rho_l} \right)^n \left( \frac{\rho_l}{\rho_g} \right)^n ), ( n = \max[0.583 - 0.18\beta^2 - 0.57\beta e^{-0.8F_{rg}/H}, 0.392 - 0.18\beta^2] )</td>
<td>( 0.4 \leq \beta \leq 0.75 ) ( ), ( 0 \leq X_{LM} \leq 0.3 ) ( D \geq 2^n ) ( ), ( DR \geq 0.02 ) ( ), ( F_{rg}/\beta^{2.5} &gt; 3 )</td>
<td>Venturi meter, Natural gas, nitrogen, argon, steam / water, Stoddard, exxsol D80, decane</td>
</tr>
</tbody>
</table>

As mentioned above the functional form of eq. (10) and (11) are often referred to as the ‘general’ over-reading equation, since its functional form can be derived theoretically, with the value of the exponent \( n \) differing depending on (at least) the type of flow regime. It can be shown that:

- \( n = 0 \) for stratified “minimum energy” flow (i.e. \( OR = \sqrt{1 + 2X_{LM} + X_{LM}^2} = 1 + X_{LM} \)).
- \( n = 0.5 \) for homogenous mist flow (i.e. the Homogeneous model).

In principle, these flow cases serve as the upper and lower limits for the over-reading at a given \( X_{LM} \) parameter and density ratio. The general over-reading equation tends towards the correct value as both the dense phase condition, and the zero-liquid content limit is approached. It is therefore
not a surprise that several of the most popular over-reading correlations are based on this equation. What primarily differentiates these correlations is the expression for \( n \);

- \( n = 0.25 \) in the Chisholm model (derived for orifice plate meters).
- \( n = \max[0.606(1 - e^{-0.746F_{rg}}), 0.41] \) in the de Leeuw correlation.
- \( n = \max[0.583 - 0.18\beta^2 - 0.578e^{-0.8F_{rg}/H}, 0.392 - 0.18\beta^2] \) in the RHG correlation.

Chisholm’s model is directly in between the two theoretical limits. Based on the assumptions used to derive the model it is expected to work best for stratified flow. In the de Leeuw correlation \( n \) varies as a function of the \( Fr_g \) number. The range of \( n \) for the de Leeuw correlation is 0.41-0.606. The correlation can surpass the theoretical homogeneous limit of \( n=0.5 \) for annual mist flow. This may be due to deficiencies with the model, the data which the correlation is based on, or due to other flow effects, which prevents the model assumptions from being met. In the RHG correlation the value of \( n \) depends on both the \( Fr_g \) number, the \( \beta \)-ratio and the type of fluid. All of which are known to influence the over-reading, possibly due to differences in the flow pattern. Within the \( \beta \)-ratio range of the RHG correlation (\( \beta=0.4-0.75 \)) the minimum value of \( n \) is approximately 0.29 at \( \beta=0.75 \) and the maximum value is approximately 0.55 at \( \beta=0.4 \), i.e. in closer agreement with the theoretical limit of 0.5 than the de Leeuw correlation.

Figure 1 shows the various expressions for \( n \) as a function of the \( Fr_g \) number. For the RHG correlation only liquid hydrocarbons and water at ambient temperatures have been considered. Except for the fact that the RHG correlation also varies with the \( \beta \)-ratio and the type of fluid, it is clear from the figure that \( n \), behaves similarly to the de Leeuw correlation. For a fixed venturi \( \beta \)-ratio the value of \( n \) is constant at low \( Fr_g \) numbers but starts to increase exponentially around 1.4 if the liquid is a hydrocarbon, and around 1.9 if the liquid is water. The RHG correlation also employs a wet discharge coefficient, which means that the final over-reading result from the correlation isn’t obtained by simply substituting the value of \( n \) into eq. (11). It can be shown, however, that as the dense phase condition or the zero-liquid limit are approached, the ‘wet’ discharge coefficient approach 1 (since \( Fr_g \to \infty \) or \( X_{LM} \to 0 \) at the two limits respectively) and the correlation reduce to the general over-reading equation. See Reader-Harris [27] for more detail.

Figure 2 shows the over-reading as a function of the \( X_{LM} \) parameter for all correlations as the dense phase condition is approached (\( \rho_g/\rho_l \to 1 \)). It can be seen from Figure 2 that all correlations which are based on the general over-reading equation, tend to the dense phase limit of \( OR = 1 + X_{LM} \). Stevens correlation approach a constant over-reading value of B/D since \( Fr_g \to \infty \), as the dense phase limit is approached.
To illustrate the predicted over-reading at more realistic density ratios, Figure 3 shows the predicted over-reading when $DR = 0.15$ (a) and $DR = 0.05$ (b) with $\beta=0.6$ and $Fr_g=4.5$. For the RHG correlation the liquid phase was assumed to be a hydrocarbon. The $Fr_g$ number suggests that the flow is in the mist region, thus indicating that the homogeneous model would be suitable. Indeed Figure 3 (a) shows that the de Leeuw correlation and the RHG correlation are in close agreement with the homogeneous model. In Figure 3 (b) the RHG correlation almost duplicates the predictions of the Homogeneous model, whereas the de Leeuw correlation yields significantly higher over-readings. It is likely, that this difference is because the de Leeuw correlation does not incorporate a $\beta$-ratio dependency, and that the over-reading for a venturi with a $\beta$-ratio of 0.4 is higher than for a venturi with $\beta=0.6$.

It is apparent from the Figure 3 (a) and (b) that the Murdock and Chisholm correlations begins to give substantially lower over-reading predictions than the other correlations at lower density ratios. Figure 3 (a) and (b) also illustrate, that while Stevens correlation is in much better general agreement with the other correlations, than at the dense phase condition, it does have another curvature than the other correlations. In both subfigures the RHG correlation has a stepper slope at low $X_{LM}$ values, due to the way in which the wet discharge coefficient is calculated. This is by design as discussed by Reader-Harris and Graham [9].
Figure 2 – Over-reading as a function of X_{LM} parameter for the various correlations as the dense phase limit is approached ($\rho_g/\rho_l \approx 1$). The homogeneous, Chisholm, de Leeuw and the RHS correlation all coincide at these conditions.

Figure 3 - Over-reading as a function of X_{LM} parameter for (a) a (high) density ratio of 0.15 and (b) a smaller density ratio of 0.05. In both flow cases $\beta=0.6$ and $Fr_g=4.5$. The liquid phase is assumed to be a hydrocarbon.
6 Results

6.1 Generalized Wet Gas Correlations

There are relatively few published venturi-tube wet gas data in the open literature. Nonetheless, 1045 experimental data points have been collected from references [1], [21], [28] in this work. Due to the scarcity of three component data in the public domain all the data are for two-phase flow. The gas phase primarily consists of nitrogen or argon gas and the liquid phase of water or hydrocarbon liquids such as Kerosene, Stoddard or Exxsol D80.

The data covers four β-ratios; 0.4, 0.55, 0.6 and 0.75 respectively. Most of the data is at β-ratios of 0.6 and 0.75. Density ratios range from 0.011 to 0.088 and Frg numbers are between 0.5 and 4.8, with most of the data being in the range 1.5-3.5. The experimental over-reading data is in the range 0 ≤ X_{LM} ≤ 0.3. Approximately 60% of the data points has X_{LM} ≤ 0.15, implying an almost equal share of low and high X_{LM} parameters.

To evaluate the performance of the models we use the percentage root mean square relative error (RMSE) given by eq. (20)

\[
RMSE(\%) = 100 \sqrt{\frac{1}{N_{exp}} \sum_{i=1}^{N_{exp}} \left( \frac{OR_{i}^{exp} - OR_{i}^{calc}}{OR_{i}^{exp}} \right)^2}
\]

The RMSE is a measure of the percentage standard deviation between the wet gas correlations and the experimental data. The percentage relative error, ε, is employed in plots:

\[
\varepsilon_i (\%) = 100 \frac{OR_{i}^{exp} - OR_{i}^{calc}}{OR_{i}^{exp}}
\]

All correlations, except for the RHG correlation, have been extrapolated to conditions outside their intended limits. Restricting the data to operating conditions which are within the published limits for each correlation is likely to result in improved predictions. In practice, however, correlations are used under the prevailing process conditions irrespective of the actual process limit. Part of the employed data have been used by other researchers to develop the Steven and the RHG correlation. This might incur a bias in the comparison, as some correlations will solely predict the over-reading, whereas some might be correlated to (parts of) the data, which would give artificially low errors.

Table 2 shows the RMSE, maximum absolute relative error and the number of parameters in the correlations. The RMSE results are comparable to other studies, which has also evaluated the performance of these wet gas correlations. See for instance the works of Steven [2], Lide et al. [13] and Collins et al. [23]. Only the latter of which is recent enough to employ the RHG correlation.
It is clear from Table 2 that the RHG correlation performs significantly better than the other correlations; both the RMSE and the maximum error is about three times smaller than those for the de Leeuw correlation, which has the second lowest errors. An equivalent result was obtained by Collins et al. [23]. The reduced errors, however, do come with the price of at least 6 additional adjustable parameters, which increases the risk of overfitting and adverse behaviors outside the correlation range.

<table>
<thead>
<tr>
<th>Wet Gas Correlation</th>
<th>RMSE (%)</th>
<th>Max abs ε (%)</th>
<th>No. of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td>4.8</td>
<td>19.5</td>
<td>0</td>
</tr>
<tr>
<td>Murdock</td>
<td>9.4</td>
<td>22.4</td>
<td>1</td>
</tr>
<tr>
<td>Chisholm</td>
<td>8.9</td>
<td>20.9</td>
<td>1</td>
</tr>
<tr>
<td>de Leeuw</td>
<td>4.3</td>
<td>16.3</td>
<td>2</td>
</tr>
<tr>
<td>Steven</td>
<td>4.9</td>
<td>20.0</td>
<td>12</td>
</tr>
<tr>
<td>RHG</td>
<td>1.3</td>
<td>5.7</td>
<td>8-10</td>
</tr>
</tbody>
</table>

The Murdock and Chisholm correlation has RMSEs approximately twice that of most correlations, and the difference is even higher compared to the RHG. As both correlations have been developed for orifice plate meters this strongly supports the consensus, that the correlations are specific to the meter type. Except for the RHG correlation the maximum errors are close to 20% for all correlations.

Considering its simplicity, and the fact that it has no fitting parameters, the Homogeneous model results in errors which are only marginally larger than in the de Leeuw correlation. The reason for the relatively high maximum errors for the de Leeuw correlation, may be, that most of the experimental data comes from venturi meters with high β-ratios. Nevertheless, the results appears to be in overall agreement with other investigations, e.g. [2], [13], [23].

Figure 4 shows the relative error of the correlations as a function of the XLM parameter. The figure clearly shows that the relative errors increase with increasing XLM for all correlations, except the RHG correlation. Both the Murdock correlation and Chisholm’s model are prone to underestimate the over-reading, giving a positive error bias, while the Homogeneous model typically overestimates the over-reading, giving a negative error bias.

Figure 5 shows the relative over-reading error for the Homogeneous model. The errors have been divided into ranges of the Fr_g number. As expected the model clearly gives the lowest errors when the Fr_g number is high, as the flow should be approaching an annular mist. It is not surprising that the model over-estimates the over-reading when the Fr_g number decreases, as the model is, in principle, the upper limit to the over-reading.
Figure 4 - Relative error of the correlations as a function of the $X_{LM}$ parameter. Symbols are different correlations.

Figure 5 - Relative error of the Homogeneous model as a function of the $X_{LM}$ parameter. Symbols are different $Fr_g$ ranges.
Figure 6 shows the relative over-reading error calculated for the de Leeuw correlation. The errors have been divided into β-ratios. The performance clearly depends on this ratio; The RMSE for the de Leeuw correlation is comparable to the RHG correlation when β=0.4, but the performance becomes increasingly poor as the β-ratio increase. The reason for this is undoubtedly that the parameters in the de Leeuw correlation were fitted to data from venturi meters with β=0.4, so that the effect of larger β-ratios weren’t captured, even indirectly, in the parameter estimation.

It is obvious from Figure 5 and Figure 6 that both the de Leeuw correlation and the Homogeneous model have a positive error bias at low X_{LM} parameters, but a negative bias at higher X_{LM} values. The reason for the underestimation of the over-reading at low X_{LM}, is that the correlations approach zero almost linearly as the X_{LM} parameter goes to zero, but the over-reading does not. This bias is significantly reduced in the RHG correlation due to the use of a wet gas discharge coefficient.

Figure 7 shows a similar over-reading plot for the RHG correlation, note the different scale on the error-axis compared to Figure 5 and Figure 6. The errors are low compared to the other correlations, and they do not seem to increase as the β-ratio or X_{LM} parameter increases. The uncertainty given by in ISO/TR 11583:2012 [26] agrees reasonably well with the results calculated in this work, although a 3% uncertainty in the whole X_{LM} range might be more accurate compared to ISO standard uncertainty shown in Figure 7. A few points does have negative relative errors higher than 4%, these points are from two data series with low density ratios (<0.015), where the effect of the liquid on the over-reading becomes negligible [1].
6.2 Modified Wet Gas Correlations

It is evident from the previous section, that absent any additional a priori knowledge about the suitability of a specific wet gas correlation, the RHG correlation and the de Leeuw correlation are probably the two best choices for a given application. For the de Leeuw correlation the errors are small when $\beta=0.4$, but they tend to increase for larger $\beta$-ratios. As there is a clear $\beta$-ratio effect on the over-reading, this is a quite serious defect for the model. Another issue, is that the correlation is too linear at low $X_{LM}$ values. The RHG correlation gives significantly better predictions than the de Leeuw correlation, but the cost of this is that it employs at least four times as many parameters.

To address these issues, we will investigate the effect of two modifications of the de Leeuw correlation and two modifications of the RHG correlation suggested in this work. To estimate the adjustable parameters 80% of the data points in the database were randomly drawn. The remaining 20% of the data points were used to validate the new parameters. The correlations were performed using a weighted least squares type objective function

6.2.1 Modified de Leeuw correlations

The functional form of the exponent in the de Leeuw correlation can be written as

$$ n = a(1 - e^{-1.5b}) \quad \text{for} \quad 0.5 \leq Fr_g \leq 1.5 $$

(22)

$$ n = a(1 - e^{-bFr_g}) \quad \text{for} \quad Fr_g \geq 1.5 $$

(23)
where $a=0.606$ and $b=0.745$ from the original correlation. It is further possible to consider the $Fr_g$ number at which $n$ becomes constant, but similarly to Collins et al. [23] this was found to offer negligible improvements.

One way to account for the linearity issue is to add an empirical term to the correlation, which contributes to the over-reading at low $X_{LM}$ values, but reduce to zero at higher values. Eq. (23) was found to be suitable, as it obeys the theoretical limits, and gives the desired contribution to the over-reading at low $X_{LM}$ values.

$$corr = (C - 2)X_{LM}e^{-dX_{LM}}$$

A single adjustable parameter, $d$, was needed in the correction term. A similar correction to the de Leeuw correlation was employed in [28]. Although, that correction term contained more adjustable parameters and the parameters were fitted to each mixture in each venturi meter, severely limiting the generality of the correction. The full modified over-reading equation becomes

$$OR = \sqrt{1 + CX_{LM} + X_{LM}^2 + (C - 2)X_{LM}e^{-dX_{LM}}}$$

The expression for $C$ is the one given in eq. (11), and $n$ is given by eq. (22) and (23). For a fairer comparison, and to improve the correlation for non-fixed $\beta$-ratios, the parameters in the de Leeuw correlation are refitted both with and without the correction term. That is, the model variants which we investigate are:

- Re-estimating of the parameters in the de Leeuw correlation.
- de Leeuw + correction term, eq. (24).

Table 3 shows the correlated parameters of the de Leeuw correlation with re-estimated parameters, and the de Leeuw correlation with the correction term (the modified de Leeuw correlation). It is noteworthy that the value of the $a$ parameter indicates, that the upper limit of the exponent, $n$, is close to the theoretical value of 0.5. The reason for this is probably that the parameters are now fitted to over-reading data at four different $\beta$-ratios, rather than just $\beta=0.4$.

<table>
<thead>
<tr>
<th>Wet Gas Correlation</th>
<th>$a$</th>
<th>$b$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original de Leeuw</td>
<td>0.606</td>
<td>0.745</td>
<td></td>
</tr>
<tr>
<td>Refitted de Leeuw</td>
<td>0.52</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Modified de Leeuw</td>
<td>0.51</td>
<td>0.88</td>
<td>31.8</td>
</tr>
</tbody>
</table>

Table 4 shows the RMSE and the maximum error in over-reading for the re-estimated and modified de Leeuw correlation respectively. Compared to the original correlation the RMSE and maximum error are significantly reduced for both correlations. The RMSE is similar sized for both the fitting data and the validation data, indicating that the reduced error is not caused by the models correlating the data, rather than predicting the over-reading.
Table 4 - RMSE (%) for the 80% fitting data, the 20% validation data and all data points combined for the refitted de Leeuw correlation and the Modified de Leeuw correlation.

<table>
<thead>
<tr>
<th>Wet Gas Correlation</th>
<th>80% fitting data</th>
<th>RMSE (%)</th>
<th>20% validation data</th>
<th>Max error (%)</th>
<th>All data</th>
<th>All data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original de Leeuw</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refitted de Leeuw</td>
<td>2.5</td>
<td>4.3</td>
<td>2.4</td>
<td>2.5</td>
<td>8.2</td>
<td></td>
</tr>
<tr>
<td>Modified de Leeuw</td>
<td>2.1</td>
<td>2.0</td>
<td>2.1</td>
<td>7.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compared to the original de Leeuw correlation the $a$ parameter has become smaller and the $b$ parameters has become larger. Similar trends were obtained by Collins et al. [23]. The obtained parameters in their work were closer to the de Leeuw parameters [8], probably because the majority of their over-reading data came from venturi meters with $\beta=0.55$, while the majority of our over-reading data comes from venturi meters with $\beta=0.75$. This might also explain why Collins et al. [23] does not find, that the new parameters reduces the over-reading error significantly.

Figure 8 and Figure 9 show the relative error in the over-reading for the refitted and modified de Leeuw correlation respectively. Comparing Figure 6 with Figure 8 we see that the errors still grow as the $X_{LM}$ parameter increases, and that the errors still depend on the $\beta$-ratio. In fact, the errors simply appear to be more evenly distributed around zero, due to the fact that the parameters are...
being estimated for experimental data from diverse venturi meters with different β-ratios. Figure 9 shows, that, as intended, the positive bias is nullified with the modified de Leeuw correlation, but all other errors for $X_{LM}>0.05$ are similar to the correlation with refitted parameters. Indeed, as seen from Table 4 the RMSE for the whole $X_{LM}$ range is only 0.4% lower with the modified de Leeuw correlation, but when $X_{LM} \leq 0.05$ the RMSE is 2.3% for the refitted de Leeuw correlation and 1.4% for the modified de Leeuw correlation, indicating that the improvement occurs in the low $X_{LM}$ range.

![Modified de Leeuw correlation](image)

**Figure 9 - Relative error of the de Leeuw correlation modified with the correction term. Errors are shown for all points in the database. Symbols indicate different β-ratios.**

### 6.2.2 The Simplified Reader-Harris Graham correlation

One of the arguments against using the RHG correlation is its many parameters, which increases the risk of over-fitting and consequently error in the prediction [20]. It is good practice to employ as few adjustable parameters as possible to obtain a desired fit. Such models are more likely to remain accurate when extrapolated to conditions outside that of the experimental data, and they often provide a better understanding of the principal factors governing the model. Indeed, using only three adjustable parameters the modified de Leeuw correlation doesn’t perform much worse than the RHG correlation. The errors are still smaller with the RHG correlation, but only by about 1%, and at the cost of at least five additional parameters.
Some of the parameters in the RHG correlation have almost identical values and appears somewhat arbitrary. In fact, a good approximation to the expression for \( n \) is

\[
\begin{align*}
    n = \max & \left[ a \left( 1 - e^{-bFr_g/H} \right) - g\beta^2, a \left( 1 - e^{-1.4b} \right) - g\beta^2 \right] \\
    \text{where } H = 1 + h \frac{Q_{mw}}{Q_{ml}}
\end{align*}
\]

which has four parameters. The 1.4 in the rightmost exponent is the \( Fr_g \) number at which \( n \) begins to change for a hydrocarbon in the original RHG. Changing it to 1.5 as in the de Leeuw correlation, was found to offer little to no change. It is obvious that eq. (24) is essentially a de Leeuw type correlation with an additional squared beta dependency, as well as a parameter which account for the effects of the fluid. Collins et al. [23], however, showed that the correlation is relatively insensitive to the fluid parameter, so the correlation might be further simplified by its removal.

The threshold equation for the wet discharge coefficient might be simplified to

\[
C_d = 1 - d e^{-dFr_g/\beta^{2.5}} \min \left( 1, \frac{X_{LM}}{X_{Lim}} \right)
\]

which has two parameters. In this work the combination of eq. (24) and (25) is termed the simplified RHG (sRHG) correlation. Rather than employ eq. (25), the correction term from eq. (22) could be added to the general over-reading equation. This variant is referred to as the modified sRHG correlation.

Table 5 shows the correlated parameters for the sRHG and the modified sRHG. Note that parameter \( d \) has a different meaning in the two correlations, see eq. (22) and (25).

<table>
<thead>
<tr>
<th>Wet Gas Correlation</th>
<th>a</th>
<th>b</th>
<th>d</th>
<th>g</th>
<th>h</th>
<th>X_{Lim}</th>
</tr>
</thead>
<tbody>
<tr>
<td>sRHG</td>
<td>0.64</td>
<td>0.79</td>
<td>0.038</td>
<td>0.27</td>
<td>0.21</td>
<td>0.015</td>
</tr>
<tr>
<td>Modified sRHG</td>
<td>0.62</td>
<td>0.92</td>
<td>35.1</td>
<td>0.20</td>
<td>0.33</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6 shows the RMSE and maximum over-reading error for the sRHG and the modified sRHG. The RMSEs are equal to 1.3±0.1 for either correlation, for both fitting data and validation data. These RMSEs are almost identical to the RMSE of 1.3, for the original RHG correlation. Although not shown in this work, it is interesting to note that in both correlations parameters \( g \) and \( h \), may be parameterized as a single adjustable parameter, without incurring any additional error (the RMSE would be unchanged). The same is the case for parameter \( a \) and \( b \) in the sRHG. There is no obvious reason, however, as to why an indicator for e.g. the water content should be related to the parameter scaling the \( \beta \) dependency.
Table 6 - RMSE (%) for the 80% fitting data, the 20% validation data and all data points combined for the sRHG and the modified sRHG correlation. Last column indicates the maximum absolute error.

<table>
<thead>
<tr>
<th>Wet Gas Correlation</th>
<th>80% fitting data</th>
<th>20% validation data</th>
<th>All data</th>
<th>Max error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>original RHG</td>
<td>1.3</td>
<td></td>
<td></td>
<td>5.6</td>
</tr>
<tr>
<td>sRHG</td>
<td>1.2</td>
<td>1.3</td>
<td>1.2</td>
<td>6.8</td>
</tr>
<tr>
<td>Modified sRHG</td>
<td>1.3</td>
<td>1.4</td>
<td>1.3</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Figure 10 and Figure 11 show the error in over-reading for the sRHG and the modified sRHG respectively. As expected from the RMSE results both error plots are very similar to the error plot for the regular RHG correlation (see Figure 7). The main difference between the RHG and the sRHG or modified sRHG seem to be, that there are slightly more outliers than with the original correlation. Furthermore, outliers which were already present for the RHG correlation are slightly amplified with the simplified correlations.

It might be attractive to use a wet discharge coefficient which gives a smoother transition than eq. (25). A suitable empirical function can be constructed as a combination of eq. (23) and (25) as:

\[
C_d = 1 - (C - 2)X_{LM}e^{-dX_{LM}Fr_d \beta^{2.5}}
\]

which, like eq. (23) only contains a single parameter. Use of this equation instead of eq. (25) gives RMSEs like the sRHG.
7 Conclusion

In this work the performance of six common wet gas correlations have been evaluated against a venturi meter dataset containing more than a thousand experimental points. Of the generalized correlations we can conclude, that the Reader-Harris Graham (RHG) correlation has the overall best predictions. The correlation results in an overall RMSE of 1.3% and absolute errors below 3% for more than 95% of the experimental data. Unfortunately, the correlation uses a considerable number of adjustable parameters to achieve this. The simple and well-known de Leeuw correlation predicts the over-reading quite well for the low β-ratio venturi meter data, for which it was developed, but deviations becomes substantial as the β-ratio increases.

![Figure 11 - Relative error of the modified RHG correlation. Errors are shown for the fitting data (blue stars) and the validation data (red squares).](image)

New parameters were estimated for the de Leeuw and a modified de Leeuw correlation both of which gave significantly better over-reading predictions than the original for arbitrary β-ratios. Ideally, however, the results indicate that the de the Leeuw correlation should be re-parameterized for each β-ratio which it is applied to. The modified de Leeuw correlation successfully nullified the bias at low $X_{LM}$ parameters.

The RHG have been the subject of some debate, partly due to its many parameters [20]. In this work we propose both a simplified RHG and a modified RHG correlation, both of which gives the same accuracy as the original correlation, but with fewer adjustable parameters. It is shown that for the sRHG and modified sRHG respectively, no more than 5 or 6 adjustable parameters are
required to obtain the accuracy of the original RHG, which employs at least 8 fitting parameters. Although possibly coincidental it is possible to further reduce the number of parameters to 4 without adverse effects. We hope that these results might give more confidence in the future use of the RHG or modifications thereof.

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9 Bibliography