

Realistic Pipe Prover Volume Uncertainty

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ABSTRACT

Traceability for liquid turbine metering systems is generally achieved via a calibrated pipe prover volume used to verify the meter K-factor in situ. This paper demonstrates how incompatibility between an arbitrary tolerance set for the calibration of a pipe prover and the achievable uncertainty in measurement when determining the prover volume can lead to practices which may result in measurement bias.

This paper presents a robust method of estimating the pipe prover volume uncertainty determined using the master meter/master pipe prover calibration method. The individual uncertainty components used in the estimate, and the method of combining them, are included along with a comparison of the gravimetric and volumetric calibration methods for determining the compact prover volume. The traceability chain relating to the calibration of the pipe prover and the importance of accreditation for measurements are also discussed.

The paper concludes by examining the tolerances in place in the North Sea and how the practices which have evolved to meet the tolerances may compromise good metrology and lead to measurement bias.

1. INTRODUCTION

Fiscal oil metering stations typically consist of two or more meter runs with turbine metering and associated secondary instrumentation such as temperature, pressure, density and sampling systems to allow the fluid properties and liquid composition to be determined. The oil measurement system is also usually equipped with a permanently installed pipe prover or small volume prover used as a volume calibration reference for the turbine meters.

When the pipe prover volume is too great to perform a volumetric water-draw, to directly compare the volume of the pipe prover and volumetric standard measure (or proving tank), calibration is generally carried out by the master meter/master pipe prover method. Using a compact prover as the master pipe prover, the calibrated volume of the compact prover is used to calibrate the master meter which is then used as a transfer standard for determining the volume of the (permanently installed) pipe prover. This calibration method is undertaken with the compact prover, master meter and pipe prover connected in series and allows both the master meter and pipe prover to be calibrated on operating fluids at the same pressure, temperature and flowrate.

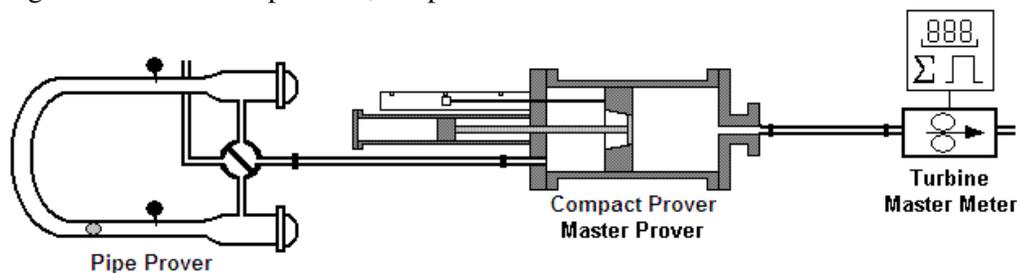


Figure 1. Master Meter/Master Prover Method

This paper investigates the pipe prover volume uncertainty determined using the master meter/master pipe prover calibration method and also compares the volumetric and gravimetric water-draw methods for compact prover volume calibration.

In the UK sector of the North Sea the regulatory DECC guidelines [1] on pipe prover volume calibration define the requirements for repeatability as $\pm 0.01\%$ with a year on year tolerance band as 0.02% and highlight that the operator must seek approval before any shift in excess of 0.02% is accepted.

2. TERMS AND DEFINITIONS

2.1 General Abbreviations

API	American Petroleum Institute
BIPM	International Bureau of Weights and Measures (translation from French)
CIPM	International Committee for Weights and Measures (translation from French)
CTE	Coefficient of Thermal Expansion
DECC	Department of Energy and Climate Change
GUM	Guide to the Expression of Uncertainty in Measurement
ISO	International Organisation for Standardisation (translation from French)
MPMS	Manual of Petroleum Measurement Standard
NASA	National Aeronautics and Space Administration
NML	National Measurement Laboratories
NMS	National Measurement System
SI	International System of Units (translation from French)
SVP	Small Volume Prover
TUR	Test Uncertainty Ratio
UK	United Kingdom
UKAS	United Kingdom Accreditation Service
VIM	Vocabulary of International Metrology

2.2 Pipe Proving Terms

A short list of pipe proving terms specific to this paper is given below, although complete lists of precise and rigorous definitions are listed within *ISO 7278* [2] and *API MPMS Chapter 4* [3]:

Calibrated Volume – also known as the ‘Base Volume’ of a pipe prover between detectors or calibrated standard measure or volumetric prover tank at standard conditions.

Compact Prover – typically a small volume prover with a piston displacer installed in a precision bored cylinder.

Detectors – optical sensors or electronic switches placed at either end of the calibrated volume section of the prover and actuated by the displacer to start and stop the pulse counters.

Displacer – generic name for the sphere or piston used to sweep the calibrated pipe prover volume.

K-Factor – number of pulses generated by a meter in relation to the volume passed.

Master Meter – a meter that serves as the reference for the proving of another meter or pipe volume.

Pass – a single movement of the displacer between detectors.

Pipe Prover – the generic name for provers either conventional pipe provers or small volume provers in which a sphere or piston is displaced to measure the passed volume.

Pulse Interpolation – technique to enhance the meter pulse count resolution, typically ‘double chronometry’.

Run – a set of passes deemed necessary to derive a single K-factor suitable for reporting.

Small Volume Prover (SVP) – a pipe prover producing less than 10,000 meter pulses per pass although capable of achieving the required repeatability and accuracy due to installation of high precision detectors and use of pulse interpolation techniques.

Water Draw – term for the operation of calibrating a pipe prover with water into a volumetric or gravimetric tank.

2.3 Uncertainty of Measurement Terms

The precise and rigorous definitions of the following terms are defined within the ISO documents, *International Vocabulary of Basic and General Terms in Metrology (VIM)* [4] and *Guide to the Expression of Uncertainty in Measurement (GUM)* [5]:

Combined Standard Uncertainty – standard uncertainty of the result of the combination of standard uncertainty components.

Covariance – measure of mutually dependent uncertainties where correlations among the input estimates affect the combined standard uncertainty of the output estimate.

Coverage Factor – numerical factor used to multiply the combined standard uncertainty to give the expanded uncertainty at a specified level of confidence.

Expanded Uncertainty – an interval about the measurement result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

Measurand – particular quantity subject to measurement.

Measurement Accuracy – closeness of the agreement between a measurement result and the true value. As the true value is not known, accuracy is a qualitative term only (not quantitative).

Precision – the closeness of agreement between independent test results obtained under stipulated conditions.

Repeatability – precision under conditions where the results of successive measurements of the same measurand are carried out under the same conditions of measurement within short intervals of time.

Reproducibility – precision under conditions where test results are obtained with the same method on identical test items under changed conditions of measurement over a longer interval of time.

Probability Distribution – a function giving the probability that the random variable takes any given value or belongs to a set of values. i.e. Gaussian (normal), rectangular (uniform), triangular, etc.

Sensitivity Coefficient – the differential change in the output value generated by the differential change in one input value divided by the change in that input.

Standard Uncertainty – the uncertainty of the result of a measurement expressed as a standard deviation.

Tolerance – the limiting or permitted range of values of a defined quantity.

Type A – evaluation of uncertainty of measured values by statistical methods.

Type B – evaluation of uncertainty by means other than statistical analysis

Uncertainty of Measurement – parameter associated with the result of a measurement that characterises the dispersion of the values that could reasonably be attributed to the measurand.

3. PIPE PROVERS

3.1 History

The first pipe provers were used in the early 1950's [6]. One of the first pipe provers was the 'mile of pipe' which used the predetermined volume of the pipe length and also tracked the position of a tightly fitted piston down the pipe flowing full of oil to increase the measurement accuracy of flow

meters. Pipe provers were first documented in the *API Standards 1101 (1960)* [7] for Positive Displacement Meters and later in *API Standards 2531 (1963)* [8] for Mechanical Displacement Meter Provers.

Pipe provers operate by displacing a known volume of liquid within a calibrated section of pipe. Repeatable displacement of fluids is achieved by an oversized sphere or piston travelling through the pipe between detectors. The first conventional pipe provers standardised by API achieved the required measurement resolution of 0.01% pulse resolution by generating no less than 10,000 meter pulses during a proving pass. The pipe prover design is such that the full flow through the metering stream being proved will pass through the pipe prover.

In 1967, the Apollo manned space program exhibited a need for precision test equipment to test altitude control rocket motors as part of the NASA test program [9]. The requirement to calibrate small flow meters to an accuracy of $\pm 0.05\%$ was met by a small manufacturer of flow meters who developed a Small Volume Prover (SVP) device which utilised an electronic pulse-counting technique known as 'double chronometry'. It was the late 1970's before modification and testing of this initial SVP design was conducted with the aim of moving the design from the laboratory into the field. The initial design used magnetic reed switches and a compressed air actuator system which comprised an air compressor and tank storage unit. These limiting components were replaced with high accuracy optical switches and a nitrogen/hydraulic system respectively. A further development was the introduction of an Invar rod with low coefficient of expansion on which the optical switches are attached and spring loaded to allow more accurate measurement of volume. These modifications made to the SVP created a portable device for use in industry similar to the present day compact prover.

3.2 Operating Principle

Pipe provers are an important part of a turbine meter fiscal oil metering station and are used to calibrate the meter K-factor periodically when flowrates or conditions change. The pipe prover is used to 'prove' the accuracy and repeatability of flow meters on actual operating fluids by measuring the volume of fluids passing through the meter in relation to the number of pulses generated by the meter.

The basic operating principle of the pipe prover is to meter the fluids swept by the displacer through a calibrated volume of pipe by counting the number of meter pulses between the start and stop detectors at either end of the calibrated volume. The displacer, either a piston or sphere, actuates the start and stop detectors and is designed to form a sliding seal which moves at the same rate as the flowing liquid. Temperature and pressure corrections are required to convert the calibrated volume at standard conditions to process conditions. The volume indicated by the meter is compared to the calibrated volume to determine a meter K-factor. Generally, meter K-factor calibration must achieve repeatability over five successive runs to within a band of 0.1% to meet the UK regulatory requirement for overall dry mass uncertainty of $\pm 0.25\%$ at the fiscal oil metering station.

International standards *ISO 7278* parts 1 to 4 [2] and American standards *API MPMS Chapter 4* sections 1 to 9 [3] are the current publications standardising pipe prover design and operation.

3.2.1 Conventional Pipe Prover

The conventional positive displacement pipe prover generates no fewer than 10,000 pulses for each proving pass to achieve a measurement resolution of 0.01% as defined in the API MPMS standards [3]. Conventional pipe provers can be constructed in a number of configurations such as uni-directional or bi-directional pipe provers with piston or sphere displacers.

The bi-directional spheroid pipe prover shown in Figure 2 utilises a four-way diverter valve to direct the fluid flow in both directions along the calibrated section of pipe and allow measurement in both directions during a proving run. Bi-directional pipe provers require 20,000 pulses for each prover round trip (ie. 10,000 pulses for each pass).

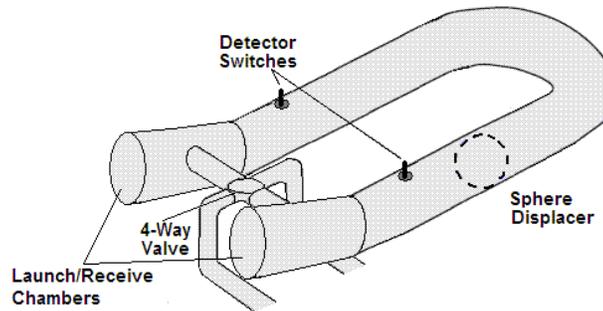


Figure 2. Bi-directional Spheroid Pipe Prover Design

The uni-directional pipe prover channels fluid in only one direction through the calibrated section of the pipe prover. The meter pulses are recorded when the displacer travels in one direction with the flow across the calibrated section of pipe. Uni-directional pipe provers also require a method of returning the displacer to its starting position.

The displacement systems in a pipe prover are either an oversized sphere which forms a seal and travel with the flow or a sealing piston within the pipe. The design of a pipe prover with sphere displacer must incorporate chambers to launch and receive the sphere. Spheroid pipe prover design is more common as the sphere can move through bends in a pipe. Uni-directional piston pipe prover design uses a piston and poppet valve to allow the fluid to pass when the piston is returning to the start position.

Since conventional pipe provers are generally large permanent constructions, traceability for the volume of the calibrated section of pipe cannot be obtained in a laboratory so must, therefore, be calibrated using a transfer standard such as a volumetric proving tank or against a master prover and master meter.

3.2.2 Small Volume Prover

As small volume provers do not have sufficient volume to generate 10,000 unaltered meter pulses, pulse interpolation techniques are employed to increase the meter pulse count resolution and can therefore operate with less than 10,000 pulses as defined in the API MPMS standards [3]. However, pulse interpolation must achieve the required resolution of 0.01%. Pulse interpolation techniques interpolate fractional meter pulses or mathematically interpolate partial pulses with the most widely used method being double chronometry.

Small volume prover design incorporates a precision bore cylinder; a displacer with means of positioning and launching the displacer upstream of the calibrated section; displacer detectors that allow fluid flow while the displacer is travelling and temperature and pressure measurement devices with meter pulse counting instrumentation (timer, counters and pulse interpolation).

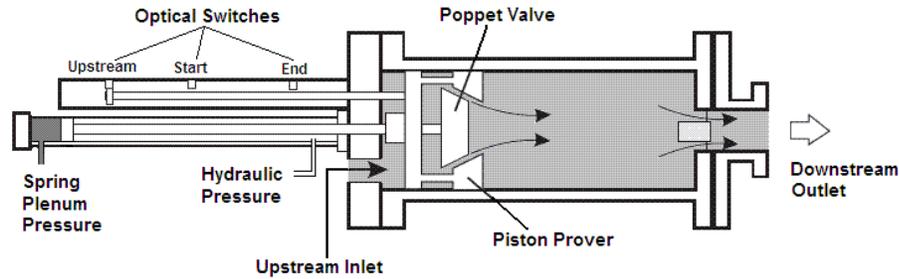


Figure 3. Uni-directional Compact Prover Design

The ‘Compact Prover’ is the name given to a typical style of mobile small volume prover. The basic design features are shown in Figure 3, and are comprised of a uni-directional pipe prover with piston and poppet valve (or flow-through valve).

Due to the compact size and portability of the small volume prover, the calibrated volume section of pipe within the precision bore chamber can be verified in a laboratory using a water draw procedure.

4. UNCERTAINTY OF MEASUREMENT

4.1 Uncertainty Evaluation

The result of a measurement is only an estimate of the value of a measurand, which means that any measurement result is only complete once a statement of uncertainty accompanies the result. Uncertainty of measurement is the doubt that exists regarding the result of a measurement and defines the level of confidence in that particular measurement. Measurement uncertainty can be estimated by quantifying the possible spread of measurements and provides the range of dispersion of results that can be reasonably associated with the measured value. Estimation of the measurement uncertainty is useful to help understand the parameters affecting the measurement; helps to define good quality measurements and allows meaningful comparison of results. A measurement result is only complete once a statement of uncertainty accompanies it.

Measurement uncertainty evaluation involves the use of a mathematical model for measurement and statistical techniques to determine the uncertainty associated with the best estimate of the value of the measurand. Each quantity significantly influencing the measurand value within the model also has prescribed uncertainties which must be accounted for within the uncertainty evaluation.

The GUM and several other documents stemming from GUM provide guidance on uncertainty evaluation. GUM enables measurements to be compared between different laboratories and provides a common approach for estimating measurement uncertainty. The GUM uncertainty framework has become the internationally accepted method for uncertainty calculation since its initial publication in 1993 but it should be noted that GUM is a guide and not a standard.

The main stages of uncertainty evaluation are given as follows:

- (i) Define the output quantity Y , to be measured.
- (ii) Determine all input quantities X_i on which the output quantity Y depends.
- (iii) Develop the mathematical model relating the input quantities X_i to the output quantity, $Y = f(X_1, X_2, \dots, X_N)$.
- (iv) The values determined for the input and output quantities are defined as x_1, x_2, \dots, x_N (input estimates) and y (output estimates), respectively.
- (v) Assign probability density functions (PDF) to the values of the input estimates x_i .

- (vi) Evaluate the standard uncertainties $u(x_i)$ by determining the estimated standard deviations for the input estimates, either by statistical means (Type A evaluation of standard uncertainty) or by other means (Type B evaluation of standard uncertainty).
- (vii) Evaluate the covariance of input estimates that are correlated. For each pair i, j for which the values of X_i and X_j are mutually dependent, calculate the estimated correlation coefficient $r(x_i, x_j)$ associated with x_i and x_j .
- (viii) Calculate the model sensitivity coefficients c_i by forming partial derivatives $\partial f/\partial x_i$ describing how the output estimate y varies with changes in the values of the input estimates x_i .
- (ix) Calculate the best estimate y of the output quantity value by evaluating the model using the input estimates x_i .
- (x) Determine the combined standard uncertainty $u_c(y)$ of the output estimate by combining $u(x_i)$, $u(x_i, x_j)$ and the model sensitivity coefficients c_i . The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$ obtained from:

$$u_c^2(y) = \sum_{i=1}^N c_i^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N c_i c_j u(x_i) r(x_i, x_j)$$

- (xi) Calculate ν (effective degrees of freedom) associated with $u(y)$ using the Welch–Satterthwaite formula (GUM G.4).
- (xii) Multiply the combined standard uncertainty $u_c(y)$ by a coverage factor k to obtain the expanded uncertainty $U = k u_c(y)$. The coverage factor k is chosen on the basis of the level of confidence required of the interval. Coverage factor would normally be $k = 2$, giving a confidence level of approximately 95%.
- (xiii) Express the result of the measurement as $y \pm U$ stating the level of confidence in the interval.

Partial Derivatives

The sensitivity coefficient c_i describes how the output estimate y varies with changes in the values of the input estimates x_i . Using the analytical method the sensitivity coefficient c_i can be obtained by partial differentiation.

$$c_i = \frac{\partial y}{\partial x_i}$$

The analytical method involves differentiating the output parameter with respect to each of the input parameters in turn. The input values are then substituted into the resultant functions and each answer provides the sensitivity coefficients appropriate to each input parameter. This is the most mathematically correct method for evaluating sensitivity coefficients.

Finite Difference

If analytical determination of the partial derivatives is complicated where the mathematical model is complex, the actual derivative can be approximated by ‘Finite Difference’, which provides a robust method for uncertainty evaluation. The partial derivative $\partial y/\partial x_i$ can be approximated numerically by the finite difference expression [10],

$$\frac{\partial y}{\partial x_i} \approx \frac{\Delta y}{\Delta x_i} = \frac{q|x_i + \Delta x_i - y| x_i}{\Delta x_i}$$

where the value Δx_i is as small as is practical, initially applying an increment which is equal to the value of uncertainty in the parameter x_i .

Where the mathematical model relates the input estimates x_i to the output quantity, $y = f(x_1, x_2, \dots, x_N)$, the value of variation $u_i(y)$ can be taken as Z_i (GUM 5.1), with corresponding sensitivity coefficient c_i as $Z_i/u(x_i)$.

$$Z_i = \frac{1}{2} [f(x_1, \dots, x_i + u(x_i), \dots, x_N) - f(x_1, \dots, x_i - u(x_i), \dots, x_N)]$$

Combined standard uncertainty $u_c(y)$ can be defined numerically as:

$$u_c^2(y) = \sum_{i=1}^N \sum_{j=1}^N Z_i Z_j r(x_i, y_j)$$

Monte Carlo Simulation

Monte Carlo simulation involves making a large number of calculations of the output estimate y , by assigning different values to each of the input estimates x_i . Each input value is generated at random from the distribution for each input quantity to form the corresponding value and distribution of the output quantity. The Monte Carlo method is a numerical approach to uncertainty evaluation and is defined in a supplement [11] to GUM published in 2006.

4.2 Traceability

The VIM defines traceability as ‘the property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties’. The traceability chain involves the calibration of a measurement artefact or measurement equipment against a reference standard of greater accuracy.

Le Système International d’Unités (International System of Units), commonly referred to as the SI System is a universally adopted self consistent international system of measurement. The SI system base units are the meter, kilogram, second, ampere, Kelvin, candela and mole, respectively, for length, mass, time, electric current, thermodynamic temperature, luminous intensity and amount of substance.

National Measurement Laboratories (NML) hold the national primary measurement standard derived from the internationally recognised standard. The SI base unit for mass is the kilogram (kg) and the international standard artefact is held by the *Bureau International des Poids et Mesures* (BIPM, International Bureau of Weights and Measures) at Sèvres in France. The artefact is a cylinder of iridium alloy which is the primary measurement standard for mass and is the only remaining artefact as all other SI base unit standards are derived from physical properties such as the wavelength of light in vacuo which can be reproduced in laboratories throughout the world.

Most countries around the world have National Measurement Laboratories which hold national measurement standards to ensure confidence and accuracy of measurement. A *National Measurement System* (NMS) is also maintained and forms the technical and organisational infrastructure that ensures a consistent and internationally recognised basis for measurement. Calibrations undertaken against the national or regional standards form the traceability chain which links back to the SI base units. The NMS ensures accuracy and traceability of measurement for use in trade, industry, academia and government. For any measurement it should be possible to demonstrate traceability to international standards via an unbroken chain of calibrations.

4.3 Measurement Accreditation

Measurement standards in the UK are managed via traceability through United Kingdom Accreditation Service (UKAS) accredited laboratories. Measurement accreditation is important as it

ensures standards, accuracy and consistency of measurement which enables consumers to compare products for sale and make informed decisions and also facilitate trade to Europe and worldwide.

Accreditation assessment is conducted by an assessor with the expertise to cover the scope of accreditation. Accreditation is verified on an annual basis by surveillance visits (audits), with a full reassessment every four years. Those laboratories which have been assessed and approved by UKAS as meeting the requirements of ISO/IEC 17025 [12] may be granted UKAS accreditation. The Laboratories meeting the ISO/IEC 17025 standard requirements for calibration and testing activities also comply with the relevant requirements of the ISO 9001 [13] standard.

The five essential requirements for measurement accreditation backed by an efficient measurement audit are (1) Staff, (2) Equipment, (3) Accommodation and Environment, (4) Documentation (Quality Manual and Measurement Procedures) and (5) Traceability.

5. UNCERTAINTY EVALUATION EXAMPLES

The five stages (gravimetric water draw) or six stages (volumetric water draw) of the traceability chain that links pipe prover volume calibration to a standard weight set are shown in Figure 4. The standard weight shown here in stage 1 is deemed to be a company standard weight although there are obviously a few stages prior to this when tracing back to regional, national and international standards.

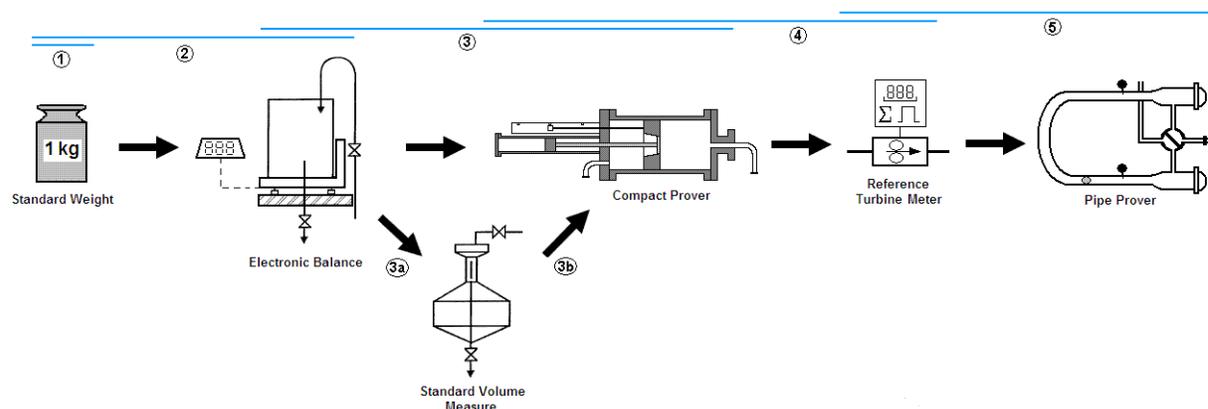


Figure 4. Pipe Prover Volume Uncertainty Traceability Chain

In stage 1, the standard weight set is calibrated in a UKAS accredited laboratory against a regional mass standard traceable to the national measurement system. Stages 2 and 3 are conducted simultaneously with the standard weights being used to calibrate the electronic weighing instrument, known as a mass comparator, by comparative methods along with the calibration of the compact prover volume by water draw. The calibration of the weighing instrument and compact prover volume is conducted in the laboratory under controlled conditions. The repeatability of 5 calibration runs should lie within a band of 0.02%. In Stage 4, the compact prover calibrated section of pipe is used to calibrate the turbine master meter k-factor. The repeatability of 5 calibration runs should again lie within a band 0.02%. Stage 5 is the final stage in which the pipe prover volume is calibrated by using the master meter. Using the K-factor obtained for the master meter, the number of pulses counted between detectors on the pipe prover calibrated section can be converted to volume. The repeatability of 5 calibration runs should again lie within a band of 0.02%. Temperature and pressure corrections are used in the calibration of the compact prover, master meter and pipe prover volume to adjust the process conditions to standard conditions of 15°C and 1.01325 bara.

The volumetric water draw traceability chain is also shown in Figure 4, although an additional stage is included whereby the volumetric standard measure is first calibrated (3a) using a mass comparator, then (3b) the standard measure is used to calibrate the compact prover. Stage 3 is broken into two which increases the number of stages to six.

An example uncertainty evaluation for pipe prover volume calibration is presented in this document. The uncertainty evaluations for the calibration of pipe prover volume using hydrocarbon fluids and water are compared. The uncertainty evaluation accounts for the correlated uncertainties that occur between successive stages. The uncertainty is evaluated numerically using the ‘Finite Difference’ method for sensitivity coefficient calculation as described in GUM.

Also presented in this document are the measurement uncertainty evaluations for compact prover volume calibration by gravimetric and volumetric methods.

5.1 Gravimetric Water Draw Compact Prover Volume Calibration

The procedure for gravimetric water draw calibration is to displace the water volume between detectors from the compact prover at a controlled flowrate into a container located on an electronic weighing instrument. The switches operate a solenoid valve which is used to divert the volume of water between optical switches into the container. The mass of the water obtained from weighing is then divided by the density of the water to obtain the volume of water between switches.

In the example a 60 litre compact prover is calibrated by gravimetric water draw method. Temperature of the water is 16°C with a pressure of 5 barg in the compact prover.

Mathematical Model

The main components of the mathematical model are shown below; all other equations are given in the Appendix.

Gravimetric Water Draw Calibration:
$$V_b = \frac{V_R \times C_{tdw}}{C_{tsp} \times C_{psp} \times C_{plp}}$$

Where:	V_b	Compact prover base volume at 15°C and zero barg [14, 15, 16]
	V_R	Compact prover indicated volume at observed temperature and atmospheric pressure
	C_{tdw}	Correction factor for the thermal expansion of water
	C_{tsp}	Correction factor for the effect of temperature on the steel of the prover
	C_{psp}	Correction factor for the effect of pressure on the steel of the prover
	C_{plp}	Correction factor for the effect of pressure on liquid at the prover

Uncertainty of Weighing, $U(M)$

Uncertainty in the balance weighing method $U(M)$ gives an expanded uncertainty [k=2] of $\pm 2g$. This uncertainty is the result of a weighing conducted by substitution method using an F1 weight set calibrated and traceable to national standards by a mass comparator weighing instrument with resolution of 0.05g. For the purpose of this uncertainty example the weighing uncertainty evaluation is not presented, although guidance on gravimetric water draw uncertainty is given in *PD ISO/TR 20461* [17]. Guidance on the uncertainty of mass comparator balance measurement can also be found in OIML D28 [18], OIML R111-1 [19] and OIML R111-2 [20]. The uncertainty components to consider in relation to the calibration of the weighing instrument are: calibration of standard weight; drift of standard weight; comparator linearity; repeatability; buoyancy correction; drift of standard; indicator resolution; temperature sensitivity; eccentricity and other influencing factors.

Uncertainty of Temperature Measurement, $U(T)$

The combined expanded uncertainty of the temperature measurement has been estimated as $U(T) = \pm 0.13^\circ\text{C}$, assuming rectangular distribution. Scale resolution is 0.1°C and it is expected that any instruments under the same conditions will differ at any time by greater than $\pm 0.1^\circ\text{C}$. All temperature indicators and probes are checked pre and post calibration by a 2 point spot check to ensure no drift in temperature measurement between instruments. The uncertainty is regarded as a constant value throughout the measuring range. Temperature calibration uncertainty has been deemed as negligible due to the covariance that exists between the temperature measurement instruments calibrated from the same reference.

Uncertainty of Pressure Measurement, $U(P)$

The combined expanded uncertainty of the pressure measurement has been estimated as $U(P) = \pm 0.13$ barg, assuming rectangular distribution. Scale resolution is 0.1 barg and it is expected that any instruments under the same conditions will not differ at any time by greater than ± 0.1 barg. The expanded uncertainty is regarded as a constant value throughout the pressure measuring range. Pressure calibration uncertainty has been deemed as negligible due to the covariance that exists between pressure measurement instruments calibrated from the same reference.

Uncertainty of the air buoyancy correction, $U(BC)$

The combined uncertainty within the buoyancy correction $U(BC)$ is calculated from the air buoyancy correction functional model by applying uncertainty estimates to the individual input values. The uncertainty components associated with buoyancy correction are: uncertainty in the density of air $U(\rho_a)$ calculated using the CIPM air density formula [21]; uncertainty in the density of reference weights $U(\rho_{sm})$; uncertainty in the density of the water sample $U(\rho_w)$. Uncertainties are given below:

Uncertainty of sample density measurement, $U(\rho_w)$

The expanded uncertainty in the sample density (water) is $U(\rho_w) = \pm 0.0468$ kg/m³ [k=2]. The uncertainty is evaluated separately for water density measurement by high precision density meter.

Uncertainty of air density measurement, $U(\rho_a)$

The expanded uncertainty in the air density measurement $U(\rho_a)$ is the combined uncertainty of the laboratory air density variation and the air density calculation uncertainty given in section C.6.3.6 of the *OIML R 111-1* [19]. The sensitivity coefficients are provided as partial derivatives for the uncertainty components for air humidity $U(H)$, air temperature $U(T)$ and air pressure $U(B)$ within the CIPM air density formula [21], although further uncertainty for draught, electrical interference and temperature variations must be considered. $U(\rho_a) = \pm 0.0031$ kg/m³.

Uncertainty of reference weights density, $U(\rho_{sm})$

The expanded uncertainty of the reference weights density of $U(\rho_{sm})$ should be known from the calibration certificate $U(\rho_{sm}) = \pm 600$ kg/m³. Standard weights density limits are specified within *OIML R 111-1* [19].

Uncertainty of Water Compressibility, $U(F_w)$

The expanded uncertainty of the water compressibility is stated by Kell [22] as being $U(F_w) = \pm 0.0003 \times 10^{-6} \text{ bar}^{-1}$, assuming rectangular distribution.

Uncertainty of Steel Area Expansion Coefficient, $U(G_{cp})$

The expanded uncertainty of the area coefficient of thermal expansion for steel is set as a default value of $U(G_{cp}) = \pm 10\%$, assuming a rectangular distribution.

Uncertainty of Invar Linear Expansion Coefficient, $U(GI)$

The expanded uncertainty for the Invar linear thermal expansion coefficient is set as a default value of $U(GI) = \pm 10\%$, assuming a rectangular distribution.

Uncertainty of Young's Modulus of Elasticity, $U(Ecp)$

The expanded uncertainty for the Young's Modulus of steel is set as a default value of $U(Ecp) = \pm 10\%$, assuming rectangular distribution.

Uncertainty of Internal Diameter Measurement, $U(IDcp)$

The expanded uncertainty of internal diameter measurement of the flow tube calibrated section is $U(IDcp) = \pm 1mm$, assuming rectangular distribution.

Uncertainty of Wall Thickness Measurement, $U(WTcp)$

The expanded uncertainty of flow tube wall thickness measurement is $U(WTcp) = \pm 1mm$, assuming rectangular distribution.

Repeatability of Calibration Process, $U(R)$

For five successive calibration runs the maximum acceptable repeatability must be within a band of 0.02%. $U(R) = \pm 0.01\%$ (0.02%), assuming rectangular distribution.

Uncertainty of Optical Switching Unit, $U(SR)$

Filling of the container is controlled by two optical detectors and a solenoid valve. Both switches are repeatable to within $\pm 0.013mm$. The uncertainty is proportional to the volume of a cylinder, $U(SR) = \pm 0.0014$ litres, assuming rectangular distribution.

Uncertainty of C_{tdw} Correction, $U(C_{tdw})$

The combined uncertainty $U(C_{tdw})$ is calculated from functional model $C_{tdw} = \rho_1 / \rho_2$, correction factor for the thermal expansion of water. Uncertainty estimates are applied to each of the individual input values. The density of water in the container $U(\rho_1)$ and the density of water in the compact prover $U(\rho_2)$ are both calculated using the Tanaka [23] equation, where the expanded uncertainty of the water density calculation is given as ± 0.00084 kg/m³. The terms in the C_{tdw} correction are correlated; therefore the covariance must be evaluated. Since the models present in the C_{tdw} correction are equal and the values of the input quantities are almost equal in magnitude and uncertainty, the correlation coefficient $r(\rho_1, \rho_2)$ may be considered to be 1.

Uncertainty of C_{tsp} Correction, $U(C_{tsp})$

The combined uncertainty $U(C_{tsp})$ is calculated from the functional model, C_{tsp} correction for the effect of temperature on prover steel, by applying uncertainty estimates to the individual input values. Since the C_{tsp} functional relationship is well known it may be assumed that the model uncertainty is confined within the uncertainty of the thermal expansion coefficients, therefore no model uncertainty is applied for the C_{tsp} correction factor.

Uncertainty of C_{psp} Correction, $U(C_{psp})$

The combined uncertainty $U(C_{psp})$ is calculated from the functional model, C_{psp} correction for the effect of pressure on prover steel, by applying uncertainty estimates to the individual input values. Since the C_{psp} functional relationship is well known it may be assumed that the model uncertainty is confined within the uncertainty of the modulus of elasticity, therefore no model uncertainty is applied for the C_{psp} correction factor.

Uncertainty of C_{plp} Correction, $U(C_{plp})$

The combined uncertainty $U(C_{plp})$ is calculated from the functional model, C_{plp} correction for the effect of pressure on the liquid at the prover, by applying uncertainty estimates to the individual input

values. Since the C_{plp} functional relationship is well known it may be assumed that the model uncertainty is confined within the uncertainty of water compressibility, therefore no model uncertainty is applied for the C_{plp} correction factor.

Uncertainty Components	Value	Expanded Uncertainty ±	Probability Distribution	Divisor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient C_i	Uncertainty Contribution $[c_i u(x_i)]^2$	Covariance	Weighting in Uncertainty
U(M)	60.000 kg	0.002 kg	normal	2	0.001	1.00174E+00	1.00348E-06		6.15974%
U(T1)	20.00 °C	0.13 °C	rectangular	1.73	0.075	1.15662E-05	7.53610E-13		0.00000%
U(T2)	16.00 °C	0.13 °C	rectangular	1.73	0.075	8.55590E-03	4.12380E-07		2.53136%
U(T3)	16.00 °C	0.13 °C	rectangular	1.73	0.075	-8.65478E-05	4.21966E-11		0.00026%
U(P1)	5.0 barg	0.1 barg	rectangular	1.73	0.075	-2.35977E-04	3.13692E-10		0.00193%
U(T4)	18.00 °C	0.13 °C	rectangular	1.73	0.075	-3.22524E-03	5.85990E-08		0.35970%
U(ρ1) Ctdw	998.946 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	6.01676E-02	6.38593E-10	-6.38592E-10	0.00000%
U(ρ2) Ctdw	998.946 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	-6.01676E-02	6.38592E-10	-6.38592E-10	0.00000%
U(ρw)	998.946 kg/m ³	0.04681 kg/m ³	normal	2	0.02340	-6.02372E-02	1.98747E-06		12.19992%
U(B)	996.80 mbar	0.15 mbar	normal	2	0.07500	6.26926E-05	2.21083E-11		0.00014%
U(H)	69 %	20 %	rectangular	1.73	11.54701	-5.51041E-06	4.04862E-09		0.02485%
U(ρsm)	8000 kg/m ³	600.00 kg/m ³	normal	2	300.00000	1.06617E-06	1.02305E-07		0.62799%
U(ρa)	1.18 kg/m ³	0.0031 kg/m ³	normal	2	0.00155	5.27211E-02	6.65078E-09		0.04083%
U(Fw)	4.6547E-05	3.0E-10	rectangular	1.73	1.7E-10	-3.00451E+02	2.70812E-15		0.00000%
U(Gcp)	0.00002160 °C-1	0.00000216 °C-1	rectangular	1.73	0.00000125	-6.01025E+01	5.61787E-09		0.03448%
U(Gl)	0.000001440 °C-1	0.000000144 °C-1	rectangular	1.73	0.0000000831	-1.80308E+02	2.24715E-10		0.00138%
U(Ecp)	1965006 bar	196501 bar	rectangular	1.73	113450	1.03011E-09	1.36576E-08		0.08384%
U(IDcp)	311.150 mm	1 mm	rectangular	1.73	0.58	-6.88103E-06	1.57829E-11		0.00010%
U(WTcp)	22.225 mm	1 mm	rectangular	1.73	0.58	9.38954E-05	2.93878E-09		0.01804%
U(R)	0.02%	0.00601 lt	rectangular	1.73	0.00347	1.00000E+00	1.20417E-05		73.91692%
U(SR)	0.0023 %	0.00140 lt	rectangular	1.73	0.00081	1.00000E+00	6.51395E-07		3.99853%
							$\sum [c_i u(x_i)]^2$	1.62909E-05	100.00000%
Combined Standard Uncertainty						$\sqrt{\sum [c_i u(x_i)]^2}$	0.0040	litres	
Expanded Uncertainty [k=2]							0.0080	litres	
Nominal Volume							60.1042	litres	
Relative Expanded Uncertainty [k=2]							0.0134	%	

Table 1. Compact Prover Volume Uncertainty (Gravimetric Water Draw Method)

Uncertainty of Compact Prover Volume, $U(V_{bm-grav})$

Table 1 shows the result of the compact prover gravimetric water draw volume calibration uncertainty evaluation. The relative expanded uncertainty $U(V_{bm-grav}) = \pm 0.0134\%$.

5.2 Volumetric Water Draw Compact Prover Volume Calibration

The procedure for gravimetric water draw calibration is to displace the water volume between detectors from the compact prover at a controlled flowrate into volumetric prover.

Mathematical Model

The main components of the mathematical model are shown below; all other equations are given in the Appendix.

Volumetric Water Draw Calibration:

$$V_b = \frac{V_R \times C_{tdw} \times C_{tst}}{C_{tsp} \times C_{psp} \times C_{plp}}$$

- Where:
- V_b Compact prover base volume at 15°C and zero barg [14, 15, 16]
 - V_R Compact prover indicated volume at observed conditions
 - Ctdw Correction factor for the thermal expansion of water
 - Ctst Correction factor for the thermal expansion of proving tank metal
 - Ctsp Correction factor for the effect of temperature on the steel of the prover
 - Cpsp Correction factor for the effect of pressure on the steel of the prover
 - Cplp Correction factor for the effect of pressure on liquid at the prover

Unless stated below, all uncertainty estimates are identical to those in section 5.1.

Uncertainty of Proving Tank Volume, $U(Vm)$

The expanded uncertainty in the proving tank volume is taken from the certificate of calibration from a regional measurement laboratory, $U(Vm) = \pm 0.01\%$ [$k=2$].

Uncertainty of Volume Expansion Coefficient, $U(Gm)$

The expanded uncertainty of the coefficient of volume thermal expansion for proving tank metal is set as a default value of $U(Gm) = \pm 10\%$, assuming a rectangular distribution.

Uncertainty in Scale Reading, $U(RD)$

The expanded uncertainty of volume reading is calculated from the scale reading resolution which has a neck gauge with 1 millilitre increments. $U(RD) = \pm 1ml$, assuming a rectangular distribution.

Uncertainty in Wetting Variance, $U(W)$

The wetting and drip variance is influenced by the liquid properties, construction of the proving tank, drip time and method. The expanded uncertainty of the wetting variance of the proving tank metal is $U(W) = \pm 0.001\%$, assuming a rectangular distribution.

Uncertainty of C_{tsm} Correction, $U(C_{tsm})$

The combined uncertainty $U(C_{tsm})$ is calculated from the functional model, C_{cts} correction for the effect of temperature on volumetric prover steel, by applying uncertainty estimates to the individual input values. As the C_{tsm} functional relationship is well known it may be assumed that the model uncertainty is confined within the uncertainty of the modulus of elasticity, therefore no model uncertainty is applied for the C_{tsm} correction factor.

Uncertainty Components	Value	Expanded Uncertainty \pm	Probability Distribution	Divisor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient C_i	Uncertainty Contribution $[c_i u(x_i)]^2$	Covariance	Weighting in Uncertainty
U(Vm)	60.000 lt	0.006 lt	normal	2	0.003	1.00000E+00	9.00000E-06		39.4569%
U(T1)	16.00 °C	0.12 °C	rectangular	1.73	0.071	8.53757E-03	3.67585E-07		1.6115%
U(T2)	18.00 °C	0.12 °C	rectangular	1.73	0.071	-8.63765E-05	3.76253E-11		0.0002%
U(T3)	16.00 °C	0.12 °C	rectangular	1.73	0.071	-6.91779E-03	2.41337E-07		1.0580%
U(P1)	5.00 barg	0.12 barg	rectangular	1.73	0.071	-3.21886E-03	5.22508E-08		0.2291%
U(Fw)	4.65E-05	1.23E-01	rectangular	1.73	1.732E-10	-2.99856E+02	2.69741E-15		0.0000%
U(Gcp)	0.00002160 °C-1	0.00000216 °C-1	rectangular	1.73	1.247E-06	-5.99836E+01	5.59566E-09		0.0245%
U(Gl)	0.00000144 °C-1	0.00000014 °C-1	rectangular	1.73	8.314E-08	-1.79951E+02	2.23827E-10		0.0010%
U(Gm)	0.00004770 °C-1	0.00000477 °C-1	rectangular	1.73	2.754E-06	5.99823E+01	2.72874E-08		0.1196%
U(Ecp)	1965006 bar	196501 bar	rectangular	1.73	113449.7	1.02807E-09	1.36036E-08		0.0596%
U(p1) Ctdw	998.94594 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	6.00485E-02	6.36067E-10	-6.3607E-10	0.0028%
U(p2) Ctdw	998.94594 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	-6.00485E-02	6.36067E-10	-6.3607E-10	0.0028%
U(IDcp)	311 mm	1 mm	rectangular	1.73	0.577	-6.86741E-06	1.57204E-11		0.0001%
U(WTcp)	22 mm	1 mm	rectangular	1.73	0.577	9.37095E-05	2.92716E-09		0.0128%
U(R)	0.02%	0.00600 lt	rectangular	1.73	0.00346	1.00000E+00	1.19941E-05		52.5833%
U(SR)	0.0023 %	0.00140 lt	rectangular	1.73	0.00081	1.00000E+00	6.51410E-07		2.8558%
U(RD)	0.001 lt	0.00100 lt	rectangular	1.73	0.00058	1.00000E+00	3.33333E-07		1.4614%
U(W)	0.001%	0.00060 lt	rectangular	1.73	0.00035	1.00000E+00	1.20000E-07		0.5261%
							$\sum [c_i u(x_i)]^2$	2.28097E-05	100.0056%
Combined Standard Uncertainty							$\sqrt{\sum [c_i u(x_i)]^2}$	0.0048 litres	
Expanded Uncertainty [k=2]								0.0096 litres	
Nominal Volume								59.9852 litres	
Relative Expanded Uncertainty [k=2]								0.0159 %	

Table 2. Compact Prover Volume Uncertainty (Volumetric Water Draw Method)

Uncertainty of Compact Prover Volume, $U(V_{bm-vol})$

Table 2 shows the result of the compact prover volumetric water draw calibration uncertainty evaluation. The relative expanded uncertainty $U(V_{bm-vol}) = \pm 0.0159\%$.

It is worth noting that any comparison between the uncertainty of the volumetric and gravimetric water-draw methods must be linked by traceability chain to the same mass reference. In the previous example, the volumetric uncertainty of the standard measure is $\pm 0.01\%$ [$k=2$] as determined by a regional measurement laboratory; therefore no comparison of uncertainty is undertaken.

5.3 Pipe Prover Volume Calibration on Water

The procedure for pipe prover calibration by master meter/master prover method is to displace a known water volume between detectors of the compact prover at a controlled flowrate and calibrate the master meter K-factor whilst simultaneously displacing the volume between the pipe prover detectors. Using the K-factor obtained for the master meter, the number of pulses counted between detectors on the pipe prover calibrated section can be converted to volume.

Uncertainty due to non-linearity and repeatability on master meter is negligible since the meter and the pipe prover are calibrated consecutively at the same stable conditions.

In the example a 60 litre compact prover and turbine master meter are used to calibrate a 1776 litre pipe prover with water as the calibration fluid. The master meter generates 23,137 meter pulses per pass and the temperature of the water is 14°C at pressure of 7 barg within the system.

Mathematical Model

The main components of the mathematical model are shown below; all other equations are given in the Appendix.

$$\text{Pipe Prover (by Master Meter):} \quad V_b = \left(\frac{N_2 \times C_{tlm} \times C_{plm}}{K_m \times C_{tsp} \times C_{psp} \times C_{tlp} \times C_{plp}} \right)$$

$$\text{Master Meter (by Compact Prover):} \quad K_m = \left(\frac{N_1 \times C_{tlm} \times C_{plm}}{V_{bm} \times C_{tscp} \times C_{pscp} \times C_{tlcp} \times C_{plcp}} \right)$$

Where:	V_b	Pipe prover base volume at 15°C and 0 barg [15, 16, 24]
	V_{bm}	Master pipe prover base volume at 15°C and 0 barg
	K_m	K-factor at master meter
	N_1	Meter pulses counted during meter K-factor calibration
	N_2	Meter pulses counted during pipe prover calibration

Unless stated below, all uncertainty estimates are identical to those in section 5.1.

Uncertainty of Compact Prover Volume, $U(V_{bm})$

The uncertainty is taken from the result of the compact prover gravimetric water draw volume calibration uncertainty evaluation in section 5.1. The relative expanded uncertainty $U(V_{bm-grav}) = \pm 0.0134\%$ [$k=2$].

Uncertainty of Water Density, $U(\rho_{15})$

The expanded uncertainty of the water standard density is taken as, $U(\rho_{15}) \pm 5.0 \text{ kg/m}^3$. It is expected that any density measurement over the 5 passes will not differ at any time by greater than $\pm 5.0 \text{ kg/m}^3$, assuming rectangular distribution.

Uncertainty of C_{tl} Correction, $U(C_{tl})$

The combined uncertainty $U(C_{tl})$ is calculated from the functional model $C_{tl} = \rho_t/\rho_{15}$, correction factor for the effect of temperature on the liquid, by applying uncertainty estimates to the individual input values. As the C_{tl} functional relationship is well known it may be assumed that the model uncertainty is confined within the uncertainty of water density calculation by Tanaka [23], therefore no model

uncertainty is applied for the C_{plp} correction factor. The expanded uncertainty of the water density calculation is given as $\pm 0.00084 \text{ kg/m}^3$.

The C_{tl} correction terms in both stages of the calibration (K-factor and pipe prover volume) are correlated; therefore the covariance must be evaluated. As the models present in the C_{tl} correction are equal and the values of the input quantities are almost equal in magnitude and uncertainty, the correlation coefficient $r(C_{tlcp}, C_{tlm})$ and $r(C_{tlm}, C_{tlp})$ may be considered to be 1.

Uncertainty Components	Value	Expanded Uncertainty ±	Probability Distribution	Divisor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient C_i	Uncertainty Contribution $[c_{iu}(x_i)]^2$	Covariance	Weighting in Uncertainty
U(Vbm)	60.10418 lt	0.00402 lt	normal	1	0.00402	3.1220E+01	1.5779E-02		55.54403%
U(F)	4.683E-05	3E-10	rectangular	1.73	1.73E-10	-1.7775E+03	9.4787E-14		0.00000%
U(lDcp)	311.15 mm	1 mm	rectangular	1.73	0.577	2.8470E-04	2.7017E-08		0.00010%
U(WTcp)	22.23 mm	1 mm	rectangular	1.73	0.577	-3.8848E-03	5.0306E-06		0.01771%
U(Ecp)	1965000 bar	196500 bar	rectangular	1.73	113449	-4.2620E-08	2.3379E-05		0.08230%
U(Gcp)	0.00002160 °C-1	0.00000216 °C-1	rectangular	1.73	1.25E-06	-8.8814E+02	1.2267E-06		0.00432%
U(Gl)	0.00000144 °C-1	0.00000014 °C-1	rectangular	1.73	8.31E-08	3.5526E+03	8.7235E-08		0.00031%
U(ρ_{15})	999.10257 kg/m ³	5 kg/m ³	rectangular	1.73205	2.89	1.5753E-13	2.0680E-25		0.00000%
U(pt1) Ctlcp, K	999.17648 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	1.2219E+04	2.6336E+01	-26.25997641	0.77706%
U(pt3) Ctlm, K	999.19091 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	-1.2183E+04	2.6184E+01	-26.25997641	
U(pt4) Ctlp, PP	999.16194 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	-1.2184E+04	2.6185E+01	-26.26035647	0.75401%
U(pt3) Ctlm, PP	999.19091 kg/m ³	0.00084 kg/m ³	normal	2	0.00042	1.2219E+04	2.6335E+01	-26.26035647	
U(Epp)	2060000 bar	206000 bar	rectangular	1.73	118934	1.1599E-07	1.9030E-04		0.66988%
U(lDpp)	307.14 mm	1 mm	rectangular	1.73	0.577	-8.2285E-04	2.2570E-07		0.00079%
U(WTpp)	8.38 mm	1 mm	rectangular	1.73	0.577	2.8209E-02	2.6524E-04		0.93368%
U(Gpp)	0.00003500 °C-1	0.00000350 °C-1	rectangular	1.73	2.021E-06	7.1052E+02	2.0614E-06		0.00726%
U(T1)	14.50 °C	0.10 °C	rectangular	1.73	0.0577	-2.2216E-01	1.6452E-04		0.57913%
U(T2)	17.00 °C	0.10 °C	rectangular	1.73	0.0577	2.5579E-03	2.1809E-08		0.00008%
U(T3)	14.40 °C	0.10 °C	rectangular	1.73	0.0577	2.5807E-01	2.2200E-04		0.78146%
U(P1)	7.0 barg	0.1 barg	rectangular	1.73	0.0577	9.5864E-02	3.0633E-05		0.10783%
U(P2)	6.0 barg	0.1 barg	rectangular	1.73	0.0577	-8.3240E-02	2.3096E-05		0.08130%
U(T4)	14.60 °C	0.10 °C	rectangular	1.73	0.0577	2.0082E-01	1.3443E-04		0.47319%
U(P3)	8.0 barg	0.1 barg	rectangular	1.73	0.0577	-1.1477E-01	4.3907E-05		0.15456%
U(R)	0.02%	0.17763 lt	rectangular	1.73	0.10255	1.0000E+00	1.0517E-02		37.02135%
U(SR)	0.0023 %	0.04139 lt	rectangular	1.73	0.02389	1.0000E+00	5.7091E-04		2.00967%
							$\sum [c_{iu}(x_i)]^2$	2.84083E-02	100.00000%
Combined Standard Uncertainty						$\sqrt{\sum [c_{iu}(x_i)]^2}$	0.1685 litres		
Expanded Uncertainty [k=2]							0.3371 litres		
Nominal Volume							1776.2724 litres		
Relative Expanded Uncertainty [k=2]							0.0190 %		

Table 3. Pipe Prover Volume Uncertainty on Water

Uncertainty of Compact Prover Volume, $U(V_{b-water})$

Table 3 shows the result of the water calibration pipe prover volume uncertainty evaluation. The relative expanded uncertainty $U(V_{b-w}) = \pm 0.0190\% [k=2]$.

5.4 Pipe Prover Volume Calibration on Hydrocarbon Fluids

The procedure is identical to the method described in section 5.3 with the exception that hydrocarbon fluid is used as the calibration medium.

Uncertainty due to non-linearity and repeatability on the master meter is negligible since the meter and the pipe prover are calibrated consecutively at the same stable conditions.

In the example a 60 litre compact prover and turbine master meter are used to calibrate a 7051 litre pipe prover with hydrocarbon fluid as the calibration fluid. The master meter generates 105,338 meter pulses per pass and the temperature of the water is 18°C at pressure of 9 barg within the system.

Mathematical Model

The main components of the mathematical model are shown below; all other equations are given in the Appendix.

Pipe Prover (by Master Meter):

$$V_b = \left(\frac{N_2 \times C_{tlm} \times C_{plm}}{K_m \times C_{tsp} \times C_{psp} \times C_{tlp} \times C_{plp}} \right)$$

Master Meter (by Compact Prover):
$$K_m = \left(\frac{N_1 \times CTL_m \times CPL_m}{V_{bm} \times Ct_{scp} \times Cp_{scp} \times Ctl_{cp} \times Cpl_{cp}} \right)$$

Where: V_b Pipe prover base volume at 15°C and 0 barg [15, 16, 24]
 V_{bm} Master pipe prover base volume at 15°C and 0 barg
 K_m K-factor at master meter
 N_1 Meter pulses counted during meter K-factor calibration
 N_2 Meter pulses counted during pipe prover calibration

Unless stated below, all uncertainty estimates are identical to those in section 5.1.

Uncertainty of Hydrocarbon Fluid Density, $U(\rho_{15})$

The expanded uncertainty of the hydrocarbon fluid standard density is taken as, $U(\rho_{15}) \pm 10.0 \text{ kg/m}^3$. It is expected that any density measurement over the 5 passes will not differ at any time by greater than $\pm 10.0 \text{ kg/m}^3$, assuming rectangular distribution.

Uncertainty of C_{pl} Correction, $U(C_{pl})$

The combined uncertainty $U(C_{pl})$ is taken from API MPMS 11.2.1M [25]. The Ctl model relative expanded uncertainty in volume is $\pm 0.03\%$ [k=2] up to 34.5 bar.

The C_{il} correction terms in both stages of the calibration (K-factor and pipe prover volume) are correlated; therefore the covariance must be evaluated. As the models present in the C_{il} correction are equal and the values of the input quantities are almost equal in magnitude and uncertainty, the correlation coefficient $r(C_{tlcp}, C_{ilm})$ and $r(C_{ilm}, C_{tlp})$ may be considered to be 1.

Uncertainty of C_{il} Correction, $U(C_{il})$

The combined uncertainty $U(C_{il})$ is taken from API MPMS 11.1 [26]. The Ctl model uncertainty in volume is $\pm 0.15\%$ [k=2] up to 65°C.

The C_{pl} correction terms in both stages of the calibration (K-factor and pipe prover volume) are correlated; therefore the covariance must be evaluated. As the models present in the C_{il} correction are equal and the values of the input quantities are almost equal in magnitude and uncertainty, the correlation coefficient $r(C_{plcp}, C_{plm})$ and $r(C_{plm}, C_{plp})$ may be considered to be 1.

Uncertainty Components	Value	Expanded Uncertainty ±	Probability Distribution	Divisor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient C_i	Uncertainty Contribution $[C_i u(x_i)]^2$	Covariance	Weighting in Uncertainty	
U(Vbm)	58.98831 lt	0.00402 lt	normal	1	0.00402	1.1954E+02	2.3134E-01		21.21343%	
U(IDcp)	742.95 mm	1 mm	rectangular	1.73	0.577	1.4370E-03	6.8828E-07		0.00006%	
U(WTcp)	9.53 mm	1 mm	rectangular	1.73	0.577	-1.9608E-02	1.2816E-04		0.01175%	
U(Ecp)	1965000 bar	196500 bar	rectangular	1.73	113449	-2.1512E-07	5.9560E-04		0.05462%	
U(Gcp)	0.00002160 °C-1	0.00000216 °C-1	rectangular	1.73	0.000001247	2.5384E+04	1.0021E-03		0.09189%	
U(Gl)	0.00000144 °C-1	0.00000014 °C-1	rectangular	1.73	0.000000083	2.4678E+04	4.2096E-06		0.00039%	
U(ρ15)	812.3 kg/m ³	10 kg/m ³	normal	2	5	5.7056E-03	8.1384E-04		0.07463%	
U(Epp)	2060000 bar	206000 bar	rectangular	1.73	118934	1.3351E-06	2.5215E-02		2.31222%	
U(IDpp)	742.95 mm	1 mm	rectangular	1.73	0.577	-3.9156E-03	5.1107E-06		0.00047%	
U(WTpp)	9.53 mm	1 mm	rectangular	1.73	0.577	2.8797E-01	2.7642E-02		2.53477%	
U(Gpp)	0.00003350 °C-1	0.00000335 °C-1	rectangular	1.73	1.934E-06	-2.4677E+04	2.2781E-03		0.20890%	
U(Ctlcp), K	0.996646862	0.00149497	normal	2	7.4749E-04	7.0753E+03	2.7970E+01	-27.94919556		
U(Ctlm), K	0.996646862	0.00149497	normal	2	7.4749E-04	-7.0700E+03	2.7928E+01	-27.94919556	0.00144%	
U(Ctlp), PP	0.996740094	0.00149511	normal	2	7.4756E-04	-7.0693E+03	2.7928E+01	-27.94919556		
U(Ctlm), PP	0.996646862	0.00149497	normal	2	7.4749E-04	7.0753E+03	2.7970E+01	-27.94919556	0.00144%	
U(Cplcp), K	1.000745317	0.000300224	normal	2	1.5011E-04	7.0463E+03	1.1188E+00	-1.118638502		
U(Cplm), K	1.000653139	0.000300196	normal	2	1.5010E-04	-7.0459E+03	1.1185E+00	-1.118638502		
U(Cplp), PP	1.000912354	0.000300274	normal	2	1.5014E-04	-7.0441E+03	1.1185E+00	-1.118638502		
U(Cplm), PP	1.000653139	0.000300196	normal	2	1.5010E-04	7.0470E+03	1.1188E+00	-1.118638502	0.00000%	
U(T1)	18.60 °C	0.12 °C	rectangular	1.73	0.071	-6.4098E+00	2.0720E-01		18.99953%	
U(T2)	18.50 °C	0.12 °C	rectangular	1.73	0.071	1.0153E-02	5.1989E-07		0.00005%	
U(T3)	18.60 °C	0.12 °C	rectangular	1.73	0.071	6.5668E+00	2.1747E-01		19.94174%	
U(P1)	8.9 barg	0.12 barg	rectangular	1.73	0.071	6.4076E-01	2.0705E-03		0.18987%	
U(P2)	7.8 barg	0.12 barg	rectangular	1.73	0.071	-5.9047E-01	1.7583E-03		0.16123%	
U(T4)	18.50 °C	0.12 °C	rectangular	1.73	0.071	6.3176E+00	2.0128E-01		18.45665%	
U(P3)	10.9 barg	0.12 barg	rectangular	1.73	0.071	-8.5712E-01	3.7049E-03		0.33973%	
U(R)	0.02%	0.70516 lt	rectangular	1.73	0.40712	1.0000E+00	1.6575E-01		15.19894%	
U(SR)	0.0023 %	0.08215 lt	rectangular	1.73	0.04743	1.0000E+00	2.2494E-03		0.20627%	
							$\sum [C_i u(x_i)]^2$	1.09053E+00		100.00000%
Combined Standard Uncertainty					$\sqrt{\sum [C_i u(x_i)]^2}$		1.0443	litres		
Expanded Uncertainty [k=2]							2.0886	litres		
Nominal Volume							7051.5760	litres		
Relative Expanded Uncertainty [k=2]							0.0296	%		

Table 4. Pipe Prover Volume Uncertainty on Hydrocarbon Fluid

Uncertainty of Compact Prover Volume, $U(V_{b-hc})$

Table 4 shows the result of the hydrocarbon fluid calibration pipe prover volume uncertainty evaluation. The relative expanded uncertainty $U(V_{b-hc}) = \pm 0.0296\% [k=2]$.

Uncertainty Components	Value	Expanded Uncertainty ±	Probability Distribution	Divisor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient C_i	Uncertainty Contribution $[c_i u(x_i)]^2$	Covariance	Weighting in Uncertainty
U(Vbm)	58.98831 lt	0.00402 lt	normal	1	0.00402	1.1948E+02	2.3110E-01		11.84727%
U(IDcp)	742.95 mm	1 mm	rectangular	1.73	0.577	2.8992E-03	2.8019E-06		0.00014%
U(WTtcp)	9.53 mm	1 mm	rectangular	1.73	0.577	-3.5632E-02	4.2322E-04		0.02170%
U(Ecp)	1965000 bar	196500 bar	rectangular	1.73	113449	-4.3403E-07	2.4246E-03		0.12429%
U(Gcp)	0.00002160 °C-1	0.00000216 °C-1	rectangular	1.73	0.000001247	3.5199E+05	1.9269E-01		9.87798%
U(GI)	0.00000144 °C-1	0.00000014 °C-1	rectangular	1.73	0.000000083	3.5199E+05	8.5640E-04		0.04390%
U(ρ 15)	812.3 kg/m ³	10 kg/m ³	normal	2	5	-1.8190E-13	8.2718E-25		0.00000%
U(Epp)	2060000 bar	206000 bar	rectangular	1.73	118934	2.4478E-06	8.4752E-02		4.34470%
U(IDpp)	742.95 mm	1 mm	rectangular	1.73	0.577	-7.1785E-03	1.7177E-05		0.00088%
U(WTpp)	9.53 mm	1 mm	rectangular	1.73	0.577	5.2795E-01	9.2910E-02		4.76289%
U(Gpp)	0.00003350 °C-1	0.00000335 °C-1	rectangular	1.73	1.934E-06	-3.5178E+05	4.6292E-01		23.73082%
U(Ctlcp), K	0.952889245	0.001429334	normal	2	7.1467E-04	7.3965E+03	2.7942E+01	-27.92086152	
U(Ctlm), K	0.952889245	0.001429334	normal	2	7.1467E-04	-7.3909E+03	2.7900E+01	-27.92086152	0.00080%
U(Ctlp), PP	0.952889245	0.001429334	normal	2	7.1467E-04	-7.3909E+03	2.7900E+01	-27.92086152	
U(Ctlm), PP	0.952889245	0.001429334	normal	2	7.1467E-04	7.3965E+03	2.7942E+01	-27.92086152	0.00080%
U(Cplcp), K	1.002277958	0.000300683	normal	2	1.5034E-04	7.0320E+03	1.1177E+00	-1.117504461	
U(Cplm), K	1.002277958	0.000300683	normal	2	1.5034E-04	-7.0309E+03	1.1173E+00	-1.117504461	
U(Cplp), PP	1.002277958	0.000300683	normal	2	1.5034E-04	-7.0309E+03	1.1173E+00	-1.117504461	0.00000%
U(Cplm), PP	1.002277958	0.000300683	normal	2	1.5034E-04	7.0320E+03	1.1177E+00	-1.117504461	
U(T1)	65.00 °C	0.12 °C	rectangular	1.73	0.071	-6.7885E+00	2.3240E-01		11.91366%
U(T2)	65.00 °C	0.12 °C	rectangular	1.73	0.071	1.0137E-02	5.1826E-07		0.00003%
U(T3)	65.00 °C	0.12 °C	rectangular	1.73	0.071	6.9410E+00	2.4296E-01		12.45507%
U(P1)	20.0 barg	0.12 barg	rectangular	1.73	0.071	8.4786E-01	3.6253E-03		0.18585%
U(P2)	20.0 barg	0.12 barg	rectangular	1.73	0.071	-8.0275E-01	3.2498E-03		0.16660%
U(T4)	65.00 °C	0.12 °C	rectangular	1.73	0.071	6.7053E+00	2.2674E-01		11.62345%
U(P3)	20.0 barg	0.12 barg	rectangular	1.73	0.071	-1.0694E+00	5.7674E-03		0.29566%
U(R)	0.02%	0.70480 lt	rectangular	1.73	0.40692	1.0000E+00	1.6558E-01		8.48830%
U(SR)	0.0023 %	0.08211 lt	rectangular	1.73	0.04740	1.0000E+00	2.2471E-03		0.11520%
							$\sum [c_i u(x_i)]^2$	1.95070E+00	100.00000%
Combined Standard Uncertainty					$\sqrt{\sum [c_i u(x_i)]^2}$		1.3967	litres	
Expanded Uncertainty [k=2]							2.7933	litres	
Nominal Volume							7048.0008	litres	
Relative Expanded Uncertainty [k=2]							0.0396	%	

Table 5. Pipe Prover Volume Uncertainty on Hydrocarbon Fluid (High Temperature)

Uncertainty of Compact Prover Volume, $U(V_{b-hc.ht})$

Table 5 shows the result of the hydrocarbon fluid calibration pipe prover volume uncertainty evaluation. The relative expanded uncertainty $U(V_{b-hc.ht}) = \pm 0.0396\% [k=2]$, at Temperature 65°C.

When a higher temperature of hydrocarbon fluids is observed during calibration the sensitivity of the cubical thermal expansion coefficient of steel at the pipe prover increases significantly. Pressure has also been increased in the example although the pressure sensitivities are not significantly increased. The increase in area expansion coefficient of steel at the pipe prover highlights that the uncertainty must reduce to maintain an uncertainty in the order of $\pm 0.03\%$ of volume.

6. HISTORIC CALIBRATION RECORDS

The results of 198 pipe prover calibrations from 14 locations have been used to compile the histogram shown in Figure 5. From the calibration results two standard deviations are calculated as $\pm 0.042\%$ of volume. The calibration data compiled in the histogram includes all results for both oil and water calibrations. It should also be noted that there has been no filtering of the data to remove volume shifts due to switch changes or equipment failures which may be related to the larger shifts in volume.

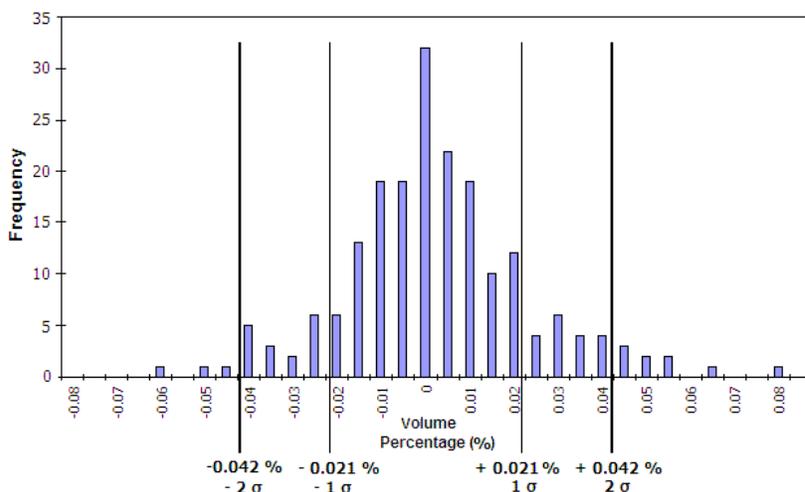


Figure 5. Pipe Prover Volume Calibration Distribution

7. RESULTS

7.1 Compact Prover Volume Calibration

Gravimetric water-draw method: $U(V_{bm-grav}) = \pm 0.013 \%$.

Volumetric water-draw method: $U(V_{bm-vol}) = \pm 0.016 \%$.

No comparison of uncertainty can be undertaken as the volumetric and gravimetric water-draw uncertainty evaluations in this document are not linked by traceability chain to the same mass reference. The volumetric uncertainty of the standard measure is $\pm 0.01\%$ [$k=2$] as determined by a regional measurement laboratory.

The gravimetric and volumetric water-draw methods both have their advantages and disadvantages. The significant difference in the methods is that the gravimetric method is mainly confined to the laboratory whereas the volumetric method can be applied on site. The gravimetric calibration eliminates the possibility of drift by allowing continual calibration of the compact prover. Also, the gravimetric method reduces the number of steps in the traceability chain, therefore having a slightly better uncertainty.

7.2 Pipe Prover Calibration

Pipe Prover Calibration with Water: $U(V_{b-w}) = \pm 0.02\%$ [$k=2$].

Pipe Prover Calibration with Hydrocarbon Fluid: $U(V_{b-hc}) = \pm 0.03\%$ [$k=2$], 18°C and 9 bar .

Pipe Prover Calibration with Hydrocarbon Fluid: $U(V_{b-hc.ht}) = \pm 0.04\%$ [$k=2$], 65°C and 20 bar .

The significant increase in uncertainty of volume with higher temperature calibrations is a result of the increase in the sensitivity of the cubical thermal expansion coefficient of the pipe prover steel. The usefulness of uncertainty evaluation can be seen from this result as it highlights the need for a reduction in the coefficient of thermal expansion (CTE) uncertainty to maintain a pipe prover volume uncertainty in the order of $\pm 0.03\%$. In the example a default value of $\pm 10\%$ is used for CTE whereas a material certificate or material testing results may reduce the uncertainty estimate of CTE.

8. DISCUSSION

Uncertainty Evaluations

From the uncertainty evaluations provided in this document the greatest sensitivity is noted for temperature measurement uncertainty, highlighting the need for accurate temperature measurement and stable operating conditions throughout the calibration.

The uncertainty evaluation for pipe prover volume calibration uncertainty with water $U(V_{b-w}) = \pm 0.0190\%$ [$k=2$], which suggests using water as the calibration fluid it may be possible to calibrate the pipe prover volume to $\pm 0.02\%$ as indicated within industry documentation [27, 28, 29, 30].

A difference in the uncertainty obtained between water and hydrocarbon fluid calibrations was also noted from the uncertainty evaluations. The difference is mainly due to greater sensitivity in relation to temperature measurement uncertainty which is greater in the hydrocarbon fluid calibration model.

In the uncertainty evaluations presented, it is clear that pipe prover cubical thermal expansion uncertainty is significantly affected by the increase in oil temperature highlighting the need for accurate determination of the coefficient of thermal expansion and hence a low uncertainty.

Pipe Prover Volume Uncertainty

There have been a number of documents published, which provide a range of what is deemed to be a sensible or achievable level of uncertainty when determining a prover volume. Alan T.J. Hayward [27] indicates that prover volume calibration accuracy can be between $\pm 0.05\%$ and $\pm 0.02\%$. API MPMS 4.9.1 [28] discusses the frequency of calibration of pipe provers and indicates a range from $\pm 0.05\%$ to $\pm 0.02\%$ on volume. API MPMS 4.1 [29] Table 3 provides some hypothetical uncertainty values within a hierarchy (traceability chain) for prover base volume as $\pm 0.03\%$, calibrated using a field standard test measure with $\pm 0.015\%$ on volume. Another source of data was found within the NFOGM Handbook of Uncertainty Calculations [30] which uses a pipe prover base volume uncertainty of $\pm 0.038\%$ (0.011m^3 in 28.646 m^3) within its fiscal turbine meter station uncertainty calculations.

From the information above it is clear that pipe prover volume uncertainty is the direct result of measurement traceability and no single generic measurement uncertainty can therefore be used to define pipe prover volume uncertainty. The measurement method and uncertainty estimates for reference instruments within the traceability chain will influence the estimate of the combined measurement uncertainty.

Pipe Prover Volume Calibration Tolerance

Within the UK sector of the North Sea pipe prover volume calibration repeatability limit is set as $\pm 0.01\%$ of the mean (or within a band of 0.02%) [1] for a set of 5 successive calibration runs. Also defined is the year to year volume shift tolerance requirement of $\pm 0.02\%$ from the previous year. Although the repeatability limit is fully achievable, the volume shift tolerance seems very narrow in relation to the uncertainty evaluations contained within this document and information presented in industry documentation [27, 28, 29, 30].

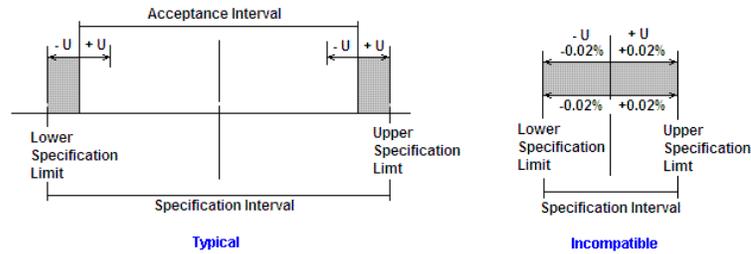


Figure 6. Tolerance Intervals

For a case where the best possible pipe prover volume measurement uncertainty is taken as $\pm 0.02\%$ and year to year tolerance interval is set as $\pm 0.02\%$, no acceptance interval [31] is available to assess the measurement system conformance, see Figure 6. As the pipe prover volume uncertainty is equal to the tolerance, conformance to specification is not possible. In fact, in some instances, the pipe prover volume calibration uncertainty may be greater than the tolerance interval.

The measurement uncertainty should generally be smaller than the tolerance in order to ensure that the tolerance is met for a given measurand. ISO 12001-1: 1993 [32] suggests minimum requirements for test uncertainty ratio (TUR) of 3:1 as being acceptable for any measurement process, although some kind of compromise is required for pipe prover calibration as it is obvious that permanently fixed pipe provers cannot be calibrated in the laboratory.

The challenge arises when a series of measurements result in values which are scattered around the tolerance limit. We know that small changes in process and environmental conditions can have a significant effect so how do we apply a rigorous method of ensuring that the final result is both eliminating errors due to the bad measurement conditions whilst maintaining objectivity by not selecting the results and possibly introducing a bias.

Statistical methods for determining the true average calibration factor may in some cases provide improvements to the conventional proving method of five successive runs as varying proving results are normally due to variations in process conditions rather than the inherent repeatability of the meter.

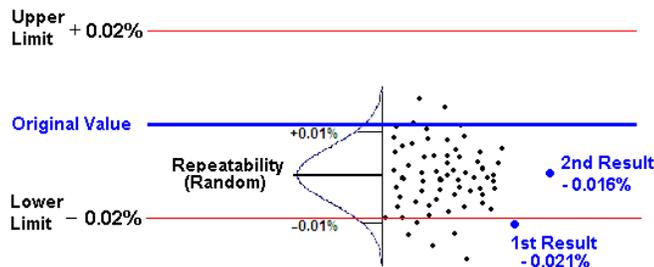


Figure 7. Theoretical Calibration Results

Consider the case given in Figure 7 where the first set of calibration runs returns a shift calculated as -0.021% . As this value is outwith the tolerance band of $\pm 0.02\%$ a second set of calibration runs are undertaken to yield a result of -0.016% shift in volume. As the verification limit between the 1st and 2nd result is set to $\pm 0.01\%$, this means the ‘as left’ calibration from the second set of runs is actually within the tolerance limit leaving considerable doubt in the measurement result.

From the information presented in this paper it may be possible to calibrate a pipe prover volume within the uncertainty limits but actually be outwith the tolerance specification. Widening the tolerance limit may be a more robust approach to pipe prover volume calibration.

Another point to consider is, why are the UK ($\pm 0.02\%$) [1] and NPD ($\pm 0.04\%$) [33] year to year pipe prover volume tolerance limits different for the same quality of measurement equipment ?

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APPENDIX

GRAVIMETRIC WATER DRAW:

$$C_{tdw} = \frac{\rho_1}{\rho_2}$$

$$C_{tsp} = 1 + [\alpha(t_1 - T) + \gamma(t_2 - T)]$$

$$C_{psp} = 1 + \frac{p}{E} \times \frac{D}{wt}$$

$$C_{pl} = [1 + (\beta) \cdot \Delta p]$$

$$M_x = M_s \frac{\left(1 - \frac{\rho_o}{\rho_c}\right)}{\left(1 - \frac{\rho_o}{\rho}\right)}$$

α	Coefficient of linear expansion of Invar rod
β	Compressibility factor of water [22]
γ	Coefficient of area expansion of steel
Δp	Differential pressure
ρ_1	Density of water at measure or mass comparator (observed conditions) [23]
ρ_2	Density of water in the compact prover (observed conditions) [23]
ρ	Density of the sample being weighed
ρ_c	Density of reference weight
ρ_o	Density of moist air [21]
C_{tdw}	Correction factor for the thermal expansion of water [34, 35]
C_{pl}	Correction factor for the effect of pressure on liquid [23]
C_{psp}	Correction factor for the effect of pressure on the steel of prover [14, 15]
C_{tsp}	Correction factor for the effect of temperature on the steel of the prover [14, 15]
D	Internal pipe diameter
E	Modulus of elasticity
M_x	Object mass [18, 36]
M_s	Mass of the sample in air
wt	Wall thickness of pipe
t	Any temperature
T	Base temperature

VOLUMETRIC WATER DRAW:

$$C_{ts} = 1 + 3\alpha(t - T)$$

α	Co-efficient of linear expansion
C_{ts}	Correction factor for the effect of temperature on the steel of the prover [14, 15]
t	Any temperature
T	Base temperature

PIPE PROVER WATER CALIBRATION:

$$K = \frac{N}{v}$$

$$C_{is} = 1 + 3\alpha(t - T)$$

$$VCF = \frac{V_T}{V_t} = \frac{\rho_t}{\rho_T} = \exp[-\alpha_T \Delta t (1 + 0.8 \alpha_T \Delta t)]$$

$$C_{il} = \frac{\rho_t}{\rho_T}$$

α	Co-efficient of linear expansion
ρ	Water density
Ctl	Correction factor for the effect of temperature on the liquid [26]
Cts	Correction factor for the effect of temperature on steel [14, 15]
K	K-factor [3]
N	Meter pulses
t	Any temperature
T	Base temperature
v	Unit volume
VCF	Volume correction factor [26]

PIPE PROVER OIL CALIBRATION:

$$C_{il} = \exp[-\alpha_T(t - T)(1 + 0.8 \alpha_T(t - T))]$$

$$\alpha_T = \frac{K0}{\rho_T^2} + \frac{K1}{\rho_T} + K2$$

$$C_{pl} = \frac{1}{1 - F(P - P_e)}$$

$$F = \exp\left(A + B.T + \frac{C}{\rho_T^2} + \frac{D.T}{\rho_T^2}\right)$$

α	Coefficient of thermal expansion of the liquid [26]
ρ_T	Hydrocarbon liquid standard density
A, B, C, D	Constants
Cpl	Correction factor for the effect of pressure on liquid [26]
Ctl	Correction factor for the effect of temperature on liquid [26]
F	Compressibility factor of the hydrocarbon liquid [26]
Kn	Hydrocarbon liquid specific constants
P_e	Vapour pressure equilibrium
P_m	Pressure at the meter
t	Any temperature
T	Base temperature