NSFMW.97

North Sea FLOW Measurement Workshop

Paper 11: 2.5

RECENT DEVELOPMENTS IN THE UNCERTAINTY ANALYSIS OF FLOW MEASUREMENT PROCESSES

Authors:
Jos G. M. van der Grinten, Nmi Certin B.V. The Netherlands

Organiser:
Norwegian Society of Chartered Engineers
Norwegian Society for Oil and Gas Measurement
Co-organiser:
National Engineering Laboratory, UK

RECENT DEVELOPMENTS IN THE UNCERTAINTY ANALYSIS OF FLOW MEASUREMENT PROCESSES

Jos G.M. van der Grinten

NMi Certin B.V., The Netherlands

SUMMARY

For the evaluation of uncertainties in flow measurement two different documents are used at present. These are the "Guide to the expression of uncertainty in measurement" [1] and ISO 5168 [2]. The Guide represents the present state of the art of uncertainty analysis. ISO 5168 is much older, is still under revision [3] and is familiar to most people in the flow community. Although the working methods for evaluating measurement uncertainties are basically the same, the two documents are based on significantly different views on measurement uncertainty [7]. So the dilemma for the experimentalist is whether to use the Guide with which he may not be familiar, or to use ISO/DIS 5168 [3] which is subject to change.
This paper discusses the developments in uncertainty analysis in past decades, the agreements and differences between the Guide and ISO 5168, and will illustrate by an example how simple uncertainty analysis can be if all unnecessary terminology can be avoided.
From the present analysis the following conclusions are drawn. Currently, the Guide is the only accepted document that is neither expired nor under revision. Uncertainty is a measure for the amount of missing information. If corrections are not applied, information is discarded and the resulting 2s-uncertainty is twice the absolute value of the deviation.

1. RECENT DEVELOPMENTS

In the past decades the view on measurement uncertainty has changed significantly which has lead to a new document on uncertainty analysis [1] shortly referred to as the Guide. Despite the fact that this document has been circulated extensively for comments, the Guide has obtained little publicity in the flow community. Instead, ISO 5168 [2,3] is the document which is more familiar to people working in the field of flow measurement. The original ISO 5168 (1978) [2] is currently being revised. The latest draft of this revised standard dates from 1989 [3]. However, the latter document and later revisions of ISO 5168 were never accepted by ISO, because these documents on uncertainty analysis were not in line with the Guide. This year a new call was made to the national standardization organisations for volunteers to write a new ISO 5168. This means that several years will be required before a new ISO 5168 will be published.
So the dilemma for the flow experimentalist whether to use the Guide [1] with which he may not be familiar, or to use ISO/DIS 5168 [3] which was rejected, is now solved. The

Guide is the only available document on measurement uncertainties. Moreover, before a new version of ISO 5168 will be accepted in the future, it must be in line with the Guide.

Since the new philosophy on evaluation of measurement uncertainty is described in the Guide, quite some reading is required before one can make an uncertainty analysis. A more readable document on measurement uncertainty is WECC Doc. 19 [8]. This document published in 1990, is based on the same principles [9] as the Guide. However, this document is also currently under revision [10] to bring it in full agreement with the Guide. Fortunately, in the field of flow measurement a few examples [4,5,7] exist at present in which the uncertainty analysis is performed according to the Guide.

In the remainder of this paper the following items will be discussed:
1. The new view on measurement uncertainty.
2. Additional uncertainties due to not correcting for known deviations.
3. Terminology and procedures to be used in uncertainty analysis.
4. A practical example of uncertainty analysis of a flow measurement process.

2. INFORMATION AND UNCERTAINTY

Missing information
The most important new view in uncertainty analysis is that uncertainty is a measure for the amount of missing information. By taking measurements we make observations of the object or system from which we want information. The more measurements we take, the more information we retrieve. Since the measurement object may be subject to change and since there is a minimum interval for taking measurements, this inherently implies that it is not possible to collect the complete information available for any system. For that reason it is useful to introduce a quantity that expresses the amount of missing information: *uncertainty*.

The definition of uncertainty used in the Guide [1] and the VIM [6] is *a parameter, associated with the result of a measurement, that characterizes the dispersion of values that could reasonably be attributed to the measurand.* In fact, this definition is the reality that we miss information when taking measurements. For that reason, in stead of a single value, a range of values can be attributed to the measurand. If we do not take measurements there is no information at all and consequently a high degree of uncertainty is experienced. If measurements are taken this uncertainty is reduced. This will be illustrated by a very simple example.

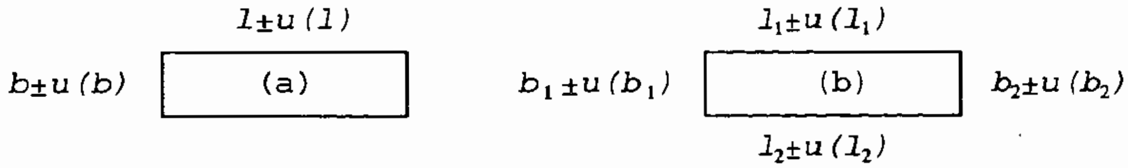

Fig. 1: Circumference of a rectangle measured by 2 independent measurements (a) and by 4 independent measurements (b).

Consider a rectangle of which we want to determine the circumference. In order to do so, a minimum of two independent measurements is required to determine length (l) and width (b) of the rectangle. In addition the opposite length and width can be measured, so there are four independent measurements. The two methods are schematically displayed in figures 1a and 1b, respectively. All dimensions are measured only once using one single instrument. The uncertainties are also indicated in figure 1. The uncertainty of l and b are denoted by $u(l)$ and $u(b)$ respectively. The circumferences are
The uncertainties in the circumferences are obtained by taking the root sum square of the

$$O_a = 2(l+b) \quad , \quad O_b = l_1+b_1+l_2+b_2 \qquad (1)$$

uncertainties of the measured lengths and widths.

$$u(O_a) = 2\sqrt{u^2(l)+u^2(b)} \quad , \quad u(O_b) = \sqrt{u^2(l_1)+u^2(b_1)+u^2(l_2)+u^2(b_2)} \qquad (2)$$

Since all dimensions are measured using one instrument, we will assume all uncertainties in l, b, l_1, b_1, l_2, b_2, to be equal to $u(l)$. Using this substitution the following result is obtained

$$u(O_a) = 2\sqrt{2}\,u(l) \quad , \quad u(O_b) = 2\,u(l) \qquad (3)$$

The result is that the uncertainty of the circumference in the second case is smaller than in the first situation. The reason is that four independent measurements contain more information than two independent measurements, since one assumption, i.e. $l_1=l_2$ and $b_1=b_2$, has been abandoned.

The relationship between information and uncertainty is not unique to metrology. In other fields, e.g. statistical physics and thermodynamics (Mach [11]), a relationship between uncertainty (standard deviation) and amount of missing information (entropy) exists.

Uncertainty due to not applying corrections

One of the basic assumptions in the Guide is that known deviations are corrected for. However, in many measurement corrections are not applied, e.g. weighing measurements in shops, domestic gas meters at home, and fuel dispensers. Since many measurements in field applications are not corrected for deviations, it is rather unfortunate that this is not discussed in the Guide. From the previous discussion it is understood that if corrections are not applied, information is discarded. So, this results in an additional uncertainty.
The formal derivation of the relationship between uncertainty and the deviation not corrected for, is given in the appendix to this paper. The result is given here.

If a known deviation is not corrected for, this results in an extra uncertainty equal to twice the absolute value of the deviation.

The factor 2 is the consequence of the fact that uncertainties are represented as the equivalent of double standard deviations.

3. TERMINOLOGY TO BE USED IN UNCERTAINTY ANALYSIS

The difference in terminology used in the Guide [1] and ISO 5168 [2,3] is the result of the difference in philosophy between the two documents. An extensive comparison of these documents is made by Van der Grinten [7]. A brief summary will be given here.
For ISO 5168 the concept of true value plays an important role. The *true value* is an ideal value which is assumed to exist and which would be obtained if all imperfections of the measurement process could be eliminated. The *error* is the difference between the measured and true values of the quantity measured. Please, note that by this definition the true value cannot be measured which implies that the error is unknowable.
Instead of the true value the Guide uses the *reference value*, i.e. the value which is obtained by using a calibrated instrument. The *deviation* is the difference between the value measured and a reference value. Please, note that the latter two definitions are more practical than the definitions used by ISO 5168.
In practice, most people use the word true value in the sense of reference value and the word error in the sense of measured deviation.
In the view of the Guide *uncertainty* is a measure for the information missing in the measured value. In fact the uncertainty analysis as presented in the Guide is based on the notion of maximizing the amount of missing information. On the other hand ISO 5168 aims at maximization of the known information. These starting points are implied by the definitions used for uncertainty. According to the Guide the uncertainty interval is the range of values which could be attributed to the measurand. According to ISO 5168 the uncertainty interval is the range of values in which the true value of the measurand lies.
So the terminology needed for the uncertainty analysis of practical measurements, consists of four concepts: measured value, reference value, deviation and uncertainty (in the sense of missing information).

The evaluation of uncertainties has to be performed in two ways. Type A evaluation of uncertainties is the method by which the available information is reduced by statistical means. Here averaging or regression methods lead to a calculable standard deviation. The result of data reduction is that information is thrown away. As a result uncertainty is returned. Type B evaluation of uncertainties involves the application of all other means. Here a thorough knowledge of the measurement process and often the theoretical skill of the experimentalist is required to estimate equivalent standard deviations. In fact this is the result of the question "What mount of information is missing?".

Throughout this paper all uncertainties are represented as equivalents of double standard deviations, a method preferred by the Guide.

4. PROCEDURES FOR EVALUATING UNCERTAINTIES

The Guide evaluates uncertainties using the following procedure which is very much equal to ISO 5168. Differences in the procedures followed by the Guide and ISO 5168 are discussed by Van der Grinten [7]. The procedure contains the following steps:

1. Express mathematically the relationship between the measurand and the input quantities. This relationship should contain all quantities, including corrections and correc-

tion factors, that can contribute significantly to the uncertainty of the result of measurement.

2. Determine estimated values of all input quantities, either based on statistical analysis or by other means.
3. Evaluate the uncertainty estimate of all input estimates, both by statistical means (type A) and by other means (type B). Document all uncertainty sources for each input estimate.
4. Evaluate the covariances associated with any input quantities that are correlated. This step can be avoided by complete substitution of all variables in the mathematical model.
5. Calculate the result of measurement from the functional relationship, using the estimates of all input quantities.
6. Calculate all sensitivity coefficients i.e. the partial derivatives of the measurand with respect to each of the input quantities. Combine all uncertainties. The easiest way is to put everything in a table as shown in the example later in this paper.

The procedure for computing propagation of uncertainties is briefly reviewed here.
Assume, the dependency of a quantity measured y of m variables x_i ($i=1..m$) is described with a function f:

$$y = f(x_1, x_2, \ldots, x_m) \tag{4}$$

The absolute uncertainty in each of the quantities x_i is $u(x_i)$. If all quantities x_i are mutually independent the uncertainty in the quantity y is described by

$$u^2(y) = u_0^2(y) + \sum_{i=1}^{m} c_i^2 \, u^2(x_i) \tag{5}$$

where c_i is the sensitivity factor [1,2,3,7,8,10], the partial derivative of f with respect to x_i

$$c_i = \frac{\partial f}{\partial x_i} \tag{6}$$

The uncertainty $u_0(y)$ is the result from repeated observations of the measurand.
For convenience later on, the quantity $u_i(y)$ is defined which is the uncertainty contribution in y due to the individual quantity x_i.

$$u_i(y) = c_i \, u(x_i) \tag{7}$$

Of course the uncertainty $u(y)$ is the root sum square of the uncertainty contributions $u_i(y)$.

5. MATHEMATICAL MODEL OF THE MEASUREMENT PROCESS

In order to demonstrate the use of uncertainties in a practical application an example will be discussed: application of a turbine gas meter for custody transfer purposes. In order to convert the line volume into a normal volume V_n (1013,25 mbar and 0°C) a flow computer is used with temperature and pressure input. The set-up is schematically displayed in figure 2. Calibration curves of the instruments are programmed into the flow computer. This means that corrections are made for known deviations. For calculation of the real gas factor the S-GERG algorithm is used.

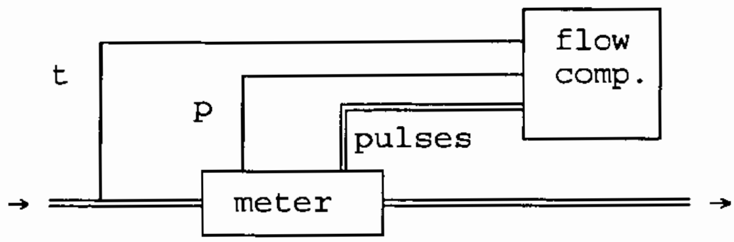

Fig. 2: Schematic set-up of a custody transfer metering system for high-pressure natural gas.

In the procedure of uncertainty evaluation, a mathematical model is set up. Since metering flow is a continuous process, split up in many small time intervals, all process conditions are stationary. Also it is assumed that the piping and the meter casing are completely rigid and dimensions do not vary with the changes in temperature and pressure. So, conversion from actual volume increment to an increment in normal volume is based on conservation of mass in each time interval:

$$\rho_1 \, \Delta V_1 = \rho_2 \, \Delta V_2 \tag{8}$$

in which ρ is the density and ΔV is the volume increment. The indices 1 and 2 refer to actual conditions and normal conditions, respectively.
Densities are obtained using the state equation of a real gas:

$$P_i = Z_i \rho_i R_a T_i \qquad (i = 1, 2) \tag{9}$$

P is the absolute pressure, T the absolute temperature, R_a the specific gas constant for air and Z_i the real gas constant or super compressibility, depending on P, T and the gas composition.

The quantity determined during calibration of the instruments, is the relative deviation of the turbine gas meter e_m. This quantity is defined as the relative difference of the quantity indicated by the instrument and the quantity at the instrument:

$$e_m = \frac{\Delta V_m}{\Delta V_1} - 1 \tag{10}$$

where the volume increment ΔV is obtained from the number of pulses ΔN collected in the time interval and the impulse factor of the meter I:

$$\Delta V_m = \frac{\Delta N_m}{I_m} \tag{11}$$

All temperatures are measured in °C, so

$$T_1 = T_0 + t_1 \quad , \quad T_2 = T_0 \tag{12}$$

Another assumption is that pressure and temperature readings are corrected for know deviations. This means that t_1, P_1 and P_2 can replaced by t_m, P_m and P_n, respectively. Substitution of equations (10)-(12) into the mass balance equation (8) leads to:

$$\Delta V_n = \frac{P_m}{P_n} \frac{Z_n}{Z_m} \frac{T_0}{T_0 + t_m} \frac{\Delta N_m}{I_m} \frac{1}{1 + e_m} \tag{13}$$

The total converted volume is the sum of all increments.

6. UNCERTAINTY EVALUATION

All instruments are calibrated in a standards laboratory. Calibration certificates issued by this laboratory state for all instruments measured deviations and the associated 2s-uncertainty, i.e. the uncertainty represented as the equivalent of a double standard deviation. The uncertainty statement of the calibration laboratory refers to calibration conditions. If an instrument is used in the field additional uncertainty sources may be present, e.g. due to varying environmental conditions.

The calibration uncertainty of the flowmeter is 0,27%. During initial verification of the flow computer it is observed that deviations of V_n are smaller than 0,15%. This means that there are known deviations not corrected for, that lead to an additional uncertainty of 0,30%. The uncertainty of the S-GERG algorithm is 0,10% [12].
Due to the temperature and pressure dependency of the real gas factor Z, covariances are introduced. These covariances are neglected for the following reason. In the ideal gas law there are linear contributions of P and T. In the real gas law Z is a correction to the ideal gas law. Values of Z vary between 0,7 and 1,0. The direct influence of temperature and pressure uncertainties via P and T in the real gas law will be much higher than the additional influence via Z.

In order to evaluate the uncertainty of the calibration result the uncertainty sources mentioned in the Guide [1] are listed below. The letters used to indicate the uncertainty source, will be used later in this section to make reference to. In the Guide the following uncertainty sources are distinguished:

a. incomplete definition of the measurand;
b. imperfect realization of the definition of the measurand;
c. nonrepresentative sampling - the sample measured may not represent the defined measurand;

d. effects of environmental conditions inadequately known or measured imperfectly
e. personal bias in reading analogue instruments;
f. finite instrument resolution or discrimination threshold;
g. inexact values of measurement standards and reference materials;
h. inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm;
i. approximation and assumptions incorporated in the measurement method and procedure;
j. variations in repeated observations of the measurand under apparently identical conditions.

The following uncertainty sources are present during the metering process described:
1 (d) additional uncertainties due environmental conditions differing from calibration conditions;
2 (g) uncertainty resulting from instrument calibration as stated on the certificate;
3 (j) fluctuation of pressures and temperatures in a single time interval;
4 (b) temperature gradients in the cross section of the pipe where the temperature is measured;
5 (f) pulses can be missed, however, this problem is avoided by using double pulse lines;
6 (i) uncertainty of the S-GERG algorithm;
7 uncertainty due to not applying corrections (in which the linear interpolation between the programmed calibration points is included).

For this high pressure gas flow metering process the uncertainty estimates are listed by variable in Table I. For each variable the measured value, sensitivity coefficient, applied uncertainty sources, and estimated uncertainties are listed. The uncertainty sources are indicated by numbers and letters which correspond to the uncertainty sources mentioned above. From this uncertainty analysis two dominant uncertainty sources appear, i.e. the deviation of the flow meter and the uncertainty due to not applying corrections.

7. CONCLUSIONS

From the previous discussion the following conclusions can be drawn.

- Currently, The Guide is the only available document to be referenced to for uncertainty analysis. Other documents must be in line with the Guide and are either under revision or not accepted as a standard. So, the experimentalist who want to make an uncertainty analysis of his measurement process, can better start using the Guide today than wait for other documents to be published.
- Uncertainty is a measure for the amount of missing information. This is the fundamental aspect in which the Guide differs from ISO 5168.
- One of the basic assumptions in the Guide is that known deviations are corrected for. If corrections are not applied, information is discarded. This results in an additional uncertainty equal to twice the absolute value of the deviation. The factor 2 is the consequence of the uncertainty representation chosen: the equivalent of double standard deviations.

Table I: Uncertainty estimates based on the Guide. For each variable the value, the sensitivity coefficient, the uncertainty of the input estimate and the uncertainty contribution to the deviation of the meter are listed.

variable x_i	value $[x_i]$	sensitivity coefficient c_i	c_i $[x_i]^{-1}$	source	$u(x_i)$ $[x_i]$	$u_i(\Delta V_m)$ $[m^3]$	$u_i(\Delta V)/\Delta V$ [-]
P_m (bar)	36,25	ΔV_n / P_m	2,8	1 (d) 2 (g) 3 (j)	0,0181 0,0181 0,0036	$7{,}2 \cdot 10^{-2}$	0,07%
P_n (bar)	1,01325			-			
Z_n	0,9997	ΔV_n / Z_n	$1{,}0 \cdot 10^{+2}$	6 (i)	0,000999	$1{,}0 \cdot 10^{-1}$	0,10%
Z_m	0,9752	$-\Delta V_n$ / Z_m	$-1{,}0 \cdot 10^{+2}$	6 (i)	0,000975	$1{,}0 \cdot 10^{-1}$	0,10%
T_0 (°C)	273,15			-			
t_m (°C)	15,35	$-\Delta V_n$ / (T_0+t_m)	$-3{,}5 \cdot 10^{-1}$	1 (d) 2 (g) 3 (j) 4 (b)	0,05 0,05 0,05 0,02	$3{,}1 \cdot 10^{-2}$	0,03%
ΔN_m	19384,63	ΔV_n / ΔN_m	$-2{,}8 \cdot 10^{-2}$		0	0	0,00%
I_m (m^{-3})	6646,16			-			
e_m	+0,25%	$-\Delta V_n$ / $(1+e_m)$	$-1{,}0 \cdot 10^{+2}$	2 (g)	0,0027	$2{,}7 \cdot 10^{-1}$	0,27%
ΔV_n (m^3)				7	0,303	$3{,}0 \cdot 10^{-1}$	0,30%
ΔV_n (m^3)	101,02					**$4{,}4 \cdot 10^{-1}$**	**0,43%**

- The terminology needed for uncertainty analysis consists of four concepts: measured value, reference value, deviation and uncertainty (in the sense of missing information).
- In the example of the high pressure gas metering system two dominant uncertainty sources appear, i.e. the deviation of the flow meter and the uncertainty due to not applying corrections.

ACKNOWLEDGEMENT

The author wishes to thank his colleague Charles M.E.E. Peters, member of ISO/TAG4/WG3, for his theoretical support and the stimulating discussions.

NOTATION

c	sensitivity coefficient $\partial f/\partial x_i$	
e	relative deviation	[-]
f	functional relationship between x_i and y	
g	distribution function representing our knowledge of x (appendix)	
I	impulse factor of a gas meter	[m^{-3}]
N	number of pulses counted	[-]
P	absolute pressure	[bar]
R_a	specific gas constant	[$Jkg^{-1}K^{-1}$]
T	absolute temperature	[K]
t	celsius temperature	[°C]
u	absolute uncertainty	
u_s	standard uncertainty, i.e. uncertainty represented as the equivalent of a single standard deviation	
V	volume	[m^3]
x	measured quantity	
x_i	input quantity	
y	measurand	
y	uncorrected measurement result (appendix)	
Z	real gas factor or (super)compressibility factor	[-]
z	expectation value, corrected measurement result (appendix)	

Greek

Δx	increment of quantity x	
ρ	mass density of the fluid	[kg/m^3]

Subscripts

1	at line conditions
2	at reference conditions
i	rank number of the input quantity
m	indicated at the metering site
n	at normal conditions, 1,01325 bar and 0°C

REFERENCES

1 BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML (1993): Guide to the expression of uncertainty in measurement, first edition, ISO 1993.

2 ISO 5168 (1978): Measurement of fluid flow - Estimation of uncertainty of a flow rate measurement, International Organization for Standardization, 1978.

3 ISO/DIS 5168 (1989): Measurement of fluid flow - Estimation of uncertainty of a flow rate measurement, International Organization for Standardization, revision of ISO 5168 (1978).

4 Grinten, J.G.M. van der (1993): The primary standard for gas flow measurement in the Netherlands, *in:* Proceedings of the 6th international conference on flow measurement, FLOMEKO '93, edited by S.D. Park and F.C. Kinghorn, pp 42-50.

5 Grinten, J.G.M. van der and Charles Peters (1994): Trend analysis, general model and application to high pressure gas flow standards, *in:* Proceedings of the IMEKO XIII conference, September 1994, Turin, Italy, pp 1161-1164a.

6 BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML (1993): International vocabulary of basic and general terms in metrology, second edition, ISO 1993.

7 Grinten, J.G.M. van der (1994): A comparison of the methods for uncertainty analysis based on ISO 5168 and the Guide prepared by ISO/TAG4/WG3, *in:* proceedings of the Flomeko '94, Flow measurement in the mid 90's, June 1994, NEL, East Kilbride, Scotland.

8 WECC Doc. 19 (1990): Guidelines for the expression of the uncertainty of measurement in calibrations, Western European Calibration Cooperation.

9 Kaarls, R. (1981): Procès-verbaux du Comité International des Poids et Mesures, 49, pp. A1-A12.

10 EAL-R3 (1997): Guidelines for the expression of the uncertainty of measurement in calibrations, Revision of WECC Doc. 19 - 1990, at present under consideration.

11 Mach E. (1919): Die Prizipien der Wärmelehre, Minerva GMBH, Leipzig, p 296.

12 Jaeschke, M., S. Audibert, P. van Caneghem, A.E. Humphreys, R. Janssen, Q. Pellei, J.A. Schouten, J.P.J. Michels and C.A. ten Seldam High accuracy compressibility factor for natural gasses and similar mixtures by use of a truncated virial equation, GERG technical monograph TM2 (1988).

CONTACT POINT

Jos G.M. van der Grinten
NMi Certin B.V., P.O. Box 394, 3300 AJ DORDRECHT, The Netherlands
Phone: +31 78 63 32 332, Fax: +31 78 63 32 309, E-mail: JvanderGrinten@NMi.nl

APPENDIX: UNCERTAINTIES DUE TO NOT APPLYING CORRECTIONS FOR KNOWN DEVIATIONS

In this appendix the additional uncertainty will be computed which is due to not correcting for known deviations.

The definition of the expectation value z is:

$$z = \int x\, g(x)\, dx \tag{A1}$$

Analogous to the statistical definition of standard deviation, the standard uncertainty in z, $u_s(z)$, is defined by:

$$u_s^2(z) = \int (z-x)^2\, g(x)\, dx \tag{A2}$$

in which $g(x)$ is the distribution function that represents our knowledge of x.

Assume that instead of z the uncorrected measurement result y is taken. Like in formula (A2) the standard uncertainty in y, $u_s(y)$, will be defined by.

$$u_s^2(y) = \int (y-x)^2\, g(x)\, dx \tag{A3}$$

Expanding formulas (A2) and (A3) results in

$$u_s^2(y) = \int (y^2 - 2yx + x^2)\, g(x)\, dx \tag{A4}$$

$$u_s^2(z) = \int (z^2 - 2zx + x^2)\, g(x)\, dx \tag{A5}$$

The difference between the squared standard uncertainties is:

$$u_s^2(y) - u^2(z) = \int (y^2 - 2yx - z^2 + 2zx)\, g(x)\, dx \tag{A6}$$

Substituting formula (A1) results in

$$u_s^2(y) - u_s^2(z) = y^2 - 2yz - z^2 + 2z^2 \tag{A7}$$

so that

$$u_s^2(y) = u_s^2(z) + (z-y)^2 \tag{A8}$$

In other words, not applying corrections for known deviations results in an extra standard uncertainty which equals the absolute value of the deviation. If the uncertainties are represented as the equivalent of double standard deviations, equation (A8) becomes

$$u^2(y) = u^2(z) + 2\,(z-y)^2 \tag{A9}$$

in which the factor 2 is the result from the uncertainty representation chosen. This uncertainty contribution is added to the other uncertainty contributions by means of root sum square summation.
Note that this result is generally valid, since no specification of the knowledge distribution function $g(x)$ is required.