INFLUENCE ON THE VELOCITY OF SOUND ON THE ACCURACY OF GAS DENSITY TRANSDUCERS

by

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INFLUENCE OF THE VELOCITY OF SOUND ON THE ACCURACY OF GAS DENSITY TRANSDUCERS.

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Much discussion about the accuracy of Solatron equipment. Stansfeld of Solatron defended this

INTRODUCTION

Although the title on the paper implies that gas density transducers in general will be included in this presentation, the paper has been prepared to examine in some detail only the models from Solartron Transducer Group, namely 7810 and 7811 which appears to be in most frequent use in all North Sea metering applications. In addition a description of the experimental work carried out by Dantest on behalf of TOM and EAN is included.

Finally it is assumed that the basic operation/calculation of the density-meter is familiar to the audience and only those aspects related to the velocity of sound correction will be discussed.

VIBRATING ELEMENT TRANSDUCERS, 78 SERIES.

The basic principle of the gas density meter is based on the fact that th surrounding gas is brought into oscillation by the vibrating cylinder and contributes to the mass term in equation 1. This means that the natural frequency, Wo, decreases with increasing gas density, but unfortunately this is not always the case.

Any physical quantity which contributes to undesireable changes in the mass, M, or spring stiffness K, of the vibrating system will cause an undesirable change in the maintained oscillation frequency and systematic errors will occur. Below is a summary of the most important factors.

- Vibrating cylinder stiffness

  Changes in the spring rate may be caused by changes in the stress of the vibrating element. A stress effect on the sensing element can arise if there is a pressure difference between the inside and the outside of the cylinder wall. However, the gas is passed, both inside and outside of the cylinder eliminating any stress effects due to the pressure of the gas.

  Changes in the spring rate can also arise, caused by variations in the elasticity of the vibrating element for different temperatures. In most cases materials can be chosen to give a very low thermoelastic coefficient. A nickel/iron alloy called Ni-Spanc has this property, and if the material is cold worked out and then carefully heat treated, changes in elasticity with temperature will be very small.(1)

- Maintaining circuit

  Regarding the closed loop magnetic maintaining circuit it must be emphasised that this in no way will affect natural frequency, but only cancells the viscous damping and other damping mechanisms.

However, because of the power supply to the coils a self-heat generation is possible, but a high mechanical Q minimises the energy required to maintain system oscillations which reduce heat generation and any attendant errors.

Another aspect is that the vibrating element must be manufactured from magnetic material and as mentioned above, must be as stable as possible, exhibiting the same basic characteristic under differing environmental conditions. Ni-Span C has both these properties, but it loses its magnetic properties above 160°C.(1) This means that it cannot be used at temperatures higher than 100°C to 125°C dependent upon the shape, size and general characteristics of the vibrating element. At higher temperatures materials such as FV 520 can be used but there is a considerable loss of stability and some form of temperature correction becomes more necessary.

- Relation of sound waves

Sound waves are generated by the vibration of the solid body in contact with the fluid medium. When sound waves are produced in a region completely enclosed by walls, rigid or otherwise, all wave motion is standing-wave motion and the acoustic pattern is determined by the nature of its geometry. For our case the acoustic picture will be very complicated, due to the shape and vibrating mode of the cylinder, but some general aspects will be discussed.

A cavity filled with fluid which is brought into oscillation will have many resonance frequencies. Those resonances depend on the geometry of the cavity and velocity of sound of the fluid.

In the frequency range before resonance, outgoing and reflected sound waves will be in the same phase. This means that the pressure will be increased both by outgoing and reflected sound waves. Sound waves will be reflected in a different manner and give a different acoustic picture outside and inside the cylinder wall due to the different shape of the reflecting walls and different distances. Thus a pressure difference will arise which will affect the cylinder stiffness and turn the vibrating system into a stiffness loading system. The natural frequency will increase instead of decrease with increasing fluid density due to the increased stiffness.

When a resonance is passed, outgoing and reflecting sound waves are not in phase and in the frequency range well above a resonance frequency, pressure difference is not dominating. The vibrating system becomes a mass loading system. Thus, the basic principle which the gas densitymeter is based on exists.

The first three resonance frequencies of this system arise when:

1. The distance between the cylinder wall and the reflecting wall is a half wavelength.

2. The cylinder circumference is one wavelength.

3. The cylinder length is one wavelength.

A combination of the three mentioned, will give a number of other resonances and the vibrating system will have a very complex acoustic picture. What must be remembered is that the vibrating system can be both a mass loading or a stiffness loading system dependent on the frequency range. In the frequency range before resonance it will always be a stiffness loading system, then it will turn into a mass loading system every time a resonance is passed. For this particular system the three first resonances will not represent a great problem, due to the mode of cylinder vibrations. The cylinder vibrates in a radial mode (see Fig.1) and the volume inside and outside is not changed. This makes the system less sensitive to stiffness loading.

- Effective oscillating mass

When the frequency and the geometry is adapted in a way which make the vibrating system a mass loading system, the fundamental principle of the vibrating cylinder densitymeter exists. However, the gas mass which is brought into oscillation, the effective oscillating mass, is not only dependent on the gas density, but will be affected by the transport characteristic of the gas too.

The vibration of the cylinder is shown greatly exagerated in Fig. 1. When the cylinder vibrates in its simplest radial hoop mode. The shaded areas represent the amount of gas which moves in a oscillatory manner over a distance 1. Of course not all the gas in the shaded areas moves directly around the circumference, there must be a circumferential fluid movement outside the cylinder as well as an axial and radial fluid movement both inside and outside the cylinder.

However, in order to develope a mathematical equation describing the motion of the gas, a simplified model is needed. Solartron describes in their technical data sheet IDS-105 (2) a simplified model shown in figure 1 which is considered to be an reasonable approximation for the actual movement of the gas along the cylinder wall. Fig. 3 shows the cylinder, spoolbody and liner "opened out" to a linear situation. On this drawing it is perhaps easier to see the analogy to a piston movement used by Solartron to develope their mathematical equation.

Refering back to figure 2, a tube filled with gas has a piston at each end moving, in synchronism, with displacement acoswt, and a distance, 1, apart. An element of gas next to each piston will move substantially with the piston and with displacement, acoswt, but for elements away from a piston face, the elementary thin cross section, δx , will move with less amplitude. The displacement, γ , is a function of distance and time, t, and velocity of sound, C, and is given by the wave equation.

$$\frac{\partial^2 \gamma}{\partial x^2} = \frac{1}{C} \frac{\partial^2 \gamma}{\partial t^2}$$

In the technical note Solartron uses the concept of energy change to find a solution to the wave equation. The kinetic energy at the point of maximum velocity is transferred to potential energy at the point of maximum deflection. Hence the maximum value for kinetic energy occurs when : $\left(\frac{\partial \delta}{\partial t}\right)_{Max}$

Equation 4 describes the general expression for the kinetic energy of a standing wave. Following the mathematical manipulation suggested in Solartrons technical note, one will hopefully agree with the expression in equation 5. This shows that any system which measures the kinetic energy of a vibrating gas column as a means of deducting its density will arrive at a value described by equation 6.

Refering back to the simplified drawing in Fig. 1 it is seen that the value of l is equal to a quarter of the circumference. However, since the movement of gas in the real situation is more comprehensive the actual value of l to be used in the correction will be different.

Finally the quality of the simplified theoretical model and the resulting equation can only be judged when compared to experimental data.

The accuracy of the correction has of course been investigated by Solartron and also by the Dutch company, Gasunie. And for the low range model 7810 (0 - 16 Kg/M^3) sufficient data are available to form the accuracy of the correction. A recent report published by the Danish national center for testing and verification, Dantest, also supports the Solartron velocity of sound correction. However, the "magnitude of the accuracy" if such an expression can be used, is still very much discussed. But again refering to the Dantest report an uncertainty in the order to $\pm$ 0,1% is indicated.

For the high pressure model 7811 (0 - 400 Kg) experimental data on natural gas mixtures is not available in the high pressure range and Solartron "proves" the relevance of the correction using the ethylene IUPAC tables, i.e. comparing the error between published ethylene data and measurements (argon calibration) to the curves for velocity of sound for ethylene.It can be shown that the minimum error occurs when ethylene has the same velocity of sound as the argon used for the calibration i.e. about 320 metres/sec. This supports the theory of the velocity of sound correction in an elegant way, but the accuracy over the entire density range has not been demonstrated.

The issue of velocity of sound correction is further complicated by Solartron Transducer Ltd. by issuing two different calibration certificates for example, the high pressure model 7811 is issued with two calibration certificates, commonly refered to as:

1. User Gas certificate (ref. equation 7)

2. Calibration gas certificate using the "user gas offset" equation (ref. equation 8).

Finally if a Solartron flow computer is purchased and connected to the densitymeter the velocity of sound correction will automatically be calculated for you. The three approaches will as you may have guessed give rise to the three different answers, so it is hard to speak of accuracy in the correction unless experimental data is available to justify the selected approach.

It can be shown that equation 8 is a simplification of equation 7.

For an argon gas calibration Solartron Transducer Group calculate the term Gc/Tc+273 equal to 0,00282. This assumes the velocity of sound for argon is constant over the entire pressure and temperature range. Further G is defined as gas specific gravity/ratio of specific heat.

Now the specific heat of a gas is as you know dependent upon both composition, pressure and temperature. But since most flow computers do not calculate this value for you, the densitymeter user will probably program a constant value in flow computations.

With the above statements we hope to have succeded in pointing out our view that one cannot talk about accuracy of correction without having accurate experimental data as reference. The most favourable of the available methods to minimize metering errors can then be selected.

For this reason Total Oil Marine Ltd. and Elf Aquitaine Norge initiated a cooperation with Dantest to carry out experiments using natural and synthethic gas, and the objective was to show the absolute accuracy of the various equations so far discussed.

DANTEST EXPERIMENT

The density meters were calibrated by determining true density using the real gas law:

$$\rho = \frac{P}{Z \times R \times T}$$

Having obtained true density a regression curve between the period of the density meters and true density was established. The parameters were measured as follows:

P : The pressure was stabilized and measured with a Desgranges and Hout deadweight tester 5201S with a relative accuracy of 0,01%.

T : The temperature was held constant within 0,1°C by placing the density meters in series in a thermal cabinet. A resistance thermometer (PT100) was attached to each density meter. The stated accuracies was 0,05°C.

Z : The compressibility factor was measured using a Desgranges and Hout Z meter type 60000. This instrument had been previously calibrated using the NBS-tables. It is possible to establish Z with an accuracy of 0,1%.

R : The gas constant is determined from chromatographic analysis of the gas and is equal to the universal gas constant divided by molecular weight.

A sample of Frigg gas was analysed by Dantest while the premixed synthetic Frigg gas had previously been certified by the Department of Energy in Leicester.

TEST PROCEDURE

Two 7811 and two 7810 density meters were placed in the thermal cabinet and connected in series. A PT100 temperature element was placed in each position of the densitymeter.

The set up was leak tested with vacuum and with nitrogen at 20 bar.

With the thermal cabinet at approx. 35°C the density meters were calibrated with nitrogen, natural gas and the synthetic gas. During calibration, the density meters were filled with gas and stabilized during measurement. Due to equipment limitation data points were limited to the range 0 to 80 bar. From the data points, calibration certificates were calculated which enabled comparison to be made between original certificates issued by Solartron. The experiment was repeated at 3°C which is similar to the operating temperature at St. Fergus Plant.

RESULTS

One of the main objectives for EAN was to try and illustrate the accuracy of the argon calibration including the data calculated on user gas certificate using equation 7. Table I shows the results obtained. In August this year Dantest informed us that they had discovered a small leak in the hydraulic system of the dead weight tester which could have affected the measurements carried out in January this year. Dantest further indicated that this could at the most have affected the measurements by 0,2%. As such one should not put too much emphasize on the value of the discrepancy but a closer matching had been expected.

In St. Fergus the complete equation 8 is programmed into the flow computers and the similarity in densities obtained using the equation compared to Dantest results was naturally the subject of detailed examination. As mentioned previously the errors will be very much dependent upon how the specific heat is calculated. However, Total Oil Marine Ltd. uses a constant value of Cp/Cv calculated by using average parameters and properties of the Frigg gas. The results showed that this approach gave satisfactory matching to the experimental results over the relevant operating range.

Finally in order to indicate the integrity and accuracy table II is included. This table compares the Solartron nitrogen calibration with the nitrogen calibration carried out by Dantest. As seen the difference is less than the uncertainty of the data points.

Refering back to table I it should be mentioned that the composition of the synthetic gas is not exactly the same as the Frigg gas composition used by Solartron when computing the user gas certificate. However, the inadequacy of the argon calibration is further illustrated in table III.

In this table, Dantest's nitrogen calibration is compared to the density calculated with Solartron constants obtained with argon calibration and corrected to nitrogen by velocity of sound correction. The error is larger than the claimed accuracy of the density cell.

SUMMARY

Although one cannot draw any decisive conclusion from the information presented in this paper, some trends are nevertheless illustrated:

These are:

1. Argon calibration of the density cell should be avoided. Both because uncertainty in the velocity of sound data for argon and that the resulting correction for velocity of sound is rather large in natural gas application. For Frigg gas in the order of 0,8%.

2. If equation 8 is used in low pressure application (densiters 0 - 60 Kg/M^3) it is probably more accurate to use a constant value for Cp/Cp than calculating the true on line valve.

3. For densities in the range 0 - 60 Kg/M^3 calibrated with nitrogen, Solartron velocity of sound correction introduces an additional uncertainty in the density measurement of about $\pm$ 0,1%, which have to be added to basic calibration error of the instrument.

4. The basic theory of the velocity of sound and the overall accuracy is well documented in the low density range.

IMPROVEMENT IN DENSITY METER CALIBRATION

Due to equipment limitations maximum working pressure at the Dantest laboratory was 80 bar when the experiments were conducted. At the time of writing this paper the equipment is being redesigned to be able to operate in the pressure range 0 - 150 bar.

Elf Aquitaine Norge and Total Oil Marine will then be able to repeat the previous experiments over a wider range of pressures. Some of the experiments carried out during Phase I will be repeated in order to confirm the trends already reported in this paper.

The goal to obtain better accuracy in calibration of density meter can be obtained by either of the following two paths:

1. When reliable experimental data is available, it is a simple task to modify the relevant parameters in equation 7 or 8 to minimise error in density measurement.

2. The densitymeter constant can be established on a synthetic gas with similar properties to the relevant natural gas, thus eliminating the need for any velocity of sound correction.

When the Dantest Phase II experiments have been completed EAN and TOM in cooperation with British and Norwegian authorities will make the necessary adjustments to the exisitng calibration procedures used in the Frigg natural gas transportation system.

TABLE I

Deviation between density as determined by Dantest and user gas certificate.

User Gas Certificate	Dantest	Deviation 1 - 2 / 1
59,09	59,38	0,49%
51,00	51,24	0,47%
43,08	43,28	0,45%
35,36	35,51	0,42%
27,85	27,96	0,4 %
20,55	20,63	0,39%
13,47	13,53	0,45%
6,60	6,65	0,75%

TABLE II

Solartron nitrogen calibration compared with Dantest nitrogen calibration for 7810 density meters.

Density meter	Nominal pressure bar, abs.	Period micro sec.	1 Density Dantest N_2 3 °C kg/m^3	2 Density Solartron N_2 20 °C kg/m^3	3 Density Solartron N_2 3 °C kg/m^3	Error
A	10	224.1815	12.260	12.271	12.250	-0.08
	50	263.3800	61.95	62.02	62.00	-0.08
B	10	222.7444	12.257	12.278	12.260	-0.02
	50	260.9917	61.93	61.99	61.98	-0.08

TABLE III

Comparison between density calculated with Solartron constants obtained with argon calibration and corrected to nitrogen by sound of velocity correction and Dantest's nitrogen calibration at 35°C.

Density meter	Nominal pressure bar, abs.	Period micro sec.	Density Dantest N_2 35°C kg/m^3 (1)	Density Solartron Ar_2 20 °C kg/m^3 (2)	Density Solartron N_2 35°C kg/m^3 (3)	Error
C	10	485.2711	10.979	10.888	10.950	-0.26
	80	623.9000	76.48	75.98	76.21	-0.35
D	10	485.9037	10.979	10.867	10.928	-0.46
	80	621.8700	76.46	76.00	76.232	-0.30

References:

1. Green, D.R. "Resonant Frequency, Measurement tool of the Seventies". Control & Instrumentation, 7. 40-43. 1975.

2. Solartron Technical Data Sheet TDS 105: Sound velocity effect on the vibrating cylinder density Transducers.

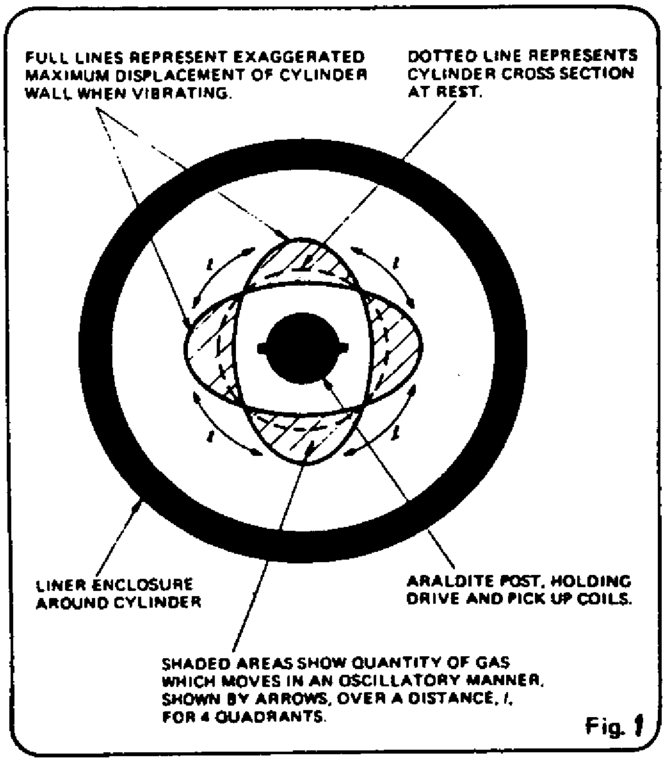

Fig. 1

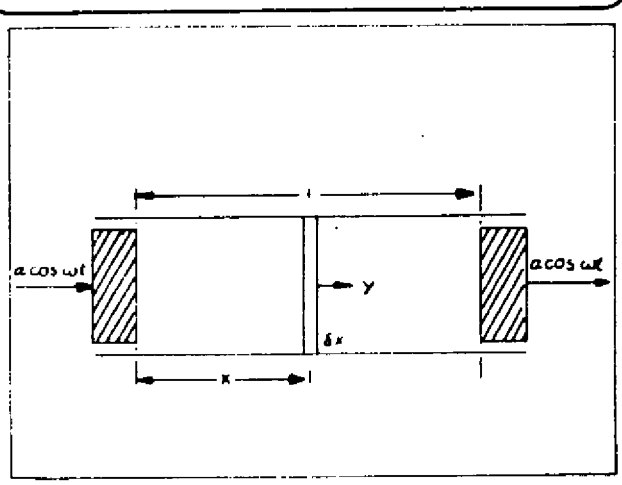

Fig. 2 A tube filled with gas and piston at each end

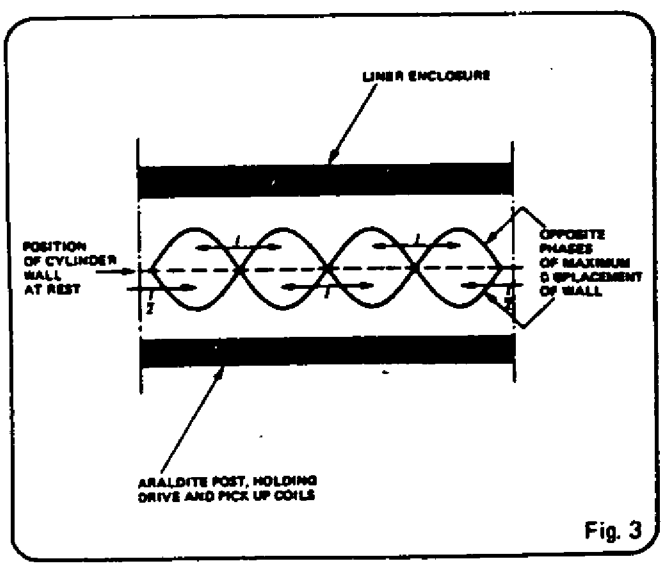

Fig. 3

$$\omega = \frac{1}{2}\pi\sqrt{\frac{K}{M}} \qquad \text{EQ. 1}$$

WHERE ω = RESONANT FREQUENCY

K = STIFFNESS OF THE CYLINDER

M = MASS OF THE SYSTEM

$$\rho_G = K_0 + K_1\frac{1}{f} + K_2\frac{1}{f^2} \qquad \text{EQ. 2}$$

WHERE ρ_G = GAS DENSITY

f = OSCILLATION FREQUENCY

K_0, K_1 AND K_2 = CALIBRATION CONSTANTS

SOUND WAVE EQUATION:

$$\frac{\partial^2 \gamma}{\partial x^2} = \frac{1}{C}\ \frac{\partial^2 \gamma}{\partial t^2} \qquad \text{EQ. 3}$$

WHERE ∂x = ELEMENTARY CROSS SECTION

γ = DISPLACEMENT

X = DISTANCE

t = TIME

C = VELOCITY OF SOUND

$$\frac{1}{2} m v^2 = \frac{1}{2} \rho l \, (\omega \cdot a)^2 \qquad \text{EQ. 4}$$

Where $1/2mv^2$ = general kinetec energy

ρ = gas density

l = half wavelenght of the cylinder motion

ω = frequency

a = constant

$$K.E = 1/2 \rho l \, (\omega \cdot a)^2 \cdot 1 + 1/6 \left(\frac{\omega l}{2c}\right)^2 \qquad \text{EQ. 5}$$

Where C = velocity of sound

$$\rho = \rho_i \left(1 + \tfrac{1}{6}\left(\frac{\omega l}{2c}\right)^2\right) \qquad \text{EQ. 6}$$

Where ρ_i = Indicated density

ρ = Actual density

$$\rho = \rho_i \frac{\left(1 + \tfrac{1}{6}\left(\frac{\omega l}{2c_c}\right)^2\right)}{\left(1 + \tfrac{1}{6}\left(\frac{\omega l}{2c_a}\right)^2\right)} \qquad \text{EQ. 7}$$

$$\rho = \rho_i \left(1 + \frac{k_3}{\rho_i + k_4}\left(\frac{G_c}{T_c + 273} + \frac{G_a}{T_a + 273}\right)\right) \qquad \text{EQ. 8}$$

Where ρ_i = Indicated density

ρ = Actual density

K_3, K_4 = Constants

G = $\frac{\text{Gas specific gravity}}{\text{Ratio of specific heats}}$

T = Temperature in °C