THE EFFECTS OF REYNOLDS NUMBER, WALL ROUGHNESS, AND PROFILE ASYMMETRY ON SINGLE- AND MULTI- PATH ULTRASONIC METERS

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1. ABSTRACT

A simple "power law" velocity profile is used to study the effects of pipe roughness and Reynolds Number (*Re*) on ultrasonic meter performance. Considered here are two measurement methods. The first utilize a single bounce path through the pipe center, and is common to multipath as well as single path meters. The second employs four parallel chordal paths, (e.g. Daniel SeniorSonic).

The single path centerline measurement requires typical corrections from 4% to 8% for changes in velocity profile due to variations in pipe roughness and *Re*. A correction for *Re* can be calculated from additional information on fluid density and viscosity, but changes of roughness with time cannot be measured, and hence cannot be corrected. It will be shown that the four path meter does an excellent job of integrating the velocity profile to give the correct flow rate over a wide range of both *Re* and roughness.

A mathematically generated profile similar to that from a single bend is used to study the effects of asymmetry on the performance of both measurements. The centerline measurement varies over about 1.5%, depending on the path orientation relative to the asymmetry. Unfortunately applying a typical *Re* correction then creates a bias error of about 4%. In contrast, the four path meter accurately integrates the velocity profile to give an answer within 0.2% of the flow irrespective of orientation. Furthermore the four-path meter is capable of recognizing the changes in operating conditions.

2. EFFECTS OF REYNOLDS NUMBER AND WALL ROUGHNESS ON SINGLE PATH CENTER LINE ULTRASONIC METERS

Power Law Velocity Profile

The velocity profile in pipe flow is quite complex, but a simple power law is a good approximation everywhere except at the pipe center, where the velocity gradient $(dv/dy)_R$ exhibits a discontinuity. This is not too serious because there is no flow through the exact center of the pipe, since the area is zero.

The power law profile is defined by

$$\frac{v}{V_{max}} = \left[\frac{y}{R}\right]^{\frac{1}{n}}$$

where

$\quad V_{max}$ = the velocity at the meter axis
$\quad y$ = the distance from the meter wall
$\quad n$ = the 'power'
$\quad v$ = the velocity at y
$\quad R$ = the meter radius

By integrating over the pipe area, one obtains the average flow velocity V_{avg} from

$$Q = \pi R^2 V_{avg} = \int_0^R v\, 2\pi r\, dr = 2\pi \int_0^R V_{max}\left(\frac{y}{R}\right)^{\frac{1}{n}}(R-y)\, dy$$

where

$\quad Q$ = the volumetric flow rate
$\quad r$ = the radial location of v $(r = R - y)$

After integration this yields

$$V_{avg} = V_{max}\frac{2n^2}{(n+1)(2n+1)}$$

By integrating along the pipe diameter, one obtains the average center chord velocity V_{chd} from

$$V_{chd}R = \int_0^R v\, dy = \int_0^R V_{max}\left(\frac{y}{R}\right)^{\frac{1}{n}}dy$$
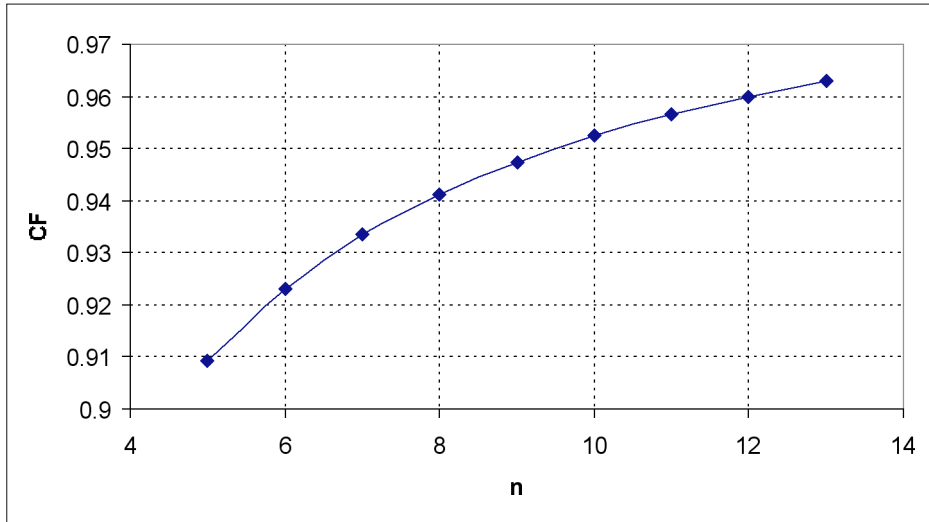
which after integration yields

$$V_{chd} = V_{max}\frac{n}{n+1}$$

A correction factor CF can then be defined as the factor used to bring the center chordal velocity to the true average velocity.

$$CF = \frac{V_{avg}}{V_{chd}} = \frac{2n}{2n+1}$$

It is plotted in Figure 1 as a function of the power law exponent.

Wall Roughness, Reynolds Number, and the Power Law Profile
To be able to use this correction factor one needs to find a way of expressing n in real world terms of Re and Wall Roughness WR. Fortunately, Prandtl and Nikuradse [Ref.1] sorted this out in the 1930's when considering the friction loss in pipe flow. The friction factor (f) depends upon Re and the relative Wall Roughness WR/d (to the pipe diameter d), but can also be related to the velocity profile through the shear stress at the pipe wall. The friction loss is proportional to the wall shear stress, which in turn is proportional to the velocity gradient at the pipe wall.

$$n = 1.74 - 2LOG\left(\frac{2WR}{d} + 18.7\frac{n}{Re}\right) \qquad \text{and} \qquad n = \frac{1}{\sqrt{f}}$$

The relationship is given by:

$$n = 1.74 - 2LOG\left(\frac{2WR}{d}\right)$$

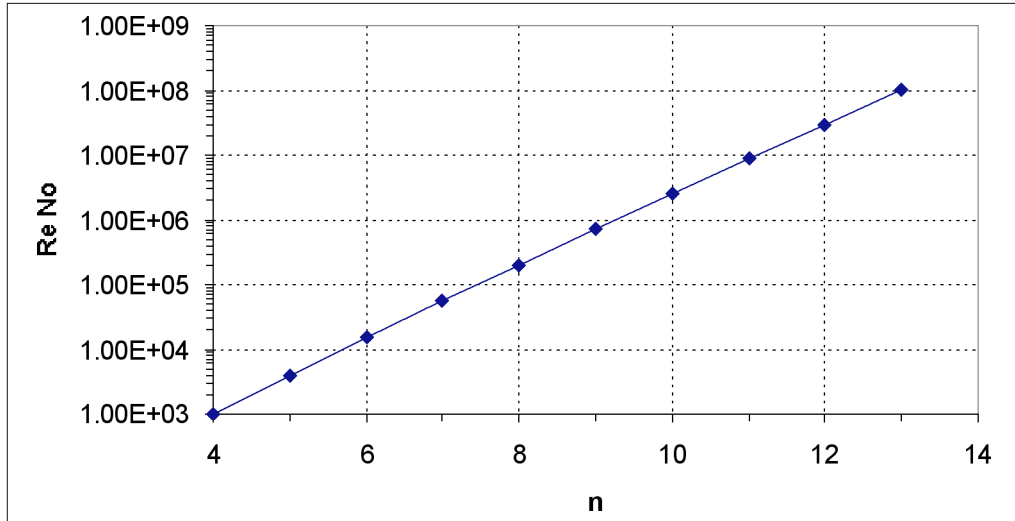This simplifies for the two special cases. For hydraulically smooth pipe flow with WR = 0, This is shown in Figure 2

Figure 2. Relation of n to Reynolds number in smooth pipe flow.

Then for fully turbulent rough pipe flow, as Re V→∞

$$n = 1.74 - 2LOG\left(\frac{2WR}{d}\right)$$
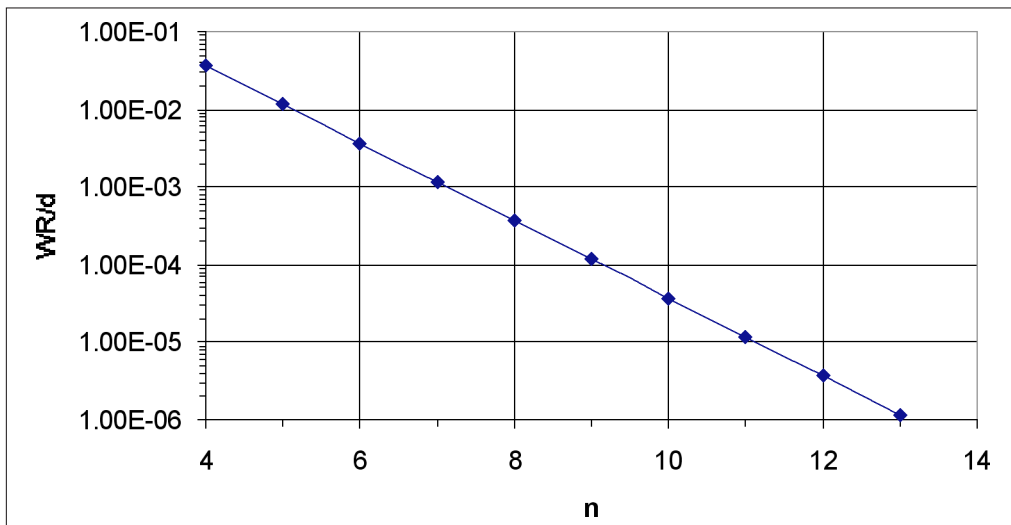
This is shown in Figure 3.

Figure 3. Relation of n to wall roughness in fully turbulent flow.

Practical Implications
This work shows that a meter using a single path through the center of the pipe will overestimate the flow by between 4% and 9% if no correction factor is applied. The overestimate is due to the high velocity in the center of the pipe affecting the line integral disproportionate to the small area represented.

It is quite straightforward to calculate the Reynolds Number (*Re*) from:

$$Re = \frac{vd\rho}{\mu}$$

Measuring the pressure and temperature allow reasonable estimates of the density r and viscosity m. Then the correction factor CF can be calculated from this live value of Re.
If the relative Wall Roughness WR/d is known it is easy to calculate a correction factor. Although WR/d might be known for a new pipe, there is no measure of how it changes with time. Hence it is not possible to make a real-time profile correction for WR/d.
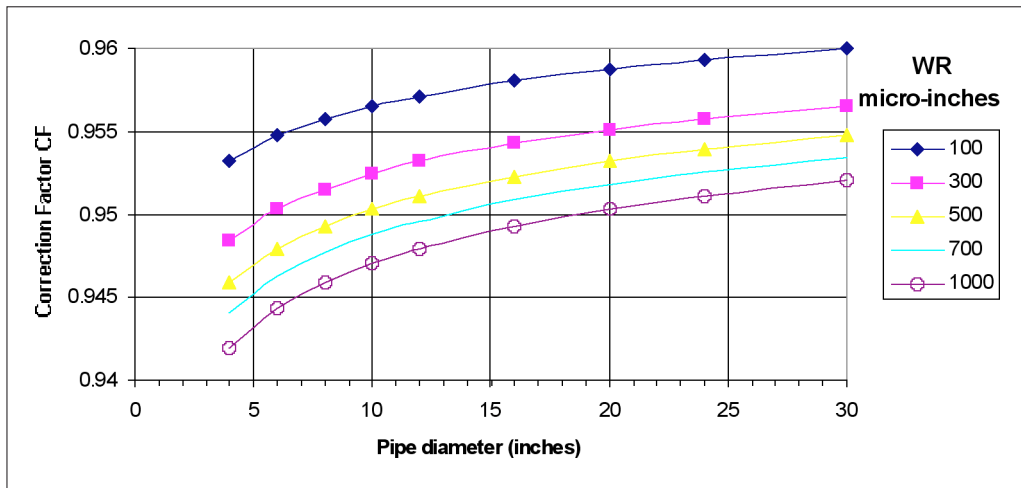The effects of changing roughness are shown in Figure 4.

Figure 4. Variation of CF with pipe roughness and diameter.

Thus it is quite easy to produce 0.5 to 1% error with time as the pipe roughness changes, without any means of correction.

3. EFFECTS OF REYNOLDS NUMBER AND PIPE ROUGHNESS ON A FOUR-PATH ULTRASONIC METER

The Four-Path Ultrasonic Meter Geometry
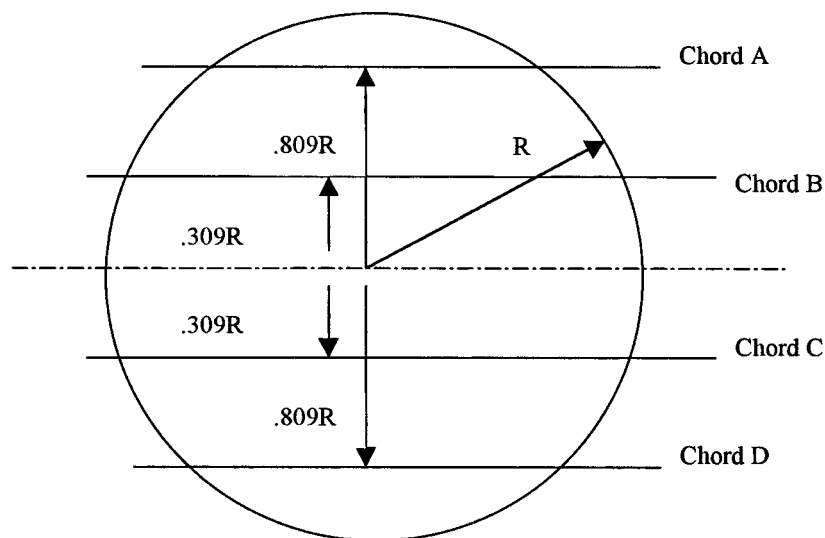An arrangement of four chordal paths proposed by British Gas is that shown in Figure 5.

Figure 5. Ultrasonic multipath geometry proposed by British Gas.

A Gauss-Jacobi numerical integration [Ref.2] gives the average flow velocity V_{avg} over the area of the pipe from the weighted sum of the line averages along the chords V_i

$$V_{avg} = \sum_{i=1}^{i=4} W_i V_i = 0.1382 V_A + 0.3618 V_B + 0.3618 V_C + 0.1382 V_D$$

where the weighting factors, W_i, are determined for the specific chord locations. Note that

$$\sum W_i = 1$$

It is significant that integration of a uniform velocity profile, using this method would give the correct answer. Another interpretation of the weighting factors W_i is the proportion of pipe area associated with each chord to obtain the flow, the sum of which must be 1 unity.

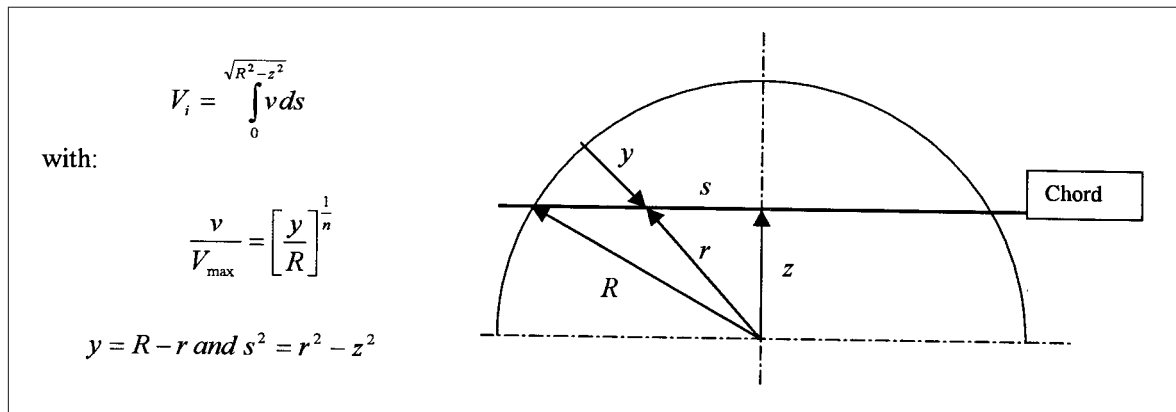The average chord velocity V_i is obtained as shown in Figure 6.

$$V_i = \int_0^{\sqrt{R^2 - z^2}} v \, ds$$

with:

$$\frac{v}{V_{max}} = \left[\frac{y}{R} \right]^{\frac{1}{n}}$$

$$y = R - r \text{ and } s^2 = r^2 - z^2$$

Figure 6. Evaluation of chordal integration.

This is integrated numerically by dividing the integration path **s** into 10 or more parts. Care is required with the segment next to the wall because of the very steep velocity gradient.

Chordal Integration
Numerical integration of the chord velocities for the four-path configuration has been performed for a range of n in the power law distribution. A typical example for $n = 7$ and $n = 12$ is shown in Figure 7.

Note, these represent the same average flow with $V_{avg} = 1$, hence if the velocity increases in the center of the pipe it must decrease near the wall to maintain continuity. In this example the velocity on the centerline increases from 1.04 to 1.07, which is the error made by a meter using a single path through the centerline. However, with the four-path meter when the velocity on Chord-B increases, that on Chord-A decreases. The decrease on Chord-A is greater than the increase on Chord-B, but this is compensated for by the weighting factors to give virtually the same flow.
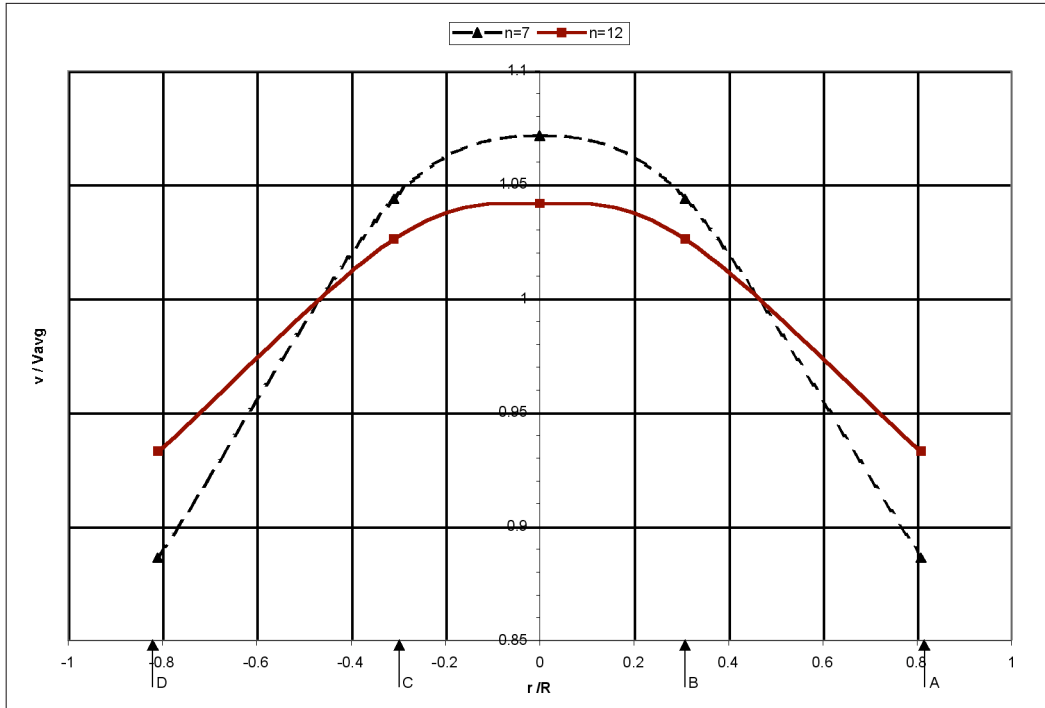
Figure 7. Velocity profile with four-path meter for two values of n.

The numerical integration process is shown in the table below:

n	V_B, V_C	V_A, V_D	$W_1 (V_B + V_C)$	$W_2 (V_A + V_D)$	V_{avg}	V_{BC} / V_{AD}
7	1.0437	0.8861	0.7552	0.2449	1.0002	1.1778
12	1.0260	0.9329	0.7424	0.2579	1.0003	1.0998

Furthermore there is information to show that the velocity profile has changed, by the ratio V_{BC} / V_{AD}, due to either Reynolds No or pipe roughness.

This process is repeated for values of *n* from 4 to 14, and the four-path meter (SeniorSonic) is compared to the single path through the centerline (JuniorSonic) with the result shown in Figure 8.

The four-path meter does an excellent job of integrating a power law profile, with a maximum error of 0.06% at *n* = 4, and from *n* = 5 to 14 the error is reduced to 0.03%. Thus the meter should be immune to normal changes in Reynolds Number and pipe roughness, whereas the meter which depends on a single path through the center shows a 7% variation over the complete range.
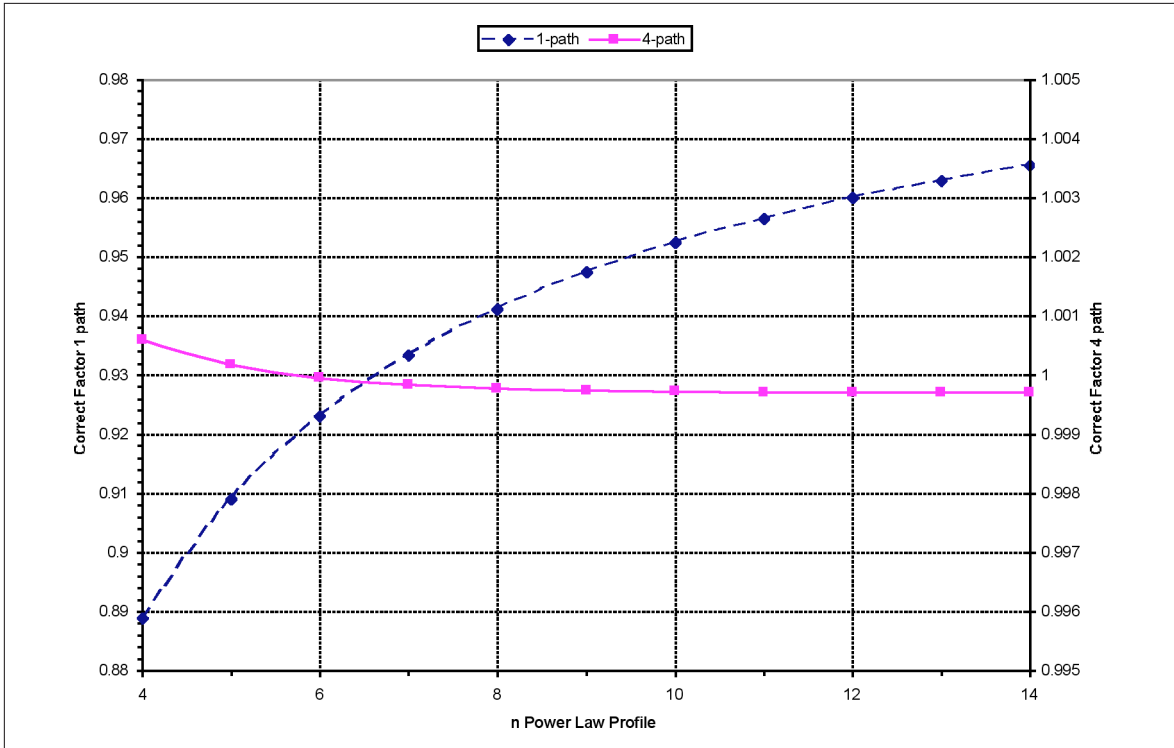
Figure 8. Effect of changing power-law profile on performance of centerline and Daniel four-path meters.

We can also make use of the ratio $V_{BC} / V_{AD} = (V_B + V_C)/ (V_A + V_D)$ to examine the velocity profile that exists in the pipe, and use the ratios V_B / V_C and V_A / V_D to check for symmetry. In reasonably well developed axi-symmetrical flow conditions, we would expect $V_B / V_C = 1$, $V_A / V_D = 1$, and V_{BC} / V_{AD} to range from *1.10 to 1.18,* as shown in Figure 9.

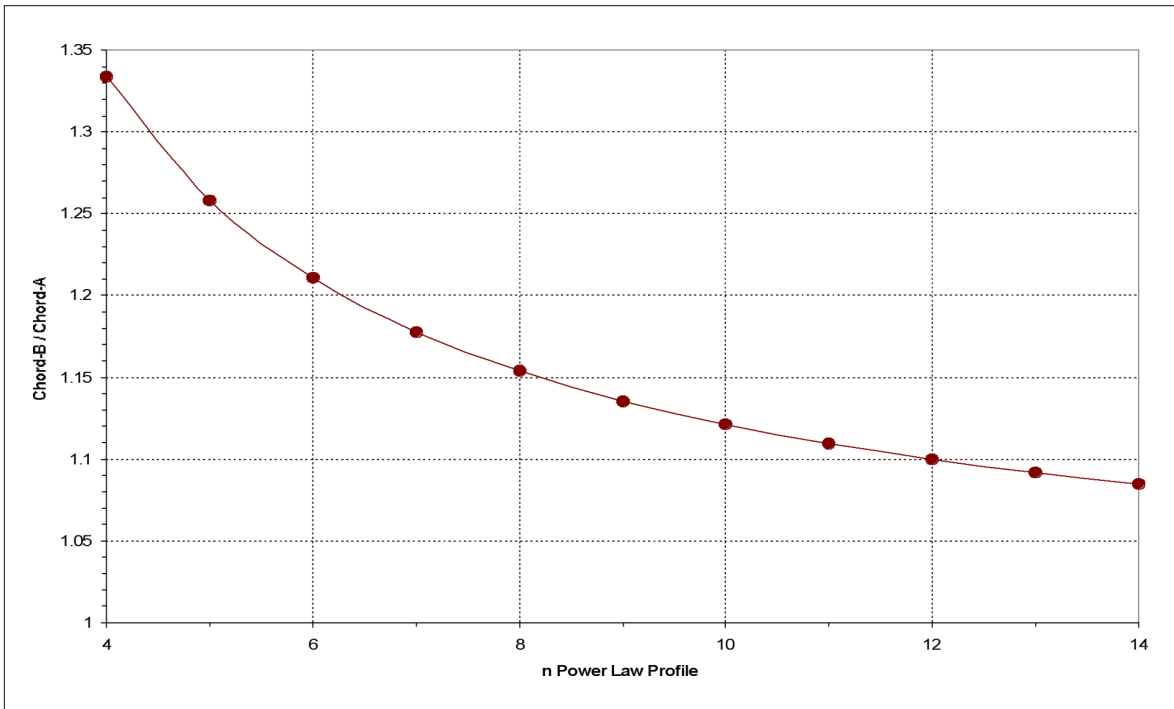*Figure 9. Variation of velocity ratio VBC/VAD with power law profile for the Daniel four-path meter.*

4. THE EFFECT OF FLOW PROFILE ASYMMETRY ON SINGLE AND MULTI PATH ULTRASONIC METERS

The profile

For this study, a mathematical model of the velocity profile downstream of a 90° elbow was used [Ref. 3]:

$$V(r,\theta) = \frac{2}{\pi^5} * r(1-r)^{1/4} * \theta^2 (2\pi - \theta)^2 + (1-r)^{1/9}$$

In cylindrical coordinates, r is the radial position from 0 at the center to 1 at the meter wall, and q is the angle in radians. This profile is plotted as velocity contours in Figure 10.

Note that when r = 0 (center), V = 1, and when r = 1 (wall), V = 0. The maximum asymmetry occurs when $q^2 (2p - q)^2$ is a maximum i.e. when q = p.
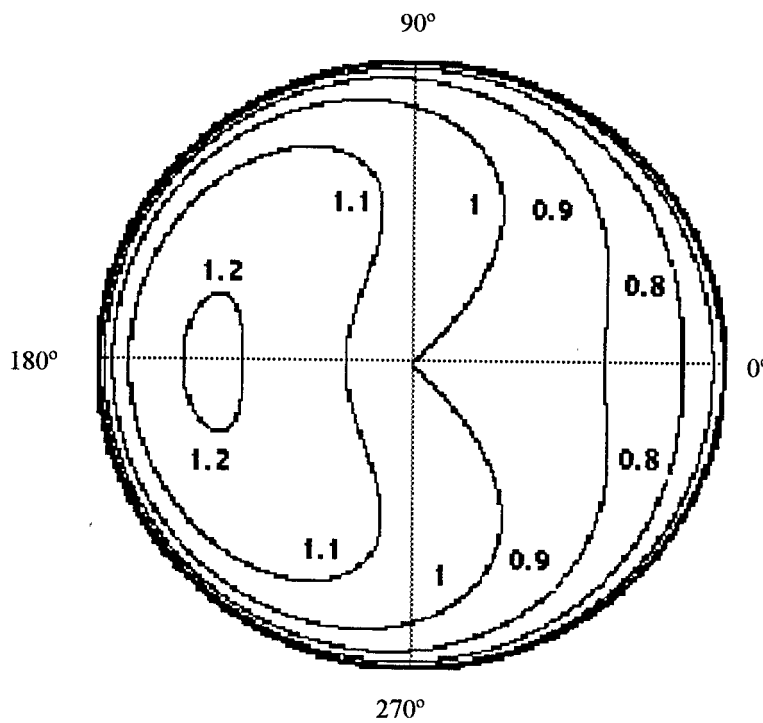

Figure 10. Mathematical model of velocity profile downstream of a single 90° elbow.

Centerline Meters

For a single path through the centerline the results shown in Figure 11 are obtained, normalized for an average flow velocity = 1.
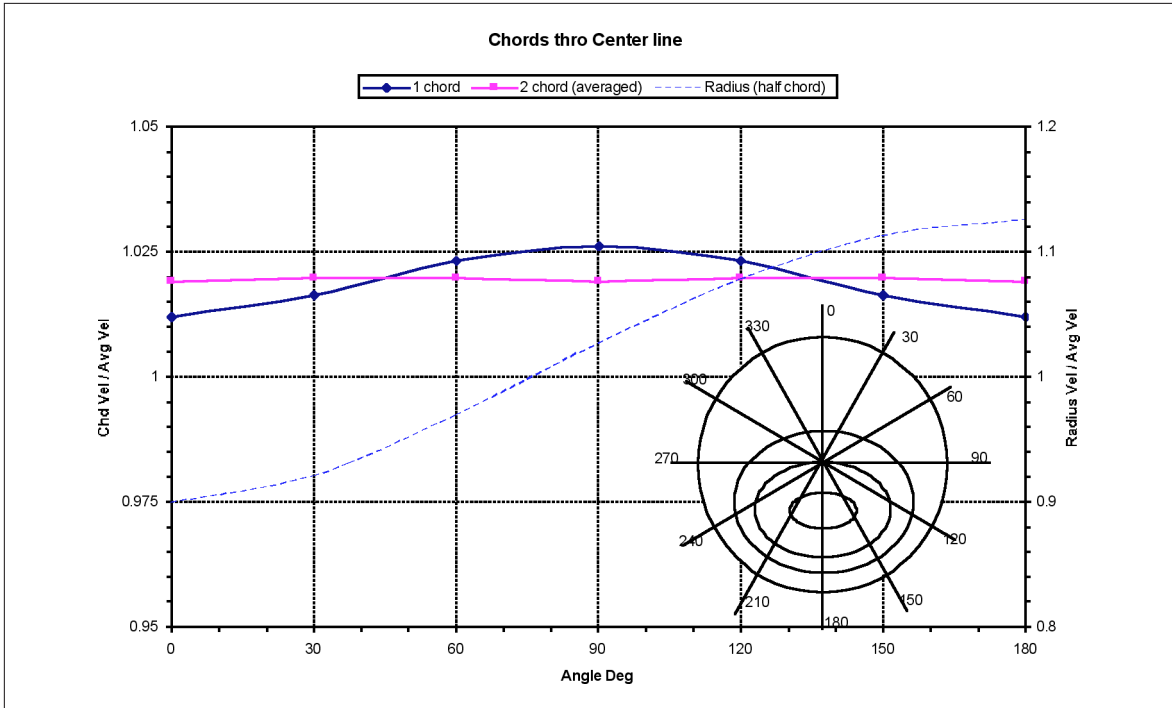
Figure 11. Velocities measured along centerline chords at various angles.

To give an idea of the asymmetry, the velocity which would be measured along a radius (half a chord) is shown by the broken line and second y-axis. The minimum velocity at 0° is 0.9 and the maximum at 180° is 1.125 i.e. a 25% variation.

The variation of velocity with the angular position (θ) of the diameter is much less: 1.012 at 0 ° and 1.026 at 90° or 1.4% and the actual value of the velocity is quite close the average flow velocity of 1. However, since it is normal practice to make a Reynolds number correction (CF) for a single path through the centerline as shown in Figure 1, with n = 9 the power law velocity profile CF = 0.947. Thus the meter output would vary from 0.958 to 0.972, or a 3 – 4% error.

If two diameters are used at right angles to one another, made up from 0° and 90°, 30° and 120°, etc., the output is virtually constant at 1.02 making the two path meter independent of angular position. Once again the Reynolds number correction will turn this +2% error into a –3% error and is not good in disturbed flows.

Four-path SeniorSonic

Integrating along the chords of the four-path meter, located at +/- 0.309 R and +/- 0.809 R gives the results shown in Figure 12.
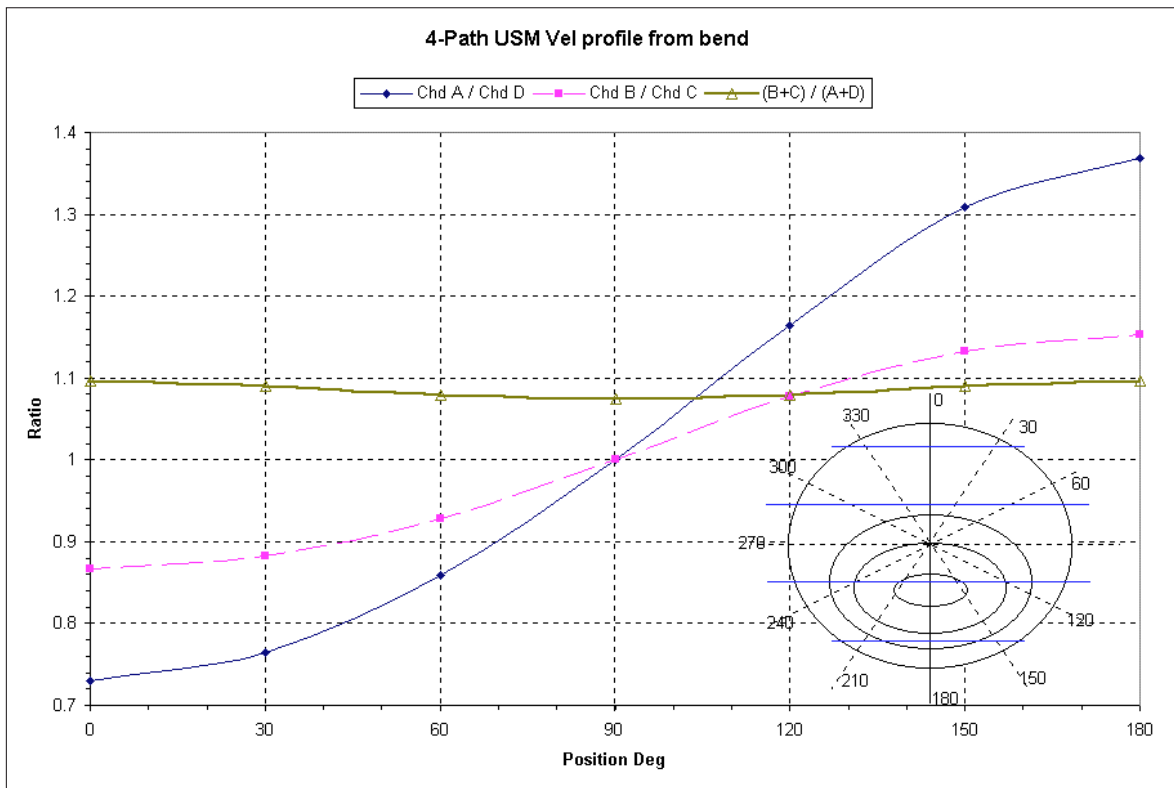

Figure 12. Chord measurements from four-path USM downstream from a bend.

The profile is not axi-symmetric, so the four-paths reflect the asymmetry in all cases except at the 90° position, where the paths and profile are symmetric about the 0° position. This is a mathematical profile which would never perfectly exist in practice, but which can be used to simplify integration.

The ratios of chord velocities can again be used as a measure of asymmetry as shown in Figure 13. In fully developed flow the velocities of the outer chords are 0.89 and the inner chords are 1.042, so $V_A / V_D = V_B / V_C = 1$ and $V_B / V_A = V_C / V_D = 1.042 / 0.89 = 1.171$

When the profile looks symmetrical at 90°, $V_{BC} / V_{AD} = 1.075$ which is 10% off normal (1.171) and suggests that the profile is too flat.

The outer and inner chord velocities are shown in Figure 14, together with the results of the integration for the average flow velocity. The four-path integration technique does a very good job on this asymmetric profile, and is 0.2% high virtually independent of the angular orientation of the chords (shown on second Y axis).

The mean chord data gives $V_{AD} = 0.94$ and $V_{BC} = 1.03$, which again appears more flat than the normal fully developed flow.
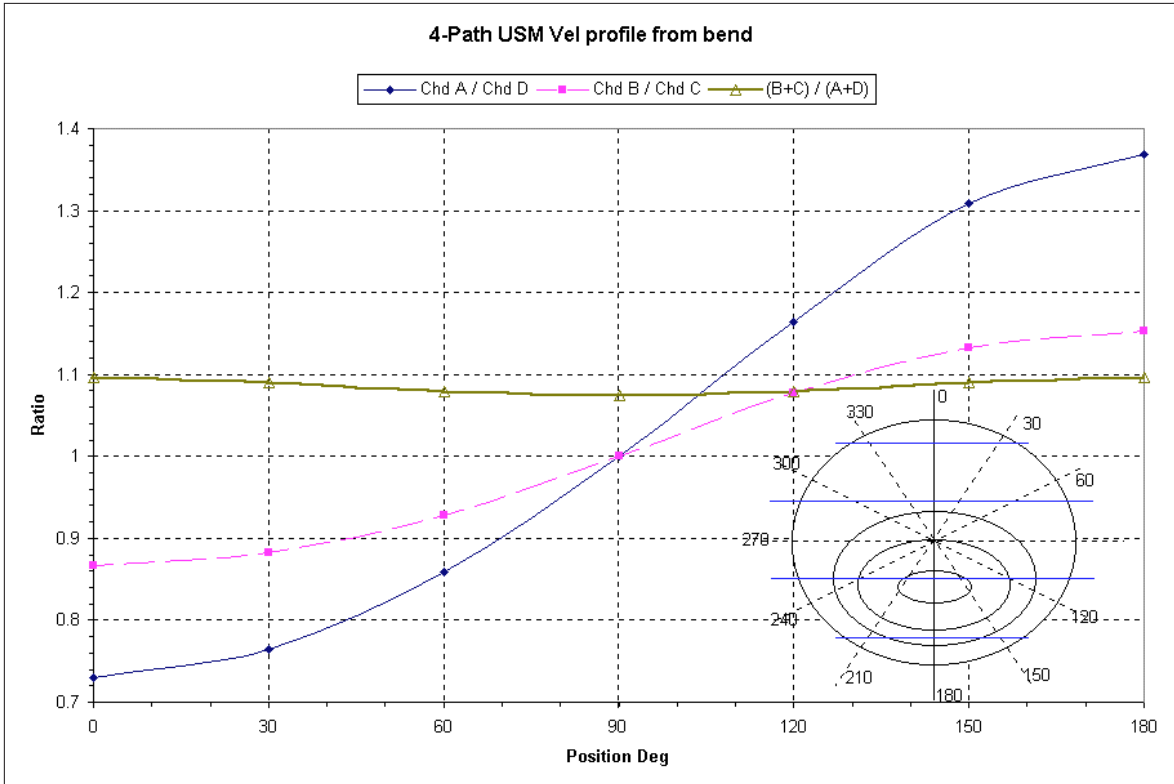
Figure 13. Velocity ratios from four-path ultrasonic meter downstream of a bend.
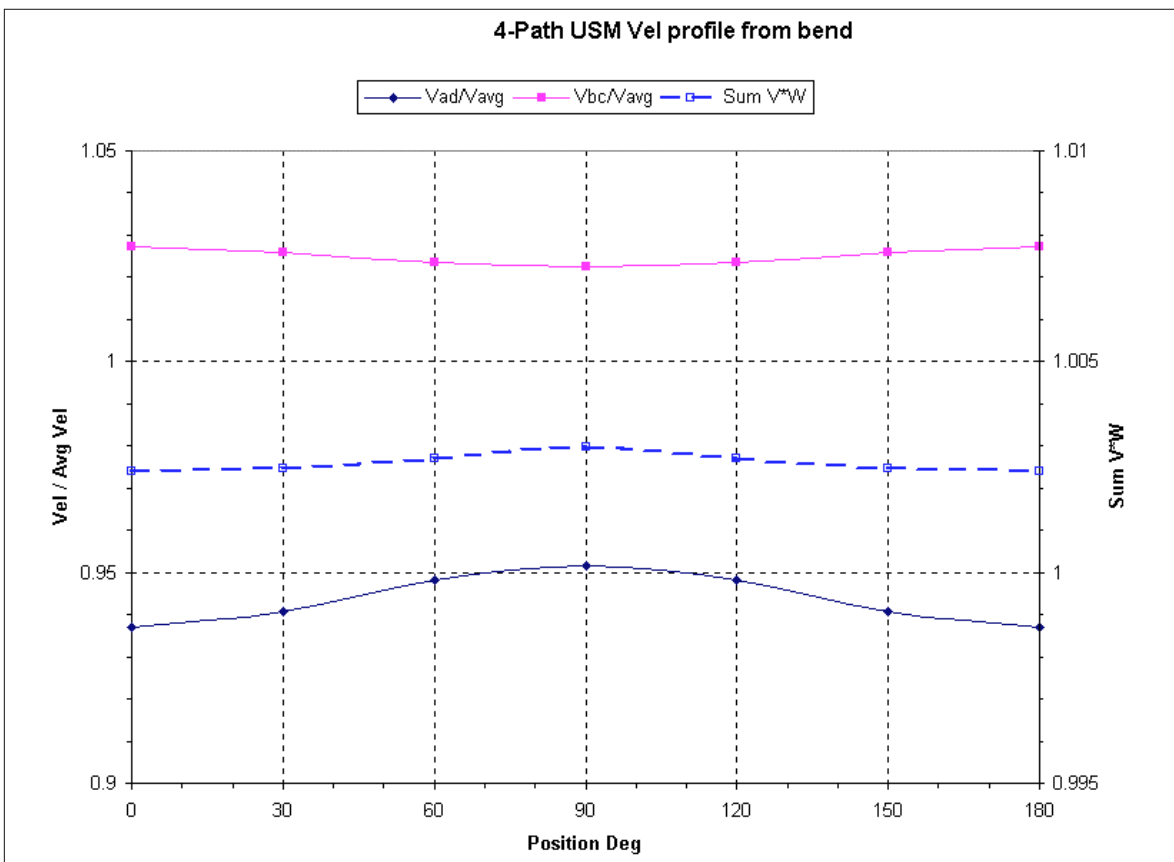

Figure 14. Inner and outer chord velocity contributions to the average flow velocity for the four-path ultrasonic meter downstream of a bend.

5. CONCLUSIONS

For centerline measurements in fully developed flow, corrections of 4 – 8% are necessary to correct for Reynolds number and roughness variations. *Re* variation can be corrected dynamically from knowledge of *r* and *m*, but variations in roughness can cause uncompensated errors of as much as 1%.

In asymmetric flow, variations of 1.5% can occur due to the orientation of the centerline chord relative to the asymmetry. An error of +1.5% will then become – 4% when the normal *Re* correction is applied. One can thus conclude that Reynolds number correction is in appropriate in disturbed flow, but only applies strictly to fully developed flow.

A flow conditioner might help improve the disturbed profile, but at the expense of additional pressure loss, cost, blockage and potential fouling. Furthermore, bi-directional flow would require two flow conditioners with separate forward and reverse calibrations.

Turning now to a four-path meter of the Daniel Senior Sonic type, in fully developed flow, corrections of only 0.06% account for all *Re* and roughness effects. In the asymmetric flow downstream of a single elbow, the integration error has been shown to be only 0.2%, irrespective of orientation. This is at least an order of magnitude less than the errors from a single path centerline meter.

Fixed upstream pipe-work would normally be expected to produce a fixed velocity profile, however with "T" pieces or headers the velocity profile can change with the proportion of flow in the respective branches. The four-path meter would detect and measure such changes through the velocity profile, while the integration technique would still give the correct flow rate. In fact four velocity measures of the profile are a very useful diagnostic tool provided by the meter.

The better accuracy and superior diagnostic ability of the four-path meter justify the claim that it is suitable for fiscal measurement, while the single path centerline meter is clearly not suitable.

A final interesting observation from the flow asymmetry work is that the velocity profile seen by the four chords is not unique for different orientations in the same flow. Hence it would be extremely difficult to devise a better integration technique (e.g. using variable weighing factors) that makes use of this velocity profile information. In fact it is obviously unnecessary, as the Gauss-Jacobi integration method works remarkably well.

REFERENCES

1. "Boundary Layer Theory" by H. Schlichting, McGraw-Hill Book Co. 1960

2. ISO/TR 12765:1997(E), Measurement of Fluid Flow in Closed Conduits – Methods Using Transit Time Ultrasonic Flowmeters.

3. Franc, S., Heilmann, C. and Siekmann, H. E. "Point Velocity Method for Flow Rate Measurement in Asymmetric Pipe Flow" Flow Meas. Instrum. Vol. 7 No. 314 1996