

## **Paper 1**

# **Investigations of Density Transducers for Natural Gas Metering**

*F. Weber, Physikalisch- Technische Bundesanstalt, Germany*

## **Investigations of Density Transducers for Natural Gas Metering**

Fritz Weber, Physikalisch-Technische Bundesanstalt, Bundesallee 100,  
D-38116 Braunschweig, Germany.

Fritz.Weber@ptb.de

In two ring tests two different sets, each consisting of four density transducers for natural gas metering were calibrated in the range up to  $90 \text{ kg}\cdot\text{m}^{-3}$  at  $20 \text{ }^\circ\text{C}$ . The density transducer is a vibrating cylinder or fork surrounded by the gas whose density determines the resonance frequency of the vibration. The differences between the results obtained by the different participants in the ring tests amounted to 0,04 percent. They were larger than was expected from the uncertainty of the instruments which had been employed for the calibration. Additionally, three different correction equations for the VoS effect are discussed. Density transducers calibrated with nitrogen and corrected for the VoS effect in methane still deviated by up to  $0,05 \text{ kg}\cdot\text{m}^{-3}$ . The deviations due to the VoS effect in two investigated mixtures amounted up to 0,15 percent. The relatively small temperature dependence of the transducers is compared with that obtained in the experiment. An overall uncertainty of 0,2 percent is estimated.

### **1 Introduction**

When large quantities of natural gas are transported through pipelines, the operating density in the gas in motion is measured with the aid of densitometers. The transducer in the densitometer usually is a mechanically oscillating element whose resonant frequency is a measure of the density of the surrounding gas. The manufacturers use oscillating cylinders or forks which are electromagnetically excited and whose resonant frequency is electrically measured. The relationship between resonant frequency and operating density is determined by calibration with a reference gas at a specified temperature. The operating density is then obtained from the resonant frequency measured or the period of oscillation. The frequency is measured with a small relative uncertainty of approximately  $10^{-6}$ . The corresponding uncertainty of the calibrated density transducer is, however, considerably higher than is deduced from this figure. When the density transducer is used in volume correctors for gases, the test error limit prescribed in legislative measurements of 0,3 percent generally is not, however, exceeded in the preliminary test [1].

Density transducers are used in a wide pressure and temperature range. This leads to systematic deviations in case the operating and calibration conditions differ considerably. The

most important influence is exerted by the gas type. Deviations of approximately 3 percent and 1 percent occur, for example, when a transducer calibrated in methane is used to measure the density in carbon dioxide or nitrogen [2]. Such systematic deviations due to the so-called gas type effect are largely eliminated by the VoS correction (VoS – velocity of sound). The velocity of sound required is additionally measured automatically in situ or calculated from equations of state for the gases used. Systematic deviations, however, remain as the approximative equations reflect the true physical behaviour of the density transducers only inadequately.

The PTB (Physikalisch-Technische Bundesanstalt) Testing Instructions [1] contain a guideline for the calibration of density transducers in volume correctors. They provide that density transducers for density determination in natural gases are to be calibrated with methane as the test gas. Such density transducers are, however, also calibrated with nitrogen. In the performance of such calibrations, differences far beyond the measurement uncertainty expected arose between different calibration centres in North Germany. At the proposal of the "Gas Measurement" Working Committee of the Plenary Assembly for Verification Matters, the PTB performed two intercomparisons to identify the causes of these deviations.

## **2 Experimental prerequisites of the intercomparison**

For calibration, the measurement conditions, i.e. the interconnection of the transducers, the measuring instruments and the reference gases, ought to correspond to the normal operating conditions of the individual laboratories. Temperature and pressure of the calibration points were specified in agreement with all participants.

To allow for the interest encountered in North Germany, four density transducers exclusively from one manufacturer (Solartron, type 7812) were calibrated in the first intercomparison not only with methane as the prescribed reference gas but also with nitrogen. About one year later, four density transducers were investigated in a second intercomparison; this time two transducers each were made available by two different manufacturers (RMG Messtechnik, type DG08, and Bopp und Reuther Messtechnik, type DIMB 104.2). This time, only methane was used for calibration. The range of the period of oscillation measured depends on the type of the oscillating body and amounted to around 500  $\mu$ s in the first and to around 1000  $\mu$ s in the second intercomparison. Participants besides PTB were test centres recognized as reference laboratories and verification centres of verification authorities. In the first

intercomparison, three laboratories from North Germany, in the second intercomparison four laboratories from South Germany participated in addition to the PTB.

The density transducers all work on the same principle: An oscillating body (oscillating fork, oscillating cylinder) is electromagnetically excited to resonant oscillations in the surrounding gas. The period  $\tau$  of the oscillation is a measure of the operating density  $\rho$  of the gas in which the body is located. The calibration approximates the density by equation

$$\rho_{DT}(\tau) = K_0 + K_1 \cdot \tau + K_2 \cdot \tau^2 \quad (1)$$

A calibration aims at determining the coefficients  $K_0$ ,  $K_1$  and  $K_2$  at a specified temperature  $T$ .

The calibration procedure is carried out according to PTB testing instructions. The temperature of the test gas during the calibration is kept constant at 20 °C, while the pressure is varied from high to low values.

The coefficients  $K_0$  to  $K_2$  are determined so that the square sum of the deviations between the "true" density  $\rho(p, T)$  which is calculated with the aid of an equation of state (EoS) [3] from pressure and temperature, and the approximated density  $\rho_{DT}(\tau)$  is minimized. To rule out systematic deviations introduced by the equation of state, the PTB had made the same equation of state available to all participants in an application file of a spreadsheet program [4]. This file also comprised the example of an evaluation performed by the PTB. Numerical results, such as the covariance matrix, are furnished by an additional matrix calculation which can be realized rather easily with the aid of computer programs which are also available [5]. The covariance matrix will be needed to estimate the uncertainty of the calibrated transducer as will be shown below. The coefficients  $K_j$  ( $j = 0, 1, 2$ ) are calculated from the fitted curve through  $(m+1)$  measuring points for density  $\rho_i$  and period of oscillation  $\tau_i$  ( $i = 0, 1, \dots, m$ ). With the matrices  $R_i = \rho_i$  and  $T_{i,j} = \tau_i^j$  the least squares method provides the coefficients in the form of a single-column matrix (transposed matrices are marked by a prime) [6]

$$K = (T' \cdot T)^{-1} \cdot T' \cdot R. \quad (2)$$

The two procedures, conventional or matrix analysis, of course furnish the same coefficients.

### 3 Measurement uncertainty of a calibrated density transducer

The measurement uncertainty is calculated in accordance with the EA Guidelines [7]. For this purpose and to determine the measurement uncertainty  $u(\rho)$  for the density, the law of the

ideal gas is regarded as an appropriate approximation of the equation of state  $\rho(p, T)$  and thus used as model function. Equation (1) is the model function for the third influence quantity, the period of oscillation  $\tau$ . The uncertainty in the course of the calibration is

$$u(\rho)^2 = \left(\frac{\rho}{p}\right)^2 \cdot u(p)^2 + \left(\frac{\rho}{T}\right)^2 \cdot u(T)^2 + \left(\frac{\rho}{M}\right)^2 \cdot u(M)^2 + (K_1 + 2 \cdot K_2 \cdot \tau)^2 \cdot u(\tau)^2. \quad (3)$$

The third term of the sum in equation (3) takes the impurities in the gas into account which are formally calculated through the uncertainty for the molar mass  $u(M)$ . It can be seen from this equation that – except for the last term which will be negligible anyway (cf. Table 1) – the relative uncertainty of the density increases proportionally to the relative uncertainty of the individual measurands. For constant relative input quantities, a constant relative uncertainty can be expected for the output quantity "density".

The measurement uncertainty upon calibration  $u(\rho)$  is passed on to the user with the uncertainty of the coefficients  $K_0$  to  $K_2$  by a covariance matrix  $U_K$ . According to DIN 1319-4 [8] the measurement uncertainty of the calibrated density transducers is obtained as

$$u(\rho_{DT})^2 = |\Theta^T \cdot U_K \cdot \Theta| + (K_1 + 2 \cdot K_2 \cdot \tau)^2 \cdot u(\tau)^2 \quad (4)$$

with the  $3 \times 3$  matrix  $U_K = u(\rho)^2 \cdot (T^T \cdot T)^{-1}$  und  $\Theta_j = \tau^j$ . Due to the very small measurement uncertainty  $u(\tau)$ , the first term of the sum, i.e. the uncertainty from the calibration, predominates in equation (4).

Table 1 shows the estimated measurement uncertainties for pressure, temperature, period of oscillation and the uncertainty resulting for the density. The values approximately agree with the values stated by all participants in the first intercomparison in the range up to  $90 \text{ kg} \cdot \text{m}^{-3}$  and for methane up to 80 bar. In the second intercomparison with measurements up to  $50 \text{ kg} \cdot \text{m}^{-3}$  only, the uncertainty for the pressure is somewhat smaller. The Type A standard uncertainty  $u(\rho)$  calculated from the deviations of the density from the fitted curve is approx.  $0,005 \text{ kg} \cdot \text{m}^{-3}$ , i.e. it is of the same order of magnitude as the value of  $0,008 \text{ kg} \cdot \text{m}^{-3}$  from the table. By comparison typical uncertainties given by the manufacturers are  $0,5 \text{ kg} \cdot \text{m}^{-3}$  for liquids and  $0,1 \text{ kg} \cdot \text{m}^{-3}$  for gases. An example of the coefficients and the associated covariance matrix is shown in Table 2. The calculation according to equation (4) shows that the ratio  $u(\rho_{DT})/u(\rho)$  varies for the calibrated density transducers within the scope of the intercomparison between 0,5 in the centre of the calibration range and 0,9 at the ends (see Fig. 1). The estimated uncertainty of the density transducer  $u(\rho_{DT})$  within the calibration range

thus is smaller than the uncertainty  $u(\rho)$  with which the calibration was carried out. Outside of the calibration range the uncertainty increases considerably.

## 4 Results

### 4.1 Comparison of calibrations in two intercomparisons

For the density transducers used in the intercomparison, the fitted curves instead of the individual measuring points (Figures 2 and 3) were compared with one another. The points plotted identify the measuring points; the bars or rectangles plotted for the reference curve in Figure 2 represent the standard uncertainty of the measurement deviations from the fitted curve. It is striking that in both intercomparisons the curves are usually so wide apart from each other that the bars do not overlap. The deviations are thus larger than is derived from the measurement uncertainty of the state variables. There exist, consequently, uncertainties which cannot be explained by the statistical variations observed. They may be caused by the following systematic deviations:

- a. physically instable behaviour of the resonant frequency of the density transducer used,
- b. systematic errors in the calibration of the test equipment for pressure, temperature and period of oscillation,
- c. contamination of the gas.

Case (a) is improbable as the deviations of one participant show the same systematic tendency for all density transducers, i.e. they are mainly determined by the characteristics of the measuring set-up and not by those of the particular density transducer. Case (b) occurs if the same systematic deviations are identified for both test gases within the scope of the intercomparison. In the first intercomparison, this could be observed only in the case of one participant.

Provided the participants gave the correct uncertainties for their test equipment, the main cause of the large deviations seems to be deviation (c). For nitrogen, a purity of 99,999 percent and for methane a purity of 99,995 percent were the proposed requirements which were accepted by all participants. The systematic deviations with large negative values are a hint that the relatively light gas methane might be contaminated with heavy gas molecules: Upon calibration, a density calculated from pressure and temperature and assigned to the oscillation period, is smaller than the density actually prevailing in the system. Methane with the purity 3.5 instead of 4.5, which is mainly contaminated with nitrogen causes a systematic

deviation of -0,04 percent. Even if the gas was originally available with the purity required, it may be contaminated in the course of the measurement. In all tests, the reference gas comes into contact with the oil seal of the pressure balance. Gases dissolved in the oil may diffuse into the test gas. In the second intercomparison it was investigated whether the gas composition is influenced to a measurable extent if the connection to the pressure balance is open over a prolonged space of time. It turned out that after three days, the deviations for the highest pressure actually amounted to  $0,15 \text{ kg}\cdot\text{m}^{-3}$ , i.e. to 0,3 percent. In the repeat measurement performed at the PTB ("PTB1" and "PTB2" in Figure 3, DT 25177 and DT 25595), care was therefore taken to ensure the connection to the pressure balance being open only during the measurement.

The general finding of the intercomparison is that handling of the test gas may lead to contamination and concurrent deviations in the calibration. It is advisable to clear the test pockets of the density transducers and the piping from extraneous gases before the test gas is filled in. Due to the lack of vacuum pumps, the system is often cleaned only by rinsing. This includes the risk of certain regions not being sufficiently rinsed and later contaminating the test gas. The relatively large maximum deviations of one participant in each intercomparison – 0,04 percent in the first and 0,16 percent in the second intercomparison – are probably due to these causes. In both intercomparison, the deviations lie – with one exception in the second intercomparison – within a range of  $\pm 0,04$  percent. The repeat measurements performed at the PTB on density transducers 25177 and 25595 (Figure 3) show, however, that the theoretical value of about 0,01 percent stated in Table 1 can be reached if great care is taken to prevent contamination.

#### **4.2 Investigation in the vacuum**

For the linear regression analysis only measuring points beyond the vacuum point were taken into account. Nevertheless the PTB proposed measurements in the vacuum in the course of the intercomparisons. They are useful because these measurement results are independent of the gas and test equipment used for pressure and temperature and are, therefore, a reliable criterion for the stability of the oscillating body. Mechanical deformations due to rude handling or deposits when operated in strongly contaminated media will influence the oscillation. It is common practice to measure the resonant frequency – if not in the vacuum so at least under environmental conditions - and to record it as a characteristic of the density transducer.

Only in the first intercomparison could the results in the vacuum be evaluated for the four density transducers. The stability was so good that over the period of about four months until the density transducers were returned to the PTB at the end of the intercomparison, the period of oscillation had changed by 0,004  $\mu\text{s}$  at most. For a sensitivity of  $0,5 \text{ kg}\cdot\text{m}^{-3}\cdot\mu\text{s}^{-1}$ , this corresponds to a change in density of  $0,002 \text{ kg}\cdot\text{m}^{-3}$ .

The values measured in the vacuum, compared with the other measuring points, were always clearly offset and would, therefore, have led to a systematic falsification of the regression analysis. In both intercomparisons, there was an offset between  $-0,09$  and  $-0,04 \text{ kg}\cdot\text{m}^{-3}$ . So a deviating behaviour was observed in the range of smaller densities. In accordance with the physical model, the resonant frequency is determined by the vibrating gas mass [9]. As the experiment has shown that the resonant frequency depends only on the density of the gas, the corresponding vibrating gas volume should be constant. The leap close to the zero density can be explained if either the vibrating volume is greater for small densities than for high densities or the density is higher close to the surface than in the rest of the space. In the latter case, an absorbed layer might first have formed on the surface of the density transducer during gas inlet before the physical behaviour characterized by equation (1) becomes effective at higher densities.

### 4.3 Correction of the gas type effect (VoS correction)

For the calibration of the density transducers, reference gas and gas to be measured should have similar thermodynamic properties. The firm of Solartron is marketing density transducers which are calibrated with nitrogen as the test gas and recommended for density measurements of natural gases [10]. The intercomparison has shown that measurement deviations of up to 0,5 percent occur if density transducers calibrated in nitrogen are used in methane. These differences are reduced by the VoS (velocity of sound) correction. Equations (6) to (8) show the conventional VoS corrections:

$$\rho_{\text{Cor}} = \rho_{\text{DT}} \cdot \left[ \frac{1 + \left(\frac{K}{w_c}\right)^2}{1 + \left(\frac{K}{w}\right)^2} \right] \quad (6)$$

$$\rho_{\text{Cor}} = \rho_{\text{DT}} \cdot \left[ \frac{1 + \left( \frac{L}{\tau \cdot c} \right)^2}{1 + \left( \frac{L}{\tau} \right)^2} \right] \quad (7)$$

$$\rho_{\text{Cor}} = \rho_{\text{DT}} \cdot \left[ 1 + \frac{K_3}{\rho_{\text{DT}} + K_4} \cdot \left( \frac{SG_c}{\gamma_c \cdot T_c} - \frac{SG}{\gamma \cdot T} \right) \right] \quad (8)$$

The meaning of the symbols used is explained in Appendix A. The corrections for a specific density transducer type exclusively depend on the gas to be measured and the calibration gas. Equation (6) can be found in the PTB Testing Instructions [1]. Equation (7) is the form of the VoS correction generally acknowledged as the best form which may be completed by an additional term [11] to take a slight dependence on viscosity into account. The constant  $L$  (with the dimension of a length) is constant for one transducer type and stated by Solartron to be  $2,10 \cdot 10^4 \mu\text{s} \cdot \text{m} \cdot \text{s}^{-1}$  for the density transducer used in the first intercomparison ( see Annex E2 of [10]).

Equation (6) and equation (7) assume that the velocity of sound is known for calibration gas and gas to be measured as a function of pressure or density, and temperature. It is either calculated or additionally measured in the field. Equation (8) is recommended by Solartron and is an approximation of equation (7). For the determination of constants  $K_3$  and  $K_4$ , the velocity of sound is used only indirectly by requiring that the corrections of both equations must agree at the ends of the calibration range (see Annex E5 of [10]). This is why only the values at the ends of the range are required for the velocity of sound. The ratio of the specific heats  $\gamma$  and  $\gamma_c$  required in addition is assumed to be density-independent. In reality, it varies, for example for methane, between 1,3 and 1,8 in the range between 20 and 90  $\text{kg} \cdot \text{m}^{-3}$ . In this density range for a type 7812 density transducer of Solartron make, the two VoS corrections (according to equation (7) and equation (8)) differ, however, by only 0,02 percent.

Equation (8) also contains a small temperature dependence of the correction. When the temperature is increased to 40 °C, the results of the two equations differ by 0,07 percent.

As density transducers calibrated with nitrogen are also used in natural gases, it was investigated which deviations remain when these transducers are used with different VoS corrections for the measurement in methane. As methane is an essential component of natural gases (mole fraction at least 80 percent), comparable results are also to be expected for the

behaviour of the density transducers used in natural gases. The velocity of sound necessary for gases and their mixtures is available in the form of PC programs [12] [13].

Figure 4 shows the deviations between calibrations performed in nitrogen and in methane for four different transducers measured in the course of the first intercomparison. According to this Figure, the density transducer calibrated in methane systematically indicates values which at the lower end of the range are too small up to 0,5 percent. This deviation is corrected by the VoS correction as shown in Figure 5 for density transducer 12081. The amount of the correction  $\Delta\rho_{\text{Cor}}$  was determined with equation (6) and equation (7). The remaining difference between the corrected and the true value results from the distance between the relevant curves. With the usual coefficients given in the legend of Figure 5, systematic deviations of 0,2 percent or 0,1 percent, respectively, remain. With other constants  $L$  and  $K$  the respective curves are shifted in parallel (Figure 6). The correction will be optimal when the curve  $\Delta\rho_{\text{Cor}}$  intersects the curve to be fitted approximately in the center of the calibration range. The equation (7) usually used today with its somewhat steeper slope seems to be better suitable. The two corrections with the adjusted coefficients are also plotted in Figure 4. Equation (6) contains only the density and is therefore independent of the individual transducer. Due to the additional dependence on the period of oscillation, equation (7) is slightly influenced by the individual characteristics of the density transducer. Even for the correction according to equation (7), deviations of up to  $0,05 \text{ kg}\cdot\text{m}^{-3}$  result over the whole range. At the lower end of the range, this results in a relative deviation of approx. 0,2 percent.

#### 4.4 Investigation in mixtures

At the end of the intercomparison, the availability of the density transducers at the PTB was used to measure the density of two gas mixtures with different calorific values in the range from  $8$  to  $50 \text{ kg}\cdot\text{m}^{-3}$  (Figure 7). Gas 97008 has a nitrogen fraction of 4 percent, in the case of gas 19, it amounts to approx. 14 percent. In the intercomparison, the density transducers had been calibrated with methane. The reference value  $\rho$  was calculated from the specified composition of the gas mixtures in accordance with the Technical Rule G 486 [13]. Due to the different sound velocities [12] in the two gas mixtures, two different corrections result which are, however, considerably smaller than shown in Figures 4 to 6. As an example, the effect of the two corrections is shown in Figure 7, last diagram, for density transducer 12801. With the value adjusted above for component  $L$ , the correction amounts to -0,15 percent for gas 97008 and to -0,12 percent for gas 19.

According to Figure 7, a relatively uniform behaviour of all density transducers is measured, i.e. the deviation between measured value and theoretical value  $\Delta\rho(\rho)$  shows approximately the same curve shape for all density transducers. The standard uncertainty of a calibrated transducer amounts to 0,04 percent. The double value approximately corresponds to the width of the range in which the measuring points of this gas type can be found. This also applies to gas 97008. An exception is density transducer 120739. As normal behaviour is observed in the other gas, this does not seem to be a special feature of the density transducer. Some irregular conditions must have occurred during this special measurement. Determination of the real causes would require a repeat measurement.

The maximum deviation is greater for gas 97008, the gas with the lower nitrogen fraction, than in gas 19. The difference shown relates to a calculated value. For calculations performed on the basis of the equations of state of the NIST - which will not be dealt with in detail here – the resulting density deviations were larger by approximately 50 percent. For mixtures, this comparison does not furnish any statement regarding the uncertainty of the density transducers or the quality of the VoS correction. For an exact comparison it would be necessary to determine the density of the mixture by experiment.

#### 4.5 Temperature influence

The temperature dependence of the resonant frequency is essentially determined by the elastic properties of the oscillating body and expressed by [10]

$$\rho_{DT}(\tau, T) - \rho_{DT}(\tau, 20\text{ °C}) = [K_{18} \cdot \rho_{DT}(\tau, 20\text{ °C}) + K_{19}] \cdot (T - 20\text{ °C}). \quad (9)$$

It is presupposed that the density transducers have been calibrated at 20 °C. The coefficient  $K_{19}$  represents the parallel shift of an isotherm, whereas  $K_{18}$  represents the slope in the diagram. Both coefficients are stated on the data sheet of Solartron. A procedure is known which is used to calculate these coefficients for density transducers from the resonant frequencies at two different temperatures in the vacuum. Table 3 shows the coefficients calculated according to this procedure from the period of oscillation at 5 °C and 30 °C.

As the transducer types probably are each made from identical material, the same coefficients are to be expected for all transducers. The coefficients in Table 3 are, however, different.  $K_{18}$  varies between  $-1,5 \cdot 10^{-5}$  and  $-2,9 \cdot 10^{-5} \text{ K}^{-1}$ ,  $K_{19}$  between  $6,6 \cdot 10^{-4}$  and  $-9,5 \cdot 10^{-4} \text{ kg} \cdot \text{m}^{-3} \cdot \text{K}^{-1}$ . Figure 8 shows that these coefficients reflect the results obtained by experiment relatively well. Calibration at 20 °C serves as the reference. For 20 °C, the individual values obtained by

experiment are additionally plotted instead of the fitted values so far used to show the random deviations from the fitted curve. It can be seen that the temperature influence is relatively small and of an order of magnitude which is comparable to these deviations. The coefficients originally furnished by the manufacturer (last column) clearly differ from the coefficients determined in the experiment. As the density transducers have been used for several years, it cannot be ruled out that the properties have changed. In view of this discrepancy, it is advisable for the user to determine the coefficients himself.

## 5 Conclusions

- Handling of the test gas may lead to contamination and to concurrent systematic deviations.
- The measurement point in vacuum should be excluded from the calibration.
- A density transducer calibrated in nitrogen shows values which are too small by up to 0,5 percent if used without correction for the measurement in methane. Application of the usual VoS correction reduces the deviations to below 0,2 percent.
- For two synthetic natural gases (mixtures), the deviations of a density transducer calibrated in methane are approx. 0,2 percent after the VoS correction has been applied.
- The temperature influence on the vibrating element is comparatively small.

## Literature

- [1] PTB-Prüfregeln (PTB Testing Instructions); Band 20; Elektronische Mengenumwerter für Gas. Braunschweig und Berlin 1993.
- [2] Jaeschke, M.; Guo, X.Y.; Kleinrahm, R. and Wagner, W.: A New Accurate Method for Correcting the VoS Effect on Vibrating Gas Density Transducers. Proc. of the Int. Gas Research Conf.: Cannes 1995; p. 871-880.
- [3] Wagner, W. and Span, R.: Special Equation of State for Methane, Argon, and Nitrogen for the Temperature Range from 270 to 350 K at Pressures up to 30 MPa. Int. J. Thermophysics 14 (4), 699-725 (1993).
- [4] e.g. Microsoft, Excel 97; 1997.
- [5] MathSoft, Mathcad, Version 5.0, 1994.

- [6] Kutner, M.H.; Nachtsheim, C.J.; Wasserman W. and Neter, J.: Applied Statistical Models. 4. Ausgabe: McGraw-Hill/ Irwin 1996. p. 227.
- [7] Expression of the Uncertainty of Measurement in Calibration; European Co-operation for Accreditation (EA); Publication EA-4/02; December 1999; Section 4.
- [8] DIN 1319-4. Grundlagen der Meßtechnik. Part 4: Auswertung von Messungen, Meßunsicherheit. Edition February 1999.
- [9] Stansfeld, J.W.: North Sea Metering Workshop. Glasgow 1986; Paper 2.2; p. 1 -15; Velocity of Sound Effect on Gas Density Transducers - The Theory, Measurement Results and Methods of Correction.
- [10] (a) Betriebshandbuch, Schwing Verfahrenstechnik; 1997.  
This is a translation of  
(b) 7812 Gas Transducer Manual, Fa. Schlumberger; Dezember 1992.
- [11] Jaeschke, M.; Boden, R.; Brandt, O; Gomez Mellado, B.; Kleinrahm, R. and Wagner, W.: An improved evaluation procedure for density transducers used in natural gas pipelines: IGRC 98, Proc. of the 1998 International Gas Research Conference, Vol.II Transmission and Storage, Gas Research Institute 1998, p. 551-563.
- [12] Huber, M.L. (ed.): NIST Standard Reference Database 4, Thermophysical properties of hydrocarbon mixtures. Computer programme SUPERTRAPP (STRAPP), Version 3.00 (1999).
- [13] Technische Regeln DVGM (Deutscher Verband des Gas- und Wasserfachs) (Technical Rules DVGS (German Association of the Gas and Water Trade)) - Arbeitsblatt G 486, August 1992. Available as PC program kzahl.exe.

**Table 1** Uncertainty  $u(p)$ ,  $u(T)$  and  $u(\tau)$  of the individual measurands  $p$ ,  $T$  and  $\tau$ .

$u(p)$ in mbar	3 ... 8
$u(T)$ in K	0,02
$u(\tau)/\tau$ (Gate time: 1s)	$10^{-6}$
$u_{\max}(\rho)$ in $\text{kg}\cdot\text{m}^{-3}$	0,008

**Table 2** Coefficient matrix  $K$  and covariance matrix  $U_K$  of a typical density transducer. For the numerical evaluation the units  $\mu\text{s}$  and  $\text{kg}\cdot\text{m}^{-3}$  for period and density are applied.

j	$K$	$U_K \cdot u(\rho)^{-2}$		
0	-353,78	$1,1 \cdot 10^1$	$-2,3 \cdot 10^{-2}$	$1,2 \cdot 10^{-5}$
1	$-5,357 \cdot 10^{-2}$	$-2,3 \cdot 10^{-2}$	$4,7 \cdot 10^{-5}$	$-2,4 \cdot 10^{-8}$
2	$4,6372 \cdot 10^{-4}$	$1,2 \cdot 10^{-5}$	$-2,4 \cdot 10^{-8}$	$1,2 \cdot 10^{-11}$

**Table 3** Temperature coefficients  $K_{18}$  and  $K_{19}$  of the pressure transducers determined by PTB from the period in vacuum at 5 °C, 20 °C and 30 °C measured in different laboratories.

Nr.		PTB	TN3	TN1	Solartron
DT120401	$K_{18} \cdot 10^5$ in $\text{K}^{-1}$	-1,5	-1,5	-1,5	-0,6
	$K_{19} \cdot 10^4$ in $\text{kg}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	6,6	6,6	7,0	17
DT120427	$K_{18} \cdot 10^5$ in $\text{K}^{-1}$	-1,7	-1,7	-1,7	-11
	$K_{19} \cdot 10^4$ in $\text{kg}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	4,3	4,9	4,7	11
DT120739	$K_{18} \cdot 10^5$ in $\text{K}^{-1}$	-2,9	-2,9	-2,9	-2,1
	$K_{19} \cdot 10^4$ in $\text{kg}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	-9,5	-8,8	-8,7	0,01
DT120801	$K_{18} \cdot 10^5$ in $\text{K}^{-1}$	-2,5	-2,5	-2,4	-1,8
	$K_{19} \cdot 10^4$ in $\text{kg}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	-4,4	-4,2	-3,8	3,2

## Figure Captions

- Fig. 1 Uncertainty of a calibrated density transducer (Bopp & Reuther, Typ DIMB 104.2, Ser.Nr. 25595). The inset rectangle indicates the calibration range.
- Fig. 2 Difference between the calibrations of the participants in the first ring test and the calibration of PTB. The calibrations are carried out with the test gas methane at 20 °C.
- Fig. 3 Difference between the calibrations of the participants in the second ring test and the calibration of PTB. The calibrations are carried out with the test gas methane at 20 °C.
- Fig. 4 Difference between the calibrations with the test gas methane and the calibrations with the test gas nitrogen at 20 °C.
- Fig. 5 VoS correction according to equations (6) and (7) of a transducer (DT 120801) for the measurement in methane. The transducer has been calibrated in nitrogen. The correction is carried out with the general constants  $K = 53,4 \text{ m}\cdot\text{s}^{-1}$  and  $L = 2,1\cdot 10^4 \text{ }\mu\text{s}\cdot\text{m}\cdot\text{s}^{-1}$ .
- Fig. 6 VoS correction according to equations (6) and (7) of a transducer (DT 120801) for the measurement in methane. The transducer has been calibrated in nitrogen. The correction is carried out with the specific constants  $K = 40 \text{ m}\cdot\text{s}^{-1}$  and  $L = 2,45\cdot 10^4 \text{ }\mu\text{s}\cdot\text{m}\cdot\text{s}^{-1}$ .
- Fig. 7 Relative deviation of the indicated density from the "true" density in two different natural gases. The "true" density was calculated from temperature, pressure and composition. The transducer was calibrated in methane at 20 °C. The lower curves in the last diagram have been corrected according to equation (7).
- Fig. 8 Difference between the calibrations at 10 °C and 30 °C and the calibration at 20 °C as the reference. For the reference curve the deviations of the individual measuring points from the regression curve are shown. The transducer was calibrated in methane at 20 °C.

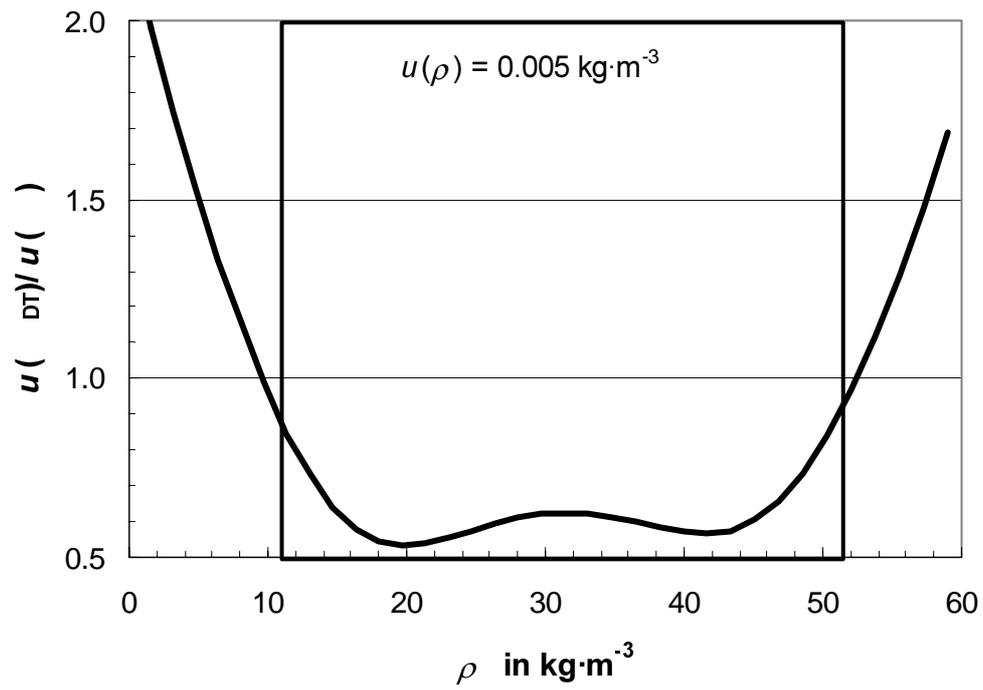
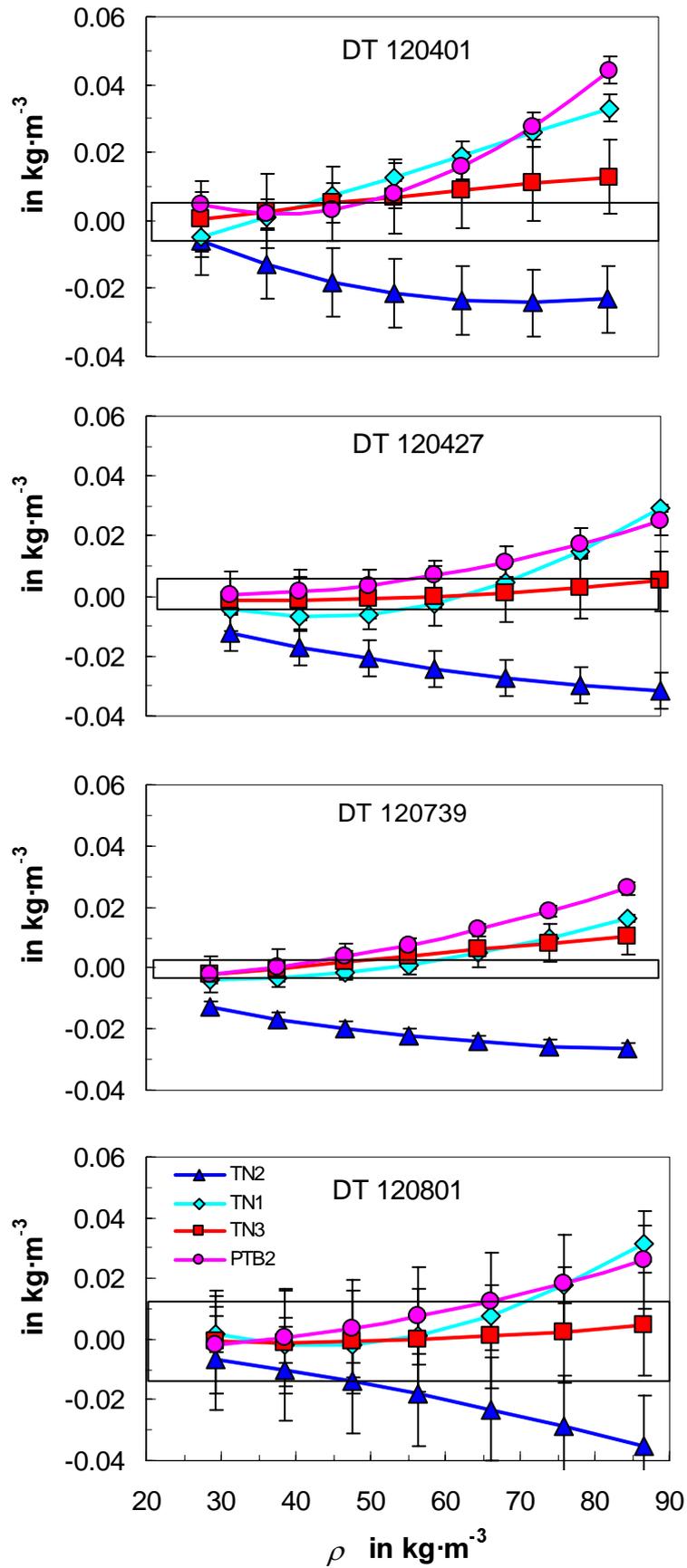
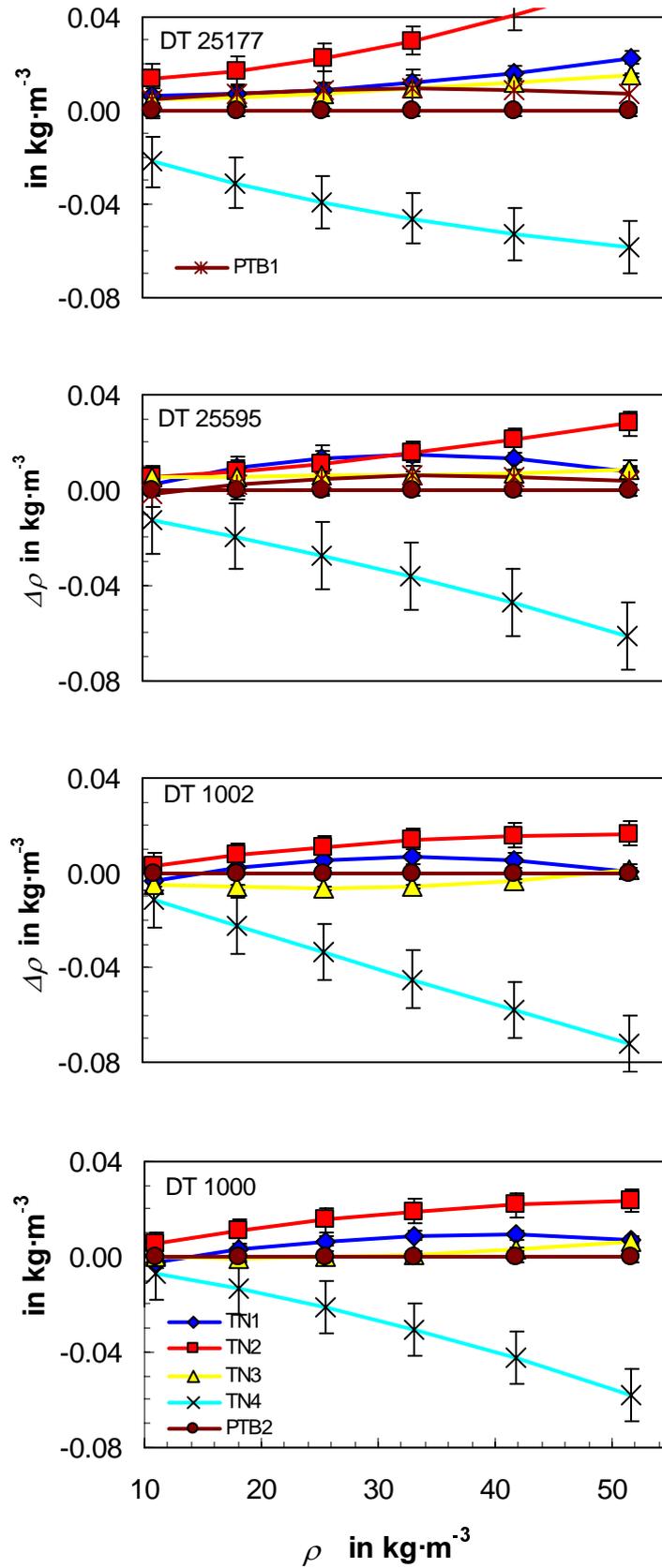


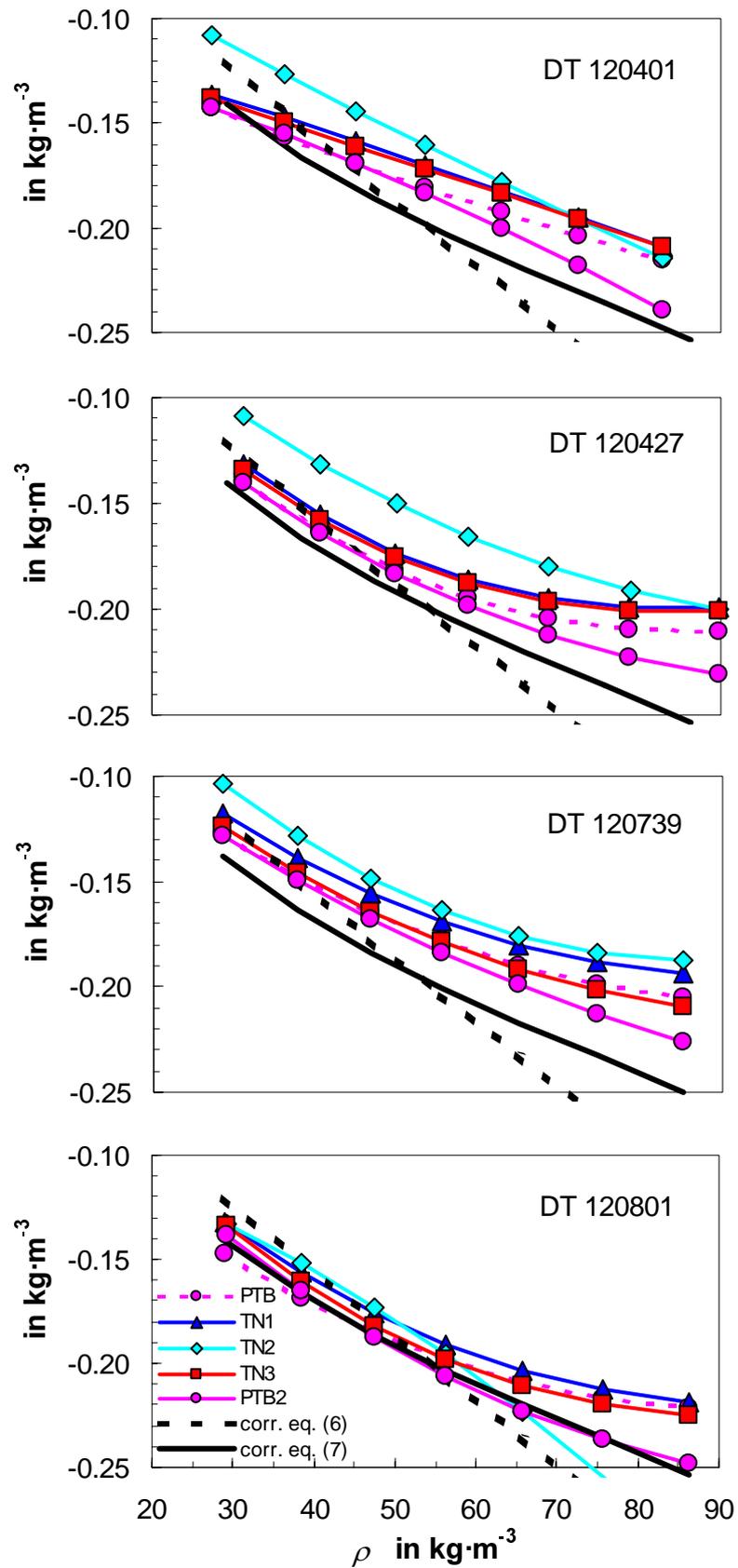
Fig. 1 Uncertainty of a calibrated density transducer (Bopp & Reuther, Typ DIMB 104.2, Ser.Nr. 25595). The inset rectangle indicates the calibration range.



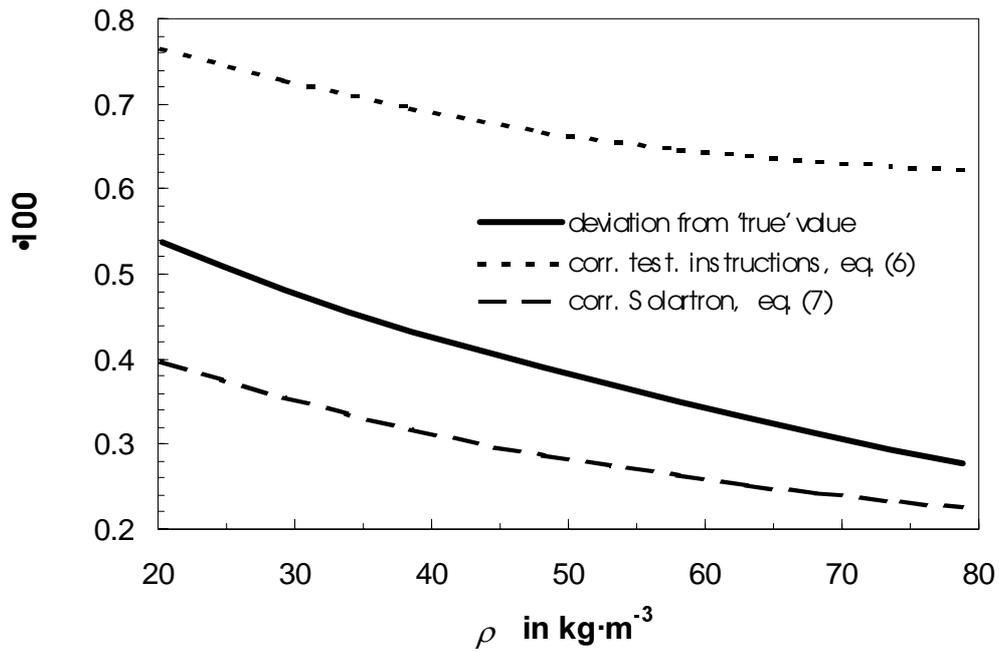
**Fig. 2** Difference between the calibrations of the participants in the first ring test and the calibration of PTB. The calibrations are carried out with the test gas methane at 20 °C.



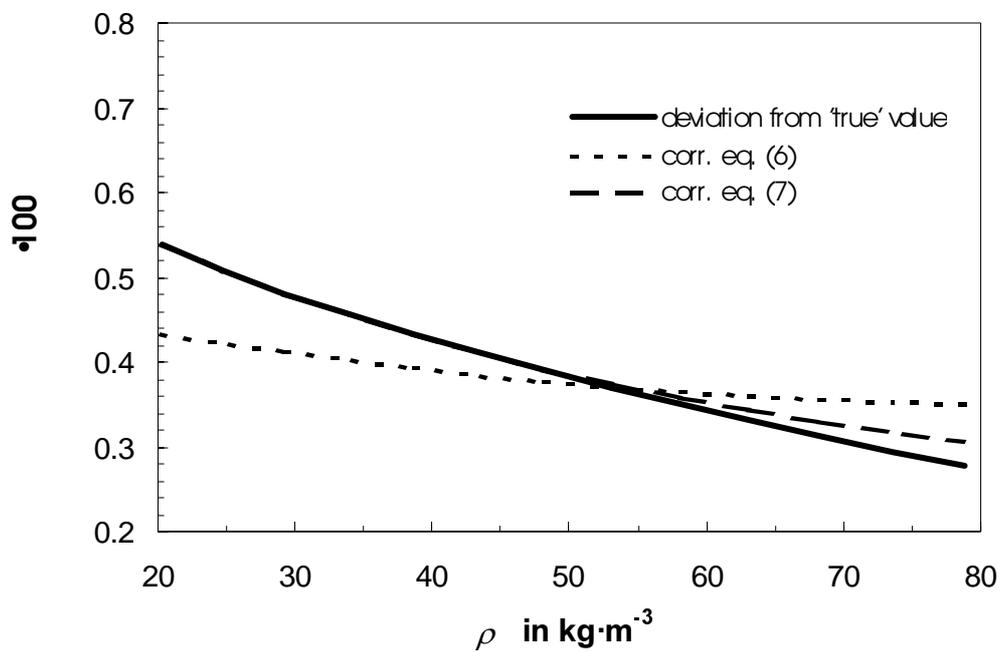
**Fig. 3** Difference between the calibrations of the participants in the second ring test and the calibration of PTB. The calibrations are carried out with the test gas methane at 20 °C.



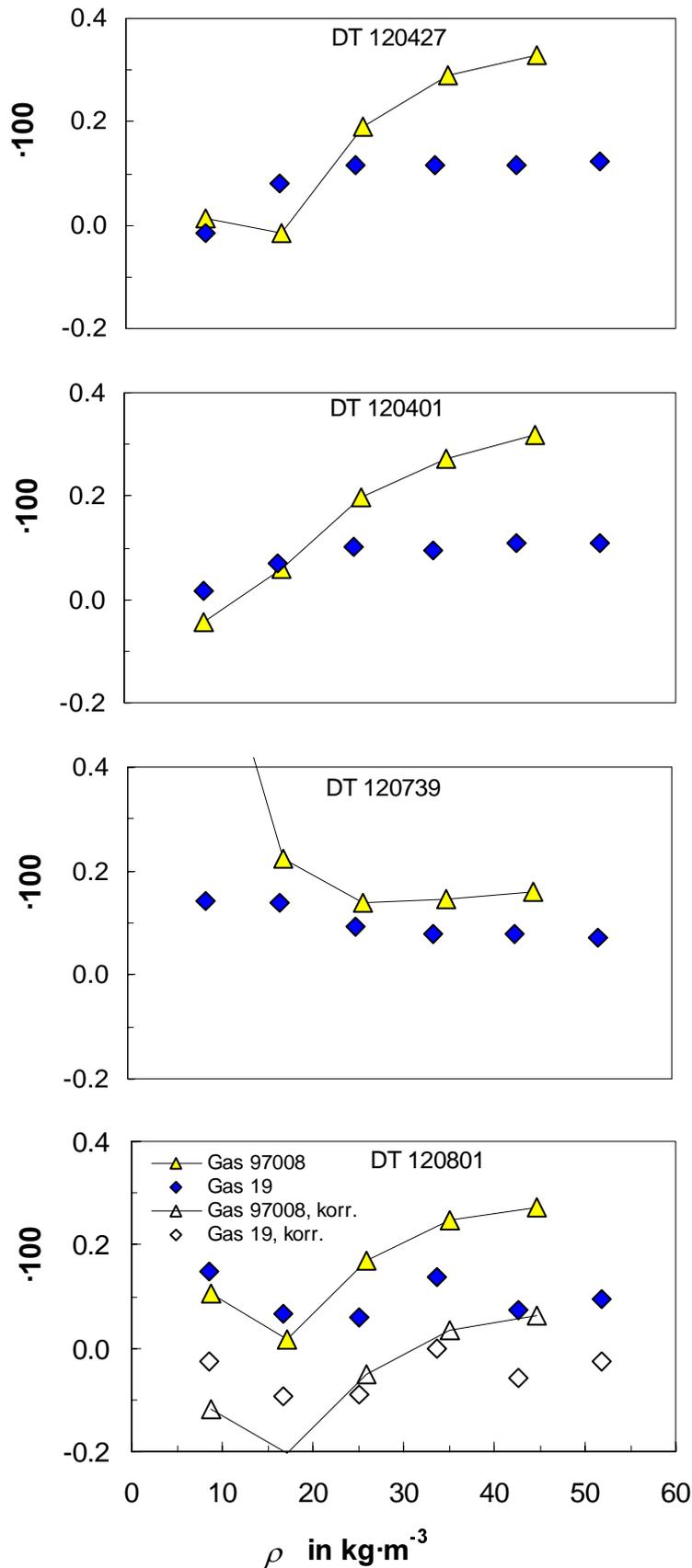
**Fig. 4** Difference between the calibrations with the test gas methane and the calibrations with the test gas nitrogen at 20 °C.



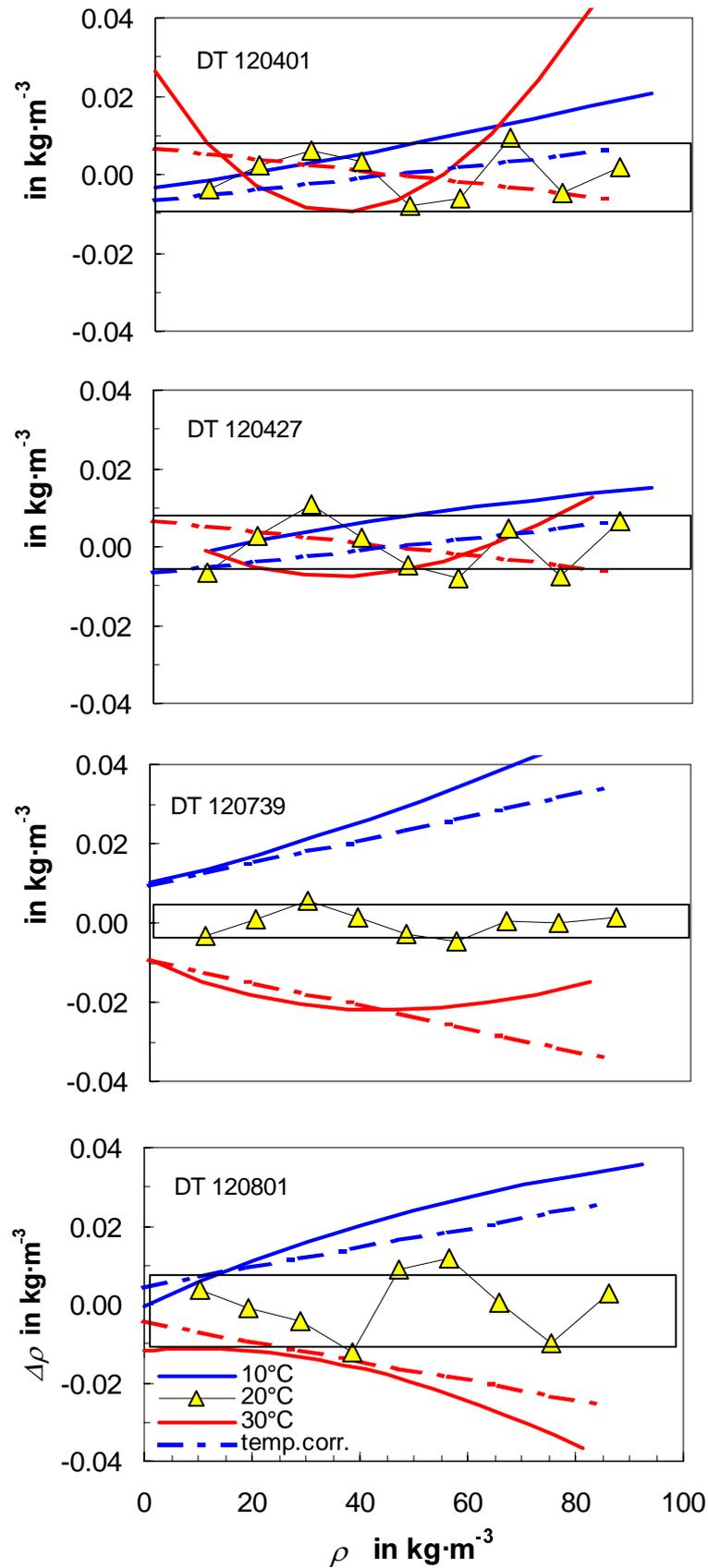
**Fig. 5** VoS correction according to equations (6) and (7) of a transducer (DT 120801) for the measurement in methane. The transducer has been calibrated in nitrogen. The correction is carried out with the general constants  $K = 53,4 \text{ m}\cdot\text{s}^{-1}$  and  $L = 2,1\cdot 10^4 \mu\text{s}\cdot\text{m}\cdot\text{s}^{-1}$ .



**Fig. 6** VoS correction according to equations (6) and (7) of a transducer (DT 120801) for the measurement in methane. The transducer has been calibrated in nitrogen. The correction is carried out with the specific constants  $K = 40 \text{ m}\cdot\text{s}^{-1}$  and  $L = 2,45\cdot 10^4 \mu\text{s}\cdot\text{m}\cdot\text{s}^{-1}$ .



**Fig. 7** Relative deviation of the indicated density from the "true" density in two different natural gases. The "true" density was calculated from temperature, pressure and composition. The transducer was calibrated in methane at 20 °C. The lower curves in the last diagram have been corrected according to equation (7).



**Fig. 8** Difference between the calibrations at 10 °C and 30 °C and the calibration at 20 °C as the reference. For the reference curve the deviations of the individual measuring points from the regression curve are shown. The transducer was calibrated in methane at 20 °C.

**Appendix A** List of symbols.

$\gamma$	ratio of the specific heats $c_p/c_v$
$\rho$	density of gas
$\rho_{DT}$	indicated value of a calibrated density transducer
$\Delta\rho_{DT}$	$= \rho_{DT} - \rho$
$\rho_{Cor}$	corrected value of the indicated density
$\Delta\rho_{Cor}$	$= \rho_{Cor} - \rho_{DT}$
$SG$	ratio of the molar mass of a gas to the molar mass of air ("specific gravity")
$\tau$	period of vibration of the vibrating element in a transducer
$T$	absolute temperature
$w$	velocity of sound in the gas
$K, L,$	
$K_0, K_1, K_2,$	
$K_3, K_4, K_{18}, K_{19}$	parameters of the transducer (see text)

The index c is used for the velocity of sound in the calibration gas.

