## accord ${ }^{(s)}$

# Pipeline Allocation and How to Win the Lottery 

## Oil and Gas Focus Group

$7^{\text {th }}$ June 2018

## Inspiration

## $26{ }^{\text {n }}$ International North Sea Flow Measurement Workshop

$21^{\text {st }}-24^{\text {th }}$ October 2008

# Features of Allocation Systems Incorporating Long Pipelines 

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## 1 INTRODUCTION

There are two main approaches to systems of allocation that include long pipelines. The first accounts for each user's hydrocarbons within the pipeline itself. The second method ignores the transit time in the pipeline and allocates the metered quantities exiting the pipeline based on the metered quantities input into the pipeline on the same day; using this approach parties will not be allocated precisely what they input to the pipeline on a day, but over a period of time there is an expectation that any daily gains and losses will even themselves out.

This paper examines instances when this is not necessarily true depending on the allocation equations employed. It demonstrates, using simple models and results from a real allocation system, how parties can be systematically under and over allocated hydrocarbons due to the uexped and subtle bias in the allo stability of the equations and approches to eliminate alloction bias.

It also discusses the wider implications for allocation systems in general, particularly in terms of how the assumptions, equations and logic of a system should be tested at the conceptual development stage to prevent problems occurring.

In Section 2 a simple model is used to describe an allocation system associated with a pipeline. This model illustrates the basic process and presents the main features of the allocation methodology. Data from an analogous real system is presented to highlight a analyse the allocation system behaviour without the obfuscating effects of measurement uncertainty in the real data.

## 2 PIPELINE ALLOCATION SYSTEM DESCRIPTION

A simple system incorporating a long pipeline is presented below and this is used as a basis to describe allocation issues associated with a real system.

### 2.1 Process Description

Consider two offshore platforms exporting gas to an onshore gas plant via a long pipeline
$35^{\text {th }}$ International North Sea Flow Measurement Workshop 24. - 26. October 2017

## Systematic bias in pro rata allocation schemes

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## ABSTRACT

Misallocation due to allocation uncertainty may result in increased exposure to economic risk for owners or stakeholders in hydrocarbon fields. It is often assumed that allocation errors are random and that they will "even out" over time, irrespective of the system setup and allocation uncertainty. In this paper, we show that this is normally not the case, even for simple allocation systems using standard pro rata allocation. For instance, a two-field pro rata allocation setup with a high measurement uncertainty for one of the meters compared to the other, causes the field with the highest allocation uncertainty to be systematically under-allocated. We show that this misallocation is inherent to the allocation system, and will occur even without any systematic measurement error present.
Since pro rata allocation systems are widely used, either as general allocation principle or as part in a multi-tier allocation, this inherent misallocation should be of particular interest to the industry. The financial loss associated with systematic misallocation can only be evaluated based on a correct quantification of the misallocation. Therefore, it is important to be aware of how systematic misallocation may be a direct consequence of the setup of a pro rata allocation system and the maintenance scheme of the different metering stations.
The objective of our work is to quantify the systematic misallocation in pro rata allocation setups, and identify in which cases this effect is economically significant. Furthermore, the aim is to establish some usefil "rules of thumb" that may be used to evaluate if an allocation setup is subject to systematic misallocation.
We explain the mechanisms behind systematic misallocation, illustrating the effect with a few simple examples. Then we analytically show how the statistical expected value in pro rata allocation differs from the actual production rate. As it may be practically unmanageable to express the systematic misallocation analytically for more complex systems, we show how this can be done using numerical methods instead.
Finally, we demonstrate the calculation of systematic misallocation for a realistic measurement setup and allocation scenario in a multi-field setting based on experience from industrial projects.
Our work shows that the pro rata allocation principle inherently leads to systematic misallocation, particularly in cases where there is a significant difference between the uncertainties of the allocated fields. This misallocation is systematic and does not cancel out over time. Therefore, pro rata allocation systems should always be evaluated for any inherent systematic misallocation.

## What I'm going to talk about

## 1

Is there an allocation problem?
2
3
4
Why does it occur?
Real world?
Are there any solutions?
5 Conclusions
6 How to

## Is there an allocation problem?

Why does it occur? Real world? Are there any solutions? Conclusions How to win the lottery!

## Long Pipeline

Field A

Field B


Field A


## Steady state



Inlet

## How should the inlet be allocated?

Field A
$100 \quad I_{A}=\left(\frac{E Q}{E Q+E Q}\right) \AA 30$
$100 \boldsymbol{E}_{\boldsymbol{B}}$
But what happens
when it's not steady state?
Field B



Field A

## It will even out over time, won't it?



Inlet
Field B

## Total flow out equals sum of flow in on a day Inlet composition varies



Field B

## Cumulative Difference Between Export and Allocation

Field A Cumulative Delta
Allocated Inlet - Export

60

40

20
0
$-20$
$-40$
$-60$
$\begin{array}{rrrrrrrrrr}-80- & 11 & 21 & 31 & 41 & \begin{array}{c}51 \\ \text { Day }\end{array} & 61 & 71 & 81 & 91\end{array}$
NFOGM 2018 Allocation

## Cumulative Difference Between Export and Allocation

Field B Cumulative Delta
Allocated Inlet - Export
$80-$
60

40

20
0
$-20$
$-40$
$-60$
$\begin{array}{rrrrrrrrrr}-80- & 11 & 21 & 31 & 41 & \begin{array}{c}51 \\ \text { Day }\end{array} & 61 & 71 & 81 & 91\end{array}$
NFOGM 2018 Allocation

## What happens if one flow is more variable?

Field A


Field B
Inlet

## Field A Varying 0 to 100/day Field B Constant at 100/day

Field A Cumulative Delta
Allocated Inlet - Export

60
40
20

0
$-20$
$-40$
$-60$
$\begin{array}{cccccccccc}-80- & 11 & 21 & 31 & 41 & \begin{array}{c}51 \\ \text { Day }\end{array} & 61 & 71 & 81 & 91\end{array}$
NFOGM 2018 Allocation

## Field A Varying 0 to 200/day Field B Constant at 100/day

Field B Cumulative Delta
Allocated Inlet - Export


## What happens if compositions vary?

Field A


Inlet
Field B

## Field A \& B Constant at 100/day C1 $\pm 10 \%$

Field A Cumulative Delta
Allocated Inlet - Export
$80-$

60

40

20
0
$-20$
$-40$
$-60$
$\begin{array}{rrrrrrrrrr}-80- & 11 & 21 & 31 & 41 & \begin{array}{c}51 \\ \text { Day }\end{array} & 61 & 71 & 81 & 91\end{array}$
NFOGM 2018 Allocation

## What happens if one composition varies?

Field A


Inlet
Field B

## Field A \& B Constant at 100/day Field A C1 $\pm 10 \%$

Field A Cumulative Delta
Allocated Inlet - Export
$80-$
60

40

20
0
$-20$
$-40$
$-60$
$\begin{array}{cccccccccc}-80- & 11 & 21 & 31 & 41 & \begin{array}{c}51 \\ \text { Day }\end{array} & 61 & 71 & 81 & 91\end{array}$
NFOGM 2018 Allocation

## Field A \& B Constant at 100/day Field A C1 $\pm 20 \%$

Field A Cumulative Delta
Allocated Inlet - Export
80 -

60

40

20
0
$-20$
$-40$
$-60$
$\begin{array}{cccccccccc}-80- & 11 & 21 & 31 & 41 & \begin{array}{c}51 \\ \text { Day }\end{array} & 61 & 71 & 81 & 91\end{array}$
NFOGM 2018 Allocation

## 1

## Is there an allocation problem?

 Why does it occur?Real world? Are there any solutions? Conclusions

## $4^{\text {th }}$ Dimension Time



Field A


Field B

Field A
In total exported 60+80 =140 Allocated $74.3+65=139.3$ Export systematically>inlet


Field B

## Chance Winning UK Lottery



## Chance Winning UK Lottery

| Match | Probability | Prize |
| :--- | :--- | :--- |
| 6 balls | 1 in $45,057,474$ | $£ 3,000,000$ |
| 5 balls | 1 in 144,415 | $£ 50,000$ |
| 4 balls | 1 in 2,180 | $£ 100$ |
| 3 balls | 1 in 97 | $£ 25$ |
| 2 balls | 1 in 10.3 | $£ 2$ |

Expected winnings = £0.89 Less cost of ticket $=£ 2.00$ Expected value $=-£ 1.11$ or $45 \%$ return

## Fixed Export Total Flows Varying composition

Field A


Inlet
Field B

## Expected value Field A's allocated inlet:

Field A

$$
E\left[I_{A}\right]=\int_{I_{A 1}}^{I_{A 2}} I_{A} P\left(I_{A}\right) d I_{A}
$$




$$
E\left[I_{A}\right]=\int_{E_{B 1}}^{E_{B 2}} \int_{E_{A 1}}^{E_{A 1}} \int_{I_{1}}^{I_{2}} I\left(\frac{E_{A}}{E_{A}+E_{B}}\right) P(I) P\left(E_{A}\right) P\left(E_{B}\right) d I d E_{A} d E_{B}
$$

$$
I=E_{A}^{\prime}+E_{B}^{\prime}
$$

$$
E\left[I_{A}\right]=\int_{E_{B 1}}^{E_{B 2}} \int_{E_{A 1}} \int_{E_{B}^{\prime} E_{11}}^{E_{B 2}} \int_{E_{A 1}}^{E_{A A}}\left(E_{A}^{\prime}+E_{B}^{\prime}\right)\left(\frac{E_{A}}{E_{A}+E_{B}}\right) P\left(E_{A}^{\prime}\right) P\left(E_{B}^{\prime}\right) P\left(E_{A}\right) P\left(E_{B}\right) d E_{A}^{\prime} d E_{B}^{\prime} d E_{A} d E_{B}
$$



$$
E\left[I_{A}\right]=\frac{\left(E_{A 1}+E_{A 2}+E_{B 1}+E_{B 2}\right)\left(\boldsymbol{F}\left(\boldsymbol{E}_{\boldsymbol{A} 2}, \boldsymbol{E}_{\boldsymbol{B} 2}\right)-\boldsymbol{F}\left(\boldsymbol{E}_{A 2}, \boldsymbol{E}_{\boldsymbol{B} 1}\right)-\boldsymbol{F}\left(\boldsymbol{E}_{\boldsymbol{A} 1}, \boldsymbol{E}_{\boldsymbol{B} 2}\right)+\boldsymbol{F}\left(\boldsymbol{E}_{\boldsymbol{A} 1}, \boldsymbol{E}_{\boldsymbol{B} 1}\right)\right)}{8\left(E_{A 2}-E_{A 1}\right)\left(E_{B 2}-E_{B 1}\right)}
$$

## Where:

$$
\boldsymbol{F}\left(\boldsymbol{E}_{A}, \boldsymbol{E}_{\boldsymbol{B}}\right)=\left(E_{A}-E_{B}\right)\left(2\left(E_{A}+E_{B}\right) \ln \left(E_{A}+E_{B}\right)-E_{A}+E_{B}\right)
$$

## Field A \& B Constant at 100/day Field A C $1 \pm 10 \%$



## Field A \& B Constant at 100/day Field A C1 $\pm 20 \%$



## But equations also predict a bias for this case...



## What about flow variation?



## Field A C1\% Varies Field B C1\% constant at 60\%



## Field A C1\% Varies Around 70\% Field B C1\% constant at 70\%



## Field A C1\% Varies Around 70\% Field B C1\% constant at 50\%



## 1

## Is there an allocation problem?

Why does it occur? Real world?

Are there any solutions? Conclusions

## Real System



## Real System

Field A appears to be over-allocated...

## Similar flows

field A exhibits less variation than field $B$


## Is there an allocation problem?

Why does it occur?
Real world?
Are there any solutions?


## Allocation Solutions (1)

## Track pipeline contents

More feasible with liquid systems

## Allocation Solutions (2)



## Allocation Solutions (3)



Incorporate feedback mechanism

Use of pipeline stock

## Allocation Over Extended Period (4) How long is a long pipeline?



Residence time $=0.01$ days

# Yes - difference in variability - 

 proved mathematically. Not accounted for the time lag Yes - effects subtle - comparable with measurement noiseYes - presented 4
How many systems affected?
How to win the lottery!


NFOGM 2018 Allocation

## Chance Winning Massachusetts State Lottery (2005)

| Match | Probability | Prize |
| :--- | :--- | :--- |
| 6 balls | 1 in 9.3 million | $\$ 1,000,000$ |
| 5 balls | 1 in 39,000 | $\$ 4,000$ |
| 4 balls | 1 in 800 | $\$ 150$ |
| 3 balls | 1 in 47 | $£ 5$ |
| 2 balls | 1 in 6.8 | $£ 2$ |

## Expected winnings $=\$ 0.80$ Less cost of ticket $=\quad \$ 2.00$ Expected value = -\$1.20 or $40 \%$ return

## Chance Winning Massachusetts State Lottery (2005)

| Match | Probability | Prize |
| :--- | :--- | :--- |
| 6 balls | 1 in 9.3 million | $\$ 3,000,000$ |
| 5 balls | 1 in 39,000 | $\$ 4,000$ |
| 4 balls | 1 in 800 | $\$ 150$ |
| 3 balls | 1 in 47 | $£ 5$ |
| 2 balls | 1 in 6.8 | $£ 2$ |

Expected winnings = \$1.01 Less cost of ticket =<br>\$2.00<br>Expected value =<br>-\$0.99 or $51 \%$ return

## Chance Winning Massachusetts State Lottery (2005)

| Match | Probability | Prize |
| :--- | :--- | :--- |
| 6 balls | 1 in 9.3 million | $\$ 1,000,000$ |
| 5 balls | 1 in 39,000 | $\$ 50,000$ |
| 4 balls | 1 in 800 | $\$ 2,385$ |
| 3 balls | 1 in 47 | $£ 60$ |
| 2 balls | 1 in 6.8 | $£ 2$ |

Expected winnings $=\$ 5.94$
Less cost of ticket $=\quad \$ 2.00$
Expected value = $+\$ 3.94$ or 197\% return

## UK Lottery Jan (2016)



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Mathematics There's never been a better day to play
the lottery, mathematically speaking

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In Saturday's draw the 'expected value' of a ticket is greater than its cost. Don't get too excited: the odds of hitting the jackpot with six balls are still 1 in $\mathbf{4 5 m}$


A It's not time to break out the bubbles just yet. The likelihood of winning the jackpot with six balls remains 1 in 45 million. Photograph: Ady Kerry/Camelot/PA

## Lottery of Life



## 1 in 40 million

1 in 400 quadrillion

