18th NORTH SEA FLOW MEASUREMENT WORKSHOP 2000

TRADITIONAL UNCERTAINTY ANALYSIS AND IP GUIDELINE

Mr R Paton, NEL

1 INTRODUCTION

Within the oil industry, measurement has always been of prime importance in the transfer of product both onshore and offshore. When calculating duty payable or allocation, the accuracy in measuring the quantity of oil is vital. It can be argued that when trading oil, consistency and agreement in measurement between buyer and seller is more important then accuracy (or even the correct value of the quantity!). However, to achieve consistency, you have to measure accurately, and to judge if you have consistency you have to know the uncertainty.

Throughout the production and distribution chain the accuracy of the measurements has always been important. The term **accuracy** is easily understood and comes into all specifications of measurements across all fields of metrology. It is, by definition, a qualitative term. Accuracy will usually have a number attached but will not define the level of confidence of the measurement. In recent years the oil industry has followed the general trend in metrology and recognised that ‘accuracy’ is inadequate to provide the information needed for transactions.

An estimate of **uncertainty** of measurement must include (explicitly or implicitly) the confidence in the estimate. Increasingly this demanded as a measure to accompany any result. As the need to express uncertainty increases so it has become clear that the methods and traceability chains used in the oil industry do not easily adapt to the methods of estimating uncertainty provided by the statistically based standards in the scientific and pure metrology fields. To retain consistency it is vital that estimation of uncertainty must be carried out in a statistically sound and auditable manner but in a way that the engineers in the industry can relate to. n many cases uncertainty estimation is an art rather than a science and it would be rare that two independent engineers would derive the same value. This is an untenable situation in the oil industry where, on transactions, agreement can be more important than accuracy. Unfortunately until many more uncertainty calculations are carried out and input figures agreed, no standardisation will be possible.

This paper has been prepared to give some guidance on how uncertainty estimation can be carried out, how to overcome some of the statistical anomalies found in oil industry practice, and prepare for a consistent expression of uncertainty compatible with international metrology practice. By following the principles set out in this paper it will also be possible to set the expected uncertainty for transactions where all the input measurements are within specified limits. The actual uncertainty may be better but only individual analyses would determine that.

Historically the oil industry has had to meet the practical needs of measurement in arduous conditions. As a result uncertainty calculations using a common methodology have not been a high priority. Traditionally measurement ‘accuracy’ was defined for secondary measurements such as temperature and much emphasis has been placed on the repeatability of measurements based on a very small sample of results. From this an overall view of the resultant accuracy is assumed.

Uncertainty estimation has rarely been carried out thoroughly and, using a pragmatic examination of results and agreement between parties, accuracy has been defined by agreement. Similarly many of the methods used to collect data during calibrations appear, at first sight, to make rigorous uncertainty estimation very difficult. Commercial and operational requirements generally prevent the collection of data in the quantity expected from any scientific based measurement.

The need to estimate uncertainty has become more important in recent years to improve the confidence in mass balance across systems and reliably fine tune ever faster contract specifications and regulatory requirements. It is vital that these uncertainty estimates are carried out in a consistent manner and are understood by all parties. All estimates of uncertainty must carry a confidence limit, but more importantly they must have the confidence of operators, contract staff, partners and regulators.

To comply with measurement standards from ISO etc, uncertainty should be included in the standard and this will carry over into regulation and contracts. Acceptance limits (criteria), accuracy expectations and repeatability criteria will continue to feature strongly in practical standards and procedures. Such criteria are compliant with good practical metrology and can be used as an input to the definition of an uncertainty.

The international standards bodies recognised that a statistically sound methodology was required to service the need for uncertainty estimation in metrology and hence produced the **Guide to Uncertainty of Measurement (GUM).** The introduction of the guide was met with some resistance from industry but overwhelming enthusiasm by scientific based metrologists and adopted by the standards bodies. Within ISO and IEC all new standards are encouraged to include uncertainty criteria and it is now up to industry to make these follow the guide.

What is obvious is that the GUM does not relate easily to the history and realities of practical industries. The GUM expresses uncertainty in statistical terms different to the established methods familiar to most measurement engineers. It is heavily scientifically and statistically based. The GUM is however a guide, not a standard! It is now up to individual industries to follow the methods and apply them to their own needs.

2 GUM: THE PRINCIPLES

Traditionally uncertainty was derived by combining all estimates of the magnitude of individual errors in a measurement. This provides an estimate of <u>systematic</u> error in the final quantity. To this systematic error, the estimate of <u>random</u> uncertainty is added by taking a statistical result from multiple measurements to estimate the probability distribution of the result, hence providing a confidence band.

The principles outlined in the GUM are the same but the approach and terminology different.

The fundamental concept of the GUM is to assume <u>all</u> uncertainties are equivalent to the standard deviation of the results from many repeated tests. By assuming this, all uncertainties can be assigned a probability function and hence the final uncertainty fully recognises the potential distribution of results and gives a much better confidence expression through a coverage factor or confidence limit. This concept is sound, but the application to industry where most uncertainties are not derived from a (apparent) knowledge of large numbers of tests requires some consideration.

A number of terms are used in this approach:

2.1 Standard Uncertainty

Standard uncertainty is the uncertainty of the result of measurements expressed as a standard deviation. All uncertainties are initially estimated as standard uncertainties. In deriving standard uncertainty, two types of uncertainty (type A and type B) are recognised.

2.2 Type A Uncertainties

Type A evaluations of uncertainty are those using statistical methods, specifically, those that use the spread of a number of measurements.

Any measurement can be repeated a number of times and the statistical distribution found, analysed and the standard deviation and probability limits defined. This is equivalent to the traditional random uncertainty but has been renamed in the GUM as a type A uncertainty.

2.3 Type B Uncertainties

Type B evaluation of uncertainty is one carried out by means other than the statistical analysis of a series of observations.

It is recognised that in many cases it is impractical or impossible to gather enough data in the experiment to derive a representative statistical estimate from repeated measurements. This situation leads to the derivation of a type B uncertainty. Type B uncertainties are in some ways equivalent to the traditional systematic error. To achieve an estimate of standard uncertainty, type B uncertainties must be assessed not only on their magnitude, but on the estimated probability distribution encompassed within the estimate. Within the GUM type B uncertainties are first estimated in magnitude, and then reduced to an equivalent standard uncertainty based on an assumed probability distribution. This is achieved by the application of a **divisor** to reduce the spread of uncertainty to standard uncertainty.

2.4 Expanded Uncertainty and Coverage Factor

Expanded uncertainty is computed by combining all standard uncertainties, both type A and type B, and multiplying by a coverage factor chosen to express the results encompassed by a large fraction of the probable values reasonably attributed to the measurement. Generally these confidence limits are accepted as being at 95% confidence levels. As is explained the GUM a coverage factor is a better parameter to be used. This is the factor by which the standard uncertainty is multiplied to give the expanded uncertainty.

If a normal distribution is assumed and a large number of results are assumed, k=2 approximates to a confidence of 95% (actually 95.45%) and k=3 is applied to approximate to a 99% confidence.

Expanded uncertainty is therefore the final expression of an uncertainty analysis. This is expressed formally as the measurement value with an expanded uncertainty and a coverage factor.

3 HOW TO CARRY OUT AN UNCERTAINTY ANALYSES.

The analyses of uncertainty can be divided into 5 steps, summarised below and explained in more detail later.

i The first stage of an uncertainty analysis is to define the process leading to the final result. Breaking an analysis into smaller process steps is sound advice. Decide on dimensional or non-dimensional analysis.

ii For each process define all the functions, algorithms and relationships used to define the result. List all the input quantities and assign their value for the conditions being analysed. For each input quantity or measurement all the sources of uncertainty are identified and quantified. The probability function describing each uncertainty should be defined to allow the calculation of standard uncertainty.

iii Many input measurements or parameters have a number of sources of uncertainty. All these individual uncertainties should be combined to give the total uncertainty in the measurement. This combined uncertainty can then be combined with the other uncertainties in the process. If an uncertainty is described by a function this must be defined as in iv) below .

Adding uncertainties of the same units is done by root sum square addition.

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[c_i u(x_i) \right]^2}$$

where c is the sensitivity coefficient.

iv The different uncertainties in the process are calculated by multiplying the uncertainties in the measurements by the 'sensitivity coefficients'. The sensitivity coefficient provides the link between the input quantity and its effect on the result through the function or algorithm used.

Where a number of algorithms are used, treating each one separately and combining the uncertainties found from each one in turn simplifies the process.

v The standard uncertainties are combined by root sum square addition. Multiplying this combined uncertainty by the chosen coverage factor provides the expanded uncertainty.

These steps require some clarification and are described I more detail below.

3.1 Step (i)

Define the value you wish to assign an uncertainty to. The final purpose can become lost in the process.

Define the traceability chain and calculation chain required to get to the result. List the processes involved in getting to the result back until reliable uncertainty estimates are available.

Define each process which will provide a result with an uncertainty estimate which can stand alone and act as an input to the next process.

Break the overall process into manageable and stand alone parts.

Work with dimensional or non-dimensional uncertainty estimates. These are sometimes called absolute or relative uncertainty estimates. Dimensional analyses uses uncertainties expressed in appropriate engineering units. Non-dimensional analysis quotes uncertainty in relative terms e.g. per cent or parts per million. It is instinctive to think in non-dimensional terms and simple analyses can be accomplished this way. More complex and generic analysis is better carried out in dimensional terms to avoid many numerical problems

An example of the problems with non-dimensional terms is illustrated by temperature where 0.1% uncertainty is very different in °C and °F and consider the relevance of 0.1°C uncertainty as a percentage of 0°C.

It is good practice to analyse in dimensional terms, converting percentages to dimensions as required, but expressing the final result in both dimensional and non-dimensional terms allowing an instinctive impression of the result to be retained.

3.2 Step (ii)

For each process, define all the equations and relationships used to calculate the final result. From these equations list all the measurements and input parameters involved. Assign uncertainties to each measurement.

Most measurements will have a number of uncertainty contributions. For example, the calibration uncertainty, the resolution of the readout, the uncertainty in any corrections, the uncertainty in any electrical transmission or how it drifts over time. There may well be a final contribution based on engineering judgement and this will be discussed later. Consider if any uncertainty is going to be included twice. e.g. resolution uncertainty may be included in a type A assessment of an instrument performance. In this case the rules for correlated values should be followed.

It is vital that all uncertainties be recognised.

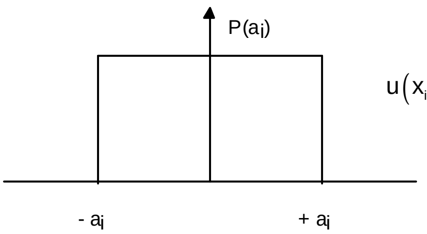
Define each uncertainty as a type A or type B estimate. Assign the appropriate divisor to reduce it to a standard uncertainty.

For type A, a large number (n) of data points will be available, and the mean and standard deviation (SD) can be calculated. The uncertainty depends on the process and whether the mean of the data (or of a similar set of data) is to be used or whether only one new example of the data set is to be used.

The uncertainty of the mean is defined $u = SD/\sqrt{n}$.
The uncertainty of a single point is defined $u = SD$.

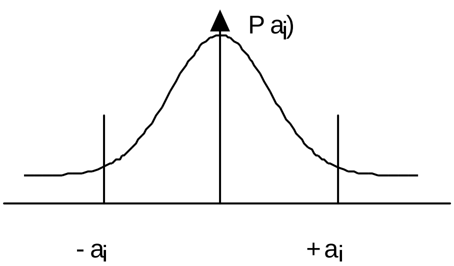
Type B uncertainties will come from other estimates. For example the manufacturer's data sheet, the resolution of the readout, historical drift of values, tolerance from the standard being followed.

Each type B uncertainty is assessed to judge the probable distribution of measurements within the bounds of the uncertainty estimate. Different distributions are commonly recognised, and choosing the distribution defines the factor (divisor) used to reduce the estimate to standard uncertainty (standard deviation).



$$u(x_i) = \frac{a_i}{\sqrt{3}}$$

The most common distribution assumed is the rectangular distribution. This distribution assumes that there is an equal probability of the measurement lying anywhere within a quoted range, and an insignificant probability of it lying outside that range.

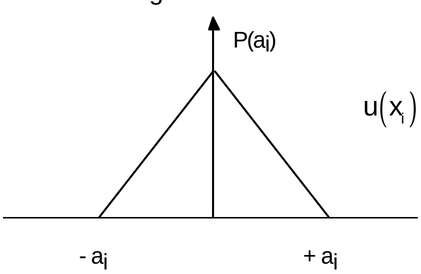


$$u(x_i) = \frac{a_i}{k}$$

If the uncertainty comes from a formal calibration, the distribution is assumed to be normal and the divisor is the coverage factor k. If 95% confidence is quoted and no other qualifying information to determine degrees of freedom, also assume k=2.

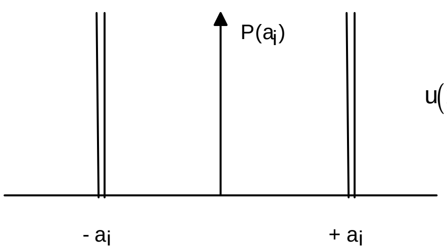
It will be rare to find any other distributions in basic oil measurement applications but two should be recognised

Triangular distribution



$$u(x_i) = \frac{a_i}{\sqrt{6}}$$

Bi-Modal distribution



$$u(x_i) = a_i$$

3.3 Step (iii)

This step requires little clarification other than to ensure all uncertainties are recognised. If an uncertainty is the result from another input and a function, this can be evaluated as described in step iv below as a self- contained process.

3.4 Step (iv)

The sensitivity coefficient is the expression of how a variation in an input parameter affects the final result of a function or algorithm. Multiplying the standard uncertainty by the sensitivity coefficient gives the standard uncertainty of the calculated parameter.

Sensitivity coefficients can be calculated in two ways, analytically or numerically.

The analytical method involves differentiating the function with respect to each of the parameters in turn. If the input values are substituted into the resultant functions, the answers provide the sensitivity coefficients appropriate to each input parameter. This is probably the mathematically correct method for evaluating sensitivity coefficients. It does require a degree of mathematical skill if the functions are complex and, if being used in a spreadsheet analysis, each differentiated function has to be programmed into the software.

The numerical method involves evaluating the function for the input values and again for the input value incremented by a small amount. The sensitivity coefficient is the difference between the two determinations of the function divided by the increment. Ideally the increment should be as small as is practical and good advice is to choose the increment approximating to the uncertainty in the parameter. The method is very suitable for spreadsheet evaluations.

3.5 Step (v)

Once all uncertainties are evaluated they can be added by the root sum square method. They are then multiplied by the chosen coverage factor to give the expanded uncertainty. Choosing the coverage factor can demand some statistical skill when very small samples are involved in determining the uncertainty or the distribution of the end result is not expected to follow a normal distribution. Otherwise K=2 is normally found to meet the needs of oil measurement. When small samples of data are used, and the standard deviation is the basis of the uncertainty, the coverage factor has to be calculated to give the equivalent of 95% confidence, this can range fro 2.5 up to 3 or 4 for small numbers of data points. This will be discussed later.

4 PARTICULAR ISSUES INVOLVED IN OIL INDUSTRY ANALYSES

4.1 Judgement

Judgement plays a key part in uncertainty analysis. It is commonly found that an underestimate of uncertainty is derived when a thorough analysis is carried out. This is invariably due to over- optimistic estimates of the input parameters. Only a careful, prudent examination of errors will lead to a satisfactory conclusion. To give an example:

A thermometer with a resolution of 0.05°C comes with a calibration compared with a standard with an uncertainty of 0.01°C. A single measurement of temperature is taken using the thermometer, in a thermal pocket, which may or may not be filled with conducting fluid. Weather is providing an unknown effect and temperature distribution across the pipe is even less well known. Sometimes even the depth of the probe in the pocket and pocket in the pipe cannot be guaranteed. The fluid is hot, viscous and slow moving. The weather is cold and windy. What is the uncertainty of the temperature of the fluid?

Two options are available to identify the missing components. The correct approach is to accurately measure, under different ambient and fluid temperatures, the temperature of the fluid using a special thermometer. This will be done in a controlled way at points across the pipe. These results will be compared with the process thermometer and either a correction error chart produced, or the uncertainty derived. Very few installations in an oil measurement system have a capability, equipment or time to do this.

The second option is most likely to be required in practice. A judgement has to be made on the uncertainty introduced by un-quantified errors. It could be suggested that in the above example the fluid temperature will not be measured to better than 1 or 0.5°C. The judgement is not based on some documented prior knowledge but is based on experience, guesswork and a confidence in the results of the measurement. What separates the scientist from the oil industry measurement technician is that the scientist will use judgement to fine tune a well documented and experimentally derived uncertainty. Oil industry engineers have frequently to use judgement to define the largest uncertainty component based on little or no experimentation.

Judgement is a cost effective solution as long as it is based on knowledge and experience and does not lead to an underestimate of the uncertainty.

The exactness of the mathematical and arithmetic part of uncertainty analysis can be completely nullified by one optimistic error estimate or by missing an error source.

4.2 Choosing the Probability Distribution

It is clear that uncertainty estimation requires an awareness of statistical techniques. The consequences of assuming a normal or rectangular distribution, where in fact a more complex distribution of results exists, can be large. On the other hand common sense must prevail and, although recognising these effects, it may be more prudent to remain with simple analysis as long as this does not lead to an underestimate. This is the pragmatic approach generally adopted.

For occasions where the distribution of results do not follow a 'normal' distribution, they may show a non-standard or skewed distribution. More advanced statistical techniques are required to account for these distributions to derive a standard uncertainty. This is explained in the GUM. To simplify such cases an alternative approach is to assume a rectangular (or normal) distribution but ensure the confidence limits are widened to include all data. This avoids complex analyses but gives rise to a slightly larger uncertainty. It does not show that the result is more likely to be higher or lower than the mean.

4.3 Small Samples

It is a feature of nearly all calibrations carried out in the oil industry that very few calibration test points are used. Three or five are the norm. Using the provisions of the GUM, this would be interpreted as a type A uncertainty, and the standard deviation used to define the uncertainty. As the mean value is used, it is the standard deviation of the mean, $u=SD/\sqrt{n}$ which is used. Based on the number of degrees of freedom and for a calibration involving 3 points, k= 3.5 will more closely represent the 95% confidence and should be used.

The method of calibration specified in many standards also ensures that serious doubts exist about the nature of the distribution of possible results. Calibrations are specified to be carried out until three consecutive tests lie within set limits. This places doubt that the three points are part of the general population of test points.

Both these factors ensure that not only is there doubt about the validity of the uncertainty, but taking the standard deviation and applying the correct coverage factor will often yield an unrealistically high uncertainty.

Closer examination of this situation shows that recognising that the tolerance limits defined in the standard provide a measure of the confidence attached to the general population of all calibrations used to define the standard allows the uncertainty to be defined as a type B with a rectangular distribution. Using this argument, the uncertainty of the mean of a small number of test points, falling within a tolerance specified in a standard can be described by

$$u=a/\sqrt{(3.n)}$$

where a is the half range acceptance criteria specified and n is the number of test points.

Note that the spread or standard deviation of the actual test points does not enter the determination. To use the actual spread either, through experience of the particular calibration, the acceptance limits are changed, or a larger number of test points used to give a large enough population to base the result on.

4.4 Comparison with History or Drift

It is common to specify that the mean result should not differ from the previous calibration by more than a specified amount.

This is an expression of the uncertainty due to drift or stability in the result with time. Again this can be based on data derived to determine the limit specified rather than on the data from the particular measurement.

In this case the standard uncertainty due to potential drift in the mean is:

$$u=b/\sqrt{3}$$

where b is half the specified tolerance range.

Note that assuming no systematic drift in the mean is perceived, the uncertainty can be improved by using the mean of all calibrations rather than the last result. This will alter the uncertainty to:

$$u=b/\sqrt{(3m)}$$

where m is the number of mean values used.

If the mean of 10 or more historical results is used the limits can be re assessed, or the uncertainty calculated directly from the data.

4.5 Double Accounting and Correlation

In any uncertainty analysis, recognition of correlated measurements will have to be made. Correlation occurs where measurement uncertainties from a common source can be seen to affect different parts of the analysis. For example a thermometer is used to measure a proving tank temperature and then used again to measure the fluid through the meter being calibrated. Two temperatures, two readings, two installations, but one thermometer and traceability are seen.

A balance has to be struck between including all uncertainties and not double accounting where measurements, instruments or calibrations are common in different parts of the process. When this occurs the sources of uncertainty are correlated in some manner either through the use of the same instrument or through the calibration of different instruments. This is another example where estimation of uncertainty employs a level of judgement not normally seen in mathematical analysis. It is up to the user to include or exclude common uncertainty factors. Both type A and type B uncertainties can be correlated. Difficulties come when it is obvious that some aspects of an uncertainty are correlated and some are not.

Correlation can be dealt with by judicious exclusion of uncertainties in different parts of an analysis if they have been included before. Also for any one parameter a judgement can be used to allocate a proportion as correlated and a proportion as independent if this is felt to best describe the application.

Always include the full uncertainty as independent and uncorrelated if there is any doubt.

A more detailed treatment is described below in Section 4.6 and in the references.

4.6 Parallel Meters (Correlated Uncertainties)

Where a number of meters are connected in parallel, or in a similar configuration, all calibrated in the same manner using the same reference, the total uncertainty of the combination has to be carefully considered. Working with dimensional uncertainty is again the preferred method and is described. The relation to non-dimensional methods will be illustrated.

Four incorrect scenarios for the combination of the uncertainties are commonly seen proposed.

a) Arithmetically add the individual uncertainties. (equivalent to maintaining the percentage the same as a single meter.
b) Add the uncertainties by root sum square (effectively reducing the percentage uncertainty)
c) Add the non-dimensional uncertainties by root sum square, effectively increasing the percentage uncertainty.
d) Add the non-dimensional uncertainties arithmetically, greatly increasing the percentage uncertainty.

These crude estimates are not representative of the correct uncertainty. To establish the correct uncertainty estimate an assessment of the whole measurement process has to be carried out to identify the correlated, uncertainties i.e. common to all meters, and separate those from the uncorrelated, i.e. inherent in each meter individually.

For example the uncertainty in the base volume of a pipe prover is correlated through to all the meters it calibrates, but the resolution error of each meter is uncorrelated as are some aspects of temperature measurement.

Unfortunately many instances are not clear cut. For instance, the calibration of the thermometer on a meter prover is correlated through to the meters, but the resolution and estimation of temperature is less clear. These contain correlated and uncorrelated components which later give rise to different effects on the measurements using them. It is clear that the resolution is uncorrelated as it affects every reading in a random manner unless some common component is recognised.

Although the separation of correlated and uncorrelated uncertainties can be carried back through a number of processes leading up to the one being analysed, it is generally the case that with some judicious selection of values that identification of correlated and uncorrelated uncertainties can be restricted to the last process in examining parallel meters. This pragmatic approach is again incorrect as many of the uncertainties carried forward have correlated components which should be separated at an earlier stage. Recognising this allows compromise on the approach which will simplify an analysis as long as judging is in favour of correlated uncertainties.

Commonly it is assumed that if all meters carry the same uncertainty overall, only one requires analysis, effectively multiplying by the number of meters at the end. If this is not true each individual meter will have its own uncertainty analysed separately.

To carry out the analysis, each uncertainty is listed and calculated as before. In addition the uncertainty is categorised as being correlated or uncorrelated. Where it is judged that an uncertainty has both components, a split is judged and a proportion allocated to each category. The two categories are listed separately. For each meter total correlated and un-correlated uncertainties are derived.

The total uncertainty is calculated as follows.

Correlated uncertainties for each meter are added arithmetically: $u_c=\Sigma u_{ci}$

Uncorrelated uncertainties for each meter are added by root-sum-of-squares: $u_u=\sqrt{\Sigma u_{ui}^2}$

Resultant correlated and uncorrelated uncertainties are combined by root sum of squares.

Clearly a lot of judgement has to be used in allocating uncertainties as correlated or not. In practice this can be fine tuned by measurement, experimentation and detailed analysis**. If in doubt always allocate to the correlated side.**

In the final analysis obviously the total absolute uncertainty for a number of meters will be larger than for each individual as of course the quantity increases.

The non-dimensional (percentage) uncertainty will reduce from that of each individual meter. If all uncertainties are correlated the percentage remains the same as a single meter but will reduces in proportion to uncorrelated component. This is based on the statistical premise that all the uncertainties have been randomised by the methods used in analysing the uncertainty.

Observing a reduction in percentage uncertainty as the number of meters increase is statistically obvious but clearly reaches some limit as the meters will start to impose larger base uncertainties as they become smaller and influence factors start to multiply.

4.7 Insignificant Uncertainties.

It is clear when analysing a process that many sources of uncertainty are seen to be insignificant relative to some dominant sources. It is acceptable practice to exclude insignificant uncertainties from the final combination and this has frequently been done to avoid the laborious hand calculation of the answer.

To be sure that an uncertainty is insignificant, it has to be recognised, quantified and its sensitivity coefficient calculated. Only then can a decision be taken that it is insignificant. This decision must be recorded.

With most uncertainty analyses now being carried out by computer and spreadsheet there is now little advantage in subsequently omitting the uncertainty from the final compilation no matter how small it may be as the computer does not see any significant advantage in not including all uncertainty sources.

If, for generic uncertainty calculations, it could be previously determined that within specified limits parameters do not contribute significantly to the overall uncertainty of the result they can then be omitted. This is acceptable as long as the reason for the decision is documented, and obvious to subsequent users of the generic analysis procedures. Again with the use of carefully designed uncertainty spreadsheet templates little is actually gained by this practice.

4.8 Monte Carlo Simulation

The mathematical theory underlying the analysis of uncertainty is based on the assumption that the uncertainties involved are small compared with the measured values (the exception being when the measurements are close to zero). This is certainly true for the standards work for which the original theories were developed and may also be true for many industrial applications. However, it cannot be said to be true for all industrial situations and where the uncertainties are large compared with the measured values the mathematical theory breaks down. In these situations the technique known as Monte Carlo Analysis can be of great value in assessing combined values of uncertainty.

Monte Carlo simulation is a method where a large number of calculations are made, in each of which different values are assigned to each of the input variables. Each input value is drawn at random from the assumed distribution for that parameter, and in this way the distribution of the output is calculated. To obtain a representative distribution for the output requires many thousands of calculations to be performed and it is only with the advent of cheap computer power that the Monte Carlo technique has become a viable method of assessing combined uncertainty. The ISO GUM does not deal specifically with large values of uncertainty and on this basis does not discuss the Monte Carlo technique; however those faced with large measurement uncertainties may find the approach of considerable value.

In essence Monte Carlo Analyses replaces the derivation of sensitivity coefficients, and the combination of uncertainties by very large number of calculations of the result. Both advantages and disadvantages can be attributed to Monte Carlo and it certainly provides a good method of analysing complex distributions. All the provisions of uncertainty identification, uncertainty distribution and judgement still have to be carried out as described in this document. Correlation is handled by linking the correlated components before selecting the random number.

5 CONCLUSION

Uncertainty analysis is not simple. The combination of statistical technique and mathematical knowledge seems daunting. This aspect is only the final combination and can be mechanistic. The difficulty is finding, recognising and judging all the sources of uncertainty. If this is done correctly, and with a good understanding of the process, the other parts of the system fall into place. Break the process down into manageable bits. Do not be ambitious and try to do everything at one go. Keep the process simple.

If you do not believe a answer, initially trust your judgement and look for the missing information. Usually it is something missed rather than a wonderful test method that gives a better than expected uncertainty.

Make use of the guides and references and adapt then to suit your problem.

8 REFERENCES

1 Guide to the expression of Uncertainty in Measurement 1995 (GUM). BIPM/ISO/OIML/IEC/IFCC/IUPAC/IUPAP.

2 Assessment of Measurement Uncertainties in the Oil Industry. A report for the Institute of Petroleum NEL and SGS Technical Services. NEL Report No 370/99, 18 November 1999, Institute of Petroleum.

3 ISO5168 WORKING DRAFT 5. Measurement of fluid flow; estimation of Uncertainties. NEL Report No 367/99. East Kilbride, Glasgow: National Engineering Laboratory, September 1999.

(Note the above has been updated within the ISO working group with addition of examples)

4 BS PD6461. International Vocabulary of Basic and General terms in Metrology (VIM), 1995 (also available form ISO, OIML,IEC).

5 A Beginners Guide to Uncertainty of Measurement, Good Practice Guide No 11. NPL, Teddington, London

ACKNOWLEDGMENTS

Denis Boam and Jane Sattary for help in providing background, statistics and editing of the paper.

Department of Trade and Industry (N.M.S.P.U.) for commissioning NEL to prepare the working draft of ISO 5168.

Institute of Petroleum for commissioning NEL and SGS Technical Services to provide a report on the Assessment of Measurement Uncertainties in the Oil Industry.

Members of the Institute of Petroleum working group in Aberdeen currently drafting a standard for assessment of uncertainty in pipe prover calibrations.

APPENDIX 1

VOCABULARY TERMS AND DEFINITIONS

To understand the treatment of uncertainty it is vital that the terms are understood and used in the correct manner. The vocabulary at the beginning of the paper will assist in this.

The terms accuracy, uncertainty and error are not the same and must be used with care.

The terminology used in the estimation of uncertainty is probably one of the main problems in understanding the process. Given here is the definitions taken form various sources, but mainly from the "Guide to the expression of uncertainty in measurement" (ISO,1995) and Vocabulary of metrology; basic and General terms BS PD6461, Part 1, 1995 (also issued by ISO/IEC/OIML.

Accuracy (of measurement)		Closeness of the agreement between the result of a measurement and a true value of the measurand. (accuracy is a qualitative concept)
Error (of Measurement)		Result of a measurement minus a 'true value of the measurand.
Repeatability:		The closeness of agreement between the results of successive measurements given by a measurand carried out under the same conditions of measurement. Repeatability may be expressed quantitatively in terms of the dispersion characteristics of the results.
Resolution:		A quantitative expression of the ability of an indicating device to distinguish meaningfully between closely adjacent values of the quantity indicated
Traceability:		The property of a measuring device enabling measurements made by it to be related to some primary standard, generally a National or International standard, through an unbroken chain of comparative measurements involving secondary standards, tertiary standards, *etc.* of stated uncertainty.
Uncertainty		Parameter, associated with the results of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand
Standard uncertainty	u	Uncertainty of the result of a measurement expressed as a standard deviation
Combined standard uncertainty	u_c	Standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariance of these other quantities weighted according to how the measurement result varies with changes in these quantities

Expanded uncertainty	$U = ku_c$	Quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand : the fraction may be viewed as the coverage probability or the level of confidence of the interval
Coverage factor	k	Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty : k is typically in the range 2 to 3
Type A evaluation		Method of evaluation of uncertainty by the statistical analysis of a series of observations
Type B evaluation		Method of evaluation of uncertainty by means other than the statistical analysis of a series of observations
Sensitivity coefficient	c_i	Change in the output estimate y produced by a unit change in the input estimate x_i.
Relative sensitivity coefficient	c^*_i	Percentage change in the output estimate y produced by a one per cent change in the input estimate x_i.

APPENDIX 2

COMMON SYMBOLS

a_i — estimated semi-range of a component of uncertainty associated with input estimate x_i , as defined in Annex B,

c_i — sensitivity coefficient used to multiply the uncertainty in input estimate x_i to obtain the uncertainty of the output estimate y,

c^*_i — dimensionless sensitivity coefficient used to multiply the dimensionless uncertainty in input estimate x_i to obtain the dimensionless uncertainty of the output estimate y,

f — functional relationship between estimates of the measurand y and the input estimates x_i on which y depends,

$\partial f/\partial x_i$ — partial derivative with respect to input quantity x_i of the functional relationship f between the measurand and the input quantities,

k — coverage factor used to calculate the expanded uncertainty U,

n — number of repeat readings or observations,

N — number of input estimates x_i on which the measurand depends,

$P(a_i)$ — probability that an input estimate x_i has a value of a_i,

$s(x_k)$ — experimental standard deviation of a random variable x determined from n repeated observations,

$s(\bar{x})$ — experimental standard deviation of the arithmetic mean $\bar{x}$,

U — expanded uncertainty associated with the output estimate y,

U^* — dimensionless expanded uncertainty associated with the output estimate y,

$u(x_i)$ — standard uncertainty associated with the input estimate x_i,

$u_c(y)$ — combined standard uncertainty associated with the output estimate y,

$u^*(x_i)$ — dimensionless standard uncertainty associated with the input estimate x_i,

$u_c^*(y)$ — combined dimensionless standard uncertainty associated with the output estimate y,

x_k — k-th observation of random quantity x,

$\bar{x}$ — arithmetic mean or average of n repeated observations x_k of randomly varying quantity x,

x_i — estimate of the input quantity x_i,

y — estimate of the measurand Y,

Δx_i — increment in x_i used for numerical determination of sensitivity coefficient, and

Δy — increment in y found in numerical determination of sensitivity coefficient.