THE RELATIONSHIP BETWEEN THE NUMBER OF PASSES AND THE ACCURACY OF A COMPACT PROVER

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SUMMARY

Compact provers are becoming increasingly attractive for use within the petroleum industry. At present their use is based on practices which were devised for conventional provers and which are not necessarily the best way of utilising the newer devices. In particular the ability of the compact prover to carry out large numbers of passes in a short period of time suggests that a statistically-based approach should be adopted. This paper discusses possible alternative methods and considers the statistics involved.

1. INTRODUCTION

It has been accepted in the petroleum industry for many years that a meter is proved in-situ by carrying out several runs against a proving device as the reference standard; from each of these a meter factor (K-factor) is determined and the process continues until a specified number (usually 3 or 5) of successive values fall within a certain percentage range of each other. The mean of these values is then taken as the meter factor.

This approach was originally developed for use with conventional "sphere-type" provers for which each proving run involves the metering of a large amount of fluid; with the advent of the compact prover the same method has continued with the modification that each run consists of several passes, the average meter factor being taken as the value for the run. The use of multiple passes is based on the fact that most meters exhibit a certain amount of intra-rotational non-linearity (IRNL); a meter factor based on metering a small amount of fluid is therefore suspect. An idea of the magnitude of the IRNL is given by the number of passes required per proving run: for a turbine meter this is usually between 1 and 5, while a positive displacement meter will generally require between 5 and 20 passes per proving run to satisfy the same criterion.

The main cause of IRNL in a turbine meter is the mechanical tolerance between the points triggering the pulses. This effect is greater when the rotor is not shrouded. In that case the blade tips are counted and the IRNL arises from the inaccuracies in the angular positions of the blades; for a shrouded rotor the pulses are usually generated by slots or buttons which are positioned to a much tighter tolerance with a consequent reduction in the IRNL. In either case the effect is cyclic within one rotation of the rotor, i.e. it is genuinely intra-rotational.

With a positive displacement meter the cause of the IRNL is generally the gear train between the metering element and the pulse transmitter. The main problem comes from non-concentricity between gears; with multiple gear trains and an adjusting mechanism involved, the cyclic effect can be over a long period, i.e. the non-linearity may not be truly intra-rotational.

Although the use of a set number of passes for each proving run has enabled the compact prover to be brought within the framework of the conventional proving technique it is not clear that this is the best way in which the compact prover can be utilised. Clearly the presence of the IRNL means that each pass of a compact prover cannot be used in the same way as a single run of a conventional prover: the amount of fluid metered is too small for this approach to succeed. However, each pass provides information which can be used if a more statistical approach is adopted; at present much of this information is discarded.

In the next section we shall consider three different proving techniques: the first is the current approach, the second a modification of it designed to make more use of the statistical information available and the third a more purely statistical approach which dispenses with the idea of a proving run consisting of a number of passes. (The statistical basis of Methods 2 and 3 is outlined in the Appendix.) Each method will be applied to the data given in Table 1, relating to a 6" turbine meter at a flowrate of 1690 GPM; the data are also shown graphically in Figure 1.

2. PROVING TECHNIQUES

2.1 Multiple pass runs

This is the system outlined at the start of the Introduction. Runs are carried out (each consisting of a number of passes) and the results compared until a number of consecutive runs produce sufficiently close figures; taking a range of 0.04% (i.e. ±0.02%) as acceptable this will occur when

$$\frac{\text{high} - \text{low}}{\text{mean}} \times 100 \leqslant 0.04 \qquad (1)$$

over the chosen number of runs (usually 3 or 5). The mean calculated from these runs is then taken as the meter factor. There is flexibility in this method over the selection of the number of consecutive runs to be considered and also over the number of passes per run. It is the latter factor which we shall consider.

2.1.1 Method 1: fixed number of passes

One way of establishing the appropriate number of passes per run would be to estimate all the uncertainties involved in the proving operation, including that due to the IRNL. Adding these in quadrature (root-sum-square method) gives the total uncertainty in the measured meter factor on each pass. The number of passes can then be selected to reduce this uncertainty to an acceptable level. This number of passes will then be sufficient to prove any meter for which the uncertainties fall within the assumed values.

Adopting this approach is possible but its value is totally dependent on the validity of the assumptions which have to be made. The IRNL is particularly difficult to estimate because although it may be regarded as part of the prover's uncertainty (owing to the small volume) its size cannot be estimated from test runs on other meters. In general it is unlikely that the use of data involving various meters can be used to provide useful estimates of the uncertainties. Such estimates can really only be used as 'worst case' figures because of the difficulty in assessing how much of the uncertainty comes from other sources, for which only upper bounds are known. Only if the total uncertainty in the results is within limits which are acceptable for the prover itself can these data be used to justify the general use of a particular number of passes per proving run.

The simplest application of this approach is to use a number of passes based on experience with the particular types of meter and prover being used, say 5 passes per run for a turbine meter and 10 for a positive displacement meter. Although this approach is very simplistic it does hold good for the majority of such meters. The disadvantage is that it ignores the inherent statistical nature of the operation because

(i) no account is taken of the number of runs carried out before achieving the ones which are sufficiently close,

and (ii) the resulting K-factor is presented with an indication of the spread of the figures from the final runs but this is not expressed in the more meaningful form of confidence levels.

Applying this method to the data in Table 1, based on 3 successive runs being within a range of 0.04%, yields a K-factor of 22.42912 with a range of ±0.011%.

2.1.2 Method 2: number of passes calculated using standard deviation

The second method of estimating the number of passes required per run is to take the prover to the meter and carry out a 'large' number of exploratory passes (20 should be sufficient). On the basis of the data thus gained it will be possible to calculate the number of passes needed per proving run to satisfy the appropriate criterion.

Once established, this number could be entered on the meter control sheet and used for future provings; it is, however, preferable to carry out the whole procedure on each occasion so that any decline in meter and/or prover performance can be detected.

Two advantages arise from this method. Firstly, it involves no assumptions about the size of the IRNL of the meter being proved (or indeed about any other meter, prover or system uncertainties) for which the appropriate adjustment is automatically made. Secondly, it gives an early indication if excessive uncertainties are present, whatever their source; the statistics of the initial exploratory passes indicate this in a way which will be discussed later in this section.

The method for estimating the number of passes indicated above is based on conventional statistical calculations, the details of which are contained in the Appendix. If e_{acc} is the acceptable error for the meter factor from each proving run, expressed as a proportion of the actual meter factor (percentage error = 100 x e_{acc}), we find from (A9) that the number of passes per run can be taken as the smallest integer M for which

$$m > \left\{ \frac{\hat{s}_n \quad t_{n-1,.975}}{e_{acc} \quad \bar{X}_n} \right\}^2 . \tag{2}$$

Here $\bar{X}_n$ is the mean meter factor from the n initial passes, $\hat{s}_n$ is the corresponding estimate for the standard deviation (given by (A2)), and $t_{n-1,.975}$ is the appropriate percentage point of the Student's-t distribution with n-1 degrees of freedom.

Applying this method to the data of Table 1 we find that $\bar{X}_{20}$=22.42954 and $\hat{s}_{20}$=.0084913; if the acceptable percentage error is ±0.02% we require, from (2), the smallest m greater than

$$\left\{ \frac{0.0084913 \times 2.093}{0.0002 \times 22.42954} \right\}^2 .$$

This results in 16 passes per proving run. Carrying out the proving runs then generates a K-factor of 22.42947 with a range of 0.006%.

[Note: To provide sufficient data values the original 20 have been repeated. In practice the initial set of passes would not form part of the proving runs at all.]

Although this technique will always provide a number of passes to be used, it is possible that the uncertainties present in meter, prover and system may be too great for proving to the required accuracy to be worthwhile. The question arises: how should such a problem be recognised? Excessive total uncertainty corresponds to s, and thus $\hat{s}_n$, being large. If the acceptable limit on s is given as s_{acc} (expressed as a proportion of the correct meter factor), we can be confident that s is not excessive if

$$\left| \frac{\hat{s}_n}{\bar{X}_n} \right| < s_{acc} . \tag{3}$$

It is important to recognise the distinction between e_{acc} and s_{acc}. The first is the accuracy to which the meter factor is required, the second indicates the level of overall repeatability of the meter, prover and system which is regarded as acceptable. In fact s_{acc} is likely to depend on e_{acc}: a greater uncertainty can be tolerated if lower accuracy is required.

Using the data from Table 1 we find that

$$\frac{\hat{s}_{20}}{\bar{X}_{20}} = 0.000378576 \ .$$

Thus the overall repeatability corresponds to a standard deviation of about 0.038% of meter factor. It may well be that this is not small enough to make it worthwhile proving the meter to an accuracy much higher than that chosen above, but the appropriate cut-off level would depend in part on the use which was to be made of the meter.

A precis of the approach is therefore:

1) Obtain data from 20 passes
2) Calculate the standard deviation estimator $\hat{s}_{20}$
3) Calculate the average K-factor $\bar{X}_{20}$
4) Check that the total uncertainty is acceptable
5) Calculate the number of passes required per proving run
6) Carry out the proving runs.

2.2 Individual passes

2.2.1 Method 3: running evaluation of meter factor

In the previous method use was made of statistical information from a number of passes to refine the existing method of meter proving. However, the use of runs, each consisting of several passes, is really not necessary at all and has only arisen so that the compact prover can be used in a similar way to the conventional prover. With a microprocessor to carry out the statistical calculations there is no reason why a series of passes should not be carried out, continuing until the average meter factor can be claimed to be sufficiently accurate at the required confidence level, i.e. until after n passes

$$\frac{\hat{s}_n \; t_{n-1,.975}}{\sqrt{n} \; \bar{X}_n} \leqslant e_{acc} \qquad (4)$$

where e_{acc} is again expressed as a proportion of meter factor. As indicated for Method 2, the value of $\hat{s}_n/\bar{X}_n$ gives the uncertainty of the meter, prover and system; the proving can then be abandoned if $\hat{s}_n$ is too great.

Applying this technique to the sample data, aiming at an acceptable error (e_{acc}) of ±0.02% of meter factor, yields the following results:

n	K-factor	$\bar{X}_n$	$100 \times \frac{\hat{s}_n}{\bar{X}_n}$	$100 \times \frac{\hat{s}_n\, t_{n-1,.975}}{\sqrt{n}\, \bar{X}_n}$
1	22.4381			
2	22.4238	22.43095	0.04508	0.40502
3	22.4334	22.43177	0.03249	0.08072
4	22.4275	22.43070	0.02818	0.04485
5	22.4276	22.43008	0.02518	0.03126
6	22.4137	22.42735	0.03737	0.03922
7	22.4401	22.42917	0.04031	0.03728
8	22.4291	22.42916	0.03732	0.03120
9	22.4309	22.42936	0.03501	0.02691
10	22.4330	22.42972	0.03340	0.02389
11	22.4211	22.42894	0.03374	0.02267
12	22.4259	22.42868	0.03241	0.02059
13	22.4326	22.42898	0.03140	0.01898

At this point the current value of X_{13} would be taken as an acceptable estimate of the K-factor.

3. COMPARISON OF RESULTS

Applying the three methods to the test data in Table 1 for a 6" turbine meter resulted in:

Method 1 : 22.42903 (range 0.011%)
Method 2 : 22.42947 (range 0.006%)
Method 3 : 22.42898 (±0.02% at 95% confidence level).

These agree to within 0.0022%.

Table 2 contains data from a 4" positive displacement meter with a pulse transmitter mounted above the accuracy adjuster in the conventional manner; the data are plotted out in Figure 2. Applying the different methods gives:

Method 1 : 39.88164 (range 0.013%)
Method 2 : 39.88305 (range 0.005%)
Method 3 : 39.88193 (±0.02% at 95% confidence level)

These agree to within 0.0035%. [Note: Only 3 successive runs have been used for Methods 1 and 2, and 5 passes per run for Method 1.]

The question remains: what happens when an attempt is made to prove a 'bad' meter. Table 3 gives data from a 6" positive displacement meter in which the coupling between meter and off-take has been deliberately off-set to give a high IRNL. Using Method 1 we find that the meter can only be proved easily if a very large error range is acceptable. (There are insufficient data for a more specific statement to be made.) Method 2 indicates that 1355 passes per proving run would be necessary for a range of 0.04%: this is a clear indication of trouble. Method 3 fails to produce a result within 20 passes unless an accuracy as large as ±0.2% is given. Methods 2 and 3 also give a standard deviation estimate of 0.35% of meter factor: this would undoubtedly be regarded as excessive, indicating problems with meter, prover or system, and proving would not continue.

4. CONCLUSIONS

Three methods of meter proving using a compact prover have been considered. The first is the current practice of using a standard number of passes (5 or 10) per proving run and continuing until successive runs are sufficiently close to satisfy an acceptance criterion. This procedure will generally work but involves throwing away a lot of valuable statistical information.

Either of the alternative methods discussed takes more account of the statistics of the operation. Method 2 represents a compromise between a strictly statistical approach and the existing practice: the result is still obtained and expressed conventionally, but the number of passes used per run is determined statistically. Additionally, a check is made to ensure that the combined repeatability of the meter, prover and system lies within acceptable bounds before the proving runs are conducted. This gives greater confidence that the final result is meaningful. Method 3 dispenses with the idea of proving runs comprising a number of passes, and simply uses the outcome of each pass directly. By this technique the proving time is reduced to a minimum, the repeatability is checked and the result can be presented in terms of an error bound and its associated confidence level. It is felt that, given the nature of the proving operation, this would be a more satisfactory approach. The use of such a method (or Method 2 if the number of passes per run is calculated afresh before each subsequent proving) requires a microprocessor to carry out the calculations involved but the program would be a simple one, incorporating a look-up table for the Student's-t values, and this is not seen as a problem.

The statistics presented in this paper are based on the assumption that successive passes can be regarded as independent, normally distributed random variables. However, Figures 1 and 2 both suggest an underlying periodicity which is not accounted for in the statistics; this has also been observed in other sets of data. The effect of this is that the theory tends to overestimate the number of passes required per proving run when Method 2 is adopted because the periodicity causes the mean K-factor to settle down more quickly than would otherwise be the case. As far as Method 3 is concerned it is unlikely that the periodicity would have a detrimental effect unless the complete set of passes occupied less than one or two periods of the oscillation.

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APPENDIX

This appendix contains the statistical basis for the discussion in the main text. It is assumed that the meter factor found from a single pass can be regarded as a normally-distributed random variable with mean μ (equal to the correct meter factor) and standard deviation s (corresponding to a variance of s^2); the passes are assumed to be statistically independent.

From n passes, estimators can be computed for μ and s, given by

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad \text{(A1)}$$

and

$$\hat{s}_n^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2 , \qquad \text{(A2)}$$

from which

$$\frac{(\bar{X}_n - \mu) \sqrt{n}}{s} \sim N(0,1) \qquad \text{(A3)}$$

and

$$\frac{(n-1) \hat{s}_n^2}{s^2} \sim \chi_{n-1}^2 . \qquad \text{(A4)}$$

N(0,1) is a normal distribution with mean 0 and standard deviation 1, χ_{n-1}^2 is a chi-squared distribution with n-1 degrees of freedom.

For Method 2, we consider an initial set of n passes followed by proving runs of m passes each, where m is determined from the initial set. Combining (A3) and (A4), with m replacing n in the former, we find

$$\frac{(\bar{X}_n - \mu) \sqrt{m}}{\hat{s}_n} \sim t_{n-1} \qquad \text{(A5)}$$

where t_{n-1} is the Student's-t distribution with n-1 degrees of freedom.

We can therefore be 95% confident that

$$\left| \frac{\bar{X}_m - \mu}{\mu} \right| < \frac{\hat{s}_n \, t_{n-1,.975}}{\sqrt{m}\,|\mu|} . \qquad \text{(A6)}$$

Now μ is not known but we have an estimate for it, given by (A1). The error introduced by substituting this for μ in the above expression is small and we can be effectively 95% confident that

$$\left| \frac{\bar{X}_m - \mu}{\mu} \right| < \frac{\hat{s}_n \, t_{n-1,.975}}{\sqrt{m}\,|\bar{X}|} . \qquad \text{(A7)}$$

Thus m can be chosen on the strength of the initial set of n passes to be the smallest integer m for which

$$\frac{\hat{s}_n \, t_{n-1,.975}}{\sqrt{m}\,|\bar{X}_n|} < e_{acc} \qquad \text{(A8)}$$

$$\Rightarrow \quad m > \left[\frac{\hat{s}_n \, t_{n-1,.975}}{e_{acc}\,\bar{X}_n} \right]^2 , \qquad \text{(A9)}$$

where e_{acc} is the acceptable error in the meter factor calculated from each proving run, expressed as a proportion of the actual meter factor (percentage error = $100 \times e_{acc}$).

For Method 3 the passes are considered individually and the statistical calculation updated after each one. Identical reasoning to that above shows that after n passes we can be effectively 95% confident that

$$\left| \frac{\bar{X}_n - \mu}{\mu} \right| < \frac{\hat{s}_n \, t_{n-1,.975}}{\sqrt{n}\,|\bar{X}_n|} . \qquad \text{(A10)}$$

Once the term on the right of (A10) falls below a predetermined accuracy figure we can be confident that the running average to that point is satisfactory as an estimate of the meter factor.

TABLE 1

6" turbine meter

Pass No	K-factor
1	22.4381
2	22.4238
3	22.4334
4	22.4275
5	22.4276
6	22.4137
7	22.4401
8	22.4291
9	22.4309
10	22.4330
11	22.4211
12	22.4259
13	22.4326
14	22.4247
15	22.4339
16	22.4397
17	22.4091
18	22.4426
19	22.4285
20	22.4355

TABLE 2

4" p.d. meter

Pass No	K-factor
1	39.8617
2	39.8629
3	39.8878
4	39.9080
5	39.8749
6	39.8693
7	39.9042
8	39.8810
9	39.8649
10	39.9020
11	39.8850
12	39.8703
13	39.8954
14	39.8959
15	39.8613
16	39.8875
17	39.8888
18	39.8739
19	39.8748
20	39.9110

TABLE 3

6" p.d. meter (misaligned)

Pass No	K-factor
1	100.1371
2	99.5347
3	99.9230
4	100.0872
5	100.4572
6	100.5970
7	100.4058
8	100.4318
9	99.5435
10	99.7330
11	99.6846
12	100.3115
13	100.0221
14	100.4590
15	100.2056
16	100.7953
17	100.2924
18	99.9690
19	100.0084
20	100.4572

FIGURE 1: 6" TURBINE METER

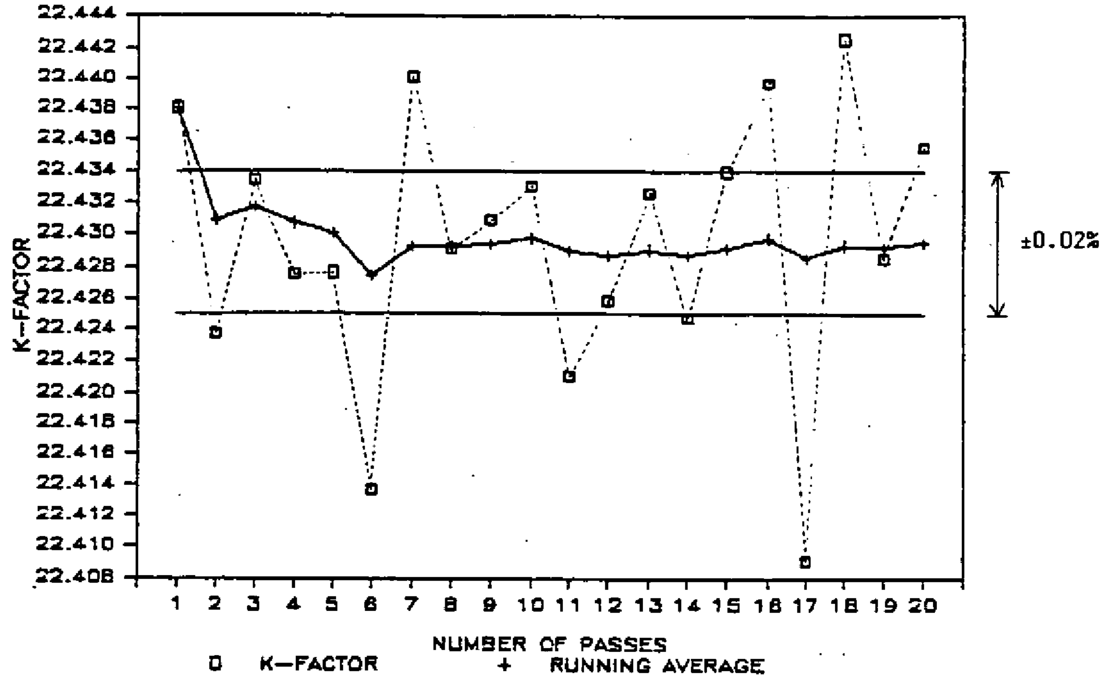


FIGURE 2: 4" PD METER

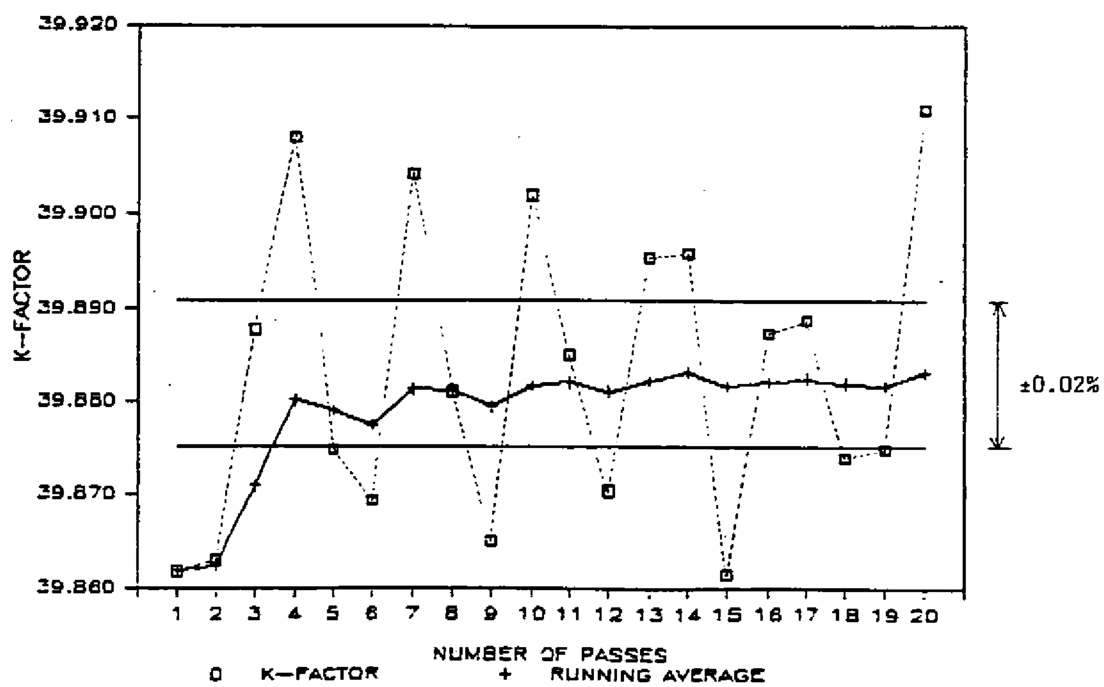
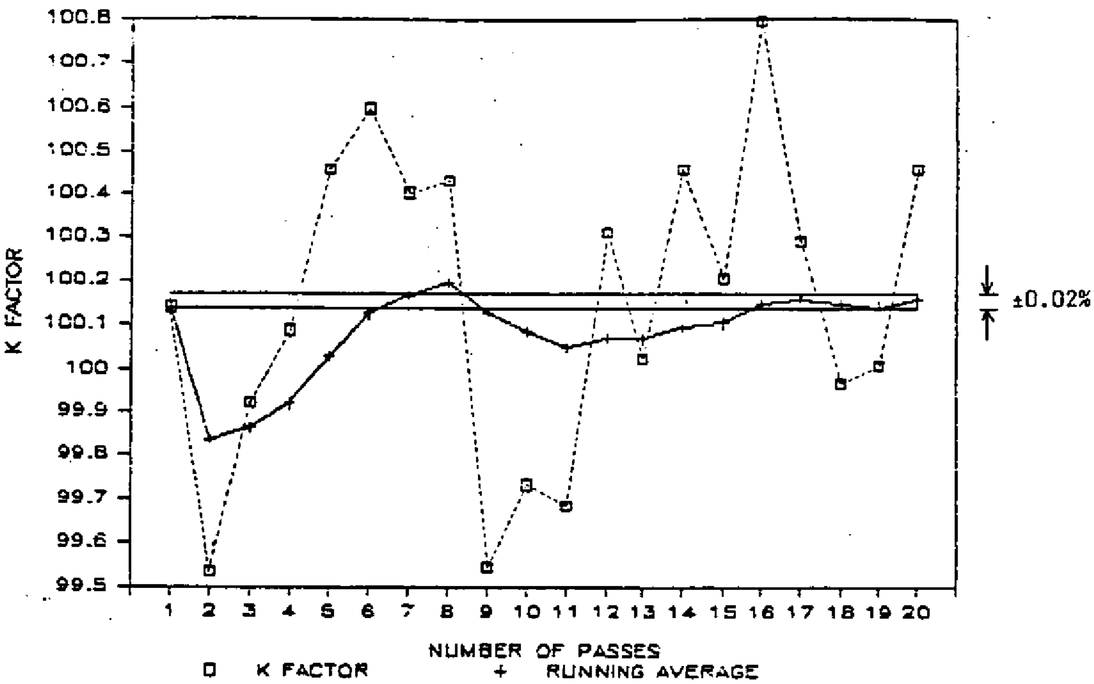


FIGURE 3: 6" PD METER (MISALIGNED)



References

[1] Paper presented at the North Sea Flow Measurement Workshop, a workshop arranged by NFOGM & TUV-NEL

Note that this reference was not part of the original paper, but has been added subsequently to make the paper searchable in Google Scholar.