North Sea FLOW Measurement Workshop

nel oil & gas services

Uncertainty Based Allocation
Work in Progress?

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1 INTRODUCTION

Recently I was asked by a customer to write a summary of frequently used reconciliation and production allocation methods. One of these methods was "Uncertainty Based Allocation", presented first at the NSFM workshop in 2002 [1] and later as an appendix in API RP 85 "Use of Subsea Wet-Gas flow meters in Allocation Measurement Systems" [2]. In 2004 a an other UBA method was presented [3], which will be considered a variation on [1] and [2].

When studying this method a number of mixed thoughts crossed my mind, the most important being

- A very interesting novelty to include the measurement uncertainty in the allocation method. – a commercial 'return' for accuracy and a penalty for inaccuracy
- What a complicated allocation formula results from the RP85 "optimum" solution.
- The method is presented in a document about subsea wet gas measurement, but nothing in the method restricts its application to that area. If the method is both general and optimum, then it should be considered for more general application.

In general I am of the opinion that there are still quite a number of queries about this method, which would justify a public debate. The objective of this paper is to make an attempt to start that discussion now.

The paper will conclude with a proposal for a different, much simpler formula for uncertainty based allocation. The formula will be based on a different "optimisation" criterion.

2 SUMMARY OF THE METHOD AS PRESENTED IN RP 85

Step a) A bulk metered quantity "Q_B" has to be allocated to the feeding points. The raw quantities of the feeding points are "Q(j)"

An imbalance ("I ") is calculated, which is the difference between the bulk quantity QB and the sum of the feeding streams

$$I = Q_B - \sum Q(i) \qquad (1)$$

For any allocation method, adjusted feeding quantities are calculated as

$$Q_{adj}(j) = Q(j) + \alpha(j) * I \qquad (2)$$

With the condition that $\sum \alpha(i) = 1$

As an example, in the conventional proportional reconciliation (or allocation) formula α(j) is:

$$\alpha(j) = Q(j) / \sum Q(i) \qquad (3)$$

This can simply be verified by substitution.

Step b)
Within the condition that the sum of α(j) equals unity, a whole series of formulas for α(j) is possible. Proportional allocation is given in the example formula (3) above. In RP85 the expression proposed for α(j) is

$$\alpha(j) = \sigma(j)^2 / (\sigma_B^2 + \Sigma\sigma(i)^2) + (1/n)*\sigma_B^2 / (\sigma_B^2 + \Sigma\sigma(i)^2) \quad (4)$$

Where σ(j) is the standard deviation of the (absolute) uncertainty of each contributing quantity Q(j) , σ_B is the standard deviation of the (absolute) uncertainty of the bulk quantity QB and n is the total number of feeding streams. The formula comprises two terms. The first represents that the allocation factors α(j) increase with increasing uncertainty of the stream that the factor belongs to, the UBA principle. The second term is a refinement. It represents that the uncertainty in the bulk measurement is allocated in a certain way to the contributing streams.

3 DISCUSSION OF VARIOUS ASPECTS OF THE RP85 METHOD

3.1 Uncertainty As Basis For Allocation

In the UBA method allocation of the imbalance “I” is not done simply on the basis of the incoming stream quantities but on the basis of the absolute uncertainty of those quantities. The intention is to allocate more of the imbalance to the less accurately metered quantities than the more accurately metered quantities. This is the great attractiveness of the principle, i.e. there is a tangible benefit for accuracy. Although the correction may still represent either a gain or a loss, better measurements mean "less dependent on measurements by others".

However to achieve a similar result, in principle many formulas and methods could be designed and used. The method presented in RP 85 is only one of these.

3.2 An "Optimum" Solution

In RP 85 the derivation of the allocation formula, is based on a very common "optimisation" method, the least square minimisation method. Many years ago when I first heard about "optimum solutions", I was in particular impressed by the suggestion that apparently mathematics were capable of calculating some kind of absolute optimum solution to a problem. Later I learned that there is nothing absolute about such a solution. The trick is in the selection of the criterion, which has to be minimised. If one chooses a different criterion one gets a different optimum. In other words the optimum is not stronger than the criterion. A good reason to examine the criterion used.

3.3 No Smooth Transition To Conventional Proportional Allocation Methods

One of the characteristics of the RP 85 method is the lack of smooth transition to the conventional proportional allocation method(s). More specifically, if all streams are measured with the same relative uncertainty one would want to see that the UBS method produces results that are not different from the proportional allocation method.
This point can best be explained with an example. Assume the case of just two feeding streams. The metering method of both streams is similar. So one assumes that the relative uncertainty of both measured streams is equal. For simplicity sake we will also neglect the uncertainty in the bulk stream.
The absolute uncertainty of stream Q(j) can be written as $\sigma(j) = \sigma_{rel}(j) * Q(j)$, where $\sigma_{rel}(j)$ is the relative uncertainty of the stream. As it is assumed now that the relative uncertainties of both streams are equal, the uncertainties vanish from the formula and (4) becomes:

$$\alpha(j) = \sigma(j)^2 / \Sigma\sigma(i)^2 = Q(j)^2 / \Sigma Q(i)^2 \quad (5)$$

Fig.1 gives a graphical comparison of the allocation coefficient according to formula (5) (all relative uncertainties having the same value) and the ones according to the proportional allocation of (3). The difference between the two results is clear. It is the authors opinion that this difference is an unwanted characteristic of the RP 85 method.

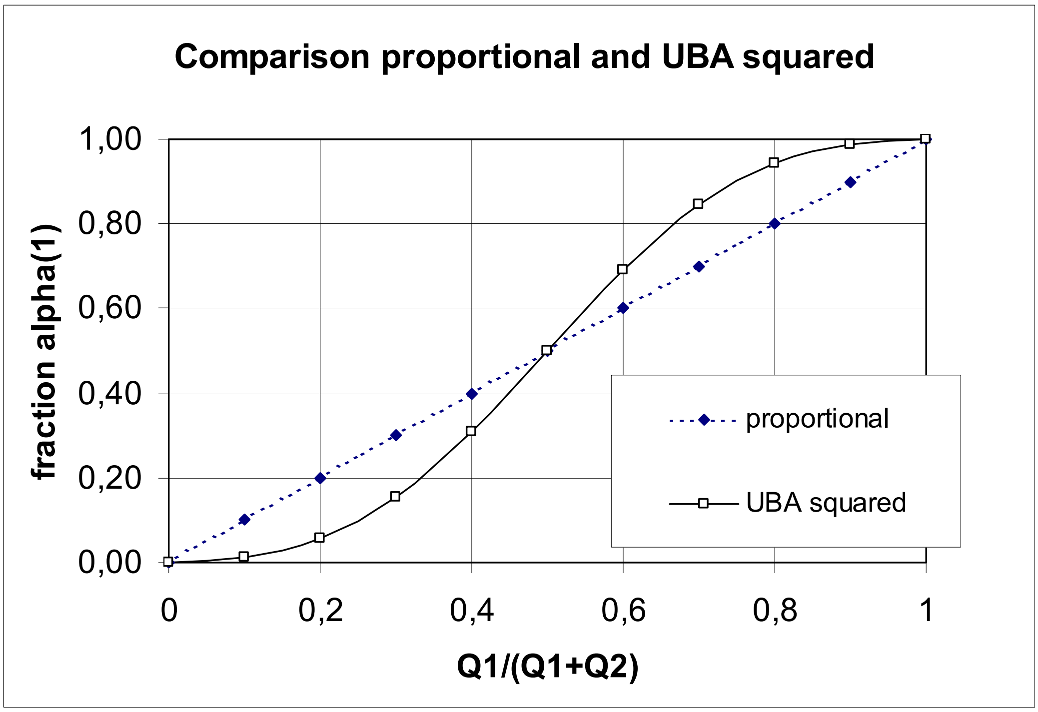

Fig.1 Comparison of proportional allocation and RP85 allocation if relative uncertainties are equal. Only α(1) is shown. α(2) = 1 – α(1)

3.4 The Uncertainty In The Bulk Quantity

Apparently the authors of RP 85 are not happy with the term that represents the uncertainty in the bulk measurement: $(1/n)*\sigma_B^2 / (\sigma_B^2 + \Sigma\sigma(j)^2)$ as it would "feel" unfair.

They propose to replace the factor 1/n by Q(j)/ ΣQ(i) and accept that the solution is sub-optimum. [2, Appendix E]

However, if one is prepared to accept a deviation from the mathematical optimum solution, then a much simpler and in my view more elegant sub-optimum formula would be obtained by simply deleting the whole term from the formula. The consequence is that ∑ α (i) is no longer unity, but that can be resolved. For example by stating that the uncertainty in the bulk measurement is zero. Which would be consistent with the fact that the bulk quantity is not adjusted anyway. (See also 3.6 below)

3.5 Random Errors

The mathematical derivation of the "optimum" solution uses a number of assumptions. One of these is that are no systematic components in the measurement errors. This assumption is used in the derivation of the expectations of the variances of the resulting individual errors i.e. the errors after the imbalance allocation has been applied.
I am of the opinion that this assumption is not correct as is explained below. As a consequence the mathematical basis under the derivation of the "optimum solution" would be absent. The arguments are the following:

It would be beyond the scope of this extended summary to enter into an argument about the definition of random and systematic errors. For those interested, the author's opinion on that subject is condensed in [4].
Fortunately a definition of random errors is not really required in the context of this discussion. For the derivation of the "optimum" solution in RP 85, it would be sufficient (and easier defined) to require that the measurement errors in the contributing streams are uncorrelated. Thus the expectation of cross-products of errors would indeed vanish.

However, especially in the case of wet gas metering, where RP 85 is primarily targeted at, the measurement errors are most likely strongly correlated. An important error source is in the wetness correction formula. Scientists are still improving this formula, taking into account more properties of the multiphase streams. Errors due to shortcomings of these models are very likely strongly correlated. Therefore the expectation of the cross-products should not be assumed to vanish.

Another assumption made is that by proper calibration the “systematic” errors in measurements can be eliminated. However this is debatable. During calibration also a measurement error will be made, which may be considered random at that moment in time. But for the period until the next calibration this error is a bias and hence systematic. It might be that this error, although systematic, is still uncorrelated with the other errors. But it would require further analysis before it can be assumed that these calibration errors are uncorrelated.

However, as said earlier, the discussion about randomness of errors is believed to be beyond the scope of this summary. In the formulas proposed in the next paragraphs for an alternative uncertainty based allocation method, the subject does not play a role.

3.6 Special Aspects of the UBS Method by Melbø et.al

As the authors of this method point out themselves, the results of the method are very similar to those of the RP85 method. The most striking difference is perhaps that in the Melbø method the consequence of assigning an uncertainty to the bulk measurement is that also the bulk measurement has to be adjusted (reconciled). In normal language it means that if the bulk measurement has a finite uncertainty, then it shall also take its share of the imbalance. From a scientific point of view this is very elegant and it leads to formulas that are very transparent. In RP85 that consequence is not accepted (for good reasons) and the part of the imbalance that should be allocated to the bulk measurement is allocated to the feeding streams. In the modified RP85 method this is done proportional to the flow rates.

Although scientifically elegant, adjusting the bulk metered value is a revolution with respect to conventional practice and will become a contractual and accounting nightmare in financial allocation systems. It is very understandable that RP85 does not accept this consequence.

4 DIFFERENT OPTIMISATION CRITERIA

As mentioned above, an optimum solution is entirely dependent on the criteria applied. Various criteria are possible in principle. The least square minimisation is only one of them.

The criteria that I consider important though, are

4.1 The less accurate a stream, the bigger its share of the imbalance (this is the basis for UBA and common with the previously discussed methods)

4.2 The solution shall have a smooth transition to conventional proportional allocation methods, as explained in section 3.3.

4.3 "Simplicity is the seal of truth" i.e. the solution shall not be more complicated than necessary to achieve the objective.

5 A SOLUTION SATISFYING THE REQUIREMENTS

There is a solution (there might be more) that satisfies all three criteria mentioned above. In that sense that solution is "optimal". This "optimum" solution is

$$\alpha(j) = \sigma(j) / \sum \sigma(i) \qquad (5)$$

Note that σ(j) represents the absolute uncertainty of stream j. If one introduces the notation $\sigma_r(j)$ for the relative uncertainty, then (5) can be rewritten as

$$\alpha(j) = \sigma_r(j)*Q(j) / \sum \sigma_r(i)*Q(i) \quad (6)$$

From this it can easily be seen that the three criteria 4.1 until 4.3 are satisfied. In the case that the streams (j) are measured with the same relative uncertainty (all $\sigma_r(j)$ have the same value), formula (6) reduces to formula (3), which was the formula for conventional proportional allocation.

6 QUANTIFICATION OF THE UNCERTAINTY

This remains a very difficult task, which is considered outside the scope of this paper. In [3] an interesting method for this is presented. An issue will remain that for an allocation contract more transparency and simplicity may be required.
In my humble opinion it may be a practical solution that in an allocation contract in which uncertainty based allocation is used, there will also be an appendix simply stating what values have been agreed between partners for the (relative) uncertainties of the various stream measurements.

7 REFERENCES

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[4] Wolff, Chris J.M. and de Wardt John P . Borehole Position Uncertainty – Analysis of measuring methods and derivation of a systematic error model. SPE 9223, 1980 and JPT (December 1981) 2339.