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A PRACTICAL EXAMPLE OF UNCERTAINTY CALCULATIONS FOR A METERING STATION - CONVENTIONAL AND NEW METHODS

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A PRACTICAL EXAMPLE OF UNCERTAINTY CALCULATION FOR A METERING STATION - CONVENTIONAL AND NEW METHODS

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SUMMARY

The principles and results of a "conventional" uncertainty analysis provided by a major supplier of metering systems, are briefly presented. Temperature, pressure, density and gross observed volume are considered. The basis of the results are real data and instrument specifications of a metering package delivery to a field development in the North Sea. Thereafter, the *Guide* procedure which is referred to as the "new" method, has been applied for calculating the uncertainty of the same quantities based on the same vendor specified instrument uncertainties and process condition at the oil export station. The results indicate that different uncertainty estimates are achieved by a conventional approach and an alternative one rested on the principles of the ISO *Guide*. The main reason for the difference is that the conventional method does not take the sensitivity coefficients of the different variables sufficiently into account, in addition to some inconsistent calculations of the relative uncertainty of some of the input quantities.

1. INTRODUCTION

A conventional fiscal oil metering station normally consists of metering runs of two or several turbine meters equipped with temperature, pressure, density and water liquid ratio measurements. The system is also equipped with a permanent installed prover or a compact prover in addition to sampling analysis equipment. The fiscal measurement of oil and gas in the North Sea must be in accordance with the regulations of the Norwegian Petroleum Directorate (NPD). An uncertainty analysis of the metering system is also required in accordance with "recognised standards" [1]. In practise different methods for evaluation of measurement uncertainties have been used. The various methods have some kind of root-sum-square calculation as the basis, but the evaluation and combination of the individual uncertainty contributions from the basic measurements differ. As a consequence different results for the estimated uncertainty of the quantity of interest, e.g. density or the standard oil volume, may be achieved.

In 1993 the International Organisation for Standardization (ISO) issued, on behalf of several national organisations, the first draft version of the "Guide to the expression of uncertainty in measurement". The document is commonly referred to as the *Guide*. A corrected and reprinted version was published in 1995 [2]. The overall objective of the *Guide* has been to establish an internationally accepted method for estimating measurement uncertainty, and to provide guidelines for the calculation procedure and the reporting of the results. In addition, the *Guide* has introduced some new terms and suppressed some traditional terminology to standardise the concepts so that "everyone speaks the same language" and agrees on how uncertainty should be quantified. For instance conventional terms like errors, accuracy, tolerance, precision, linearity and repeatability should be avoided and replaced by, or defined in terms of, standard or expanded uncertainties.

It should be noted that at present the *Guide* is an ISO recommendation and not a standard. However, the standard published this year by the European cooperation for Accreditation of Laboratories (EAL), [3], is in conformity with the *Guide*.

This paper briefly presents the principles and results from a "conventional" uncertainty analysis provided by a major supplier of metering systems for the oil and gas industry. Some quantities related to a typical oil metering station such as temperature, pressure, density and gross observed volume are presented. The results are based on real data and instrument specifications of a metering package delivery to a field development in the North Sea. Thereafter, the *Guide* procedure which is referred to as the "new" method, has been applied for calculating the uncertainty of the same quantities based on the same vendor specified instrument uncertainties and process condition at the oil export station. Finally, the results of this case study using the two methods are compared.

2. DEFINITIONS OF TERMS

In metrology and metering technology applications certain terms are frequently used in relation to expressing the quality of a measurement. In this section some important of these are briefly reviewed according to definitions given by ISO [4], [5] and NFOGM [6]. Furthermore, in the definitions of the terms the description in Appendix D of ref. [7] have been adopted since these are more in accordance with the *Guide*.

In the following the terms have been classified into two groups. Terms of most relevance in conjunction with calculating uncertainties using the *Guide* procedure are presented first, followed by other and more "conventional" terms. All terms, however, have been defined with the terminology of the *Guide* as basis.

2.1 Important "new" terms specific to the *Guide*

Measurand:
Particular quantity subject to measurement.

Result of a measurement:
Value attributed to a measurand, obtained by measurement. It is an estimated value of the measurand.

Standard uncertainty:
Uncertainty of the result of a measurement expressed as a standard deviation.

Type A evaluation (of uncertainty):
Method of evaluation of uncertainty by the statistical analysis of series of observations.

Type B evaluation (of uncertainty):
Method of evaluation of uncertainty by means other than the statistical analysis of series of observations, i.e. by engineering/scientific judgement.

Correction:
Value added algebraically to the uncorrected result of a measurement to compensate for systematic error. The result of a measurement after correction for systematic error is referred to as corrected result.

Correction factor:
Numerical factor by which the uncorrected result of a measurement is multiplied to compensate for systematic error.

Comments:
The uncertainty of a correction applied to a measurement result to compensate for a systematic effect, is a measure of the uncertainty of the result due to incomplete knowledge of the required value of the correction. The correction must be included in the functional relationship and the calculation of the combined standard uncertainty must include the associated standard uncertainty of the applied correction[1].

2.2 Other (conventional) terms

Influence quantity:
Quantity that is not the measurand, but that affects the result of the measurement.

Accuracy of measurement:
The closeness of the agreement between the result of a measurement and the value of the measurand.

Comments:
1. The value of the measurand may refer to an accepted reference value[2].
2. "Accuracy" is a qualitative concept, and it should not be used quantitatively. The expression of this concept by numbers should be associated with (standard) uncertainty[3].

Uncertainty of measurement:
Parameter, associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand[4].

Repeatability:
A quantitative expression of the closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement, i.e. by the same measurement procedure, by the same observer, with the same measuring instrument, at the same location at appropriately short intervals of time.

[1] Cf. Section 3.2.1.

[2] In some documents it also points to the "true value" or "conventional true value". However, according to the *Guide* this definition should be avoided since the word "true" is viewed as redundant; a unique "true" value is only an idealised concept [2], and "a true value of a measurand" is simply the value of the measurand.

[3] E.g. the accuracy of the velocity measurement is high; the standard uncertainty is as low as 0.002 m/s.

[4] A definition based on more traditional concepts of uncertainty may be given as follows: A measure attached to the result of a measurement which states the range of values within which the value of the measurand is estimated to lie.

Reproducibility:
A quantitative expression of the closeness of the agreement between the results of measurements of the same measurand, where the individual measurements are carried out under different specified conditions, e.g. by different methods, with different measuring instruments, by different observers, at different locations, after intervals of time that are appropriately long compared with the duration of a single measurement, or under different customary conditions of use of the instruments employed.

Error of measurement:
The result of a measurement minus the value of the measurand. In general, the error is unknown because the value of the measurand is unknown. Therefore, the uncertainty of the result of the measurement should be evaluated and used in specifications and documentation of e.g. test results.

Random error:
The result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions.

Comment:
Because only a finite number of measurements can be made, it is possible to determine only an estimate of the random error. Since it generally arises from stochastic variations of influence quantities, the effect of such variations are referred to as random effects in the *Guide*.

Systematic error[5]:
The mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus the value of the measurand.

Comment:
As pointed out in the *Guide* and also mentioned above, error is an idealised concept which cannot be known exactly. In practise, therefore the recognised effect from which the systematic error arises, can be quantified and it is generally referred to as a systematic effect.

Some additional used terms in metering applications have also been included here for the sake of completeness:

Range:
The interval between the minimum and maximum values of the quantity to be measured, for which the instrument has been constructed, adjusted or set.

Span:
The algebraic difference between the upper and lower values specified as limiting the range of operation of a measuring instrument, i.e. it corresponds to the maximum variation in the measured quantity of interest[6].

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5 The systematic error of the indication of a measuring instrument is also referred to as bias.

6 E.g. a flow metering system which covers the range 50-200 m^3/h, has a span of 150 m^3/h.

3. METHODS FOR UNCERTAINTY CALCULATIONS

3.1 Conventional approach

For a fiscal metering station using turbine meters as the prime element, there is no dedicated standard for the uncertainty calculations. However, the method adopted and referred to as the conventional approach here, is based upon the calculation of the uncertainty of (individual) parameters and combining them using the root-sum-square (RSS) method. The uncertainties of these parameters are all quantified in relative percentage terms (in some cases a quantity is first calculated as a "worst case figure" and the relative deviation to the nominal value is identified as the associated uncertainty). The uncertainty of each parameter is denoted by "E_{xi}" where "x_i" refers to the parameter whose uncertainty contributes to the total measurement uncertainty. Using the RSS method the resulting uncertainty in the different quantities are calculated [8].

The total relative uncertainty, E_y, is determined as:

$$E_y = \sqrt{(E_{x_1})^2 + (E_{x_2})^2 + (E_{x_3})^2 + \cdots + (E_{x_n})^2} \tag{1}$$

The different uncertainty parameters are weighted equally with sensitivity coefficient[7] of 1.

3.2 The *Guide* procedure

The starting point of the *Guide* in assessing measurement uncertainty is that all measurements are estimates of the quantities of interest. An estimate may be defined as a prediction that is equally likely to be above or below the actual result [9]. In many cases a measurement process can be represented by a mathematical model which the *Guide* refers to as the functional relationship [2][8]. For instance, often a quantity is determined as a result of several individual measurements which combined through a mathematical expression define the resulting measurand. Theoretically the uncertainty of the output quantity may then originate from one of the following sources:

a) Measurement uncertainties of (some of) the input variables.
b) Uncertainties due to incorrect assumptions about a model's input parameters (e.g. their estimates, or probability distributions).
c) Uncertainty of the (empirical) mathematical model itself because it is an abstraction of reality which may not include all the factors that affect the measurement.

The *Guide* procedure includes establishment of the equations for mathematically combining the standard uncertainties based on the functional relationship between the measurand and the input quantities. This means that the sensitivity of the quantity in question with respect to the different input measurements can be taken into account through calculated sensitivity coefficients. Furthermore, the *Guide* puts definite requirements to the documentation and reporting of the uncertainties. The objective has been to establish a universal method where the (standard) uncertainties are transferable so that the result of an uncertainty analysis can be used directly in a subsequent uncertainty evaluation. Ideally this make measurements taken at different times and at different places comparable [12].

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[7] Sensitivity coefficients are defined by Eqs. (5) and (10), respectively.

[8] Other important documents dealing with estimating measurement uncertainties based on, or in harmony with the Guide, can be found in references [3], [7] and [10], see also discussion in [11].

3.2.1 Procedure for evaluation of uncertainties

The *Guide* procedure for evaluating and expressing the uncertainty of the result of a measurement can be summarised as follows [2]:

1. Derive mathematically the *functional relationship* between the measurand Y and the input quantities X_i upon which Y depends: $Y = f(X_1, X_2, \ldots, X_N)$. The function f should include all corrections for systematic effects. The modelling of the measurement by the function f is further used (in step 5) to calculate the output *estimate*[9], y, of the measurand, Y, using the *estimates* $x_1, x_2, \ldots x_N$ for the values of the N input quantities $X_1, X_2, \ldots X_N$:

$$y = f(x_1, x_2, \ldots, x_N) \tag{2}$$

2. Determine the estimated value, x_i, of input quantity X_i, either on the basis of the statistical analysis of series of observations, or by other means.

3. Evaluate the *standard uncertainty* $u(x_i)$ of each input estimate x_i, either based on statistical analysis of observations (Type A evaluation of standard uncertainty), or obtained by other means (Type B evaluation of standard uncertainty).

4. Evaluate the *covariances* associated with input estimates that are correlated:

$$u(x_i, x_j) = u(x_i)\,u(x_j)\,r(x_i, x_j) \tag{3}$$

   where $r(x_i,x_j)$ is the correlation coefficient between x_i and x_j.

5. Use the functional relationship, Eq. (2), to calculate the result of the measurement y based on the estimates x_i obtained in step 2.

6. Determine the *combined standard uncertainty* $u_c(y)$ of the measurement result y from the standard uncertainties and covariances associated with the input estimates. The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$, which is given by[10]:

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j) \tag{4}$$

   The partial derivatives are referred to as sensitivity coefficients, c_i:

$$c_{y\_x_i} \equiv \frac{\partial f}{\partial x_i} \tag{5}$$

7. The *expanded uncertainty* $U(y)$ of the output estimate y can be evaluated by multiplying the combined standard uncertainty $u_c(y)$ by a coverage factor, k, on the basis of the level of

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[9] This "estimate" is also generally referred to as "the result of the measurement".

[10] Eq. (4) is frequently referred to as the law of propagation of uncertainty.

confidence required for the interval $y \pm U(y)$. Assuming a normal[11] distribution of y, and a level of confidence of close to 95%, k equals 2, the value of k used in this work[12].

$$U(y) = k \cdot u_c(y) \tag{6}$$

8. Report the result of the measurement y together with its combined standard uncertainty $u_c(y)$ or expanded uncertainty $U(y)$. This includes documentation of the value of each input estimate, x_i, the individual uncertainties which contribute to the resulting uncertainty and the evaluation method used to obtain the reported uncertainties of the output estimate as summarised in steps 1 to 7.

Of practical reasons it is useful to define a symbol for the *relative* (*standard, combined standard, or expanded*) *uncertainty* of an input/output estimate. The following definitions are used[13]:

The *relative standard uncertainty* of an input estimate, x_i:

$$\delta x_i \equiv \frac{u(x_i)}{|x_i|}, \; |x_i| \neq 0 \tag{7}$$

The *relative combined standard uncertainty* of an output estimate, y:

$$\delta y \equiv \frac{u_c(y)}{|y|}, \; |y| \neq 0 \tag{8}$$

The *relative expanded uncertainty* of an output estimate, y:

$$\delta y_U \equiv \frac{U(y)}{|y|} = k \cdot \delta y, \; |y| \neq 0 \tag{9}$$

Finally, the *relative sensitivity coefficient*, s_i, is defined as:

$$s_{y\_x_i} \equiv \frac{\partial f}{\partial x_i} \cdot \left| \frac{x_i}{y} \right| \tag{10}$$

As distinct from the sensitivity coefficient, c_i, the relative sensitivity coefficient, s_i, is dimensionless. It is a useful quantity. The relative sensitivity coefficient and the relative standard uncertainty can be used for assessing the uncertainty contribution of each individual input variable under investigation.

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11 If x is a normal variable with mean μ and standard deviation σ, $x \sim N(\mu, \sigma)$, then the probability distribution of the normal variable is given by the expression [13]: $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2}$.

12 Note that a coverage factor of $k = 2$ produces an interval corresponding to a level of confidence of 95.45% while that of $k = 1.96$ corresponds to a level of confidence of 95%. Since the calculation of intervals having specified levels of confidence is at best only approximate, the *Guide* justifiably emphasises that for most cases it does not make sense to try to distinguish between e.g. intervals having levels of confidence of say 94, 95 or 96%, cf. Annex G of the *Guide* [2]. In practice, it is therefore recommended to use $k = 2$ which is assumed to produce an interval having a level of confidence of approximately 95%.

13 It should be noted that the present version of the *Guide* [2] does not assign distinct symbols for the defined relative uncertainties. The terminology used here has been introduced by the authors.

3.3 NPD regulations

The estimated uncertainties should be reported as stated by the regulations of the NPD for fiscal metering of oil and gas (§16 in [1]). According to these regulations an uncertainty analysis should be carried out at a 95% confidence level. This means that the probability is 95% for the real measurement being confined within the specified interval, and according to the *Guide* this corresponds to a coverage factor of $k = k_{95} = 2$.

4. CALCULATION EXAMPLE - OIL METERING STATION

In the following, results of a "conventional" uncertainty calculation carried out by a major supplier of metering systems, are summarised in Section 4.2. These data are part of a metering station delivery to a field development in the North Sea. The vendor's uncertainty data of pressure, temperature, density and gross observed volume are briefly presented. In Section 4.3, the uncertainty of the same quantities are calculated based on the same vendor specified instrument uncertainties and process condition at the oil export station. These calculations have been carried out using the procedure of the *Guide*.

4.1 Oil metering station and process condition

The calculations that follow are based on vendor specified uncertainties of an oil metering station consisting of the equipment listed in Table 1 and the process condition in Table 2.

Table 1 Equipment of the oil metering station used as basis for the presented uncertainty evaluations.

Item	Type
Turbine meter	Turbine Meter 8" 300lb
Densitometer	ITT Barton 668
Pressure Transmitter	Rosemount Model 3051CG
Temperature Transmitter	Rosemount 3044CA, temperature element 13669
Flow computer	Liquid Turbine Flow Computer

Table 2 The process condition of which the presented uncertainty evaluations have been carried out.

Quantity	Condition
Service	Hydrocarbon liquid
Operating pressure	20.2 barg
Design pressure	32 barg
Operating temperature	65.4 °C ± 10 °C
Ambient temperature	-10/23 °C
Operating density	776 kg/m^3
Normal flow rate	1867 Sm^3/h

The turbine meter is a flow sensing device with a rotor that senses the velocity of flowing liquid in a closed conduit. The flowing liquid causes the rotor to move with a tangential velocity that is proportional to volumetric flow rate. A volume prover is used for calibration of the meter. Density is normally measured by densitometers mounted in the meter run or in connection with the inlet or outlet manifold. Pressure and temperature are measured close to the turbine meter, at the inlet and outlet of the volume prover and at the density measuring device.

4.2 Results from "conventional" calculation

The uncertainty in pressure, temperature, density and gross observed volume are determined as outlined in Section 3.1 [8]. The uncertainty analysis calculated by this vendor is reported to be within a confidence level of 95%, where the confidence level of 95% is equivalent to 2σ (i.e. two standard deviations). The supplier identified the following parameters as the main source contributors to the overall uncertainty in a turbine based metering system:

a) Turbine meter calibration
b) Turbine meter linearity
c) Turbine meter repeatability
d) Stream density measurement error
e) Stream and prover loop temperature and pressure measurement errors
f) Flow computer calculation error and pulse interpolation

In addition the following instrument uncertainties were included:

g) Uncertainty in pressure measurement as a result of primary measurement ("*ptx*"), calibration ("*pcal*"), stability ("*pstab*") and ambient temperature effects ("*paT*").
h) Uncertainty in temperature measurement as a result of primary temperature measurement ("*ttx*"), calibration ("*tcal*"), stability ("*tstab*") and ambient temperature effects ("*taT*").
i) Uncertainty in density measurement calculated on the basis of the data sheet of the densitometer uncertainty specification.
j) Pulse interpolation.

For this metering station the vendor has obtained the following results [8][14] :

Uncertainty of pressure measurement

$E_{pressure} = \sqrt{(E_{ptx})^2 + (E_{pcal})^2 + (E_{pstab})^2 + (E_{paT})^2} = 0.44\%$. At the operating pressure of 20.2 barg, the uncertainty in the pressure measurement equals 0.09 bar, which is within the NPD regulation of 0.2 bar.

Uncertainty of temperature measurement

$E_{temperature} = \sqrt{(E_{ttx})^2 + (E_{taT})^2 + (E_{tcal})^2 + (E_{tstab})^2} = 0.29\%$. With an operating temperature of 65°C, the uncertainty in the temperature measurement equals 0.19 °C, which is within the NPD regulation of 0.30 °C.

Uncertainty of density measurement

$E_{density} = \sqrt{(E_{pd})^2 + (E_{temperature})^2 + (E_{freq})^2} = 0.41\%$. The result is outside the NPD requirement of 0.30% of measured value. In fact the uncertainty of the raw density measurement is calculated to E_{pd} = 0.29%, thereafter this figure is RSS-combined with the temperature uncertainty to estimate the uncertainty of the temperature corrected density (the contribution from the uncertainty of the frequency reading by the flow computer, E_{freq}, is negligible). This procedure increases the uncertainty to 0.41%. However, it can be shown that the sensitivity coefficient with respect to temperature in this case is very low so the uncertainty of 0.41% is likely to be a conservative estimate.

[14] The calculations as such have not been quality checked by the authors.

Uncertainty in gross observed volume, E_{gov}

Turbine meter linearity, E_{lin} = 0.15%,
Turbine meter repeatability, E_{rep} = 0.02%,
Meter factor, E_{MKF} = 0.044%
Correction factor, E_{Ctsm} = 0.0007%
Correction factor, E_{Cpsm} = 0.0135%
Calculation, E_{cal} = 0.001%
Flow computer clock, E_{clock} = 0.0001%

$$E_{gov} = \sqrt{(E_{lin})^2 + (E_{rep})^2 + (E_{MKF})^2 + (E_{Ctsm})^2 + (E_{Cpsm})^2 + (E_{calc})^2 + (E_{clock})^2} = 0.1582\%$$

4.3 Results from "new" method; i.e. according to the ISO *Guide*

To calculate the uncertainty in pressure, temperature, density and the gross observed volume, the functional relationship of each quantity and the different associated input variables must first be established. Some of these primary measurements may again be related to other parameters. Such relations must be included as part of the functional relationship. In addition, basis documentation and certificates of equipment from the supplier which give details of the primary measurement uncertainties have been evaluated based on engineering judgement (i.e. Type B evaluation of uncertainties), and the (relative) expanded uncertainties have been calculated using the *Guide* procedure presented in Section 3.2. The detailed calculations have been separately reported [14], and only the main results are summarised here for simplicity. However, the detailed calculations of the relative expanded uncertainty of the density measurement have been included as an example and given as an attachment to this paper. The principles of the *Guide* have been used for uncertainty calculations in several projects at CMR [15], [16].

Uncertainty of pressure measurement

By using the procedure of the *Guide*, the expanded uncertainty of the pressure measurement has been estimated to: $U(P) = P \cdot \delta P_U = k_{95} \cdot P \cdot \delta P = 0.19$ barg. The expanded uncertainty of 0.19 barg ($k = 2$) is closely within the NPD requirements of 0.2 barg [1]. Note that the estimated uncertainty is higher than that given by the conventional method.

Uncertainty of temperature measurement

By using the procedure of the *Guide*, the combined standard uncertainty of T at 65.4 °C, $u_c(T)_{T=65.4}$, has been estimated to 0.13 °C. The corresponding expanded uncertainty of the temperature measurement is: $U(T)_{T=65.4} = k_{95} \cdot u_c(T)_{T=65.4} = 0.26\,°C$.

The expanded uncertainty of T as a function of the actual value, i.e. the measured temperature, throughout the temperature range 55-75 °C, is shown in Figure 1.

It is observed that the expanded uncertainty increases marginally with temperature (less than 8 millidegrees for temperatures between 55 and 75 °C). From a practical point of view the expanded uncertainty can be regarded as constant throughout this temperature range with a numerical value of 0.26 °C. The expanded uncertainty of 0.26 °C ($k = 2$) is within the NPD requirements of 0.30 °C [1]. However, note that the estimated uncertainty is higher than that reported using the conventional method.

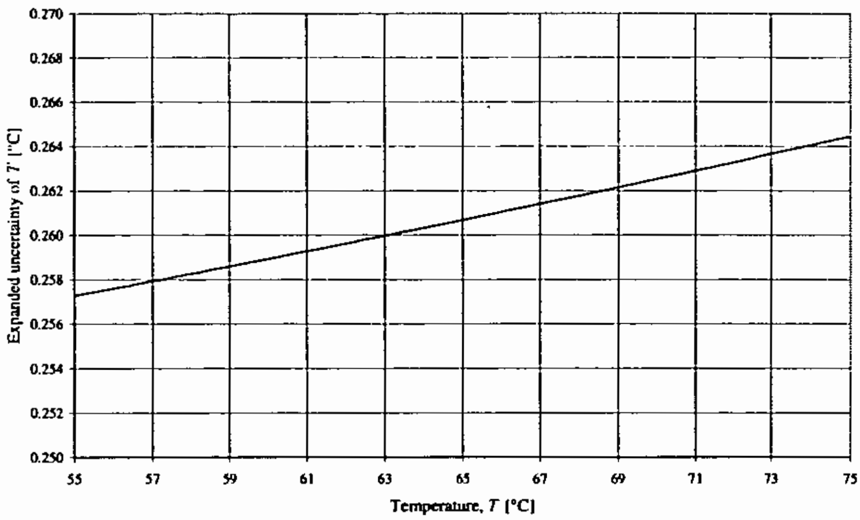

Figure 1 Expanded uncertainty of the temperature measurement as a function of actual value in the measurement range 55-75 °C. The level of confidence is 95%, $k = 2$. The span of the temperature transmitter is 100 °C.

Uncertainty of density measurement[15]

By using the procedure of the *Guide*, the expanded uncertainty of the measured density has been estimated to: $U(\rho) = k_{95} \cdot u_c(\rho) = 2 \cdot 0.843$ kg/m^3 = 1.686 kg/m^3 where $u_c(\rho)$ is the combined standard uncertainty of ρ.

The relative expanded uncertainty of ρ ($k = 2$) is: $\delta\rho_U = \dfrac{U(\rho)}{\rho} = 0.22\%$.

Note that the estimated uncertainty in this case is lower than that given by the conventional method. The uncertainty is within the NPD requirement of a relative uncertainty lower than 0.30% in the density measurement.

Uncertainty of liquid volume flow rate (Gross Observed Volume Flow rate)

The principle for calculating the uncertainty of the gross observed volume[16] in accordance with the *Guide* is presented in this section. The functional relationship of the gross observed volume, V_l, may be expressed as:

$$V_l = \frac{MR \cdot C_{tsm} \cdot C_{psm}}{MKF} + V_{calc} \tag{11}$$

where the value of the variable V_{calc} is zero, but to which associated uncertainty is assigned as a result of finite resolution of the calculation procedure in the flow computer. *MR* is the number of interpolated pulses from the turbine meter during the measurement period. *MKF* is the meter factor of the turbine meter whose unit is (pulses/m^3). Since a metering station is considered here, the *MKF* is determined using a pipe prover rather than by an external calibration. C_{tsm} and C_{psm} are correction factors. The relative expanded uncertainty of V_l is calculated based on the associated

15 The details of the uncertainty calculation of the density are included as an attachment at the end of the paper.

16 Gross observed volume is the volume measured by the turbine meter at process or line conditions. Corrections for changes in liquid volume as a result of influence of temperature and pressure, must further be taken into account to convert V_l to the volume, V_s, at standard conditions of temperature 15 °C and reference pressure of 1.01325 bar a [1]. This procedure which is not dealt with further here, includes the correction factors C_{tlm} and C_{plm}, respectively.

uncertainties of the input quantities in Eq. (11) which were first evaluated giving the following relative standard uncertainties:
$\delta V_{calc} = 0.0005\%$
$\delta MR = 0.0085\%$
$C_{tsm} = 1.001974$ and its standard and relative standard uncertainty are estimated to:
$u_c(C_{tsm}) = 5 \cdot 10^{-6}$ and $\delta C_{tsm} = 0.0005\%$, respectively.
$C_{psm} = 1.000191$ and its standard and relative standard uncertainty are estimated to:
$u_c(C_{psm}) = 2 \cdot 10^{-6}$ and $\delta C_{psm} = 0.0002\%$, repectively.

The functional relationship for the meter factor, MKF, may be given by:

$$MKF = MKF_p + MKF_{lin} + MKF_{rep} \tag{12}$$

where the values of the variables MKF_{lin} and MKF_{rep} both equal zero, but to which uncertainties are assigned associated with the linearity and repeatability of the meter factor. Using a pipe prover the meter factor (at standard conditions), MKF_p, is calculated according to the following expression:

$$MKF_p = \frac{MR_p \cdot C_{tsm} \cdot C_{psm} \cdot C_{tlm} \cdot C_{plm}}{BV \cdot C_{tsp} \cdot C_{psp} \cdot C_{tlp} \cdot C_{plp}} \tag{13}$$

where MR_p is the number of pulses from the turbine meter throughout the measurement (or "proving") period, BV is the base volume of the prover and C_{ijk} are correction factors.

The uncertainty of the calculated metering factor should be estimated according to the *Guide* based on the standard uncertainties of MR_p, BV and the 8 eight correction factors C_{ijk} using Eq. (4). The calculation includes six partial derivatives of the expressions[17] for the correction factors C_{ijk} in addition to those of C_{tsm} and C_{psm}. These calculations have not been carried out here. Instead, in this limited study, the uncertainty of MKF_p is assumed to be satisfactorily estimated by the conventional uncertainty calculation method [8], but the "new" uncertainty estimates of C_{tsm} and C_{psm} calculated in this work are accounted for. The uncertainty of the six remaining correction factors have not been evaluated, i.e. the quoted vendor uncertainties have been adopted. The uncertainty in the estimate of MKF using a pipe prover is then found as δMKF_p=0.0155%. Uncertainties due to linearity and repeatability are given directly by data sheets from vendors as $\delta MKF_{lin} = 0.075\%$ and $\delta MKF_{rep} = 0.01\%$, respectively.

Based on the relative standard uncertainties of the different quantities of which V_l depends, the relative combined standard uncertainty of the gross observed volume is estimated to:
$\delta V_l = 0.0777\%$, and the relative expanded uncertainty of V_l ($k = 2$) is: $\delta V_{l\_U} = k \cdot \delta V_l = 0.1554\ \%$.

The result shows that in this case with the individual uncertainties of the primary quantities (T, P, EH[18], ER, R, AT, t), the uncertainty in gross observed volume is mainly caused by the uncertainty in the meter factor MKF. Furthermore, if the uncertainty in the estimate of MKF as a result of the proving, δMKF_{p_U}, had been evaluated stringently according to the *Guide* giving an estimate larger than 0.031%, the relative expanded uncertainty of the gross observed volume would be

[17] The expressions for the correction factors can be found in reference [17] and [18].

[18] EH is the coefficient of the linear expansion of the meter housing, ER is the coefficient of the linear expansion of the meter rotor, R is the inner radius of the meter housing, AT is the area of the rotor in the plane of the flow and t is the wall thickness of the housing [14].

larger than the estimate reported here. The estimated relative expanded uncertainty in this work of 0.155% holds true as long as the relative expanded uncertainty in the meter factor as a result of proving, is lower than 0.031%. If this uncertainty limit of the meter proving is exceeded due to e.g. high uncertainties in pressure or temperature, a thorough evaluation of Eq. (13) should be carried out based on the individual standard uncertainty contributions of MR, base volume of the prover, BV, and the different correction factors. Such an evaluation based on the procedure of the *Guide* should be addressed in future as a coming step in further standardising measurement uncertainty evaluation of metering stations. This should also include the conversion to standard conditions, the calculation of mass rates and inclusion of water cut measurement uncertainty facilitating the evaluation of the uncertainty in the gross standard volume and the net oil observed volume.

5. DISCUSSION

The preceding calculations summarised in Table 3 below, show that different results are obtained using a conventional uncertainty calculation method compared to the results following the *Guide* procedure. The main reason for the differences is that the conventional RSS method does not sufficiently take the sensitivity of the measurand with respect to the different input variables into account. Also, when studying the details of the calculations, the method for determining the individual relative uncertainties of the input variables has, in some cases, been found to be some subtle. This may make the method "vendor or application dependent" leading to lower uncertainties than that obtained by the *Guide* method for some quantities and vice versa for others as shown by the results presented here.

By taking the uncertainty of the density measurement as an example, it is shown that the uncertainty estimate of the conventional approach is too conservative since the uncertainty of the temperature correction is given too high sensitivity by the RSS combination. In this case the uncertainty associated with the temperature correction is in fact negligible with a very low sensitivity coefficient compared to the raw density measurement itself as shown in Table 4. It should be noted that conventional method gives an uncertainty estimate of the density of 0.29% if the uncertainty contribution from the temperature correction is neglected.

Finally, it should be pointed out that in addition to giving recommendations for the uncertainty calculations, the *Guide* also puts clear requirements to the documentation of the results. This means that more effort may have to be put into the measurement uncertainty evaluation than experienced previously.

Table 3 Summarised quantitative comparison of the results of the conventional uncertainty calculation with those obtained by the method of the ISO *Guide* for a fiscal oil metering station. The process condition is characterised by a temperature of 65.4 °C, pressure of 20.2 barg and a density of 776 kg/m^3.

	Conventional method	**The *Guide***		
Quantity	Uncertainty	Expanded uncertainty (k=2)	Unit	NPD requirement
Pressure, P	0.09	0.19	[bar]	0.20
Temperature, T	0.19	0.26	[°C]	0.30
	Relative uncertainty	Relative expanded uncertainty (k=2)		
Density, ρ	0.41	0.22	[%]	0.30
Gov, V_t	0.1582	0.1554	[%]	-

6. CONCLUSIONS

This case study using an oil metering station as basis for the calculations, indicates that different uncertainty estimates are achieved by a conventional approach and an alternative one rested on the principles of the ISO *Guide*. It should be noted that this study does not include the evaluation of any systematic effects which may occur in practise and which have to be corrected by re-calibration. The main reason for the differences is that the conventional RSS method does not sufficiently take the sensitivity of the measurand with respect to the different input variables into account. Furthermore, it has been found that the individual relative uncertainties of the input variables, in some cases, have been determined by some inconsistent methods. This may make the method vendor or application dependent leading to lower uncertainties than that obtained by the *Guide* method for some quantities and larger for others.

Even though the *Guide* is frequently considered as a difficult document it may represent a more sound approach for measurement uncertainty calculation in particular with respect to the documentation of the results.

With respect to the uncertainty calculation of the gross observed volume, MKF_p is the most complicated variable to evaluate. A thorough evaluation of Eq. (13) should be carried out based on the individual standard uncertainty contributions of *MR*, base volume of the prover, *BV*, and the different correction factors[17]. The contribution from the uncertainty in any of these fundamental parameters on the resulting uncertainty in gross and net observed volume could then be evaluated for different process conditions and prover characteristics. Such an evaluation based on the procedure of the *Guide* should be addressed in future as a coming step in further standardising measurement uncertainty evaluation of metering stations.

7. ACKNOWLEDGEMENT

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CALCULATION OF THE UNCERTAINTY OF THE DENSITY MEASUREMENT

The expanded uncertainty of the liquid density measured by a Barton Model 668 densitometer [19], [20] which is temperature corrected by the flow computer [21], can be determined according to the *Guide* as follows:

The functional relationship may be expressed as:

$$\rho = A_0 + A_1 \cdot t + A_2 \cdot t^2 - A_B \cdot (T-20) + A_{B1} \cdot t^2 \cdot (T-20) + A_{B2W} \cdot t^2 \cdot (T^2 - 20^2) + \rho_{meter} \tag{14}$$

where t is the time period for one pulse (the reciprocal of frequency), T is the temperature and the variable ρ_{meter} can be expressed by:

$$\rho_{meter} = \rho_{dt} + \rho_{cal} + \rho_{res\_temp} + \rho_{res\_press} + \rho_{rep} + \rho_{model} \quad (15)$$

where the estimated values of the parameters ρ_{dt}, ρ_{cal}, ρ_{res_temp}, ρ_{res_press}, ρ_{rep} and ρ_{model} all equal zero. These six variables have associated uncertainties and affect the resulting uncertainty of the density measurement. However, the uncertainties are about the nominal parameter values of zero. Hence, the statistical expectation of the variable ρ_{meter} is zero; i.e. $\rho_{meter} = 0$.

The raw density which is not temperature corrected, can then be expressed as:

$$\rho_{rd} = A_0 + A_1 \cdot t + A_2 \cdot t^2 + \rho_{meter} \quad (16)$$

where the standard uncertainty of the raw density measurement is mathematically represented by the virtual variable ρ_{meter}; i.e. $u(\rho_{rd}) = u(\rho_{meter})$.

Calibration constants of the densitometer [20]:
A_0 = -434.43 kg/m^3
A_1 = -206.93 (kg/m^3)/ms
A_2 = 7122.46 (kg/m^3)/ms^2

Coefficients for temperature compensation of the raw density measurement:
A_B = 0.00 (kg/m^3)/°C
A_{BIC} = $-4.2 \cdot 10^{-2}$ (kg/m^3)/(°C· ms^2)
A_{BIW} = $8.7 \cdot 10^{-2}$ (kg/m^3)/(°C· ms^2)
$A_{BI} = A_{BIW} + A_{BIC}$ = $4.5 \cdot 10^{-2}$ (kg/m^3)/(°C· ms^2)
A_{B2W} = $-6.4 \cdot 10^{-4}$ (kg/m^3)/((°C)^2· ms^2)
The operating density at the metering station is: ρ = 776 kg/m^3 which corresponds to a time period and frequency of:
t = 0.427 ms
f = 2.342 kHz
ρ_{rd} = 775.84 kg/m^3
ρ = 775.76 kg/m^3 ≈ 776 kg/m^3

Assumed variations in temperature about the operating condition for calculation of uncertainty in density due to these residual temperature effects: T_{res} = 41 °C = 5 °F.

Assumed variations in pressure about the operating condition for calculation of uncertainty in density due to these residual pressure effects: P_{res} = 10 bar = 145 psi.

The level of confidence of the uncertainties is not specified in the data sheets from the suppliers. In the uncertainty estimation that follows it is assumed that the level of confidence is approximately 95%, i.e. $k = k_{95} = 2$ except for some of the uncertainty contributions which are reported as "precisions" and stated as a "± a maximum" value. In the lack of further documentation of these figures, it is assumed that the "± a maximum" values represent endpoints of the interval $-a$ to $+a$ of a uniform or rectangular probability distribution of the quantity "x_i" in question. Under these assumptions, the standard uncertainty is [2]:

$$u(x_i) = \frac{a}{\sqrt{3}} \quad (17)$$

Then the different specified uncertainties can be converted into standard uncertainties:

Uncertainty of the primary densitometer (linearity):
±1.2 kg/m^3 maximum, i.e. $u(\rho_{dl}) = (1.2 \text{ kg/m}^3)/\sqrt{3} = 0.693$ kg/m^3

Uncertainty due to the calibration reference standard:
$U(\rho_{cal})$ = 0.3 kg/m^3 (k_{95} = 2), i.e. $u(\rho_{cal}) = (0.3 \text{ kg/m}^3)/k_{95} = 0.150$ kg/m^3

Uncertainty due to residual temperature effects:

±6 kg/m^3 per 100 °F maximum, i.e.:

$$u(\rho_{res\_temp}) = \frac{6}{\sqrt{3}} \cdot \frac{T_{res}(°F)}{100} \tag{18}$$

With T_{res} = 5°F, $u(\rho_{res\_temp})$ = 0.173 kg/m^3.

Uncertainty due to residual pressure effects:

±0.2 kg/m^3 per 100 psi maximum, i.e.:

$$u(\rho_{res\_press}) = \frac{0.2}{\sqrt{3}} \cdot \frac{P_{res}(\text{psi})}{100} \tag{19}$$

With P_{res} = 145 psi, $u(\rho_{res\_temp})$ = 0.167 kg/m^3.

Uncertainty due to repeatability:

±0.3 kg/m^3 maximum, i.e. $u(\rho_{rep})$ = (0.3 kg/m^3)/$\sqrt{3}$ = 0.173 kg/m^3

Uncertainty associated with density model calibration (curve fit)[19]:

±0.6 kg/m^3 maximum, i.e. $u(\rho_{model})$ = (0.6 kg/m^3)/$\sqrt{3}$ = 0.346 kg/m^3

By assuming that the different variables are independent, the combined standard uncertainty of ρ_{meter} can be calculated.

The combined standard uncertainty of the raw density measurement:

$$\begin{aligned} u_c(\rho_{meter}) &= \\ &\sqrt{u(\rho_{dt})^2 + u(\rho_{cal})^2 + u(\rho_{res\_temp})^2 + u(\rho_{res\_press})^2 + u(\rho_{rep})^2 + u(\rho_{model})^2} = \\ &\sqrt{(0.693)^2 + (0.150)^2 + (0.173)^2 + (0.167)^2 + (0.173)^2 + (0.346)^2} = 0.843 \end{aligned} \tag{20}$$

The corresponding expanded uncertainty of the raw density measurement is:

$$U(\rho_{meter}) = k_{95} \cdot u_c(\rho_{meter}) = 2 \cdot 0.843 = 1.686\text{; i.e. } 1.686 \text{ kg/m}^3 \tag{21}$$

The relative expanded uncertainty of ρ_{rd} (k = 2) is:

$$\delta\rho_{rd\_U} = \frac{U(\rho_{meter})}{\rho_{rd}} = 0.0022 \text{ ; i.e. } 0.22\ \% \tag{22}$$

Overall uncertainty of the density measurement:

The combined standard uncertainty of the measured density, ρ, can be derived on the basis of Eq. (14):

$$u_c(\rho) = \sqrt{\left(\frac{\partial\rho}{\partial t} \cdot u(t)\right)^2 + \left(\frac{\partial\rho}{\partial T} \cdot u_c(T)\right)^2 + \left(\frac{\partial\rho}{\partial\rho_{meter}} \cdot u_c(\rho_{meter})\right)^2} \tag{23}$$

where

$$\frac{\partial\rho}{\partial t} = A_1 + 2t \cdot [A_2 + A_{B1} \cdot (T - 20) + A_{B2W} \cdot (T^2 - 20^2)], \tag{24}$$

[19] Note, the curve fit calibration results in an overall model calibration uncertainty included here which incorporates the uncertainty of the individual calibration coefficients A_0, A_1 and A_2. These calibration coefficients are therefore assumed to be fixed constants which do not have any associated uncertainties.

$$t = \frac{1}{f} \Rightarrow u(t) = \sqrt{\left(\frac{\partial t}{\partial f}\right)^2 \cdot u^2(f)} = \frac{1}{f^2} \cdot u(f) = \frac{1}{f^2} \cdot f \cdot \delta f = \frac{\delta f}{f} \qquad (25)$$

where the relative uncertainty of the flow computer frequency input is [21]:
δf_U=0.0013% (k_{95} = 2), i.e. $\delta f = 1.3 \cdot 10^{-5} / k_{95}$,

$$\frac{\partial \rho}{\partial T} = -A_B + t^2 \cdot (A_{B1} + 2 \cdot A_{B2W} \cdot T), \qquad (26)$$

$$\frac{\partial \rho}{\partial \rho_{meter}} = 1 \qquad (27)$$

and $u_c(T)$ and $u_c(\rho_{meter})$ have been estimated previously, cf. Section 4.3 and Eq. (20), respectively.

An uncertainty budget for the uncertainty of the measured density is given in Table 4.

Table 4 Orderly summary of the uncertainty estimation of the density measurement at the actual process condition of ρ = 776 kg/m^3 and T = 65.4 °C. The sensitivity coefficients are: $\partial\rho/\partial t$, $\partial\rho/\partial T$, $\partial\rho/\partial\rho_{meter}$.

Measurand: density		Value of measurand at process condition:			ρ = 776 kg/m^3
		Standard uncertainty			**Expanded** uncertainty
Parameter, x_i	Estimate	$u(x_i)$	Sensitivity coefficient,	$u_i^2(\rho)$	(k=2)
			c_i		[kg/m^3]
Time period, t	0.427 ms	0.00000278	5875.28000000	0.00026591	
Temperature, T	65.4 °C	0.13000000	-0.00706000	0.00000084	
Density, ρ_{meter}	0 kg/m^3	0.84300000	1	0.71064900	
			$u^2(\rho)$	0.71092	$U(\rho)$
	Combined standard uncertainty:		$u(\rho)$	0.8432	1.686

Hence, it is observed that the uncertainty in the temperature measurement and the resolution of the frequency measurement by the flow computer do not significantly contribute to the resulting uncertainty of the density measurement. The uncertainty of the density measurement originates from the raw density measurement.

The estimated expanded uncertainty of the measured density is:

$$U(\rho) = k_{95} \cdot u_c(\rho) = 2 \cdot 0.843 = 1.686\text{; i.e. } 1.686 \text{ kg/m}^3 \qquad (28)$$

The relative expanded uncertainty of the density measurement:

The relative expanded uncertainty of ρ (k = 2) is:

$$\delta\rho_U = \frac{U(\rho)}{\rho} = 0.0022 \text{ ; i.e. } 0.22\ \% \qquad (29)$$