



# VELOCITY OF SOUND EFFECT ON GAS DENSITY TRANSDUCERS – THE THEORY, MEASUREMENT RESULTS AND METHODS OF CORRECTION

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## SUMMARY

This paper describes the basic theory of the velocity of sound effect on vibrating cylinder gas density transducers. Measurement results are presented which confirm this effect. With this understanding the user is better able to select with confidence the most suitable correction method for a particular application. It is also hoped that this understanding will help to focus more emphasis on other application and installation details, thereby ensuring that the total measurement task is adequately performed.

## 1 INTRODUCTION

Differential pressure and gas density are the two prime quantities which are required in order to calculate the flow of gas through an orifice system. Gas density measurement is also of considerable importance when metering high pressure gas using turbine or vortex shedding meters.

Vibrating cylinder gas density transducers are now widely used for this service and when correctly applied offer very high accuracy. In order to achieve these high accuracies, it is important to consider a number of factors such as the performance of the prime calibration, installation details like pressure and temperature equilibrium, the representation of the sample gas and the systematic offsets due to temperature and the gas velocity of sound. Of these, the most difficult to understand has been the velocity of sound effect. A lack of understanding of this effect has, in a number of instances, led to bad practices and in consequence unnecessarily large measurement errors. This difficulty can give rise to conflict between the buyer and seller of the gas and should therefore be avoided.

The problem is clearly in two parts. The first is quantifying the velocity of sound effect on the sensor and the second is determining the actual velocity of sound of the calibration gas and the user gas so that, if necessary, suitable corrections can be applied.

## 2 HISTORY

Vibrating cylinder gas density transducers were first introduced in 1968. Early evaluations by Shell<sup>1</sup>, British Gas<sup>2</sup> and later Sira<sup>3</sup> clearly identified calibration offsets when changing from one gas type to another. Careful examination of this problem soon identified the velocity of sound effect and resulted in the publication of a paper by Solartron<sup>4</sup> on this topic.

This early work resulted in some design improvements for the vibrating cylinder and some application recommendations to correct for this effect.

Much of this early work was confused by the difficulties of carrying out accurate calibrations on gases and gas mixtures and by the manner in which the accuracy of some of these instruments were specified — for example, the use of 'percent full scale' against 'percent reading'. The velocity of sound effect is a 'percent reading' influence so becomes negligible at low densities, against an accuracy which is specified with respect to a large full scale reading.

A dramatic demonstration of the velocity of sound effect occurs when measuring gases such as ethylene through their critical region. Under these conditions there is a large change in the velocity of sound at constant density for small changes in pressure and temperature. For this reason much work has been concentrated in this area with impressive results. More recently, and with respect to very high pressure natural gas measurement, Dantest<sup>5</sup> and Ruhrgas<sup>6</sup> have carried out extensive tests, the results of which are now published.

During the past fifteen years a great deal of work has been done both theoretically and experimentally on the subject. Without exception this effort confirms the velocity of sound effect and in consequence gives strong indications on how corrections can best be achieved.

### 3 THEORY

Most vibrating element density transducers work on the principle of a simple mass spring system which is maintained in resonance by some electro-mechanical/electronic feedback system.

Part of the fluid, whose density is to be measured, is carried by or is in contact with the vibrating element and in consequence forms part of the total mass of the vibrating system. As the density changes, the total mass changes and hence the resonating frequency changes. This is simply illustrated by Figure 1 which shows the fluid within a container on the end of a spring. In this illustration it is assumed that the fluid is oscillating in complete sympathy with the container.

This assumption cannot be completely correct since the fluid is compressible and has a finite velocity of sound.

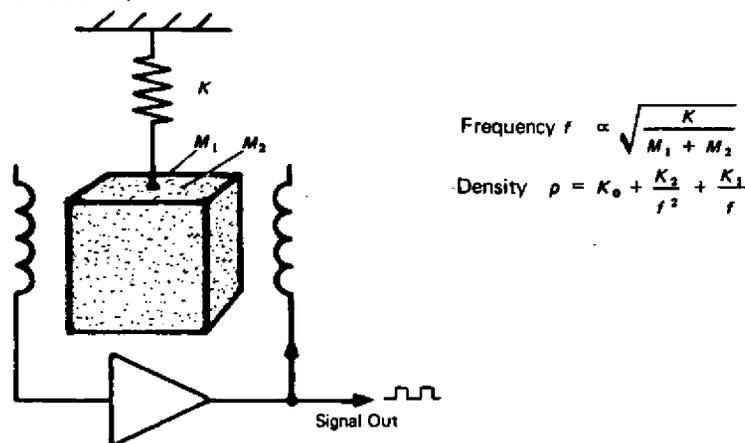


Fig. 1 Simple schematic of resonant element density transducer

A great deal of effort has gone into trying to understand and quantify the properties of a fluid, and their affect on the resonating frequency. Some of this work is still classified as confidential since its application is of significant benefit in the design optimisation of these density transducers. However, the conclusions reached after similar research by other interested parties confirm, without exception, the presence of a velocity of sound effect which causes the transducer to overread for a reduction in the fluid velocity of sound.

In general the simple approaches to solving the problem are easy to understand but will not yield accurate results. The more exact solutions are very complex but have been shown to be accurate.

One very simple model is illustrated in Figure 2. Here the resonant element is shown as having a mass of  $M_1$  and a stiffness of  $K_1$ . The fluid within the container is shown as having two components, one of mass  $M_2$  is coupled rigidly to the container while the other of mass  $M_3$  is coupled to the container by a stiffness  $K_2$ . This stiffness term  $K_2$  is a very simple representation of the properties of the fluid such as compressibility and velocity of sound.

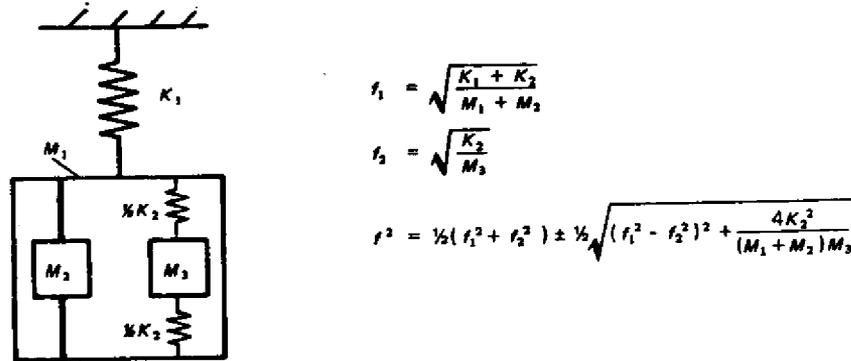


Fig. 2 Simple representation of fluid effects

This is a classical description of a resonant system with two degrees of freedom for which the solutions are illustrated in Figure 3.

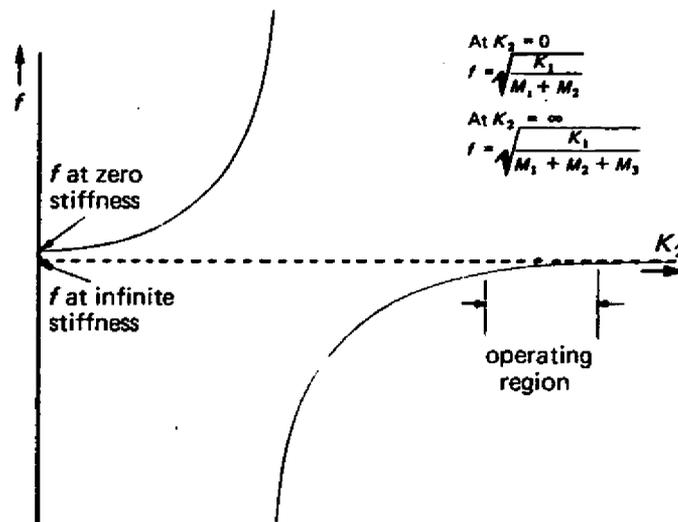


Fig. 3 Solution of system illustrated in Figure 2



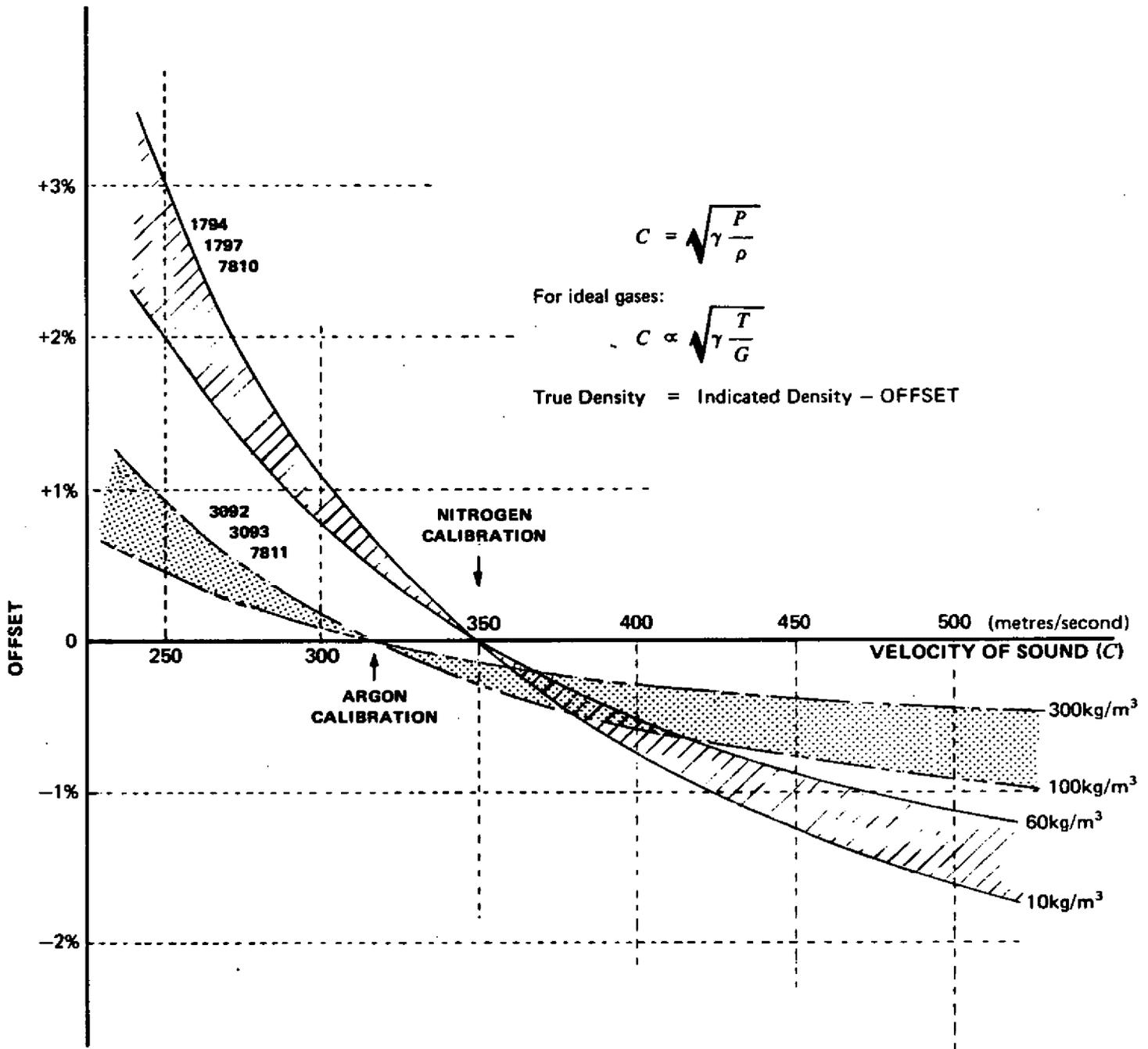


Fig. 5. Density offset due to velocity of sound effect

**Note:** For simplicity, this figure does not show the effects of changing Velocity of Sound of the calibration gas itself.

#### 4 MEASUREMENT RESULTS

Early measurements of the velocity of sound effect were based on noting the difference between calibrations performed on different gases. These calibrations used pressure and temperature measurements on certified pure gases with the density calculated from the standard gas tables. The results were good with respect to the limitations of the gas quality and the gas tables. For applications where the calibration offset is not large, such as for medium pressure natural gas measurement, these tests, supported by theory, were considered adequate.

For applications where large calibration offsets are expected, such as for ethylene in its critical region, where the velocity of sound changes dramatically and is of a low value, more comprehensive tests were considered necessary. Since accurate Pressure/Temperature/Density tables for ethylene in its critical region are not available, it was necessary to determine the density from highly accurate measurements of gas volume and mass.

Two experiments were set up, one sponsored by Shell in the Netherlands and carried out by the Dutch Weights and Measures Authority at Dordrecht<sup>7</sup>. The other was sponsored by Solartron, BP and ICI and carried out at the National Physical Laboratory (NPL)<sup>8</sup> in London. Both experiments used a pressure vessel which contained the ethylene gas and embodied the density transducer. Although the two vessels varied in their construction (Figures 6 and 7) the measurement principles applied were identical and the results were in very close agreement.

Volume was determined by weighing the vessels empty and then full of water. The pressure and temperature coefficients were also established. The vessels were then charged with gas and sealed. Gas density was determined by calculating the mass of the gas from the weighing of the vessels full and the vessels empty and then dividing the difference by the already established volume. Uncertainties of about 0.02% are claimed for this method. With the vessels charged in this manner, the temperature and hence the pressure conditions may be varied without changing the density, except as defined by the small pressure and temperature coefficients of the vessels. In this manner the performance of the density transducer can be determined at several fixed and accurately defined densities and over the required pressure and temperature ranges.

By performing these tests using argon and then ethylene, the differences can be quantified, and then examined with respect to the expected velocity of sound offsets. The benefit of this work is that some of the major systematic uncertainties of this measurement technique such as the volume determination cancel out. Tables 1 and 2 list the results from the NPL studies and they clearly show very good agreement, and that an instrument calibrated on one gas will accurately measure the density of another gas after correction for the velocity of sound effect.

In recent years much experimental work has been carried out on the effects of natural gas mixtures. Probably the most significant published work is that carried out by Ruhrgas<sup>9</sup> and Dantest<sup>5</sup>. Both of these reports clearly confirm the velocity of sound explanation for the calibration offsets.

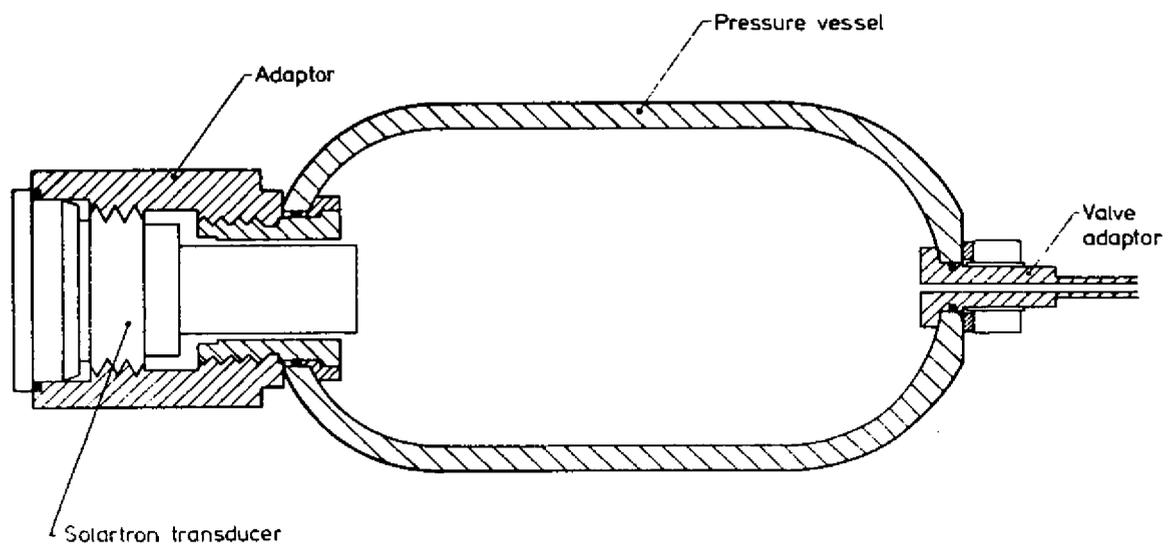


Fig. 6 Outline diagram of NPL pressure vessel (1200cc capacity)



Fig. 7 The pressure vessel used by Shell (5000cc capacity)

Table 1 NPL argon results

Temperature (T°C)	Periodic Time (τ μsec)	True Density (kg/m³)	Calc. Density (kg/m³)	ρ <sub>Cal</sub> - ρ <sub>True</sub> (kg/m³)		Vel. of Sound (m/s)	VOS Cor. Density (kg/m³)	ρ <sub>Vos/cor</sub> - ρ <sub>True</sub> (kg/m³)		Offset ρ <sub>Press</sub> (%)
				(kg/m³)	(%)			(kg/m³)	(%)	
15	1028.422	399.70	399.70	0.00		375	399.63	-0.07	-0.02	-0.04
20	1028.266	399.54	399.55	0.01		378	399.51	-0.03	-0.01	-0.02
25	1028.086	399.38	399.38	0.00		382	399.38	0.00	0.00	0.00
30	1027.925	399.22	399.22	0.00		386	399.26	0.04	0.01	0.02
45	1027.534	398.88	398.85	-0.03		397	398.98	0.10	0.03	0.07
15	895.391	279.98	279.99	0.01		348	279.90	-0.08	-0.03	-0.04
20	895.289	279.89	279.90	0.01		352	279.86	-0.03	-0.01	-0.02
25	895.173	279.80	279.80	0.00		356	279.80	0.00	0.00	0.00
30	895.041	279.70	279.69	-0.01		360	279.73	0.03	0.01	0.04
45	894.722	279.47	279.43	-0.04		371	279.58	0.11	-0.04	0.07
15	737.884	160.13	160.13	0.00		330	160.05	-0.08	-0.05	-0.05
20	737.794	160.07	160.07	0.00		333	160.03	-0.04	-0.03	-0.03
25	737.692	160.00	160.00	0.00		337	160.00	0.00	0.00	0.00
30	737.588	159.94	159.92	-0.02		340	159.95	0.01	0.01	0.01
45	737.340	159.81	159.75	-0.06		350	159.88	0.07	0.04	0.05
15	533.925	40.15	40.17	0.02		318	40.14	-0.01	-0.02	-0.02
20	533.875	40.13	40.15	0.02		321	40.13	0.00	0.00	0.00
25	533.800	40.11	40.11	0.00		324	40.11	0.00	0.00	0.00
30	533.730	40.09	40.08	-0.01		327	40.10	0.01	0.02	0.02
45	533.513	40.05	39.97	-0.08		337	40.04	0.01	-0.02	-0.02

Table 2 NPL ethylene results compared against argon calibration

Temperature (T°C)	Periodic Time (μsec)	True Density (kg/m³)	Calc. Density (kg/m³)	ρ <sub>Cal</sub> - ρ <sub>True</sub>		Vel. of Sound (m/s)	VOS Cor. Density (kg/m³)	ρ <sub>Vos/cor</sub> - ρ <sub>True</sub>		Offset ρ <sub>Eth</sub> (%)
				(kg/m³)	(%)			(kg/m³)	(%)	
15	1027.146	399.07	398.47	-0.60	-0.15	644	399.62	0.55	0.14	0.04
20	1026.863	398.88	398.20	-0.68	-0.17	656	399.37	0.49	0.12	-0.01
25	1026.624	398.70	397.97	-0.73	-0.18	663	399.16	0.45	0.11	0.04
30	1026.385	398.51	397.74	-0.73	-0.18	674	398.94	0.43	0.11	0.05
45	1026.724	398.91	398.07	-0.84	-0.21	705	398.32	0.41	0.10	0.09
15	897.063	280.10	281.39	1.29	0.46	311	280.81	0.71	0.25	0.12
20	896.534	280.01	280.94	0.93	0.33	320	279.56	0.45	0.16	0.02
25	896.140	279.90	280.61	0.71	0.25	329	280.29	0.39	0.14	0.00
30	895.826	279.79	280.35	0.56	0.20	338	280.14	0.35	0.13	-0.03
45	895.197	279.55	279.82	0.27	0.10	369	279.95	0.40	0.14	-0.01
15	740.590	159.94	161.99	2.05	1.27	236	160.17	0.23	0.14	0.12
20	740.058	159.86	161.62	1.76	1.09	241	159.95	0.09	0.05	0.02
25	739.657	159.80	161.34	1.54	0.96	247	159.84	0.04	0.02	-0.03
30	739.292	159.72	161.09	1.37	0.85	253	159.74	0.02	0.01	-0.05
45	738.452	159.58	160.52	0.94	0.58	270	159.55	-0.03	-0.02	-0.10
15	533.979	40.06	40.20	0.14	0.35	288	39.96	-0.10	-0.25	-0.22
20	533.867	40.03	40.14	0.11	0.28	291	39.93	-0.10	-0.25	-0.24
25	533.773	40.00	40.14	0.14	0.35	294	39.95	-0.05	-0.13	-0.12
30	533.668	39.97	40.05	0.08	0.20	297	39.88	-0.09	-0.23	-0.24
45	533.433	39.92	39.93	0.01	0.03	306	39.82	-0.10	-0.25	-0.22

\* Taken from NPL results: Reference NPC 3/05 Page 5

Notes:

- $\rho_{Cal} = K_0 + K_1\tau + K_2\tau^2$  where  $K_0 = -85.462$ ,  $K_1 = -0.020001$ ,  $K_2 = 4.78166E-4$   
 K factors are optimised for argon at 25°C
- $$\rho_{VOS} = \rho_{Cal} \left( \frac{1 + \left(\frac{K}{\tau c_0}\right)^2}{1 + \left(\frac{K}{\tau c}\right)^2} \right)$$

$$K = 2.62 E4$$

$$c = \text{Actual VOS}$$

$$c_0 = \text{Argon VOS at 25°C}$$
- $\rho_{Press} =$  Pressure corrected density offset  
 $= 0.0017\%/bar$  from argon pressure at 25°C  
 $= (\rho_{Vos} - \rho_{True})\% + 0.0017(P - P_0)$  where  $P =$  Vessel pressure  
 $P_0 =$  Vessel pressure with argon at 25°C
- $\rho_{Eth} =$  Pressure and temperature corrected density offset  
 $= (\rho_{Press})_{ethylene} - (\rho_{Press})_{argon}$

Table 3 shows the measurement deviation on a density transducer when calibrated with methane and nitrogen, and how this deviation is reduced after the inclusion of the velocity of sound correction.

**Table 3** *The Ruhrgas/VDI results for methane and nitrogen*

Nominal Density ( kg/m <sup>3</sup> )	Deviation Methane/Nitrogen ( % )	Deviation after VOS Correction ( % )
22	0.93	+0.01
	0.97	-0.03
35	0.83	+0.04
	0.86	+0.07
47	0.71	+0.01
	0.73	+0.03
59	0.59	-0.03
	0.60	-0.04

Figure 8 is a summary of the measurement results on a number of natural gases from the Ruhrgas tests based on a methane calibrated sensor. Figure 9 demonstrates the consequence of applying velocity of sound correction.

It is important to note that there have been reports of calibration offsets which cannot be explained by the velocity of sound effect. In most cases an explanation has later been found, either with respect to gas compositions uncertainties, measurement errors, calculation errors or sensor contamination. This clearly highlights that considerable care and skill is necessary in order to carry out these tests and, whilst these tests may be important in order to confirm an effect, they should not form the basis of routine calibrations. In general, for best performance and confidence, it is recommended that calibrations are performed on a pure gas such as nitrogen with suitable corrections for changes in gas composition and operating conditions.

## 5 METHODS OF CORRECTION

As clearly shown by theory and by measurement results, a systematic measurement offset will be experienced if the velocity of sound of the measured gas is different from that of the calibration gas. If this measurement offset is not acceptable then one of the following two procedures must be applied:

- (a) To calibrate the density transducer with the user gas at the mean operating conditions. Good calibrations using this procedure are very difficult and are limited in their application. It is often the case that better calibrations would result using pure gas (e.g. nitrogen) calibrations and a fixed estimated offset.

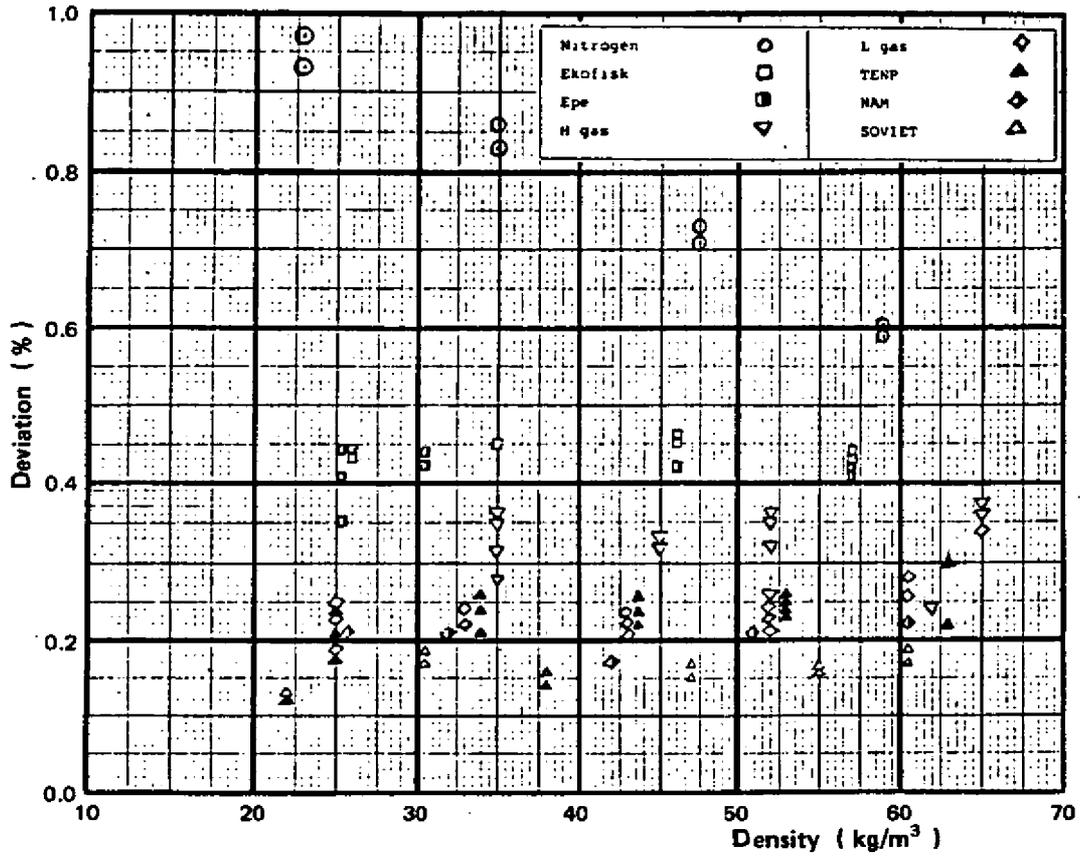


Fig. 8 Ruhrgas natural gas results - uncorrected

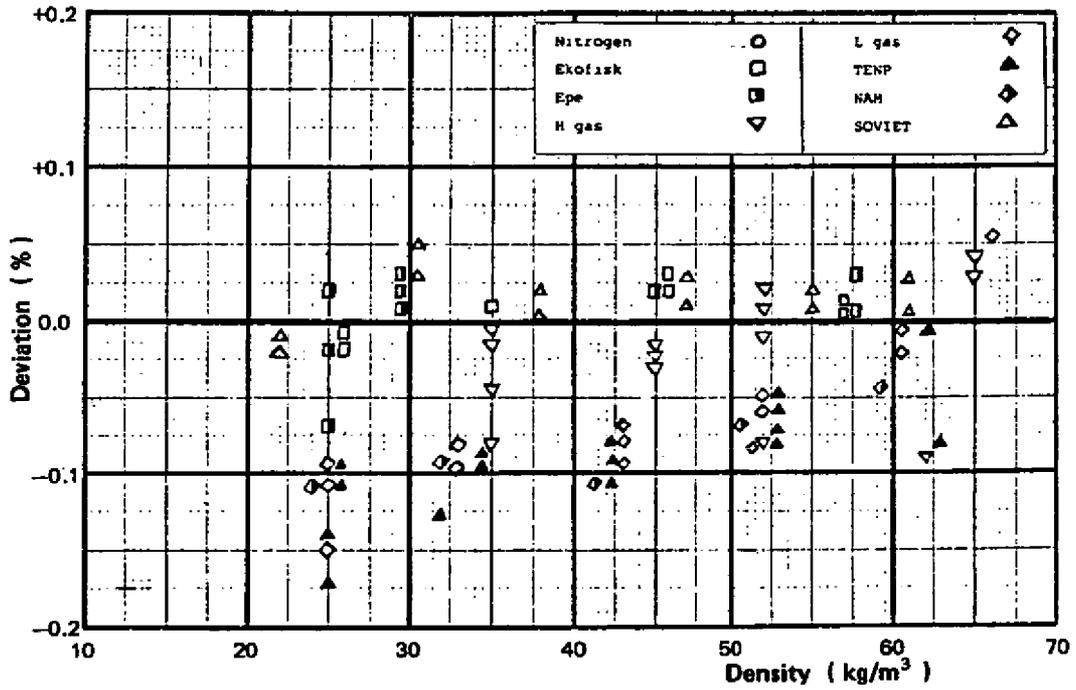


Fig. 9 Ruhrgas natural gas results - VOS corrected



- (c) Temperature/Density Composition Method
- (d) Temperature/Specific Gravity Composition Method

These methods are described in Appendix A of this paper and may be expanded or simplified as necessary in order to obtain the desired accuracy. Similarly, they may result in the application of fixed calibration offsets or they may be applied continuously from measurements of pressure, temperature and composition. Experience has shown that the Pressure/Density method, whilst being one of the simplest to apply, is also one of the most effective.

For further simplicity it is sometimes the practice to combine the sensor effect with the gas effect to form one correction equation. This is often referred to as the User Gas Equation and its derivation is described in Appendix B. Its use is somewhat limited to specific gas types over closely defined operating ranges and it is chiefly intended to highlight that for the best accuracy a gas composition related correction factor should be applied.

It follows that there is an almost unlimited variation of correction methods, depending primarily on how the VOS of the User Gas can be described from the available measurements. This wide choice does sometimes cause difficulties, but it should be stressed that the velocity of sound effect only generates a small offset and in consequence there is little justification in using complex procedures to accurately define these offsets when simple procedures are quite adequate

It is often best to consider the velocity of sound correction as part of the total calibration requirements for the density transducer and in this respect the following general recommendations should be considered:

1. Calibration:

- (a) For low density and natural gases use nitrogen at 20°C (Figure 10).
- (b) For high densities use argon at 20°C.
- (c) Only use defined gases at defined conditions if good density standards are available, if gas quality can be assured and if calibration conditions cover the full range of operating conditions.

2 Velocity of Sound Correction

- (a) If measuring a defined gas at a relatively constant temperature, use fluid offsets as provided by the 'User Gas Certificate' (Figure 11) or similar.
- (b) If measuring different gases and/or the measurement conditions are changing significantly, use continuous correction for VOS, preferably by the Pressure/Density method or alternatively by the Temperature/Specific Gravity method but as defined by available measurements.

3 Temperature Corrections (Excluding VOS Effect)

The vibrating cylinder density transducers normally have a very small temperature coefficient (typically  $0.001 \text{ kg/m}^3$ ) and correction for this is only necessary if operating at low densities and at temperatures which are far removed from the calibration temperature (20°C). Where necessary, corrections can be applied using the equation on the prime calibration certificate to either a mean fixed temperature condition or by using temperature measurements.

CALIBRATION CERTIFICATE

Farnborough Division

Solartron Transducers

7810N GAS DENSITY METER

Serial No: 100108  
 Cylinder No: 004887  
 Amplifier No: 001393  
 Cal Date: 20JAN86

Unit Pressure Tested to 375 BARS

DENSITY CALIBRATION FOR NITROGEN AT 20 DEG.C  
 (Based on Pressure-Temperature-Density Data in IUPAC Tables)

DENSITY [KG/M3]	PERIODIC TIME [uS]
0	212.277
1	213.201
2	214.102
5	216.78
10	221.161
15	225.446
20	229.639
30	237.78
40	245.519
50	253.193
60	260.522

$$DENSITY = K0 + K1.T + K2.T**2$$

K0 = -1.008782E+02  
 K1 = -1.515094E-01  
 K2 = 2.951956E-03

TEMPERATURE COEFFICIENT DATA

$$Dt = DI(1 + K18(t - 20)) + K19(t - 20) \quad K18 = -2.33E-05$$

$$K19 = -1.88E-04$$

USER GAS OFFSET DATA

Nitrogen/Methane Gas Mixture Over Density Range 10 to 60 KG/M3

$$DA = Dt \left( 1 + \frac{K3}{Dt+K4} \left( \frac{G}{t+273} - 0.00236 \right) \right) \quad K3 = 831$$

$$K4 = 59$$

where

T = Periodic Time (uS)  
 DA = Actual Density (KG/M3)  
 t = Temperature (DEG.C)  
 DI = Indicated Density (KG/M3)  
 Dt = Temp.Corrected Density (KG/M3)

$$G = \frac{\text{Gas Specific Gravity}}{\text{Ratio of Specific Heats}}$$

TESTER

QC

Ref No:-GD01/L

DATE-20JAN86

Fig. 10 Nitrogen calibration certificate

USER GAS CALIBRATION CERTIFICATE

Farnborough Division

Solartron Transducers

7810N GAS DENSITY METER

Serial No: 100100  
Cylinder No: 004887  
Cal Date: 20JAN86

NITROGEN CALIBRATION DATA AT 20 DEG.C

K0 = -1.008782E+02      K18 = -2.33E-05  
K1 = -1.515094E-01      K19 = -1.33E-04  
K2 = 2.951956E-03

USER GAS DATA AT 20 DEG.C

COMPOSITION BY % VOLUME :-      (SINGLE PHASE FLUID IS ASSUMED)

HYDROGEN	0.0000
HELIUM	0.0000
NITROGEN	0.4183
CARBON MONOXIDE	0.0000
CARBON DIOXIDE	1.8985
OXYGEN	0.0000
ARGON	0.0000
METHANE	84.6505
ETHANE	8.7505
ETHYLENE	0.0000
PROPANE	3.1074
PROPYLENE	0.0000
BUTANE	0.9684
PENTANE	0.1677
HEXANE +	0.0387
TOTAL	100.0000

VELOCITY OF SOUND AND ERROR DATA WITH USER COEFFICIENTS

DENSITY [KG/M3]	PERIODIC TIME [μS]	V.O.S [M/S]	MAX. ERROR [%density]
5	216.746	408	0.235
10	221.102	403	0.208
20	229.545	395	0.174
30	237.665	390	0.181
40	245.495	389	0.160
60	260.400	392	0.186

TESTER	USER GAS COEFFICIENTS	QC
	K0 = -1.071680E+02	
	K1 = -1.004218E-01 K2 = 2.850954E-03	

Ref No: GD02/F

DATE-20JAN86

Fig. 11 User gas calibration certificate

## CONCLUSIONS

All vibrating density sensors are influenced by changes in the velocity of sound of the fluid being measured. By understanding this effect, sensors can be designed which exhibit minimum effect but without the loss of other features. By quantifying this effect with respect to a particular measurement task, the most suitable correction method can be defined and agreed by all parties.

This characteristic of these gas density sensors should be considered alongside details of the calibration and the installation which are often of much greater significance. When correctly applied, measurement performance to 0.1% of reading can be demonstrated in the laboratory environment and to 0.2% to 0.3% of reading in the pipeline environment. This compares very favourably with alternative methods of continuous gas measurement.

## REFERENCES

- 1 J. Oostervink, W.R. Franken - *Solartron Gas Density Transducer Type NT1792* - Shell Research NV - June 1969
- 2 T. Archbold - *Evaluation Report: The Solartron Gas Density Meter Type NT1794* - Ref. ERS R 103 CICC 106 Gas Council Engineering Research Station - January 1972
- 3 N.G. Hedges, E. Komolibus - *Solartron Type NT1794 Gas Density Meter* - Sira Institute Evaluation Report E286 - November 1972
- 4 *Sound Velocity Effect on Vibrating Cylinder Density Transducers* - Solartron Technical Data Sheet TDS-105 - January 1974
- 5 Lars Rosenkilde, Marianne Tambo - *Calibration and Examination of Gas Density Meters* - Dantest, National Institute for Testing and Verification, Project No. TR 133/360.81. 368 - January 1983
- 6 H.M. Hinze - *Influence of the Type of Gas on the Indicator of Operating Density Transducers* - Ruhrgas - August 1982
- 7 H.J.M. van Rooij - *On the Inaccuracy of a Massflow Meter for Supercritical Ethylene* - Thesis for Shell/Technische Hogeschool Delft - November 1977
- 8 I.J. Lawrenson, G.B. Lewis - *A Method for the Calibration of Ethylene Gas Density Meters* - National Physical Laboratory, Report Chem. 69 - May 1977
- 9 H.M. Hinze, M.J. Jaeschke (Dorsten) - *Measurement of the Density of Natural Gases by the Weighing Method and Operating Density Pick-ups* - VDI Verlag, Series 6 Report No. 162
- 10 Petroleum Measurement Manual Part VII, Density - Sect. 2 - *Continuous Density Measurement* - Institute of Petroleum, London - November 1983
- 11 J.L. Finjord - *Principles of the Velocity of Sound Correction in Vibrating Element Natural Gas Density Transducers* - Rogaland Research Institute, Reference PRC T-1/85 - May 1985
- 12 J.L. Finjord, Haakon Norvik - *Specification and Measurement of Physical and Chemical Properties of Natural Gas* - Proceedings of the Congress of Gas Quality, Reference Page 181 - 1986
- 13 U.K. Patent GB2009931B - *Automatic Velocity of Sound Correction for Gas Density Meters*
- 14 J.W. Stansfeld - *V.O.S. Correction Methods for Natural Gas Measurement when Using Solartron Density Transducers of 0 to 60 kg/m<sup>3</sup> Range* - Solartron Engineering Report - DENS - 01121 November 1985

## APPENDIX A

### V.O.S. DETERMINATION OF GAS MIXTURES

#### Prime Influence

From the thermodynamics of a gas, the speed of sound is given by the equation:

$$c = \sqrt{\gamma \left( \frac{\delta P}{\delta \rho} \right)_T} \dots \dots \dots A1$$

where  $c$  = speed of sound ( m/s )

$\gamma = \frac{C_p}{C_v}$  Ratio of specific heats

$\left( \frac{\delta P}{\delta \rho} \right)_T$  = ratio of pressure change (in Pa) to density change (in kg/m<sup>3</sup>)  
at constant temperature

At low pressures this equation may be simplified to:

$$c = \sqrt{\gamma \frac{P}{\rho}} \dots \dots \dots A2$$

Also since

$$\frac{P}{\rho} = \frac{R T_A Z}{M} \dots \dots \dots A3$$

where  $R$  = universal gas constant

$T_A$  = absolute temperature

$M$  = molecular weight

$Z$  = compressibility factor

it follows

$$c = \sqrt{\gamma \left( \frac{R T_A Z}{M} \right)} \dots \dots \dots A4$$

#### Pressure/Density Method

The Pressure/Density method of calculating the velocity of sound of a gas is based on equation A2. However, to include for changes in the specific heat ratio and the pressure density ratio at high pressure, the equation is expanded as follows:

$$c = \sqrt{\gamma_0 \frac{P}{\rho} + K_1 \rho^2 + K_2 \rho^3} \dots \dots \dots A5$$

where  $\gamma_0$  is the zero pressure specific heat ratio.

By correct selection of  $\gamma_0$ ,  $K_1$  and  $K_2$ , this equation is more than adequate for V.O.S. determination. Furthermore, these values are not very sensitive to changes in gas composition. It is this equation which is used in Solartron Flow Computers types 7900 and 7910.

### Temperature/Density Composition Method (2nd Gas Certificate)

A procedure has been developed by Solartron which allows the velocity of sound to be calculated for a defined gas mixture, at a defined operating temperature and over a specific density range.

The velocity of sound for each component gas is determined by the following equation:

$$c = K_{00} + K_{01}x + K_{02}x^2 + (K_{10} + K_{11}x + K_{12}x^2)y + (K_{20} + K_{21}x + K_{22}x^2)y^2 + (K_{30} + K_{31}x + K_{32}x^2)y^3 \dots \dots \dots A6$$

where  $x$  = temperature ( °C )  
 $y$  = component density ( kg/m<sup>3</sup> )

The velocity of sound of a gas mixture is determined by the combination of the component velocity of sounds.

This procedure is used by Solartron to generate a Users Gas Calibration Certificate (2nd Gas Certificate)

### Temperature/Specific Gravity Method

This procedure is described in V.D.I. Report 162 and for natural gases uses the following equation:

$$c = B_0 + B_1d^{-0.5} + B_2T^2\rho^{0.5} + B_3d^{0.5}T\rho^2 + B_4dT\rho + B_5dT^{1.5}\rho + B_6d^{1.5}T^{0.5}\rho^{1.5} + B_7d^{1.5}T^{1.5} + B_8d^{1.5}T^{1.5}\rho^{0.5} + B_9d^2T^{1.5} \dots \dots \dots A7$$

where  $c$  = velocity of sound ( m/s )  
 $\rho$  = density ( kg/m<sup>3</sup> )  
 $T$  = temperature ( °K )  
 $d$  = density ratio ( Specific Gravity )

The coefficients  $B_0$  to  $B_9$  are selected from one of three tables as a function of the calorific value of the gas.

This procedure should only be considered if a pressure measurement is not available and in general is considered unnecessarily complex when compared with the Solartron User Gas Equation.

### Temperature/Specific Gravity Method (Reverse PTZ Procedure)

This procedure uses the Pressure/Density method (page A1). Equation A5, in combination with a method of calculating line pressure from temperature, density, specific gravity and the Z factor. One of several available procedures can be applied for the determination of Z such as NX19.

This procedure, like the V.D.I. procedure, should only be considered if a pressure measurement is not available and in general is considered unnecessarily complex when compared with the Solartron User Gas Equation.

## APPENDIX B

### SOLARTRON USER GAS EQUATION

The Solartron User Gas Equation is a simplified combination of the density sensor V.O.S. effect (Equation 2 on page 4 of this paper) and the gas V.O.S. definition (Equation A4). The combination method is as follows:

From Equation 2

$$\rho_A = \rho_1 \left( \frac{1 + \left(\frac{K}{T c_c}\right)^2}{1 + \left(\frac{K}{T c_g}\right)^2} \right)$$

If  $\left(\frac{K}{T c_c}\right)^2$  and  $\left(\frac{K}{T c_g}\right)^2$  are small with respect to 1.0 then

$$\rho_A = \rho_1 \left( 1 + \left(\frac{K}{T}\right)^2 \left( \frac{1}{c_c^2} + \frac{1}{c_g^2} \right) \right) \dots \dots \dots B1$$

The sensor Density/Time period relationship is approximately

$$\rho_1 = K_0 + K_2 T^2 \dots \dots \dots B2$$

also from Equation A4

$$c^2 = \gamma \frac{RT_A Z}{M} \dots \dots \dots B3$$

substituting into Equation B1

$$\rho_A = \rho_1 \left( 1 + \frac{K^2 K_2}{(\rho_1 - K_0) R} \left( \left( \frac{M}{\gamma T_A Z} \right)_c - \left( \frac{M}{\gamma T_A Z} \right)_g \right) \right) \dots \dots B4$$

This can be simplified further

$$\rho_A = \rho_1 \left( 1 + \frac{K_3}{(\rho_1 + K_4)} \left( K_5 - \frac{G}{\gamma_0 (T + 273)} \right) \right) \dots \dots \dots B5$$

Where  $K_3$  and  $K_4$  are coefficients which can be selected to give no error between a calibration gas and a user gas at minimum and maximum densities.  $K_5$  is the calibration gas factor.

- $G$  is the specific gravity of the user gas
- $\gamma_0$  is the zero pressure specific heat ratio
- $T$  is the temperature in °C.

This equation is presented on the Primary Calibration Certificate for Solartron Density sensors where the factors  $K_3$  and  $K_4$  have been selected to give no error when the sensor is used on the calibration gas and on methane.

Alternative values of  $K_3$  and  $K_4$  can be determined to give no error for any specified gas at minimum and maximum operating densities.

For most applications, this correction equation can be further simplified with only a small loss in accuracy, by assuming a constant value of  $\gamma_0$ . Hence:

$$\rho_A = \rho_1 \left( 1 + \frac{K_6}{\rho_1 + K_4} \left( K_7 - \frac{G}{(T + 273)} \right) \right) \dots \dots \dots \text{B6}$$

Where no pressure measurement is available, this method is normally recommended due to its simplicity and effectiveness.