A NEW TYPE OF FORMULA FOR THE ORIFICE-PLATE DISCHARGE COEFFICIENT

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S U M M A R Y

This paper describes a new concept, and hence form of equation, for fitting orifice plate discharge coefficients in which the physical basis of the different terms of the equation is clearer. As well as a term based on throat Reynolds number, a term based on friction factor is included. The latter makes clear the effect of pipe roughness.

The work described here is the first half of a project; further work is being performed.

N O T A T I O N

a, a_0, a_1, a_2	Coefficients at infinite Reynolds number
b, b_0, b_1	Coefficients of Reynolds number term
c, c_1	Coefficient of friction factor term
C	Discharge coefficient using corner tappings
D	Pipe diameter
k_m	Pipe roughness (as on Moody diagram)
k_s	Sand roughness
k, ℓ, m, n	Exponents
N	Number of data
Re_d	Throat Reynolds number
Re_D	Pipe Reynolds number
V	Variance
β	Diameter ratio
λ	Friction factor
σ	Standard deviation

SUBSCRIPTS

β	For a particular value of β

SUPERSCRIPTS

′	With throat Reynolds number term subtracted
*	Best fit value
#	Approximate valve
-	Mean value

1 INTRODUCTION

Both in the EEC and the USA there are current programmes of work to determine a new orifice plate discharge coefficient equation. The data have been collected and the analysis is being done. It is important to obtain an equation which truly describes the physics. The work presented in this paper demonstrates that the corner tap discharge coefficient depends on friction factor as well as on Reynolds number and that a simple equation can nevertheless be obtained.

2 A NEW TYPE OF FORMULA

Reader-Harris and Keegans[1] found that for a fixed pipe Reynolds number, Re_D, as the pipe roughness changed so the change in discharge coefficient was approximately proportional to $\beta^4\lambda$, where β is the diameter ratio and λ is the pipe friction factor. This led Reader-Harris to suppose that the form of the equation for the discharge coefficient, C, using corner tappings might be

$$C = a(\beta) + c_1\beta^m\lambda, \quad (1)$$

where m is approximately 4. Rapier[2] made experimental measurements in very smooth pipes and as a result of his work also proposed that C was best expressed as a function of friction factor. However, equations of the form of equation (1) do not give good agreement with experiment for small β, since they then give C almost constant.

Further computational work, solving the time-averaged Navier-Stokes equations with the k-ϵ turbulence model, was then undertaken at NEL to determine the effect of pipe roughness on an orifice plate of diameter ratio 0.7. Whereas in the initial work[1] the grids used had about 40 x 25 points, the new grids had about 80 x 60 points. Even with these more refined grids, which had small grid expansion ratios, and with careful attention to boundary conditions complete grid independence was not achieved except for Reynolds numbers of 3×10^6 and upwards. The computed discharge coefficients are shown in Fig. 1; multiple symbols at one Reynolds number give the calculated values for different grids (the higher values are for the more refined grids). The rough pipe values, though for different sand roughnesses, correspond approximately to a relative roughness of 0.0003 on the Moody diagram, which is within the ISO Standard[3]. It is interesting to note that the rough pipe values approximately follow the Stolz equation, although they are about 1 per cent high, whereas the smooth pipe values decrease much more rapidly than the Stolz equation for large Reynolds numbers. From the computations differences of more than 1 per cent can be caused by pipe roughness even when it lies within the ISO Standard; it is because of the magnitude of these differences that it is necessary to include a function of friction factor in the orifice plate discharge coefficient equation.

Replotting the computed points as a function of friction factor gives Fig. 2. The rough pipe points now lie approximately on the line of the smooth pipe ones. However, they fall just below the line, and this may not just be numerical error. The discrepency may be due to the need for an additional term in equation (1). To establish what that term is it is necessary to consider what type of equation would be appropriate for small β since it is for small β that equation (1) seems to be most inappropriate.

It seems reasonable that for low β the discharge coefficient should only depend on throat Reynolds number, Re_d:

$$C = a + f(Re_d). \quad (2)$$

To check this theory C was plotted against log Re_d (Fig. 3) for β less than or equal to 0.375 using all the EEC and American flange tap data available in summer 1987. So that all the data included in this analysis had been collected with the same type of tappings flange tap data from the EEC were used rather than corner tap data. In all cases the flange tap data were converted to corner tap data using the tap term formula of Reader-Harris and Cribbin[4]; it is much easier to fit corner tap data than flange tap data since optimisation of the tapping terms is then not required. The data in Fig. 3 approximately lie on a single curve (the data for $\beta = 0.375$ are slightly higher than those for $\beta = 0.2$ and the data for $\beta < 0.2$ are rather scattered). So for low β it appears that C only depends on Re_d. It then seems reasonable to suppose that for low β

$$C = a + b(10^6/Re_d)^n$$

$$= a + b(10^6\beta/Re_D)^n, \quad (3)$$

and this assumption was tested later.

Since we now have approximate equations for low and high β the most obvious general equation for C is given by

$$C = a(\beta) + b(10^6\beta/Re_D)^n + c_1\beta^m\lambda. \quad (4)$$

3 CONSTRUCTING THE EQUATION ITSELF

The first stage in constructing the equation was to attempt to determine n and thus b in equation (3). The value of n which gives the minimum standard deviation of the data about the best line fit of the form of equation (3) lies in the range 0.6-1.1 for each $\beta \le 0.375$; 0.75 was chosen. The data for $\beta \le 0.375$ and $Re_D > 4000$ have been plotted in Fig. 4 against $(10^6/Re_d)^{0.75}$. b is taken to be 0.0006, which is the approximate slope of the data in Fig. 4. Only the data for $Re_D > 4000$ were used in the subsequent fitting, in case the formula for C is different for laminar flow.

Then it is assumed that

$$C' = a'(\beta) + c'(\beta)(\lambda - 0.01), \quad (5)$$

where $C' = C - 0.0006(10^6\beta/Re_D)^{0.75}$.

$(\lambda - 0.01)$ is used rather than λ since when Re tends to infinity λ does not tend to 0 if the pipe is not hydraulically smooth (if the relative roughness on the Moody diagram, k_m/D, is 0.000 037, λ tends to 0.01 as Re_D tends to infinity).

λ can be calculated from measurements of pressure drop or estimated from measured values of roughness. For the EEC 100 mm pipe measurements are given in References 5-7 and for the 250 mm pipe the measurements are in References 8-11. Measured values of roughness for each of the American pipes were supplied by Wayne Fling[12].

k_m/D is in the range 0.000 01-0.0002. For the present work k_m/D was taken to be 0.000 02 for all pipe diameters, although for a given pipe finish k_m/D is a function of D. This value is probably too low and further work will be done in which the measured values of k_m/D are used for each pipe, where the measurements are available. Where values have not been measured approximate values based on measurements made elsewhere will be used.

Given k_m/D and Re_D, λ can be obtained from the Colebrook-White equation[13]:

$$\frac{1}{\sqrt{\lambda}} = 1.74 - 2 \log\left[2 \frac{k_m}{D} + \frac{18.7}{Re_D \sqrt{\lambda}}\right]. \tag{6}$$

Using equation (6) C' can then be plotted against λ for each β and the results are approximately linear in each case; Figs 5 and 6 show the data for $\beta = 0.57$ and $\beta = 0.74$ (since the highest β at which EEC data were obtained was 0.7503 and the highest β at which American data were obtained varied with D but was about 0.73, $\beta = 0.74$ was taken as an approximate figure). Fitting straight lines of the form of equation (5) to the data then gives best fit values $a_\beta'^*$ for $a'(\beta)$ and $c_\beta'^*$ for $c(\beta)$ as follows:

β	$a_\beta'^*$	$c_\beta'^*$	σ_β^*
0.2	0.597 14	−0.044 91	0.001 8752
0.375	0.599 86	0.113 59	0.001 4982
0.5	0.603 79	0.220 58	0.001 1593
0.57	0.603 92	0.423 49	0.001 4060
0.66	0.602 00	0.884 74	0.001 4550
0.74	0.593 29	1.552 73	0.001 8598

The data for $\beta = 0.242$ and those for $\beta < 0.2$ were excluded. σ_β^* is the standard deviation of the data about the best line fit and is given by

$$\sigma_\beta^{*2} = \frac{1}{N_\beta - 2} \sum_{i=1}^{N_\beta} (C'_{\beta,i} - a_\beta'^* - c_\beta'^*\lambda_{\beta,i})^2, \tag{7}$$

where, for each β, N_β is the number of data points and, for each i from 1 to N_β inclusive, $C'_{\beta,i}$ is a value of C for that β and $\lambda_{\beta,i}$ is the corresponding value of λ.

Equations for $a'(\beta)$ and $c'(\beta)$ are now required: the form for $c'(\beta)$ has already (equation (4)) been assumed:

$$c'(\beta) = c_1'\beta^m. \tag{8}$$

The form of $a'(\beta)$ is initially assumed to be

$$a'(\beta) = a_0' + a_1'\beta^k + a_2'\beta^\ell. \tag{9}$$

It is possible to find the best fit for each of $a'(\beta)$ and $c'(\beta)$ separately, but it is better to fit them simultaneously. Suppose $a_\beta'^*$ and $c_\beta'^*$ in the line fit are replaced by $a_\beta'^{\#}$ and $c_\beta'^{\#}$ (derived from equations (9) and (8) or similar). Then the standard deviation of the data about this line fit would be $\sigma_\beta^{\#}$, where

$$\sigma_\beta^{\#2} = \frac{1}{N_\beta - 2} \sum_{i=1}^{N_\beta} (C'_{\beta,i} - a_\beta'^{\#} - c_\beta'^{\#}\lambda_{\beta,i})^2. \tag{10}$$

Now

$$\sigma_\beta^{\#2} = \sigma_\beta^{*2} + \frac{N_\beta}{N_\beta - 2} \{a_\beta'^{*} - a_\beta'^{\#} + (c_\beta'^{*} - c_\beta'^{\#})\bar{\lambda}_\beta\}^2 + $$

$$+ \frac{(c_\beta'^{*} - c_\beta'^{\#})^2}{N_\beta - 2} \sum_{i=1}^{N_\beta} (\lambda_{\beta,i} - \bar{\lambda}_\beta)^2, \tag{11}$$

where

$$\bar{\lambda}_\beta = \frac{1}{N_\beta} \sum_{i=1}^{N_\beta} \lambda_{\beta,i}. \tag{12}$$

Suppose now that $a_\beta'^{\#}$ and $c_\beta'^{\#}$ are in fact of the form of equations (9) and (8). Then we could minimize ΔV, where

$$\Delta V = \frac{1}{6} \sum_{\beta=0.2}^{0.74} (\sigma_\beta^{\#2} - \sigma_\beta^{*2}). \tag{13}$$

A more immediately comprehensible measure of fit is $\Delta\sigma$, where

$$\Delta\sigma = \frac{1}{6} \sum_{\beta=0.2}^{0.74} (\sigma_\beta^{\#} - \sigma_\beta^{*}). \tag{14}$$

For any given m,l and k there is no problem in determining a_0', a_1', a_2' and c_1' to minimize ΔV. It is natural also to choose m,l, and k such that ΔV is at its minimum. For the data in the above table this gives m = 4.8, k = 1.4, l = 8.1, and $\Delta\sigma$ = 0.000 022. But a_0' = 0.593 56, which, from Fig. 4, appears to be too low. Inclusion of the data from $\beta \leq 0.1$ would probably prevent this, but those data are very scattered. With only a small increase in standard deviation ($\Delta\sigma$ = 0.000 027) we can, for example, obtain a_0' = 0.595 38 with m = 4.8, k = 2.2, and l = 6.9. But such solutions are rather arbitrary: perhaps the best option is to require that when the velocity profile term (which is proportional to β^m) is small $a'(\beta)$ should reduce to a constant. One simple way of securing this is to put k = m and l > m. This also ensures that the same best fit is obtained whether the independent variable (λ - 0.01) or λ is used. Then the best solution has m = 4.3 and l very near to 4.3; moreover a_1' and a_2' are large in magnitude and a_1' is approximately equal to $-a_2'$. So equation (9) is replaced by

$$a'(\beta) = a_0' + \{a_1' + a_2' \log(\beta)\}\beta^m. \tag{15}$$

Then we have the best fit equation

$$C = 0.596\ 31 + 0.0006(10^6\beta/Re_D)^{0.75} + \{5.465\ 99(\lambda - 0.01) -$$

$$- 0.840\ 15 \log(\beta) - 0.119\ 75\}\beta^{4.3}. \tag{16}$$

This has $\Delta\sigma$ = 0.000 058, which is still small compared to each σ_β^*.

Although λ may appear to be a rather inconvenient variable, it is possible to approximate it by a simple function of Re_D if a value of k_m/D is assumed. For instance if k_m/D = 0.000 02 (the assumed value for this work) λ is always within 0.0005 of $\lambda^\#$ for Re_D > 4000 if

$$\lambda^\# = 0.008\ 59 + 0.809\ 26/Re_D^{0.39}. \tag{17}$$

However, the significance of the new formula depends on the term in λ being a true pipe roughness term, not simply another Reynolds number term. It is necessary, therefore, to examine the rather limited quantity of data on the effect on the discharge coefficient of different pipe roughnesses at the same Reynolds number. The experiments included here are those of Clark and Stephens[14], Herning and Lugt[15], Spencer, Calame and Singer[16], Thibessard[17] and Witte[18]; the computations were done by Reader-Harris[1]. Values from both experiment and computation are plotted in Fig. 7, in which the product of ΔC, the change in C, and $\beta^{-4.3}$ is plotted against the change in λ. The experimental data are generally a little lower than the computed values. However, the data of Clark and Stephens are higher than the computed values. The line on the graph represents the prediction of equation (16) for constant Reynolds number. The agreement between the experimental and computed values and equation (16) is excellent and suggests that the term in λ in the equation is a true pipe roughness term.

In order to compare the fit of equation (16) with those of standard equations of the following form

$$C = a(\beta) + b(\beta)(10^6/Re_D)^n, \tag{18}$$

best fit values a_β^* and b_β^* for $a(\beta)$ and $b(\beta)$ in equation (18) were obtained for a series of values of n together with values of standard deviation σ_β^*. The values of σ_β^* for n = 0.5, 0.6 and 0.7 were as follows:

β	n = 0.4	n = 0.5	n = 0.6
0.2	0.001 854	0.001 844	0.001 853
0.375	0.001 762	0.001 627	0.001 537
0.5	0.001 182	0.001 109	0.001 138
0.57	0.001 519	0.001 409	0.001 475
0.66	0.001 947	0.001 519	0.001 518
0.74	0.002 091	0.001 916	0.002 216

The smallest values of σ_β^* were obtained for n = 0.5; for this value of n a_β^* and b_β^* were as follows:

β	a_β^*	b_β^*
0.2	0.595 89	0.000 6137
0.375	0.598 10	0.001 3501
0.5	0.602 25	0.001 7524
0.57	0.602 83	0.002 2131
0.66	0.600 81	0.003 4198
0.74	0.592 21	0.004 9383

Then it was assumed that $a(\beta)$ and $b(\beta)$ were of the following standard forms:

$$a(\beta) = a_0 + a_1\beta^k + a_2\beta^\ell \tag{19}$$

$$b(\beta) = b_1\beta^m. \tag{20}$$

Best fit values of the constants in equations (19) and (20) were obtained in the same manner as those for the friction factor fit and the best fit equation for C was as follows:

$$C = 0.597\,11 + 0.015\,47\beta^{1.8} - 0.289\,87\beta^{10.3} +$$

$$+ 0.008\,884\beta^{2.2}(10^6/Re_D)^{0.5}. \tag{21}$$

Whereas the values of $\sigma_\beta{}^*$ were very similar to those obtained with the friction factor fit, the values of $\sigma_\beta{}^{\#}$ were greatly increased by the poor fit obtainable for $b_\beta{}^*$ ($\Delta\sigma$ = 0.000 287).

Because only a poor fit for b* is obtainable of the form of equation (20) it was decided to test also an alternative form for $b(\beta)$; so it was assumed that $a(\beta)$ was of the form of equation (19) and $b(\beta)$ was of the form:

$$b(\beta) = b_0 + b_1\beta^m. \tag{22}$$

Best fit values of the constants were again obtained and the best fit equation for C was as follows:

$$C = 0.589\,31 + 0.024\,56\beta^{0.9} - 0.227\,74\beta^{8.9} +$$

$$+ (0.000\,7082 + 0.011\,887\beta^{3.5})(10^6/Re_D)^{0.5} \tag{23}$$

$\Delta\sigma$ = 0.000 072, which is small compared to each $\sigma_\beta{}^*$, but a_0 is too low.

The standard deviation of the data about the fits in equations (16), (21) and (23) (ie $\sigma_\beta{}^{\#}$) for each β were as follows:

β	Equation (16)	Equation (21)	Equation (23)
0.2	0.001 929	0.002 250	0.001 910
0.375	0.001 569	0.001 959	0.001 846
0.5	0.001 232	0.001 288	0.001 131
0.57	0.001 522	0.001 766	0.001 497
0.66	0.001 478	0.001 574	0.001 532
0.74	0.001 887	0.002 136	0.001 930

The best two equations (16) and (23) are plotted on a background of the data for 3 values of β for $Re_D > 4000$ in Figs 8-10. The good agreement between the EEC and the American data is visible as well.

4 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

Although the standard deviation of the data about the fit in equation (16) is not much lower than that about the fit in equation (23), the new form of equation suggested (equation (16)) makes clearer the physical cause for the different terms of the equation, one solely dependent on throat Reynolds number, the other on upstream velocity profile. It is, therefore, more suitable for extrapolation than formulae with less physical basis. It also makes clear the effect of pipe roughness.

A computer program is being written so that the best fit equation of a given form can be obtained by minimizing the standard deviation of all the data from that equation (including all the values of β simultaneously). It will be possible in this work to use the correct value of β (rather than an approximate one) for each set of data and to include the data for $\beta = 0.242$.

Although the data for $\beta < 0.2$ are more scattered than the other data and thus cannot be included as a whole in the simultaneous fitting it may be appropriate to follow a suggetion of Stolz and to include these data for which the orifice diameter is greater than 12.5 mm (0.5 in). It should be the case that equations of the form of equation (16) will automatically give a good fit to the data for very low β.

In the simultaneous fitting of the data the new tapping term formula which is being derived will be included. More accurate values of k_m/D will also be used in this work.

Further tests will be required to check that equations of the form of equation (16) are suitable for the data for $Re_D < 4000$. A suitable formula for λ in this range will be required; it may not matter that it will not be the true friction factor.

REFERENCES

1 READER-HARRIS, M. J. and KEEGANS, W. Comparison of computation and LDV measurement of flow through orifice and perforated plates, and computation of the effect of rough pipework on orifice plates. In Proc. of the Int. Symp. on Fluid Flow Measurement, Washington, D.C., Paper 12.1, November 1986.

2 RAPIER, A. C. The extrapolation of orifice calibrations to high Reynolds numbers. In Proc. of the Fluid Mechanics Silver Jubilee Conference, National Engineering Laboratory, East Kilbride, Glasgow, Paper 4.3, 1979.

3 INTERNATIONAL ORGANIZATION FOR STANDARDIZATION. Measurement of fluid flow by means of orifice plates, nozzles and venturi tubes inserted in circular cross-section conduits running full. ISO 5167, Geneva: International Organization for Standardization, 1980.

4 READER-HARRIS, M. J. and CRIBBIN, S. The tapping terms in the orifice plate discharge coefficient formula. Paper submitted to the EEC Orifice Plate Experts' Group, 1987.

5 Progress Report on EEC Orifice Plate Coefficients Project. Initial NEL Tests in Water on EEC 100 mm (4 in) Orifice Run for European Economic Community DGXII Research, Science and Education. Progress Report No PR2: EUEC/03. East Kilbride, Glasgow: National Engineering Laboratory, 1980.

6 Progress Report on EEC Orifice Plate Coefficients Project: 1982/83 NEL Tests in Air on Thin Orifice Plates for EEC 100 mm Assembly for European Economic Community DGXII Research, Science and Education. Progress Report No PR15: EUEC/10. East Kilbride, Glasgow: National Engineering Laboratory, 1984.

7 WILLEMSE, A. J. Dimensions of the EEC 100 mm orifice plate meter run. N. V. Nederlandse Gasunie, Department of Planning and Research, Report TP/I 83.R.203, 1983.

8 HARBRINK, B. EEC Orifice plate coefficient project: Progress Report No 1: Dimensional Measurements on the EEC 250 mm Orifice Meter run. Ruhrgas AG, Essen, 1983.

9 WILLEMSE, A. J. Second report on the dimensions of the EEC 250 mm meter run. N. V. Nederlandse Gasunie, Department of Planning and Research, Report TP/1 82.R.353, 1982.

10 NORMAN, R. and DREW, W. A. The EEC 250 mm orifice meter project. Part II Inspection of the meter run and orifice plates. British Gas Engineering Research Station, Report ERS R.3127, 1985.

11 EEC orifice plate coefficient project, final water tests on the EEC 250 mm orifice run. Delft Hydraulics Laboratory, M 1733-IV, 1986.

12 FLING, W. A. Jr. Private communication, 1988.

13 SCHLICHTING, H. Boundary layer theory. McGraw-Hill, New York, 1960.

14 CLARK, W. J. and STEPHENS, R. C. Flow measurement by square-edged orifice plates: pipe roughness effects, Proceedings of the I.Mech.E., 171, pp 895-910, 1957.

15 HERNING, Fr. and LUGT, H. Neue Versuche mit Segmentblenden und Normblenden, Brennst-Warme-Kraft, 10, Part 5, pp 219-223, 1958.

16 SPENCER, E. A., CALAME, H. and SINGER, J. Edge sharpness and pipe roughness effects on orifice plate discharge coefficients. NEL Report No 427, East Kilbride, Glasgow: National Engineering Laboratory, 1969.

17 THIBESSARD, G. Le coefficient de débit des diaphragmes, la rugosité et le nombre de Reynolds. Chaleur et Industrie, No 415, pp 33-50, 1960.

18 WITTE, R. Neue Beiträge zur internationalen Normung auf dem Gebiete der Durchflußmessung, Brennst-Warme-Kraft, 5, Part 6, pp 185-190, 1953.

LIST OF FIGURES

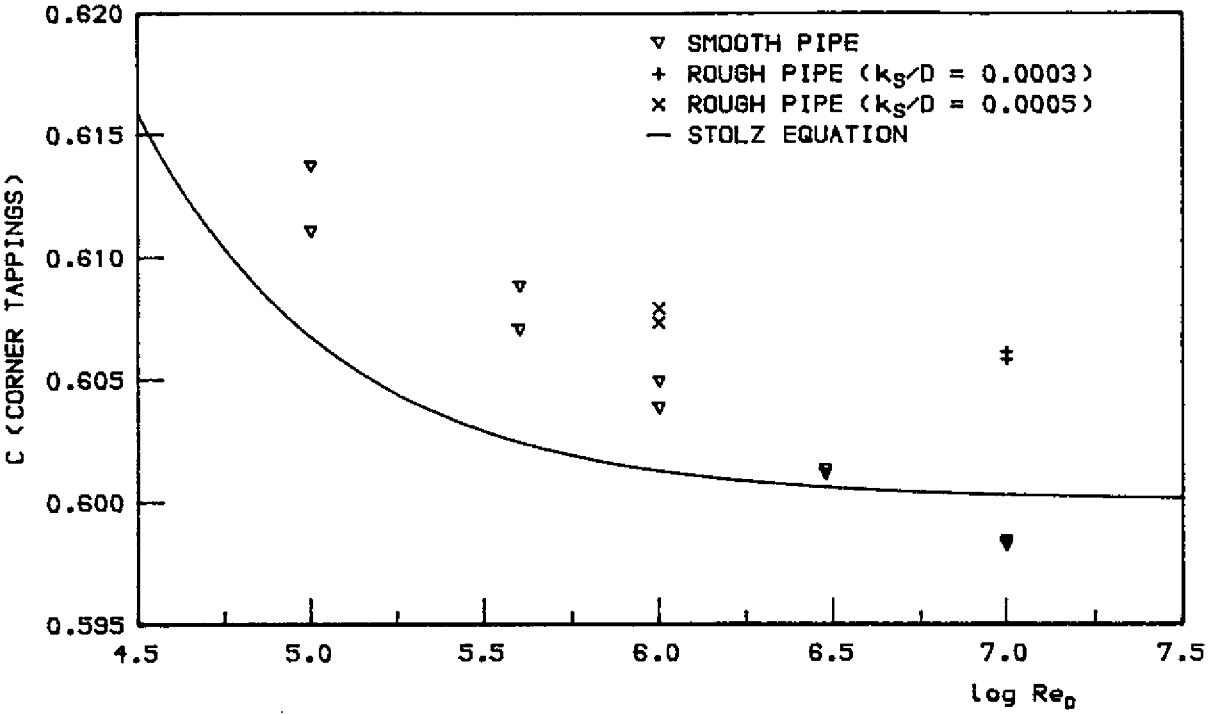


FIG 1 COMPUTED C v log Re_D FOR β = 0.7

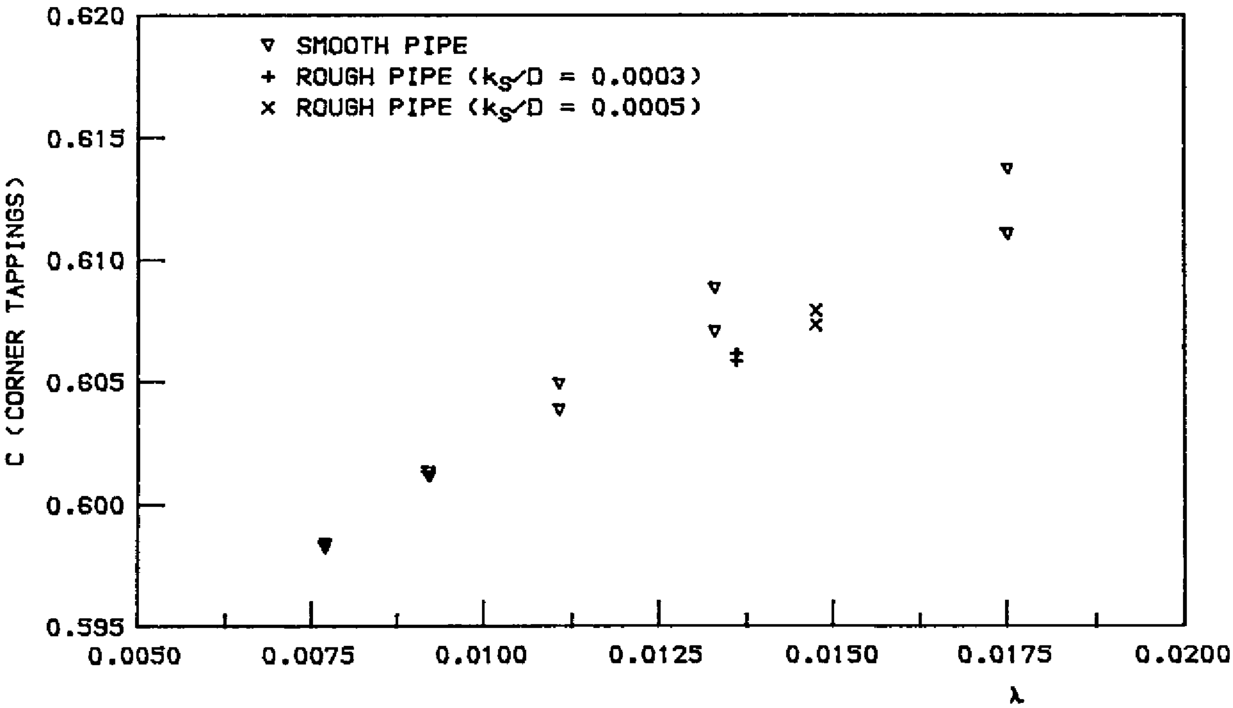


FIG 2 COMPUTED C v λ FOR β = 0.7

FIG 3 DISCHARGE COEFFICIENT V LOG(THROAT REYNOLDS NUMBER) FOR B<=0.375

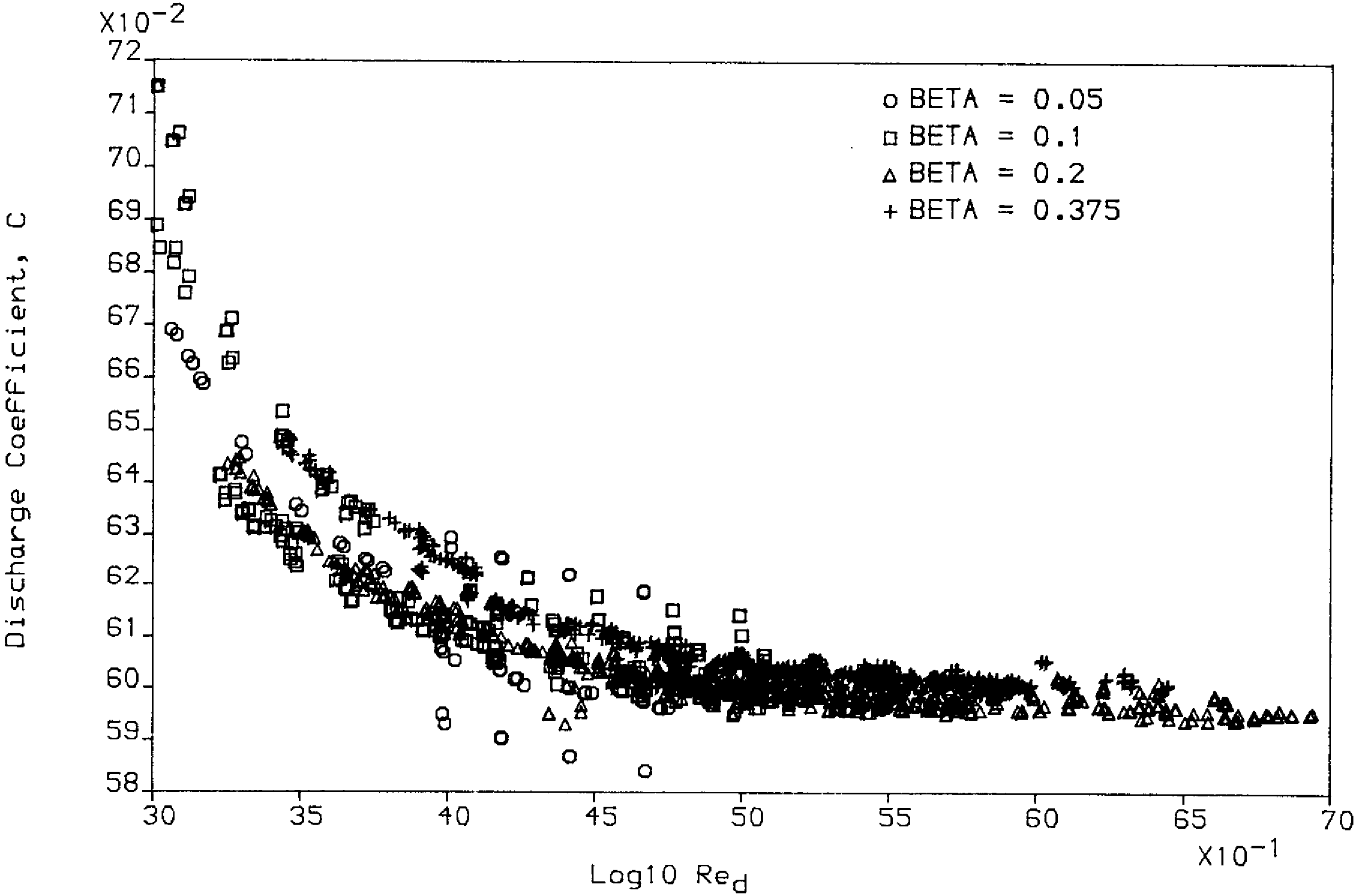


FIG 4 DISCHARGE COEFFICIENT V (10↑↑6/THROAT REYNOLDS NUMBER)↑↑0.75 FOR B<=0.375

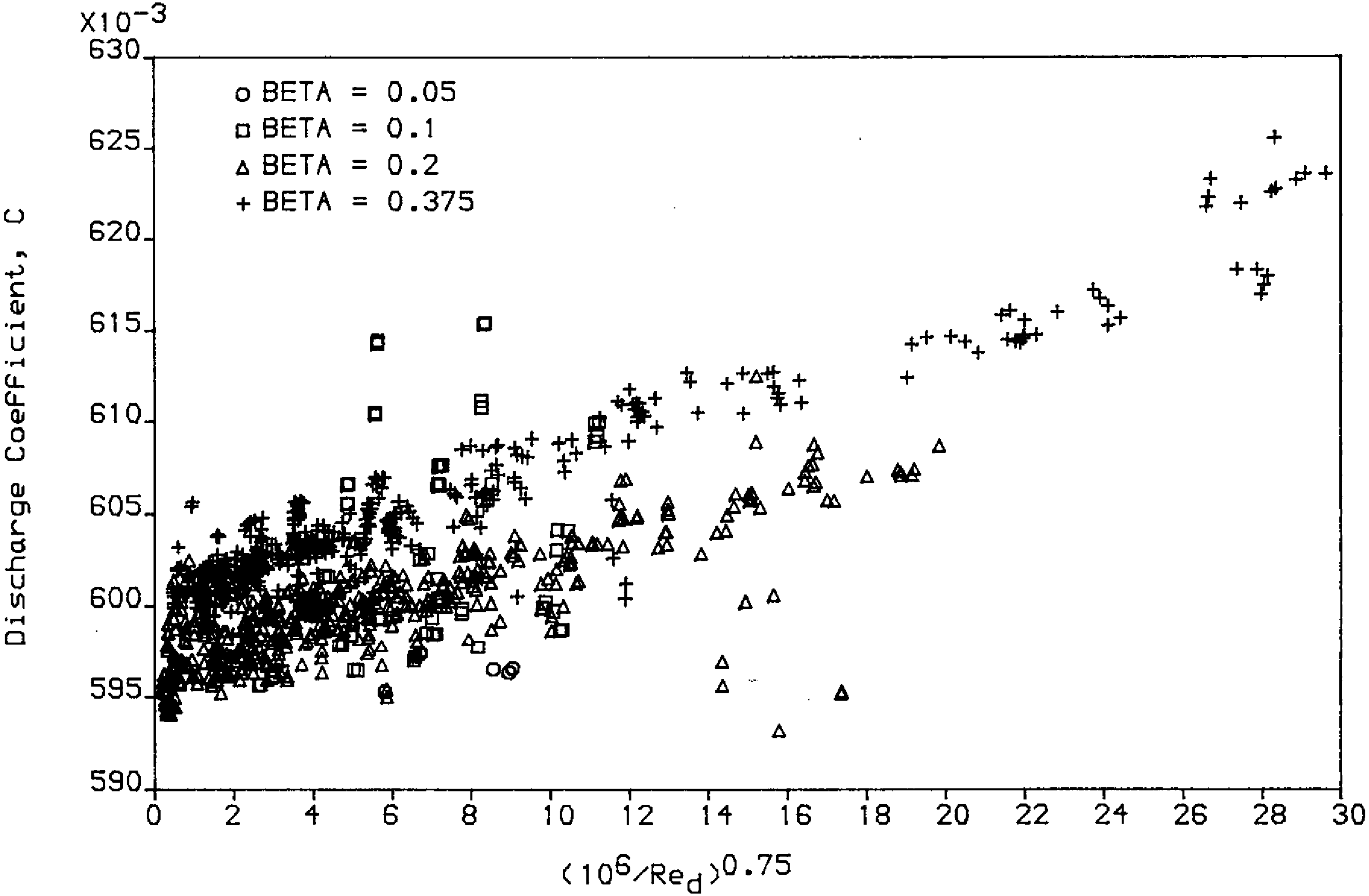


FIG 5 C' V (LAMBDA - 0.01) FOR BETA = 0.57

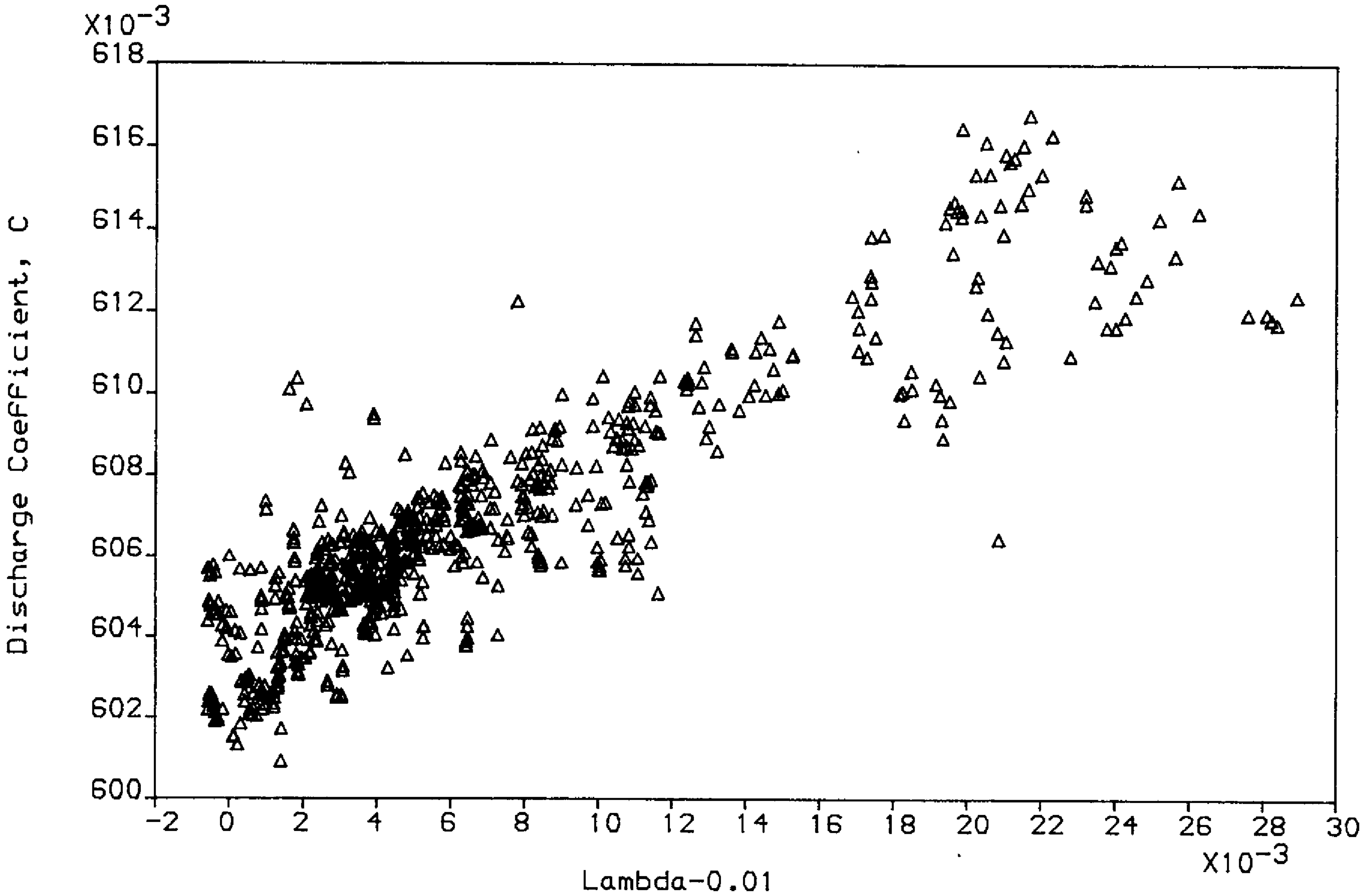
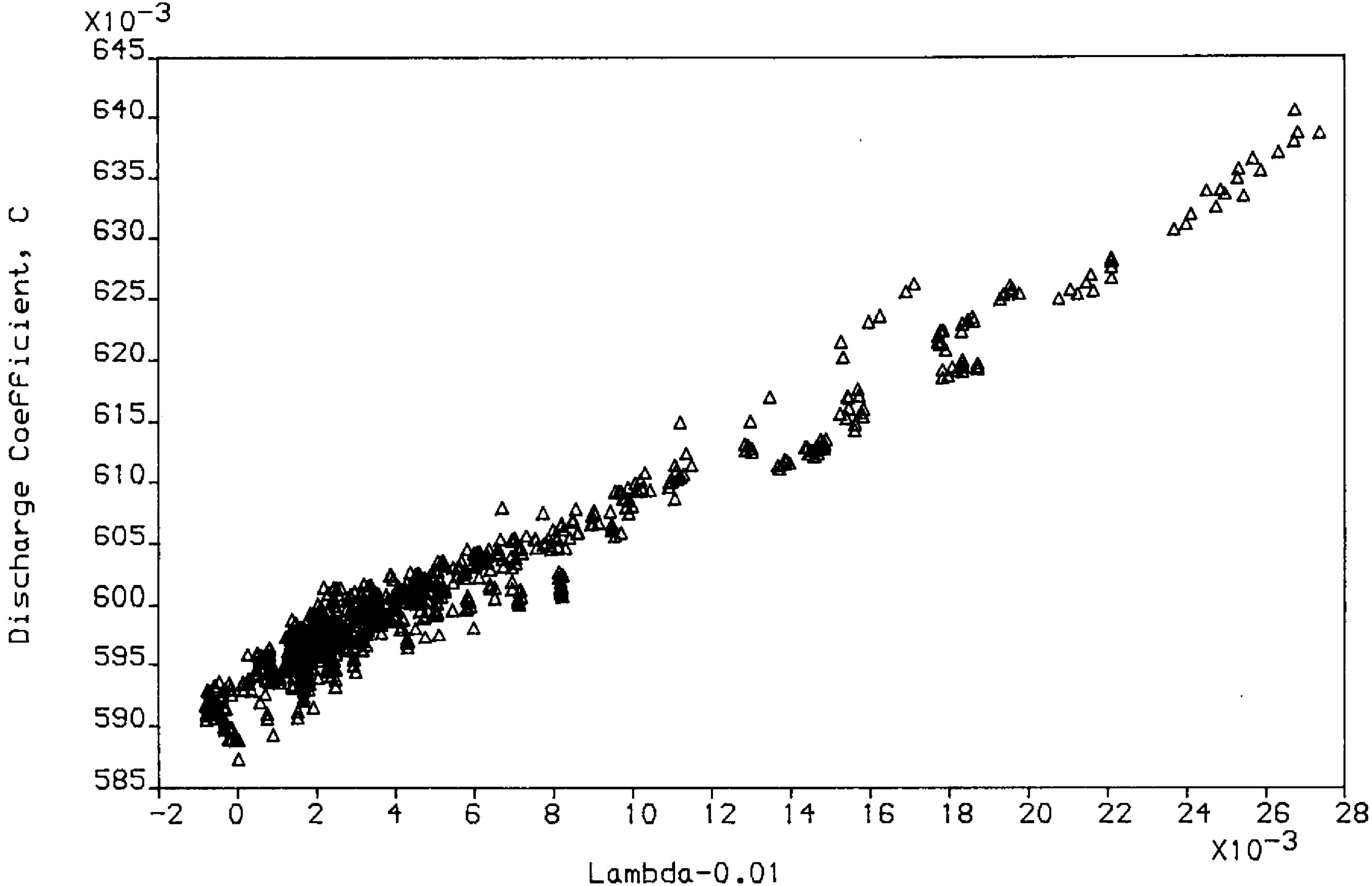


FIG 6 C′ V (LAMBDA - 0.01) FOR BETA = 0.74



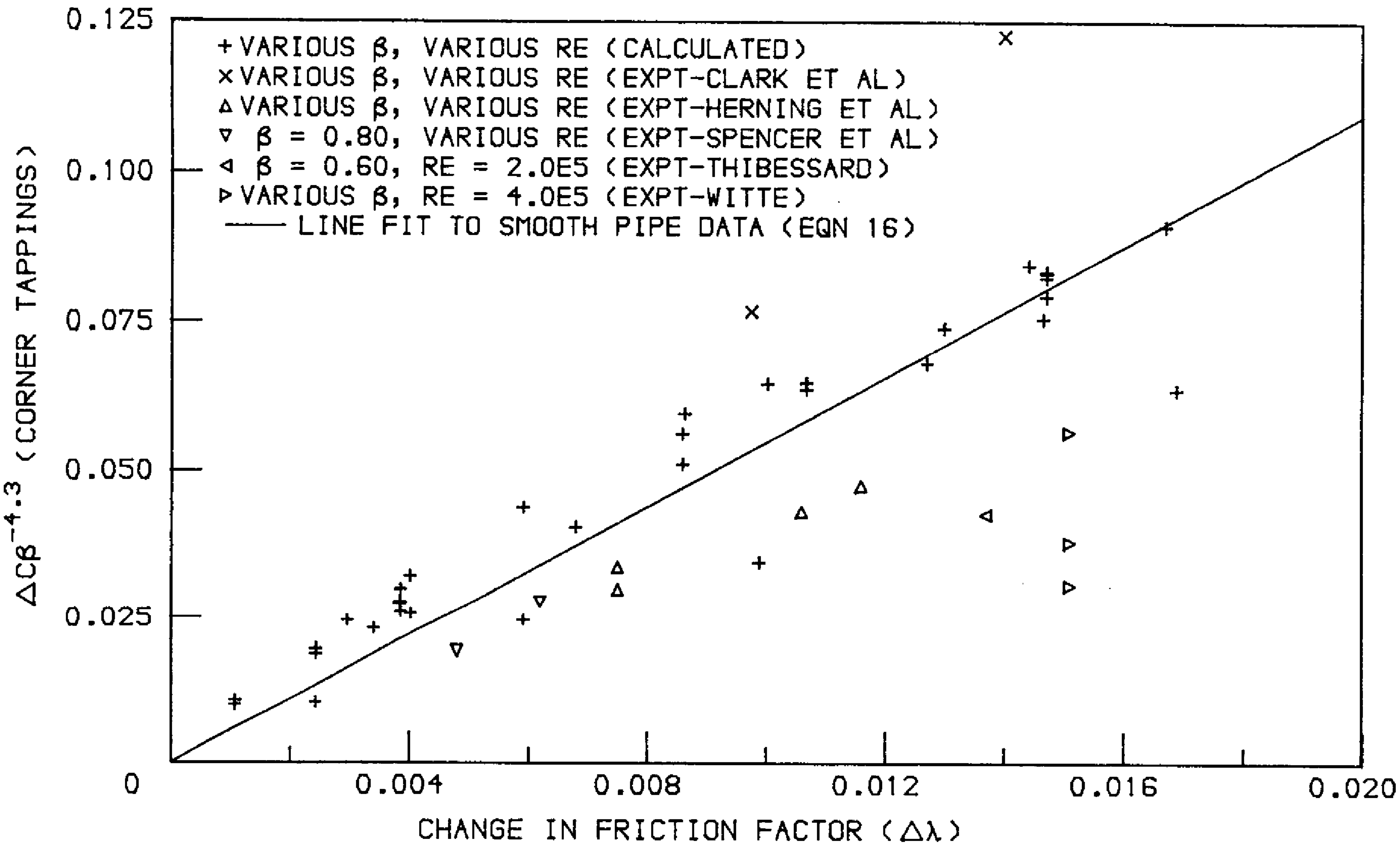


FIG 7 EFFECT OF ROUGH PIPE ON DISCHARGE COEFFICIENT
(COMPUTATION AND EXPERIMENT, CORNER TAPPINGS)

FIG 8 DISCHARGE COEFFICIENT V LOG (REYNOLDS NUMBER) FOR BETA=0.2 (NOT 2 INCH)

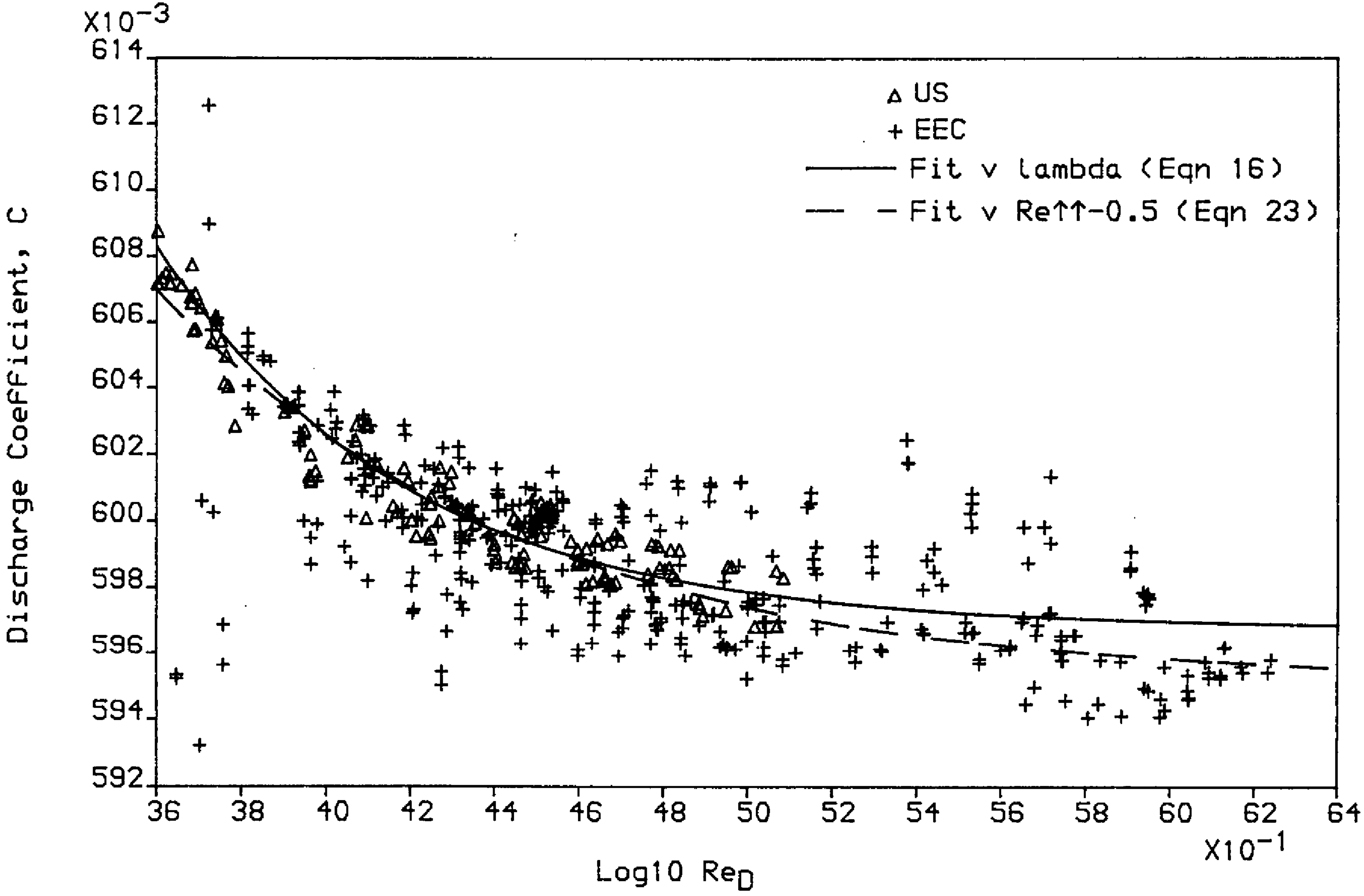


FIG 9 DISCHARGE COEFFICIENT V LOG (REYNOLDS NUMBER) FOR BETA=0.57

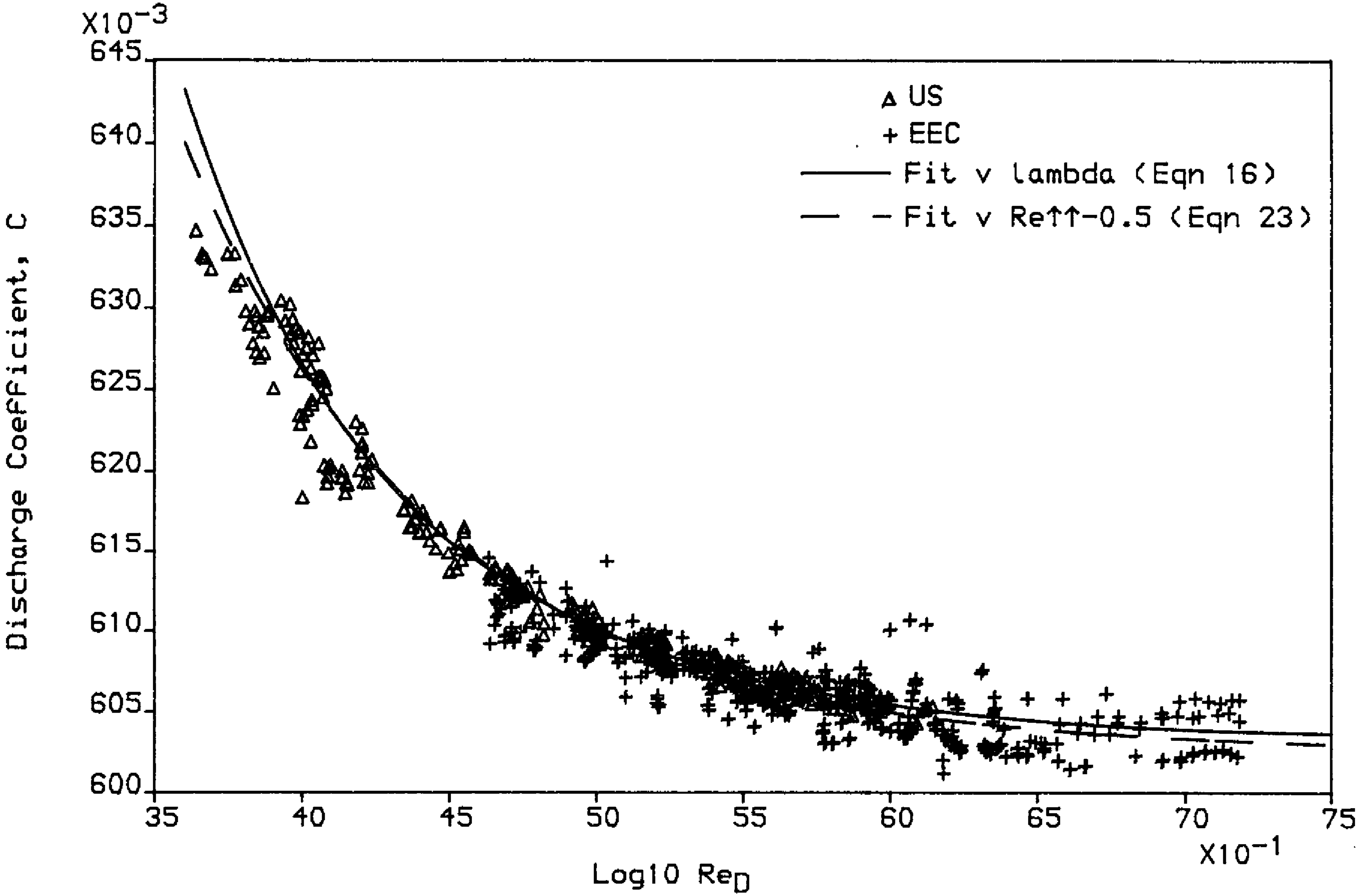


FIG 10 DISCHARGE COEFFICIENT V LOG (REYNOLDS NUMBER) FOR BETA=0.74