

THE ORIFICE PLATE DISCHARGE COEFFICIENT EQUATION - FURTHER WORK

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S U M M A R Y

This paper describes the work undertaken to derive the revised orifice plate discharge coefficient equation based on the final EEC/API database including the data collected in 50 mm and 600 mm pipes. It consists of several terms, each based on an understanding of the physics. An earlier version of this equation, based on a smaller database, was accepted at a meeting of EEC and API flow measurement experts in New Orleans in 1988, and emphasis is placed on the two principal changes to the equation: improved tapping terms for low Reynolds number have been calculated; and an additional term for small orifice diameter has been obtained, and its physical basis in orifice edge roundness given.

N O T A T I O N

A	Function of orifice Reynolds number (see equations (6) and (7))
C	Orifice discharge coefficient
C_c	Orifice discharge coefficient using corner tapplings
C_s	Dependence of C_c on Reynolds number
C_∞	C_c for infinite Reynolds number
ΔC_{down}	Downstream tapping term
$\Delta C_{down,min}$	Value of ΔC_{down} at the downstream pressure minimum
ΔC_{up}	Upstream tapping term
ΔC_{round}	Change in discharge coefficient due to edge roundness
D	Pipe diameter
d	Orifice diameter
L_1	Quotient of the distance of the upstream tapping from the upstream face of the plate and the pipe diameter
L_2	Quotient of the distance of the downstream tapping from the upstream face of the plate and the pipe diameter
L_2'	Quotient of the distance of the downstream tapping from the downstream face of the plate and the pipe diameter
M_2	Quotient of the distance of the downstream tapping from the upstream face of the plate and the dam height (equation (2))

M_2'	Quotient of the distance of the downstream tapping from the downstream face of the plate and the dam height (as in equation (2))
Δp	Differential pressure across the orifice plate
Δp_c	Differential pressure across the orifice plate using corner tappings
r_e	Edge radius
Re_D	Pipe Reynolds number
Re_d	Orifice Reynolds number
r_{em}	Mean value of edge radius
r_{max}	Maximum value of edge radius
u	Uncertainty of the pressure measurement
β	Diameter ratio
λ	Friction factor
$\Delta\lambda$	Shift in friction factor due to pipe roughness

1 INTRODUCTION

Although the orifice plate is the recognized flowmeter for the measurement of natural gas and light hydrocarbon liquids, the orifice discharge coefficient equations in current use are based on data collected more than 50 years ago. Moreover, for 20 years the United States and Europe have used different equations, a discrepancy with serious consequences for the oil and gas industry since many companies are multinational. For more than ten years data on orifice plate discharge coefficients have been collected in Europe and the United States in order to provide a new database from which an improved discharge coefficient equation could be obtained which would receive international acceptance.

In November 1988 a joint meeting of API (American Petroleum Institute) and EEC flow measurement experts in New Orleans accepted the equation derived by NEL. At that time the database contained 11 346 points, collected in pipes whose diameters ranged from 50 to 250 mm (2 to 10 inch); 600 mm (24 inch) data were being collected but had not yet been included in the database. 600 mm data have now been collected in gas and in water and extend the database both in pipe diameter and in Reynolds number. The data which were least well fitted by the equation presented at New Orleans were the 50 mm data; so additional 50 mm data have been collected in water and oil which provide additional information about discharge coefficients both for small orifice diameters and for low Reynolds numbers. All the additional data have now been included in the database and the equation refitted. This paper gives that revised equation and its derivation.

2 THE DATABASE

The final database consists of 16376 points: the diameter ratios range from 0.1-0.75, orifice Reynolds numbers from 1700 to 7×10^7 , and pipe diameters

from 50-600 mm. The data were collected in nine laboratories in four fluids: water, air, natural gas and oil. The data points for which the orifice diameter was less than 12.5mm ($\frac{1}{2}$ inch) and those for which the differential pressure across the orifice plate is less than 600 Pa are very scattered and were excluded. The American data remain as in References 1 to 3; no additional American data have been collected. The complete EEC data are tabulated in References 4 to 10; the data sets which have been accepted for analysis are indicated in Reference 11. A very small number of points (0.5 per cent of all the EEC points) was removed from the EEC data as outliers; each of those removed was identified as an outlier within its own set of data by the Grubbs' extreme deviation outlier test; details will be found in Reference 12.

3 TAPPING TERMS

The tapping terms are equal to the difference between the discharge coefficient using flange or D and D/2 tapplings and those using corner tapplings. They are expressed as the sum of an upstream and a downstream tapping term. The upstream term is equal to the change in discharge coefficient when the downstream tapping is fixed in the downstream corner and the upstream tapping is moved from the upstream corner to another position. The downstream term is equal to the change in discharge coefficient when the upstream tapping is fixed in the upstream corner and the downstream tapping is moved from the downstream corner to another position. In theory the total tapping term is the sum not only of the upstream and the downstream term but also of a product term because the discharge coefficient depends on the reciprocal of the square root of the differential pressure (Reference 13). In practice the product term is not included in the formula and, to compensate, the upstream tapping term in the formula is very slightly smaller than the true upstream term.

In the database only the total tapping term, the sum of the upstream and the downstream terms, is available. To divide the tapping term into two parts so that each can be accurately fitted, measurements of the individual tapping terms collected outside the EEC and API projects were used to indicate the form and approximate value of the upstream and downstream terms; however the constants in the formulae were obtained to fit the EEC/API database. The EEC collected data with several tapping systems; so the total tapping term could be simply obtained (References 13 and 14). Although the American data were collected with flange tapplings alone small adjustments were made to the final tapping terms in order to obtain the optimum fit to the database as a whole.

On examining the measured tapping terms it has been shown (References 13 and 14) that for high Reynolds number (orifice Reynolds number, Re_d , greater than about 10^5) the tapping terms may be considered not to vary with Re_d , but that for low Reynolds number the terms depend on Re_d . An important part of the work undertaken since the meeting in New Orleans has been to provide more accurate low Reynolds number tapping terms. Since the high Reynolds number tapping terms need to be determined first they are described here first.

3.1 High Reynolds number tapping terms

For each diameter ratio mean total tapping terms for the EEC data for D and D/2 tapplings and for flange tapplings in 50, 100, 250 and 600 mm pipe were calculated and used for determining best constants and exponents. Low

Reynolds number data were excluded. Details of the method by which the values of the total tapping terms were calculated are given in References 13 and 14.

3.1.1 Upstream term

To determine the correct form of the tapping terms, the upstream term, ΔC_{up} , was determined first. The dependence of the upstream term on $\beta^4/(1-\beta^4)$ is well established (References 13 - 16); so here it is only necessary to consider the dependence on L_1 , the quotient of the distance of the upstream tapping from the upstream face of the plate and the pipe diameter. Several forms of equation were tried, and their exponents and constants determined, and the optimum one found to be the following:

$$\Delta C_{up} = (0.043 + 0.090e^{-10L_1} - 0.133e^{-7L_1}) \frac{\beta^4}{1 - \beta^4} \quad (1)$$

This equation is physically realistic: it has the required dependence on $\beta^4/(1-\beta^4)$, is equal to 0 for $L_1=0$, does not become negative, tends rapidly to a constant once L_1 exceeds 0.5, and has a continuous derivative. Together with the downstream term it gives a very good fit to the total tapping term data. It is plotted in Fig. 1 against many sets of experimental measurements of the upstream tapping term and the quality of the fit is good. References to the work of the many experimenters who collected the data in Fig. 1 are given in References 13 and 14.

3.1.2 Downstream term

Many experimenters have measured the pressure profile downstream of the orifice plate, and, although the data are more scattered than those upstream of the plate, the pattern is clear: the pressure decreases downstream of the plate till it reaches a minimum and then quite a short distance downstream of the minimum it begins to increase rapidly. The orifice plate should not be used with the downstream tapping in the region of rapid pressure recovery.

An important step in the determination of the downstream formula was the work of Teyssandier and Husain (Reference 17) who non-dimensionalised downstream distances with the dam height rather than the pipe diameter. Instead of working in terms of L_2 , the quotient of the distance of the downstream tapping from the upstream face of the plate and the pipe diameter, it is better to use M_2 , the quotient of the distance of the downstream tapping from the upstream face of the plate and the dam height, which is given by

$$M_2 = \frac{2L_2}{1 - \beta} \quad (2)$$

L_2' and M_2' are defined identically except that in each case the distance from the downstream face of the orifice plate is used.

From consideration of data from many experimenters References 13 and 14 confirmed the advantage of non-dimensionalising with dam height by showing that the pressure minimum occurs for

$$M_2 = 3.3. \quad (3)$$

Although it is theoretically better to work in terms of M_2 , it is more convenient to work in terms of M_2' since it avoids the need to include the plate thickness in the discharge coefficient equation. Provided appropriate restrictions are placed on plate thickness, an equation for M_2' can be used without introducing significant errors (Reference 13). These plate thickness restrictions are satisfied by the EEC and API plates.

To determine the downstream tapping term, ΔC_{down} , it is desirable first to determine an appropriate dependence on β : this can be done by considering $\Delta C_{\text{down,min}}$, its value at the downstream pressure minimum. Fig. 2 gives values of measured downstream tapping terms from various experimenters (given in References 13 and 14). It is not necessary to determine the best fit, but the following, from equation (5), gives a good fit to the data:

$$\Delta C_{\text{down,min}} = -0.0101\beta^{1.3}. \quad (4)$$

Several forms of equation for the complete downstream tapping term were tried, and their exponents and constants determined, and the optimum one found to be the following:

$$\Delta C_{\text{down}} = -0.031(M_2'^{1.1} - 0.8M_2'^{1.3})\beta^{1.3}. \quad (5)$$

This equation is physically realistic: it has an appropriate dependence on β , is equal to 0 for $M_2'=0$, has a minimum, and has a continuous and finite derivative. Together with the upstream term it gives a very good fit to the total tapping term data. It is plotted in Fig. 3 against many sets of experimental measurements of the downstream tapping term.

3.2 Low Reynolds number tapping terms

When data were taken by NEL in oil in 50 mm pipe in 1990 for inclusion in the EEC database not only discharge coefficients but also direct measurements of pressure profile were made: the pressure rise to the upstream corner from tappings at distances D , $D/2$, $D/4$ and $D/8$ upstream was measured, where D is the pipe diameter, as well as the pressure drop from the downstream corner to tappings at distances $D/8$ and $D/4$ downstream of the downstream face of the orifice plate and to the downstream $D/2$ tapping. Whereas for high Re_d the tapping terms do not depend on Re_d , the tapping terms at the Reynolds numbers obtained in oil are significantly different. Previous work by Johansen (Reference 18) had shown that the upstream tapping term at the upstream pressure minimum decreases as Re_d decreases. Similarly the downstream tapping term at the downstream pressure minimum decreases in magnitude as Re_d decreases. Data from Witte and Schröder (quoted in References 19 and 20), who only measured the upstream tapping term, agree with Johansen. So the equation presented at New Orleans reflected these data on the reasonable assumption that the tapping terms, though decreasing in size as Re_d decreases, retain the same shape as a function of distance from the plate, since data were not available except at the pressure minima.

However, the data collected on tapping terms by NEL in 50 mm pipe (Reference 21) show that the reality is more complex than the previous

analysis, but also provide revised tapping terms which correspond much better to the tapping term collapse found in the database as a whole than the New Orleans tapping term did. Fig. 4 shows all the data for $\beta \approx 0.74$: it can be seen that all the data collapse on to one another as Re_d decreases. However, the amount of data makes it difficult to see that the flange data collected in 50 mm, 100 mm and 150 mm pipes collapse on to one another at a higher Reynolds number than that at which the corner tapping data collapse on to the other data. The collapse of the flange tapping data on to one another can be seen clearly in Fig. 5 which shows the US data for $\beta \approx 0.74$: the approximately constant values of the tapping terms for high Re_d can also be seen. Figures 4 and 5 confirm the need for tapping terms which are functions of Reynolds number, but also show that the simple dependence on Re_d used in the New Orleans equation is insufficient.

The main features of the tapping term data collected in 50 mm pipe (in Figs 83-100 of reference 21) are as follows: the upstream tapping term for D and for D/2 tapplings decreases with decreasing Re_d as expected from the work of Johansen and of Witte and Schröder, although the NEL data decrease slightly more slowly with Re_d ; the upstream tapping term for D/4 tapplings remains approximately constant; the upstream tapping term for D/8 tapplings increases with decreasing Re_d . The dependence of the downstream tapping terms on Re_d depends on β : for $\beta > 0.7$ they decrease in magnitude with decreasing Re_d ; otherwise they are constant. At the bottom of the Re_d range the uncertainties in the data become large, especially for the upstream D/2 tapping data.

It is interesting that downstream of the orifice plate the data in Reference 21 are apparently inconsistent from those of Johansen: one possibility is that it is only near the pressure minimum that the magnitude of the downstream tapping term decreases with decreasing Re_d , through the pressure minimum becoming closer to the orifice. For, where the values of the downstream tapping term deduced from the data in Reference 21 were taken at the pressure minimum, that is for $\beta > 0.7$, they decrease in magnitude like those of Johansen; elsewhere the two sets of data are not directly comparable: the data in Reference 21 were not taken at the pressure minimum, whereas Johansen's were.

It is important to see the pattern in the tapping term data: to do this it is necessary to do an analysis of the uncertainty of these data. It is then possible to analyse all the upstream tapping term data simultaneously, and, in particular, to verify that the dependence on $\beta^4/(1-\beta^4)$ which characterizes the data for high Re_d continues to apply for low Re_d . Fig. 6 shows the change in discharge coefficient due to moving the upstream pressure tapping from the upstream corner for the 4 values of L_1 for which measurements were made. Where the data are multiplied by $(1-\beta^4)/\beta^4$ they fall on to a single curve for each value of L_1 . Data are only plotted if $(1-\beta^4)u/(\beta^4\Delta p_c) < 0.05$, where u is the uncertainty of the pressure measurement at that point and Δp_c is the pressure differential across the orifice plate using corner tapplings.

Various possible forms of the upstream tapping term, ΔC_{up} , for low Re_d data were tried, and the best one found to be the following:

$$\Delta C_{up} = (0.043 + (0.090 - aA)e^{-10L_1} - (0.133 - aA)e^{-7L_1})$$

$$(1 - bA) \frac{\beta^4}{1 - \beta^4}, \quad (6)$$

where

$$A = \left(\frac{2100\beta}{Re_D} \right)^n$$

and a, b, and n are to be determined.

This equation has the same behaviour for $L_1 = 1$ as the equation accepted in New Orleans, and is equal to 0 for $L_1 = 0$, but is significantly different for intermediate values of L_1 . With this form of equation the best fit to the upstream tapping term data included in Fig. 6 was obtained. Since the product term is not included in the final formula for the tapping terms, the fitted upstream term is adjusted to make allowance for it: following the argument in Reference 13, $\Delta C_{up} (1 + 3\Delta C_{down}/C_c)$, where C_c is the discharge coefficient using corner tappings, is fitted instead of ΔC_{up} itself. Allowance was also made for the fact that especially for $L_1 \leq 0.125$ the measured tapping term (adjusted to allow for the absence of the product term) even for $Re_d \approx 100000$ is not equal to the high Re_d value of the equation; the fitted equation was therefore calculated based on data points shifted so that for each L_1 the mean value of the data for $Re_d > 80000$ (adjusted to allow for the absence of the product term) agrees with the high Re_d version of the equation.

The best fit value for n was 0.925, but for simplicity this was rounded to 0.9, and with n = 0.9 the other constants were

$$a = 0.833$$

$$\text{and } b = 1.307.$$

However, these values were adjusted to give a better fit to the database: the best fit of the complete database gave a larger value of a than the fit to the upstream tapping term data: a compromise value was obtained as follows: from the Figures in Reference 21 it appears that the data for $L_1 = 1$, those for $L_1 = 0.25$ and those for $L_1 = 0.125$ cross at $Re_d \approx 13\,000$. Since in equation (6) the three curves representing the three values of L_1 do not intersect at a single point, a further simplification is to consider the intersection of the curve for $L_1 = 0.167$ (corresponding to flange tappings in 6-inch pipe) with the curve for $L_1 = 1$: this occurs for a = 1.03. This constant is then rounded to 1. Equation (6) with a = 1, b = 1.307, and n = 0.9 is then plotted in Fig. 6 for comparison with the data. This equation describes a change in the pressure profile upstream of the orifice in which, as Re_d decreases, the upstream tapping term at D decreases but the gradient of the tapping term near the corner increases.

It is unnecessarily complicated to construct a downstream tapping term which decreases in magnitude with decreasing Re_d for very large β but is constant for smaller β ; since the upstream term is significant for large β , but very small for small β , this downstream Re_d effect is incorporated in the upstream term by reducing b from 1.307 to 1. The final upstream

tapping term (incorporating a small downstream effect) is therefore

$$\Delta C_{up} = (0.043 + (0.090 - A)e^{-10L_1} - (0.133 - A)e^{-7L_1})(1 - A)\frac{\beta^4}{1 - \beta^4}, \quad (7)$$

where

$$A = \left(\frac{2100\beta}{Re_D} \right)^{0.9}.$$

With this upstream formula no change in the downstream formula is required for $Re_D > 4000$. However, from examination of the data for $Re_D < 4000$ in Reference 10 it can be seen that in this region the discharge coefficient using corner tapplings becomes increasingly larger than that using flange or D and D/2 tapplings as Re_D decreases; since this applies even for small β this can best be represented by the downstream tapping term being modified, although both upstream and downstream tapping terms change with Re_D . The model used was as follows:

$$\Delta C_{down} = -0.031(M_2' - 0.8M_2')^{1.1} \{1 + c \max(\log_{10}(T/Re_D), 0.0)\} \beta^{1.3}, \quad (8)$$

where c is a constant and T is the pipe Reynolds number at which transition to fully turbulent flow occurs. T varies, as would be expected, from one set of data to another, but a reasonable estimate of the range of values encountered in the database is 3000 - 4500, and $T = 3700$ has been used for both the tapping term and the slope term. With this value for T c is determined by fitting the data in Reference 10: using the difference between flange and corner tapplings only, $c = 8.20$; using the difference between D and D/2 and corner tapplings only, $c = 7.88$; using all the data, $c = 8.04$. The agreement between the values of c obtained using flange and D and D/2 tapplings is very good, and the downstream tapping term used in the final equation is as follows:

$$\Delta C_{down} = -0.031(M_2' - 0.8M_2')^{1.1} \{1 + 8 \max(\log_{10}(3700/Re_D), 0.0)\} \beta^{1.3}. \quad (9)$$

4 SMALL ORIFICE DIAMETER TERM

An additional term for small orifice diameters has been added to the equation accepted at New Orleans as a result of collecting the NEL 50 mm data which include measurements of edge sharpness. The problem here is that it is extremely difficult to obtain a sufficiently sharp edge where the orifice diameter is small: Fig. 7, which includes averages of measured edge radii from the plates used in the EEC tests, in which D was in the range 50-600 mm, shows that for orifice diameter, d , less than 50 mm the plates rarely meet the requirements of ISO 5167-1 (Reference 22). It is clear that for $d \leq 25$ mm there will be large shifts in C . When the edge radii themselves are plotted as in Fig. 8, it appears that the edge radius, r_e , (in mm) increases as d decreases from 50 mm, whereas to meet the standard it needs to decrease fairly rapidly.

The change in discharge coefficient due to edge roundness, ΔC_{round} , has been measured by Hobbs (Reference 23) as a function of change in edge radius, Δr_e , and can be expressed approximately as

$$\Delta C_{round} = 3.33 \Delta r_e / d. \quad (10)$$

It seems reasonable to suppose that the mean value of r_e , r_{em} , for $d < 50$ mm is given by

$$r_{em} = 0.01 + B(50 - d), \quad (11)$$

where B is a constant. This is linear with d and gives r_{em}/d equal to 0.0002 where $d = 50$ mm. Given that the discharge coefficient equation for large d is based on r_{em}/d being equal to 0.0002, the additional term for $d < 50$ mm will be

$$\Delta C_{round} = 3.33(r_{em}/d - 0.0002), \quad (12)$$

which on substituting from equation (11) becomes

$$\Delta C_{round} = 3.33(B + 0.0002)(50/d - 1). \quad (13)$$

When this term is determined by fitting the database a good approximation to the term is

$$\Delta C = 0.0015 \max(50/d - 1, 0), \quad (14)$$

which corresponds to $B = 0.00025$ and to

$$r_{em} = \max(0.0225 - 0.00025d, 0.0002d). \quad (15)$$

The maximum value of r_e , r_{max} , is $2r_{em}$, which is equal to

$$r_{max} = \max(0.045 - 0.0005d, 0.0004d), \quad (16)$$

and from Fig. 8 it can be seen that all the plates lie within this limit. Clearly this term gives rise to an increase in uncertainty for $d < 50$ mm.

5 C_∞ AND SLOPE TERMS

Given the tapping and small orifice diameter terms it is possible to determine the C_∞ and slope terms. C_∞ is the discharge coefficient using corner tappings for infinite Reynolds number, and the slope term, C_s , gives the dependence of the discharge coefficient using corner tappings on Reynolds number, so that the discharge coefficient using corner tappings is given by $C_\infty + C_s$. These terms are of the same form as in previous work (References 14 and 24) and are described there, but, in brief, the basis of these terms is as follows: C_∞ increases with β to a maximum near $\beta = 0.55$ and then decreases increasingly rapidly. Thus an appropriate form for C_∞ is

$$C_\infty = a_1 + a_2\beta^{m_1} + a_3\beta^{m_2}. \quad (17)$$

The slope term consists of two terms, an orifice Reynolds number term and a velocity profile term. For low β C_s depends only on orifice Reynolds number, and a simple expression as a reciprocal power of Re_d is appropriate:

$$\begin{aligned} C_s &= b_1(10^6/Re_d)^{n_1} \\ &= b_1(10^6\beta/Re_D)^{n_1}. \end{aligned} \quad (18)$$

This term is inadequate to describe C_s for large β : for large β there is

also a velocity profile term which can be derived using the fact that, for a fixed Reynolds number, as the pipe roughness changes the change in the discharge coefficient is approximately proportional to $\beta^{-1}\Delta\lambda$, where $\Delta\lambda$ is the change in the pipe friction factor and $1 \approx 4$ (Reference 24). A simple integral of this expression together with the orifice Reynolds number term gives

$$C_s = b_1(10^6\beta/Re_D)^{n_1} + b_2\beta^1\lambda. \quad (19)$$

This term is adequate for high Reynolds number but for practical use it requires three further enhancements. There is no data on the effect of rough pipework on the discharge coefficient for low Reynolds number, and a better fit to the database is obtained by including an additional term proportional to A , on the basis that, as the tapping terms begin to change, so may the dependence on friction factor:

$$C_s = b_1(10^6\beta/Re_D)^{n_1} + (b_2 + b_3A)\beta^1\lambda. \quad (20)$$

λ is an inconvenient variable with which to work, but for the pipes used in collecting the data in the EEC/API database a typical pipe roughness as a function of Re_D can be determined; so typical values of λ as a function of Re_D can be calculated, and λ can be approximated by a constant (which leads to a term to be absorbed into C_∞ and a small term which is neglected) together with a reciprocal power of Re_D :

$$C_s = b_1(10^6\beta/Re_D)^{n_1} + (b_2 + b_3A)\beta^1(10^6/Re_D)^{n_2}. \quad (21)$$

It is also necessary to make provision for transition from turbulent to laminar flow since, except for very low β , the gradient of the discharge coefficient as a function of a reciprocal power of Re_D is very different below a transition point in the range $Re_D = 3000 - 4500$ from that above it. This change of gradient occurs because the velocity profile changes very rapidly as the flow changes from turbulent to laminar, and when the velocity profile term is extended so that it can be used below the fully turbulent range the slope term becomes:

$$C_s = b_1(10^6\beta/Re_D)^{n_1} + (b_2 + b_3A)\beta^1 \max\{(10^6/Re_D)^{n_2}, c_1 - c_2(Re_D/10^6)\}. \quad (22)$$

It remains to determine the constants and exponents in equations (17) and (22). m_2 is equal to 8 in both the Stolz equation (Reference 22) and that adopted in New Orleans (Reference 14) and using this high value gives a good representation of the rapid decrease in C_∞ for high β . Previously m_1 has been taken to be 2, but the optimum value of m_1 , in terms of the lowest standard deviation of the data about the equation, lies between 1.2 and 1.3: the same value as the exponent of β in the downstream tapping term (equation (5)) is used. $n_1 = 0.7$ and $n_2 = 0.3$ give the optimum fit to the complete database. 1 is taken to be 3.5 rather than 4 because it gives a better fit to the complete database: in terms of the dependence of the effect of rough pipework on β an exponent of 3.5 is as acceptable as 4 (Reference 12). As stated in section 3.2 the mean Re_D at which the flow becomes fully turbulent was taken to be 3700. This gives c_1 in terms of c_2 . c_2 is obtained by trying appropriate values in turn and obtaining the best overall fit: $c_2 = 4800$ gives an excellent overall fit. Given the tapping terms in equations (7) and (9) and the small orifice diameter term in equation (14) a least-squares fit of the complete database was performed: on rounding the constants, the C_∞ and slope terms become

$$C_{\infty} + C_s = 0.5934 + 0.0232\beta^{1.3} - 0.2010\beta^8 \\ + 0.000515(10^6\beta/Re_D)^{0.7} \\ + (0.0187 + 0.0400A)\beta^{3.5}\max\{(10^6/Re_D)^{0.3}, 23.1 - 4800(Re_D/10^6)\}. \quad (23)$$

6 THE FINAL EQUATION AND ITS QUALITY OF FIT

The complete equation can be brought together from equations (7), (9), (14) and (23) and is as follows:

$$C = 0.5934 + 0.0232\beta^{1.3} - 0.2010\beta^8 \\ + 0.000515(10^6\beta/Re_D)^{0.7} \\ + (0.0187 + 0.0400A)\beta^{3.5}\max\{(10^6/Re_D)^{0.3}, 23.1 - 4800(Re_D/10^6)\} \\ + (0.043 + (0.090 - A)e^{-10L_1} - (0.133 - A)e^{-7L_1})(1 - A)\frac{\beta^4}{1 - \beta^4} \\ - 0.031(M_2' - 0.8M_2'^{1.1})\{1 + 8\max(\log_{10}(3700/Re_D), 0.0)\}\beta^{1.3} \\ + 0.0015\max\left(\frac{50}{\beta D} - 1, 0\right), \quad (D: \text{mm}) \quad (24)$$

where

$$M_2' = \frac{2L_2'}{1 - \beta},$$

and

$$A = \left(\frac{2100\beta}{Re_D}\right)^{0.9}.$$

L_1 is the quotient of the distance of the upstream tapping from the upstream face of the plate and the pipe diameter, and

L_2' is the quotient of the distance of the downstream tapping from the downstream face of the plate and the pipe diameter.

Its quality of fit is quantified in Tables 1 to 4. Table 1 gives a description of the meaning of the different lines in Tables 2 to 4. These tables give the deviations of the data about equation (24) as a function of β , D , Re_D and pair of tappings used and of certain combinations of these. The tappings described as Corner (GU) are tappings in the corners which were designed by Gasunie and are simpler to make than those in ISO 5167-1. They are described in Reference 7. The standard deviation of all the data (including those for $Re_D < 4000$) about the equation is 0.269 per cent and the deviations are well balanced as functions of β , D , Re_D and pair of tappings used. Tables giving deviations as a function of other combinations of independent variables and for each laboratory which collected the data are given in Reference 12. It can be seen that the standard deviation increases for small d , large β , and small Re_D . If the

8515 data points for $0.19 < \beta < 0.67$, $Re_d > 30000$ and $d > 50$ mm are analysed the standard deviation of the points about the equation is 0.208 per cent.

7 CONCLUSIONS

The orifice plate discharge coefficient equation has been revised in the light of the complete EEC/API database including the data collected in 50 mm and 600 mm pipes. There are two principal changes to the equation accepted at New Orleans: using the additional data collected in 50 mm pipes improved tapping terms for low Reynolds number have been calculated; and an additional term for small orifice diameter has been obtained, and its physical basis in orifice edge roundness given. The revised orifice plate discharge coefficient equation is given as equation (24): the deviations of the database from the equation have been tabulated and shown to be well balanced as functions of β , D , Re_d and pair of tappings used.

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T A B L E 1

GENERAL INFORMATION ABOUT THE ANALYSIS OF RESIDUALS IN TABLES 2 TO 4

For each cell, line 1 - Mean per cent error
line 2 - Per cent standard deviation
line 4 - Number of observations
line 5 - Per cent standard deviation to model.

For the i^{th} point in a cell Per cent error, $P_i = \frac{(C_{im} - C_{ie})}{C_{im}} \times 100$,

where C_{im} is the measured discharge coefficient of the i^{th} point
and C_{ie} is the corresponding discharge coefficient from equation
(24).

$$\text{Mean per cent error, } \mu = \frac{\sum_{i=1}^N P_i}{N}$$

where N is the number of points in the cell.

$$\text{Per cent standard deviation} = \left(\frac{\sum_{i=1}^N (P_i - \mu)^2}{N - 1} \right)^{1/2}$$

$$\text{Per cent standard deviation to model} = \left(\frac{\sum_{i=1}^N P_i^2}{N} \right)^{1/2}$$

Statistics for the entire population appear in the bottom right hand cell.

T A B L E 2

RESIDUALS FROM EQUATION (24) AS A FUNCTION OF β AND D

D (mm) β	50	75	100	150	250	600	Summary by β
0.100 (0.0991 to 0.1028)	0.000 0.000 - 0 0.000	0.000 0.000 - 0 0.000	0.000 0.000 - 0 0.000	-0.060 0.230 - 81 0.237	0.060 0.270 - 79 0.274	0.000 0.000 - 0 0.000	-0.001 0.257 - 160 0.256
0.200 (0.1982 to 0.2418)	0.055 0.462 - 507 0.465	-0.435 0.125 - 57 0.452	-0.125 0.226 - 645 0.258	0.094 0.106 - 111 0.141	-0.020 0.186 - 714 0.187	0.176 0.238 - 394 0.296	-0.005 0.305 - 2428 0.305
0.375 (0.3620 to 0.3748)	0.184 0.303 - 444 0.354	-0.197 0.099 - 106 0.220	-0.012 0.259 - 429 0.259	0.123 0.254 - 133 0.281	-0.119 0.159 - 439 0.198	0.022 0.113 - 591 0.115	0.015 0.241 - 2142 0.242
0.500 (0.4825 to 0.5003)	0.065 0.294 - 398 0.300	0.027 0.055 - 69 0.061	0.099 0.198 - 285 0.221	0.040 0.100 - 109 0.107	0.001 0.164 - 392 0.163	-0.101 0.119 - 526 0.156	0.004 0.205 - 1779 0.205
0.570 (0.5427 to 0.5770)	-0.024 0.386 - 348 0.386	0.050 0.076 - 72 0.091	0.010 0.232 - 992 0.233	-0.049 0.109 - 136 0.120	0.059 0.258 - 1123 0.264	-0.110 0.120 - 567 0.162	0.001 0.249 - 3238 0.249
0.660 (0.6481 to 0.6646)	-0.025 0.287 - 498 0.287	0.119 0.102 - 64 0.156	-0.007 0.236 - 627 0.236	-0.120 0.128 - 92 0.175	0.072 0.190 - 823 0.203	-0.123 0.151 - 643 0.195	-0.015 0.224 - 2747 0.225
0.750 (0.7239 to 0.7509)	-0.023 0.322 - 866 0.322	0.114 0.105 - 101 0.155	0.105 0.312 - 971 0.329	0.097 0.203 - 130 0.225	-0.005 0.315 - 1478 0.315	-0.238 0.359 - 336 0.430	0.005 0.326 - 3882 0.326
Summary by D	0.031 0.353 - 3061 0.354	-0.045 0.210 - 469 0.215	0.013 0.266 - 3949 0.266	0.027 0.192 - 792 0.194	0.011 0.250 - 5048 0.251	-0.063 0.217 - 3057 0.226	0.001 0.269 - 16376 0.269

T A B L E 3

RESIDUALS FROM EQUATION (24) AS A FUNCTION OF D AND PAIR OF TAPPINGS

Tappings D (mm)	Corner (ISO)	Flange	D&D/2	Corner (GU)	Summary by D
50	0.080	0.013	0.020	0.000	0.031
	0.403	0.310	0.383	0.000	0.353
	-	-	-	-	-
	728	1605	728	0	3061
	0.410	0.311	0.383	0.000	0.354
75	0.000	-0.045	0.000	0.000	-0.045
	0.000	0.210	0.000	0.000	0.210
	-	-	-	-	-
	0	469	0	0	469
	0.000	0.215	0.000	0.000	0.215
100	-0.003	-0.002	0.061	0.000	0.013
	0.224	0.254	0.323	0.000	0.266
	-	-	-	-	-
	1084	1932	933	0	3949
	0.224	0.254	0.328	0.000	0.266
150	0.000	0.027	0.000	0.000	0.027
	0.000	0.192	0.000	0.000	0.192
	-	-	-	-	-
	0	792	0	0	79
	0.000	0.194	0.000	0.000	0.19
250	0.042	-0.027	0.006	0.056	0.011
	0.237	0.210	0.266	0.305	0.250
	-	-	-	-	-
	1155	1841	1167	885	5048
	0.241	0.212	0.266	0.310	0.251
600	-0.153	0.025	-0.061	-0.084	-0.063
	0.213	0.171	0.217	0.256	0.217
	-	-	-	-	-
	828	876	1130	223	3057
	0.262	0.173	0.226	0.269	0.226
Summary by Tappings	-0.006	-0.001	-0.003	0.028	0.001
	0.281	0.242	0.296	0.301	0.269
	-	-	-	-	-
	3795	7515	3958	1108	16376
	0.281	0.242	0.296	0.302	0.269

T A B L E 4

RESIDUALS FROM EQUATION (24) AS A FUNCTION OF Re_D AND β

Re_D β	100 to 4000	4000 to 10^4	10^4 to 10^5	10^5 to 10^6	10^6 to 10^7	10^7 to 10^8	Summary by β
0.100 (0.0991 to 0.1028)	-0.111 0.183 - 52 0.213	0.069 0.243 - 49 0.250	0.038 0.293 - 59 0.293	0.000 0.000 - 0 0.000	0.000 0.000 - 0 0.000	0.000 0.000 - 0 0.000	-0.001 0.257 - 160 0.256
0.200 (0.1982 to 0.2418)	-0.003 0.552 - 237 0.551	-0.026 0.370 - 238 0.370	-0.082 0.246 - 1190 0.259	0.030 0.167 - 447 0.170	0.249 0.169 - 316 0.301	0.000 0.000 - 0 0.000	-0.005 0.305 - 2428 0.305
0.375 (0.3620 to 0.3748)	0.444 0.493 - 125 0.662	0.207 0.286 - 133 0.352	-0.012 0.199 - 748 0.199	-0.092 0.147 - 671 0.174	0.033 0.085 - 325 0.091	0.066 0.083 - 140 0.106	0.015 0.241 - 2142 0.242
0.500 (0.4825 to 0.5003)	0.027 0.769 - 33 0.758	-0.055 0.247 - 83 0.252	0.065 0.182 - 436 0.193	0.042 0.167 - 773 0.173	-0.128 0.136 - 205 0.186	-0.095 0.086 - 249 0.128	0.004 0.205 - 1779 0.205
0.570 (0.5427 to 0.5770)	-0.507 0.965 - 18 1.066	-0.320 0.361 - 59 0.480	-0.020 0.238 - 502 0.238	-0.009 0.196 - 1420 0.196	0.064 0.272 - 776 0.279	0.008 0.225 - 463 0.225	0.001 0.249 - 3238 0.249
0.660 (0.6481 to 0.6646)	-0.030 0.116 - 5 0.108	-0.296 0.391 - 35 0.486	-0.069 0.283 - 466 0.291	0.035 0.203 - 1110 0.206	0.045 0.187 - 471 0.192	-0.087 0.180 - 660 0.200	-0.015 0.224 - 2747 0.225
0.750 (0.7239 to 0.7509)	0.000 0.000 - 0 0.000	0.353 0.443 - 78 0.565	-0.067 0.339 - 615 0.345	0.042 0.299 - 1668 0.302	-0.018 0.310 - 1045 0.310	-0.038 0.358 - 476 0.360	0.005 0.326 - 3882 0.326
Summary by Re_D	0.086 0.593 - 470 0.599	0.027 0.392 - 675 0.393	-0.039 0.257 - 4016 0.260	0.013 0.225 - 6089 0.226	0.037 0.260 - 3138 0.263	-0.043 0.239 - 1988 0.243	0.001 0.269 - 16376 0.269

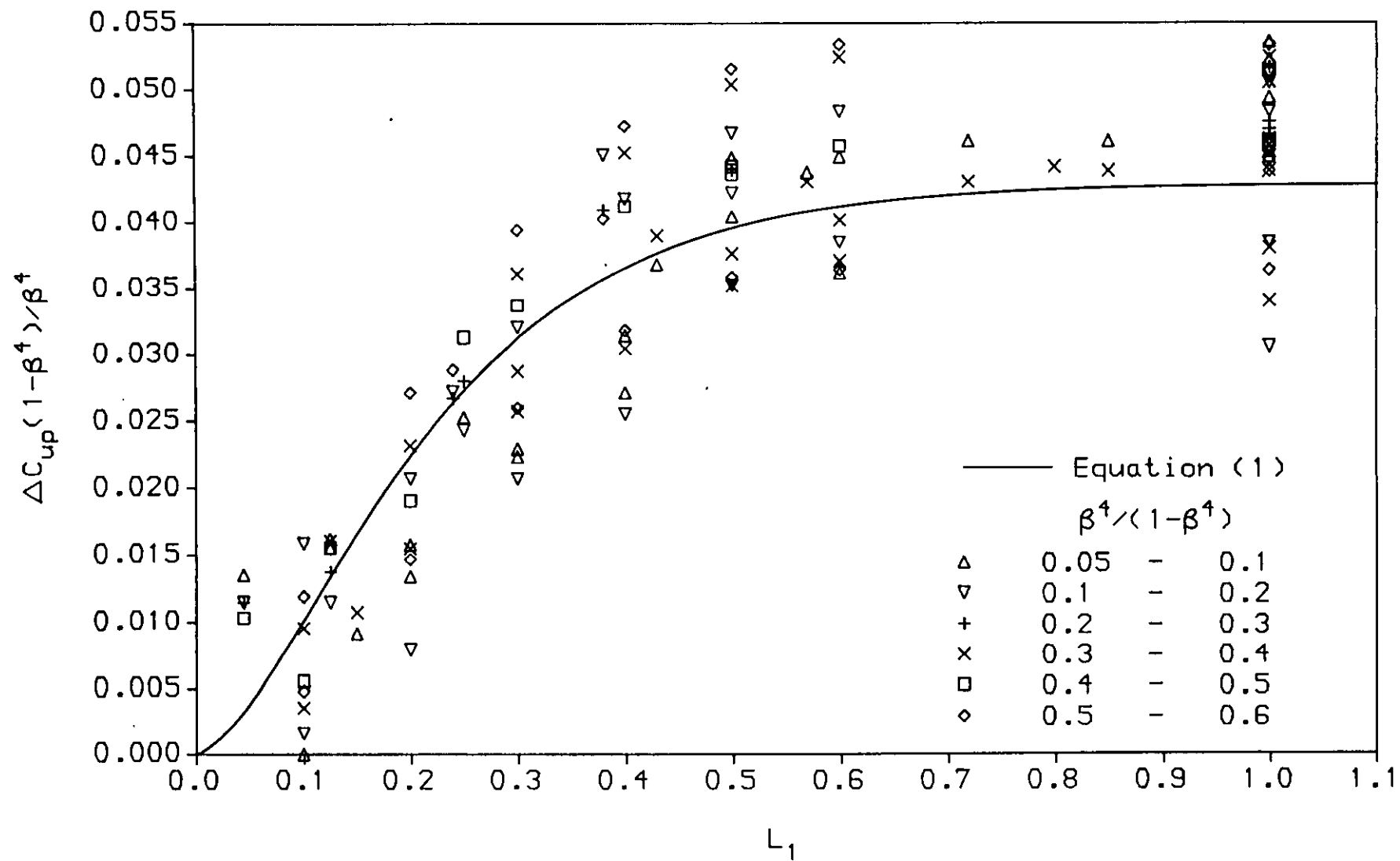


FIG 1 Upstream tapping term as a Function of L_1

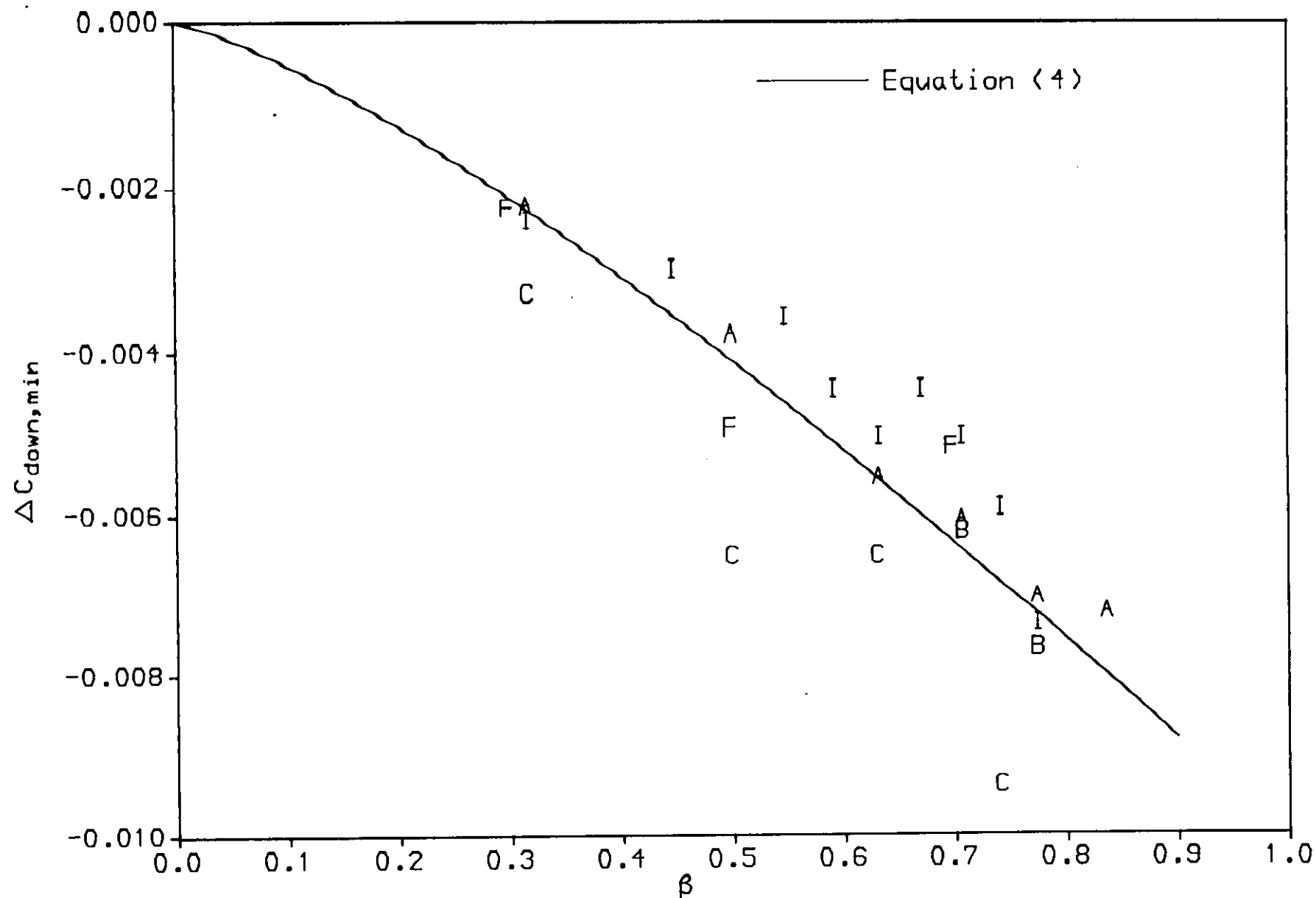


FIG 2 Downstream tapping term at the pressure minimum

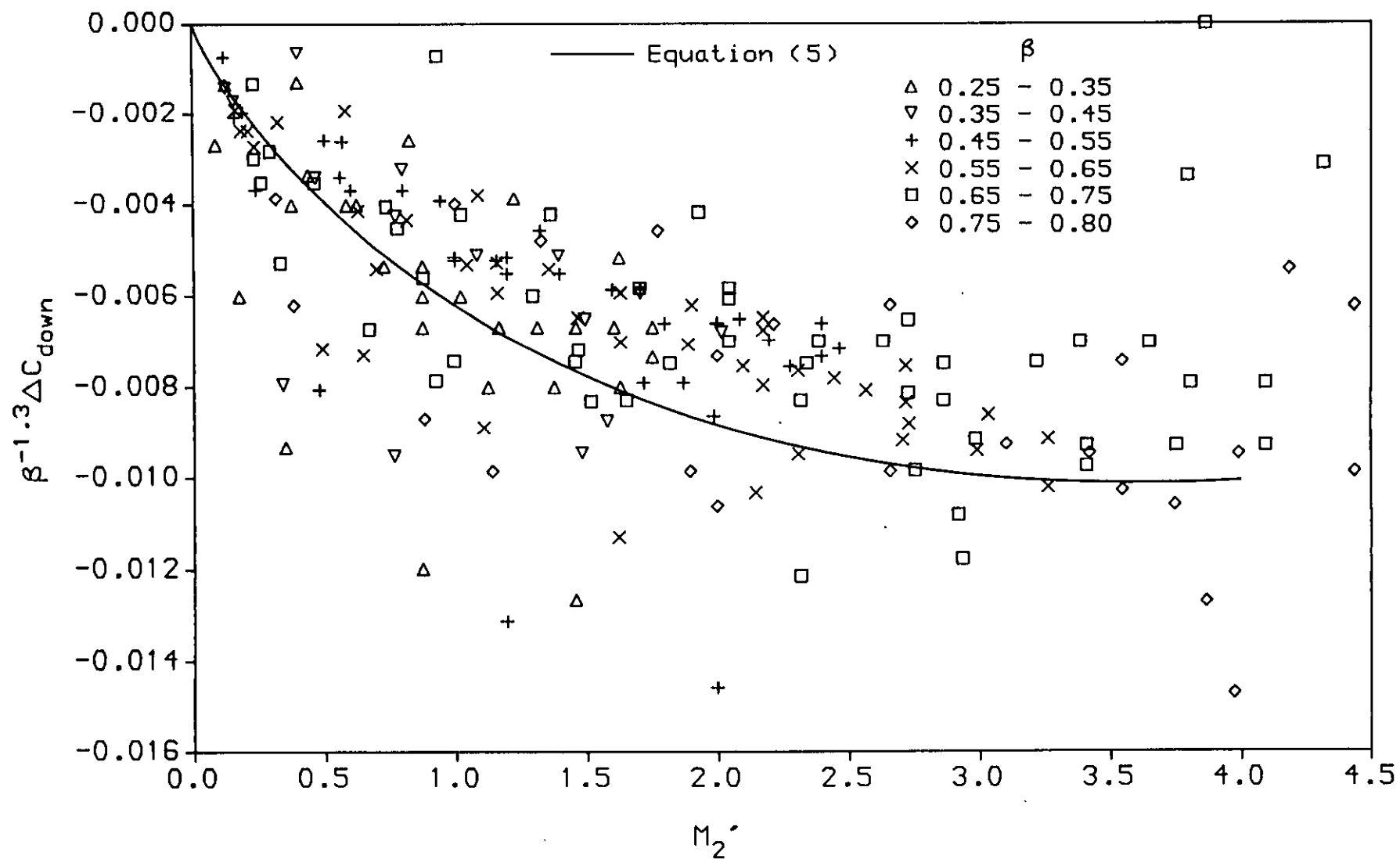


FIG 3 Downstream tapping term as a Function of M_2'

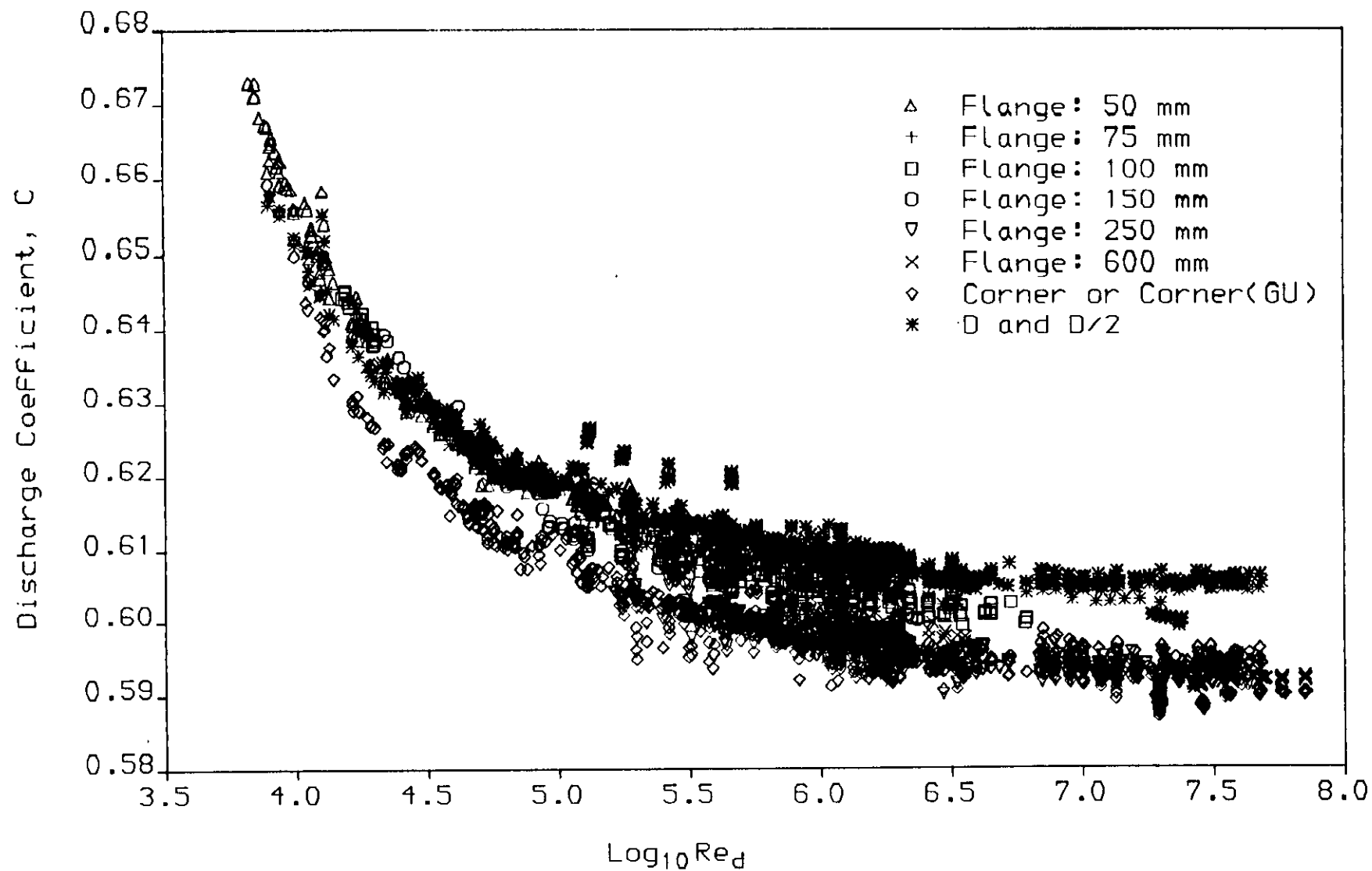


FIG 4 All data for $\beta = 0.74$

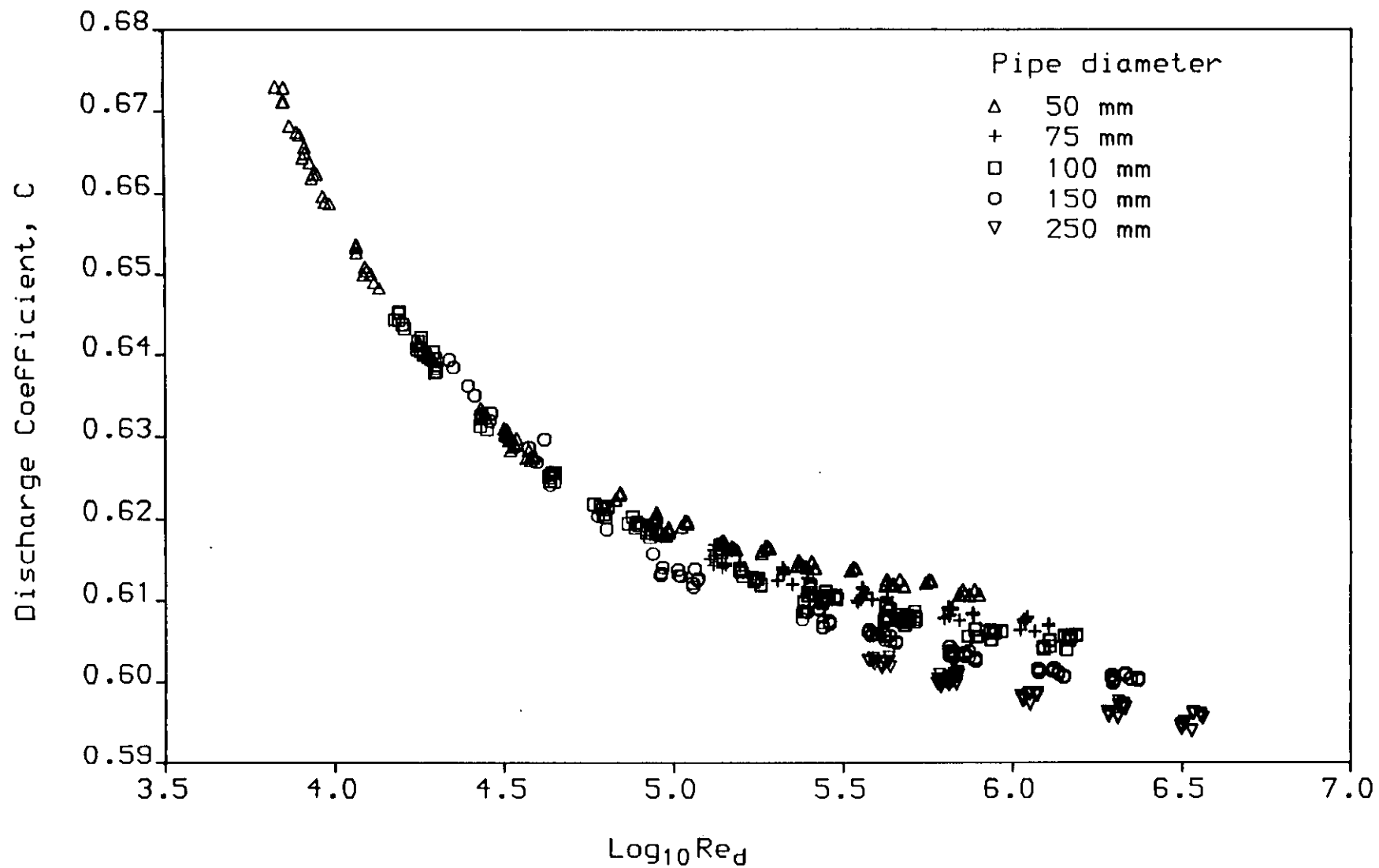


FIG 5 US data (Flange tappings) for $\beta = 0.74$

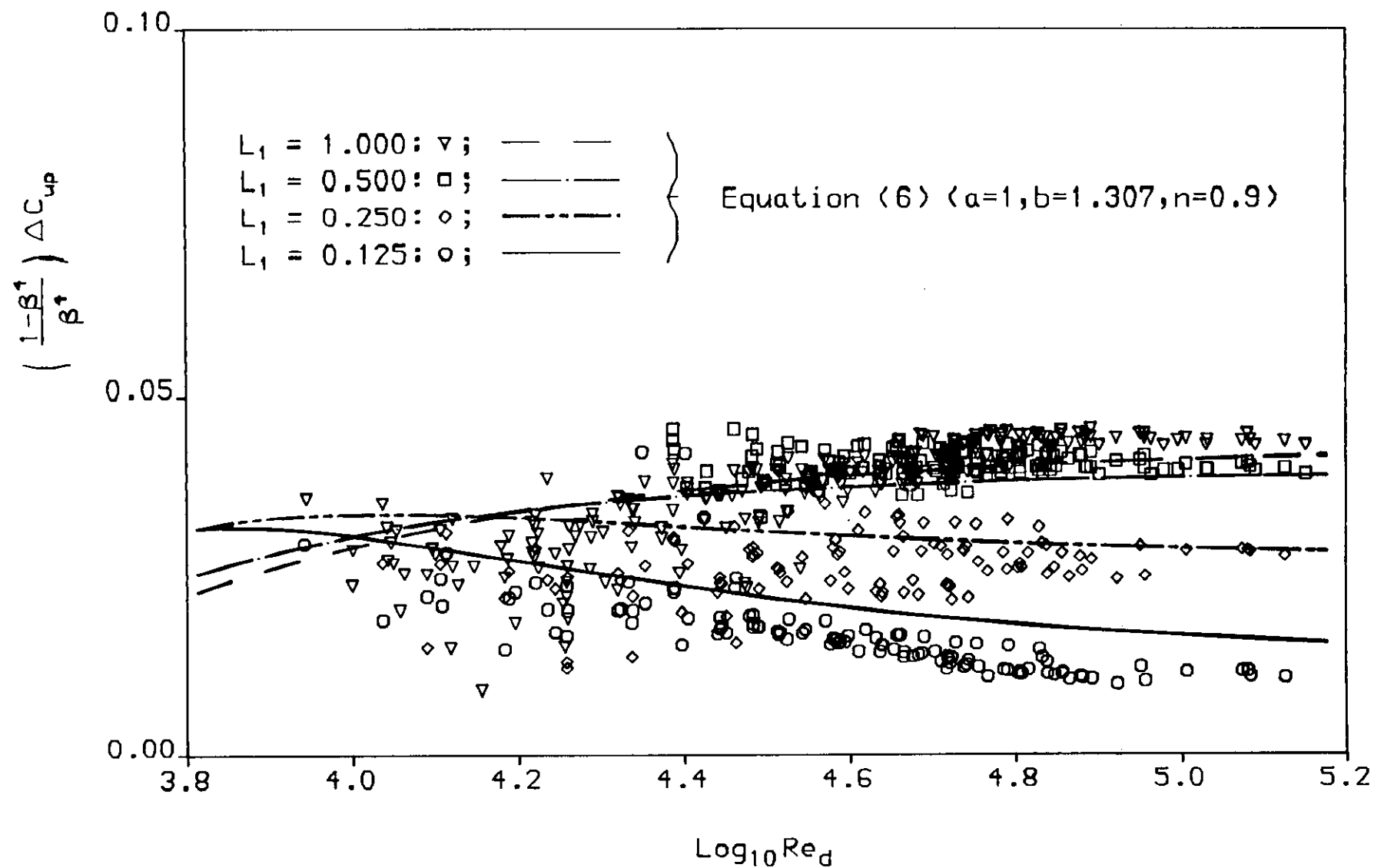


FIG 6 Upstream Tapping Term for low Re_D

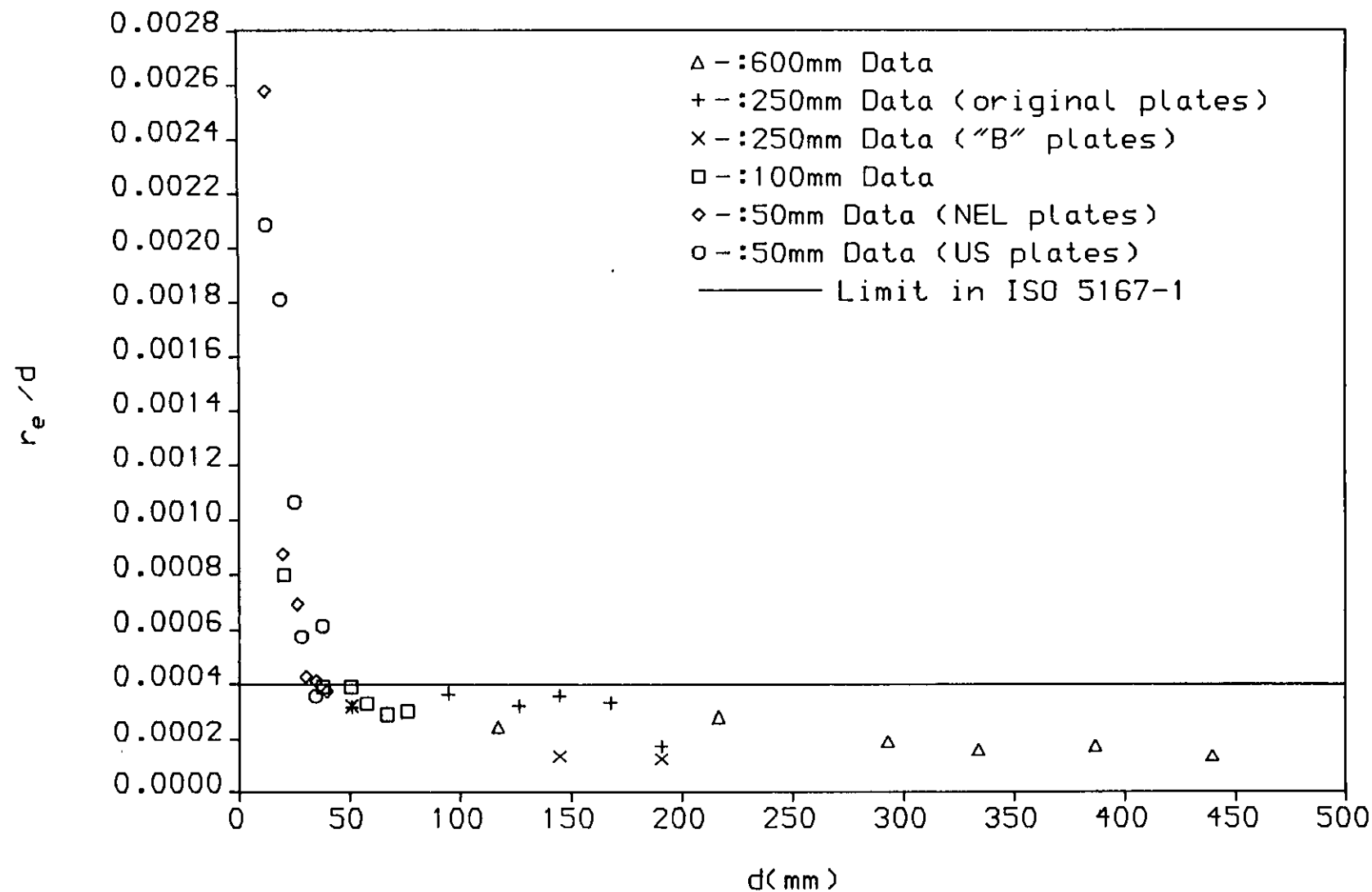


FIG 7 r_e/d as a function of d

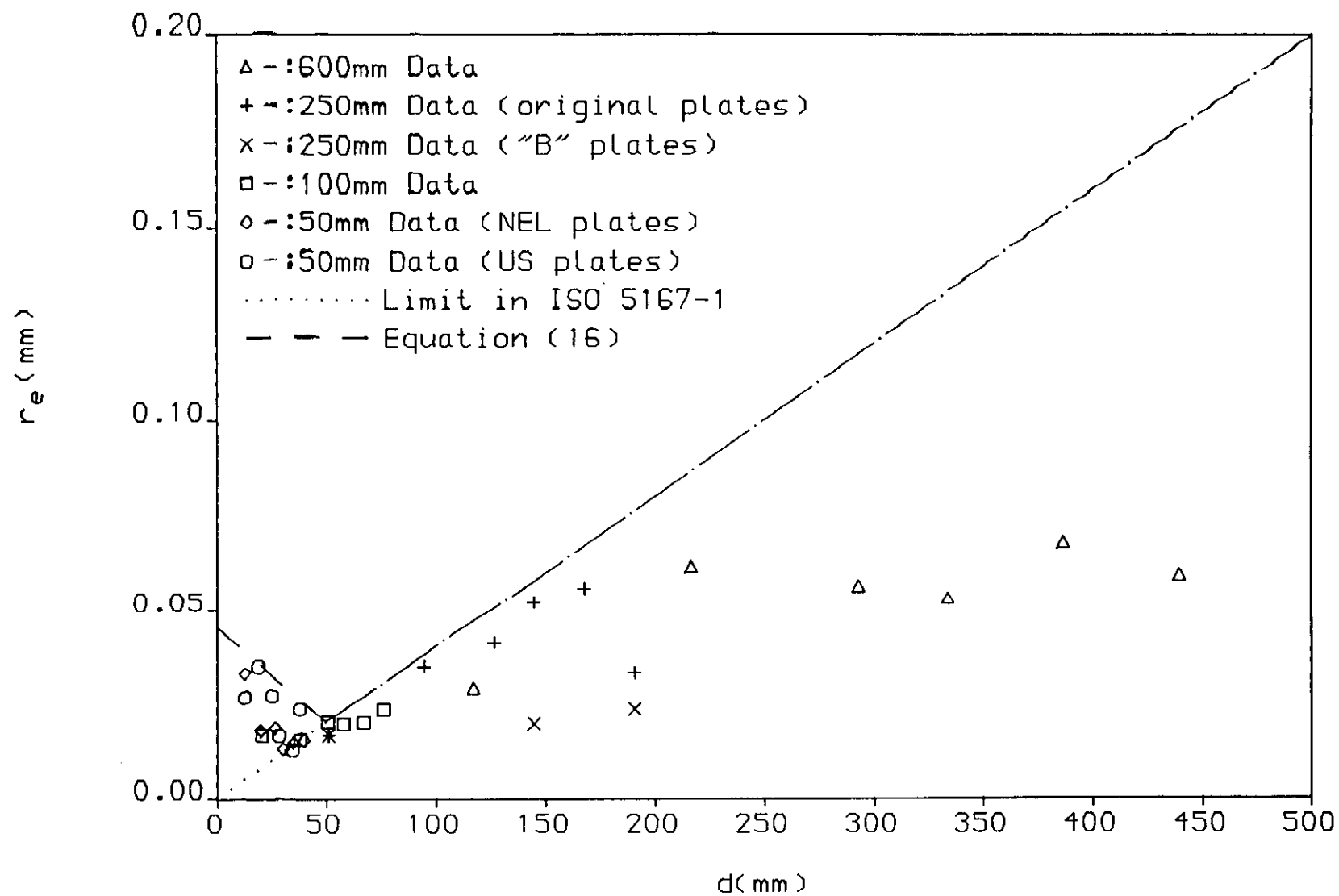


FIG 8 r_e as a Function of d

References

[1] Paper presented at the North Sea Flow Measurement Workshop, a workshop arranged by NFOGM & TUV-NEL

Note that this reference was not part of the original paper, but has been added subsequently to make the paper searchable in Google Scholar.