

Poster Paper

Handbook of Uncertainty Calculations: Fiscal Oil Metering Stations Based on Ultrasonic Duty and Master Meters

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Handbook of Uncertainty Calculations: Fiscal Oil Metering Stations Based on Ultrasonic Duty and Master Meters

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ABSTRACT

A "Handbook of uncertainty calculations - Ultrasonic fiscal oil metering stations" [1] is being developed in a cooperation between NFOGM, NPD, Tekna and CMR, addressing fiscal metering of oil using multipath ultrasonic transit time flow meters (USM). The many different approaches to calculating the uncertainty of ultrasonic oil metering stations have been a source of confusion; - varying practice in this respect has definitely been experienced. The intention of the present initiative has been that a handbook together with a Microsoft Excel program *EMU - USM Fiscal Oil Metering Station*, based upon the principles laid down in the "Guide to the expression of uncertainty in measurement (GUM)" [2] and ISO 5168:2005 [3], would satisfy the need for a modern method of uncertainty evaluation in the field of ultrasonic fiscal oil measurement.

Three metering system scenarios are addressed in [1]: (A) a USM duty meter which is *in-situ* proved using a large volume prover (at specified time intervals, such as typically every 4th day), which is *in-situ* flow calibrated using a portable small-volume prover system (typically once a year), (B) a USM duty meter which is *in-situ* proved using a USM master meter (at specified time intervals), which is *in-situ* flow calibrated using a portable small-volume prover system (typically once a year), and (C) a USM duty meter which is *in-situ* proved using a turbine (TM) master meter (at specified time intervals), which is *in-situ* flow calibrated using a portable small-volume prover (typically once a year).

Scenario A has been addressed in ref. [4]. In the present paper, Scenario B with a USM duty and a USM master meter is analyzed. The situation with use of master meter in addition to the prover, and that the duty and master meters are of the same type (USM), necessitates dedicated treatment of correlated and uncorrelated uncertainty contributions to the metering system. The uncertainty model for Scenario B necessarily differs from the Scenario A and C uncertainty models. However, synergies between the three different scenarios can be exploited in the formulation of the uncertainty models.

Calculation of the expanded uncertainties of the following three measurands are addressed: the actual volumetric flow rate, the standard volumetric flow rate and the mass flow rate. The uncertainty model accounts for metering station instrumentation such as pressure transmitters, temperature elements and transmitters, density measurement (vibrating element densitometer), an *in situ* flow calibrated ultrasonic gas flow master meter (USM_{MM}) and an *in-situ* proved multipath ultrasonic gas flow duty meter (USM_{DM}). The expanded uncertainties of each of these measurands and instruments can be calculated and analyzed, isolated and combined, for the complete metering system (station). The basis for the handbook and the Microsoft Excel uncertainty evaluation program is described, together with an example of a metering station uncertainty evaluation.

1 INTRODUCTION

1.1 Uncertainty Evaluation of Ultrasonic Fiscal Metering Stations

Regulations relating to fiscal measurement of oil and gas [5-8] require that the overall measurement uncertainty is documented to be within defined limits. However, the different methods used have given different results. Consistent and standardised methods of uncertainty evaluation have been required, so that different measurement systems could be directly and reliably compared.

On this background, a series of handbooks on uncertainty calculation of fiscal metering stations for oil and gas is being developed in a cooperation between the Norwegian Society for Oil and Gas Measurement (NFOGM), the Norwegian Petroleum Directorate (NPD), Tekna and Christian Michelsen Research (CMR). This series includes handbooks of uncertainty calculations for ultrasonic fiscal gas metering stations [9,10] and for fiscal orifice gas and turbine oil metering stations [11]. The series is being extended by a handbook of uncertainty calculations for ultrasonic fiscal oil metering stations [1]. These handbooks are developed in conformity with the ISO "Guide to the expression of uncertainty in measurement" (commonly referred to as the "Guide" or the "GUM") [2], which provides general rules for evaluating and expressing uncertainty in measurement, intended for a broad scope of measurement areas.

1.2 The Handbook

The *Handbook of uncertainty calculations - USM fiscal oil metering stations* [1] (for convenience here referred to as the "*Handbook*") consists of the *Handbook* document itself and the Microsoft Excel program *EMU - USM Fiscal Oil Metering Stations* for performing uncertainty calculations of fiscal oil metering stations based on multipath ultrasonic transit time flow meters (USM), and individual instruments of such stations (cf. Section 2).

The USM fiscal oil metering stations addressed in the *Handbook* are primarily taken to be built and operated according to NPD regulations [5], however, this is not a prerequisite for the study. For USM fiscal metering of oil, the NPD regulations refer to e.g. the NORSOK I-105 national standard [6] and the API standard on ultrasonic liquid hydrocarbon meters [7] as recognised standards ("accepted norms"). The NPD regulations require hydrocarbon metering stations to be in conformity with the requirements stated by the European Union's "measurement instrument directive" (MID) [8]. Both the NPD regulations and the NORSOK I-105 standard refer to the *GUM* [2] as the "accepted norm" with respect to uncertainty analysis.

The *Handbook* and the accompanying computer program *EMU - USM Fiscal Oil Metering Stations* are based primarily on the recommended procedures in the *GUM*, in conformity with ISO 5168:2005 [3] (also based on the *GUM*).

With respect to uncertainty evaluation and documentation, refs. [3,5] state that the expanded uncertainty of the metering station shall be specified at a 95 % confidence level, using a coverage factor $k = 2$. Consequently, for output expanded uncertainties $k = 2$ is set as a fixed value in the program. For input expanded uncertainties, however, k is set by the user for each input uncertainty value (depending on the confidence level of the input uncertainty in question).

The uncertainty model for the USM oil metering station used here is based on an analytical approach. That is, the uncertainty models involving the USM_{DM}, USM_{MM}, small-volume prover system (SVP)¹, pressure transmitter, temperature element/transmitter, densitometer, etc., are fully analytical, with expressions given and documented for the model and the sensitivity coefficients. The model is treated at a sufficiently generic level so that all relevant USM types for oil measurement are covered (cf. e.g. [12-15]), irrespective of path configuration, including non-reflecting path as well as reflecting path USMs. The intention has been to meet as far as possible manufacturer data specified today with respect to instrument uncertainties, including the USM. The work builds on earlier developments in this field [16-20,9-11].

The *Handbook* [1] is intended to provide a practical approach to the field of uncertainty calculations of ultrasonic fiscal oil metering stations. It is primarily written for experienced users and operators of fiscal oil metering stations, manufacturers of ultrasonic oil flow meters, engineering personnel as well as others with interests within the field. It has been the intention that the *EMU* Excel program may be run without needing to read much of the *Handbook*, such as the theory part. However, Chapter 5 in the *Handbook* which gives an overview of the program, as well as Chapter 4 which - through an uncertainty evaluation example - provides some guidelines for specifying input parameters and uncertainties to the program, may be useful to read together with running the program for the first time.

The present paper follows closely the description given in [4] for Scenario A (including some of the text, whenever relevant and useful), so that the similarities and differences between the uncertainty models of the Scenario A and B type of oil metering stations (defined below, cf. Table 1 and Fig. 1), addressed in [4] and the present paper, respectively, may be emphasized.

¹ For definition of these abbreviations, cf. Section 2.

2 USM FISCAL OIL METERING STATIONS

The types of fiscal oil metering stations considered in the *Handbook* consist basically of a USM "duty meter", in some applications a "master meter" (which may be a USM or a turbine meter (TM)), in some applications a large volume stationary prover (LVP), a small-volume portable prover system (SVP), a flow computer, and instrumentation such as pressure transmitters (for measurement of the pressure, P), temperature element and transmitters (for measurement of the temperature, T), and a vibrating element densitometer (for measurement of the line density, ρ).

With respect to proving and flow calibration of the USM "duty meter" and the "master meter" (USM or TM), three different operational scenarios A, B and C are considered in the *Handbook* [1], cf. Table 1 and Fig.1.

Table 1. Characteristics for the 3 scenarios A, B and C of the types of USM fiscal oil metering stations addressed in the *Handbook* [1]. (See also Fig. 1.)

Operational scenario	Instrumentation for flow calibration and proving
Scenario A	A USM "duty meter" (USM _{DM}) which is <i>in-situ</i> proved using a large-volume prover (LVP), typically every 4th day, at a single flow rate ("proving point"). The large-volume prover (LVP) is <i>in-situ</i> flow calibrated using a portable small-volume prover system (SVP), typically once a year.
Scenario B	A USM "duty meter" (USM _{DM}) which is operated together with a USM "master meter" (USM _{MM}). The USM "duty meter" is <i>in-situ</i> proved vs. the USM "master meter", typically every 4th day, at a single flow rate ("proving point"). The USM "master meter" is <i>in-situ</i> flow calibrated vs. a portable small-volume prover system (SVP), typically once a year, at multiple flow rates (typically 5-10 "calibration points").
Scenario C	A USM "duty meter" (USM _{DM}) which is operated together with a turbine "master meter" (TM _{MM}). The USM "duty meter" is <i>in-situ</i> proved vs. the turbine "master meter", typically every 4th day, at a single flow rate ("proving point"). The turbine "master meter" is <i>in-situ</i> flow calibrated vs. a portable small-volume prover system (SVP), typically once a year, at multiple flow rates (typically 5-10 "calibration points").

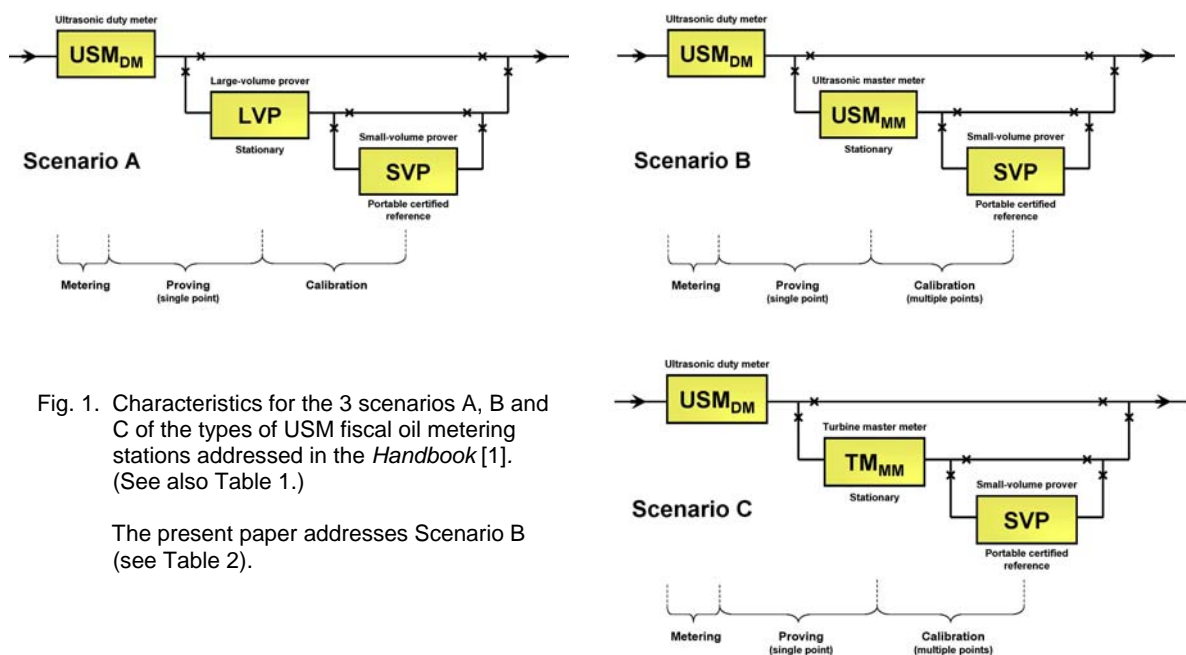


Fig. 1. Characteristics for the 3 scenarios A, B and C of the types of USM fiscal oil metering stations addressed in the *Handbook* [1]. (See also Table 1.)

The present paper addresses Scenario B (see Table 2).

Scenario A has been addressed in [4]. The present paper addresses Scenario B, involving a USM duty and a USM master meter, denoted by USM_{DM} and USM_{MM}, respectively. A portable small-volume prover system is used as a certified and traceable reference for the

USM_{MM} and thus the USM_{DM} measurements. The small-volume prover system may in practice consist of e.g. a compact prover and a turbine transfer meter, or other possible calibration system solutions. The functionality of this calibration system "package" is of no importance for the treatment of uncertainties in the present *Handbook*, and is for simplicity and convenience referred to as the "small volume prover system", denoted SVP. In the present *Handbook*, the accredited uncertainty of the SVP (i.e. the calibration system) is used as the reference uncertainty. In the calibration certificate of the USM_{MM} the reference uncertainty is usually given for M calibration flow rates, together with M K-factors for the USM_{MM}.

The situation with use of a master meter in addition to a small-volume prover, and that the duty and master meters are of the same type (USM), necessitates dedicated treatment of correlated and uncorrelated uncertainty contributions to the metering system. The uncertainty model for Scenario B necessarily differs from the Scenario A and C uncertainty models. However, synergies between the three different scenarios can be exploited in the formulation of the uncertainty models [1].

For the analysis of Scenario B, consider a fiscal oil metering station equipped with instrumentation as specified in Table 2. Proving of the USM_{DM} versus the USM_{MM} is typically made at a single flow rate ("proving flow rate") which is normally relatively close to the metering flow rate in question. A significant deviation between the proving and metering flow rates normally triggers a new proving. Flow calibration of the USM_{MM} versus the SVP is typically made at multiple flow rates covering the range of expected metering flow rates (M "calibration flow rates"). However, for generality and harmonization of notation, the theory given here will cover both single-point and multi-point proving and calibration. That is, the number of proving flow rates² is taken to be an integer $L \geq 1$, and the number of calibration flow rates is $M \geq 1$.

Pressure and temperature measurements are made at the USM_{DM} (denoted P_m and T_m , respectively), the USM_{MM} (P_M and T_M), the SVP (P_p and T_p), and the densitometer (P_d and T_d).

Table 2. Equipment specified as default instrumentation for **Scenario B** of the USM fiscal oil metering station addressed in the *Handbook* [1]. Included is also example instrumentation used for uncertainty evaluation of a fiscal oil metering station in Section 5.

Measurement	Instrument
Ultrasonic "duty meter" (USM _{DM})	Multipath ultrasonic transit time flow meter (USM). Otherwise not specified (arbitrary type and fabrication). <i>In-situ</i> proved by use of a USM master meter (USM _{MM}), typically every 4 th day, at a single flow rate ("proving point").
Ultrasonic "master meter" (USM _{MM})	Multipath ultrasonic transit time flow meter (USM). Otherwise not specified (arbitrary type and fabrication). <i>In-situ</i> flow calibrated by use of a portable small-volume prover system (SVP), typically once a year, at multiple flow rates (typically 5-10 "calibration points").
Small-volume prover system (portable reference) (SVP)	Not specified (arbitrary type and fabrication). Serves as the certified and traceable reference, with accredited calibration certificate.
Flow computer	Not specified (arbitrary type and fabrication).
Pressure (static), P	Not specified. Example: Rosemount 3051P Reference Class Pressure Transmitter [21]. 4 pressure measurements: P_m , P_M , P_p and P_d (see text).
Temperature, T	Not specified. Example: Pt 100 element: according to EN 60751 tolerance A [6]. Rosemount 3144 Smart Temperature Transmitter [22]. 4 temperature measurements: T_m , T_M , T_p and T_d (see text).
Density, ρ	On-line (by-pass) installed vibrating element densitometer. Otherwise not specified. Example: Solartron 7835 Liquid Density Transducer [23].

² In practice, proving is made at a single flow rate, so that $L = 1$. The uncertainty model of the fiscal oil metering station needs to account for the uncertainty introduced when the metering flow rate is different from the proving flow rate. To formulate the uncertainty model, it is found convenient to use a description of multipoint proving ($L \geq 1$), similar to multipoint calibration ($M \geq 1$). It is however noted that this description also covers the case of single point proving ($L = 1$), which is the case used in the uncertainty evaluation example, cf. Sections 3.7 and 5.

For fiscal oil metering stations, three types of flow rate measurements are in question [6]:

- Actual volumetric flow rate (i.e. the axial volumetric flow rate at metering (line) conditions), q_v ,
- Standard volumetric flow rate (i.e., the axial volumetric flow rate at standard reference conditions, 1 atm. and 15 °C), Q_v , and
- Mass flow rate, q_m .

For Scenario B, these are given as³ [1]

$$q_v = 3600 \cdot C_{prov} \cdot C_{cal} \cdot q_{USM} \quad [\text{m}^3/\text{h}], \quad (1a)$$

$$Q_v = \frac{\rho}{\rho_0} q_v = q_v \cdot [C_{ilm} \cdot C_{plm}]_{metering} \quad [\text{Sm}^3/\text{h}], \quad (1b)$$

$$q_m = \rho q_v = \rho_0 Q_v = Q_v \cdot \frac{\rho_d}{[C_{ild} \cdot C_{pld}]_{metering}} \quad [\text{kg}/\text{h}], \quad (1c)$$

respectively, where subscript "metering" means "at metering conditions".

In Eq. (1a), q_{USM} is the axial volumetric flow rate indicated by the USM_{DM} during metering, before the correction factors C_{cal} and C_{prov} are applied. The "calibration correction factor"

$$C_{cal} \equiv f_{cal}(C_1^{cal}, C_2^{cal}, \dots, C_M^{cal}, q_v) \quad (2)$$

is some function, f_{cal} , of the M calibration meter factors,

$$C_j^{cal} = \frac{K_{DFC}^{UMM}}{K_{cal,j}^{UMM}}, \quad j = 1, \dots, M. \quad (3)$$

where $K_{cal,j}^{UMM}$, $j = 1, \dots, M$, are the calibration K-factors of the USM_{MM} obtained by calibration of the USM_{MM} vs. the SVP at M different calibration flow rates ("calibration points") (cf. e.g. Table 4) [1]. Typically, M is 5 - 6 (sometimes up to 10), and C_{cal} is flow rate dependent.

Similarly, the "proving correction factor"

$$C_{prov} \equiv f_{prov}(C_1^{prov}, C_2^{prov}, \dots, C_L^{prov}, q_v) \quad (4)$$

is some function, f_{prov} , of the L proving meter factors,

$$C_\ell^{prov} = \frac{K_{DFC}^{USM}}{K_{DFC}^{UMM}} \cdot \frac{K_{cal,\ell}^{UMM}}{K_{prov,\ell}^{USM}} = \frac{K_{DFC}^{USM}}{K_{prov,\ell}^{USM}} \bigg/ \frac{K_{DFC}^{UMM}}{K_{cal,\ell}^{UMM}}, \quad \ell = 1, \dots, L, \quad (5)$$

where $K_{prov,\ell}^{USM}$, $\ell = 1, \dots, L$, are the proving K-factors of the USM_{DM} obtained by proving of the USM_{DM} vs. the USM_{MM} at L different proving flow rates ("proving points"). $K_{cal,\ell}^{UMM}$ is the calibration K-factor of the USM_{MM} at proving flow rate no. ℓ , obtained by interpolation of the M calibration K-factors in-between the calibration flow rates according to the interpolation method used to calculate C_{cal} .

³ Symbols are defined at the end of the paper.

The DFC (digital-to-frequency converter)⁴ factors K_{DFC}^{USM} and K_{DFC}^{UMM} are used to let the ultrasonic meters (the USM_{DM} and USM_{MM}, respectively) deliver pulses per second as their output quantities (instead of m³/s), and are specified as fixed values (e.g. 6000 pulses/m³, cf. Table 4) by the USM manufacturer [1].

Methods for calculation of the correction factors C_{prov} and C_{cal} from the meter factors C_{ℓ}^{prov} and C_j^{cal} (cf. Eqs. (3) and (5)) include various types of single-factor and multi-factor corrections⁵.

The functional relationships of the USM, pressure transmitter, temperature element/transmitter and liquid densitometer, which form the basis for the uncertainty model described in Section 3, are described in [1]. Due to space limitations these are not given here.

3 UNCERTAINTY MODEL OF THE OIL METERING STATION

For the three measurands given in Eqs. (1), the relative combined standard uncertainties are given as [1]

$$E_{qv}^2 = (E_{liq}^{\Delta c})^2 + (E_{qv}^B)^2, \quad (6a)$$

$$E_{Qv}^2 = (E_{liq}^{m\Delta c})^2 + (E_{qv}^B)^2, \quad (6b)$$

$$E_{qm}^2 = (E_{liq}^{\Delta m\Delta c})^2 + (E_{\rho d})^2 + (E_{qv}^B)^2, \quad (6c)$$

where

$$(E_{qv}^B)^2 = E_{cal}^2 + E_{prov}^2 + E_{USM}^2 + E_{comm}^2 + E_{flocm}^2. \quad (6d)$$

The various terms involved in Eqs. (6) are explained below. The corresponding relative expanded uncertainties at a 95 % confidence level are obtained by multiplying with the coverage factor $k = 2$ (assuming a normal probability distribution).

The derivation underlying the uncertainty model given by Eqs. (6) is rather comprehensive, and for details it is referred to [1]. The model has been obtained by a detailed analysis of correlated and uncorrelated effects in the USMs, such as for the USM_{DM} in duty operation (metering) vs. proving, the USM_{MM} at proving vs. calibration, between the various acoustic paths of the two USMs, between upstream and downstream signal propagation in a given acoustic path (correlated and uncorrelated transit time contributions) in each of the two USMs, etc. Thus, elimination of systematic effects in the USMs by proving and calibration is accounted for, with the intention that these do not contribute to the metering station uncertainty. The analysis is made in compliance with the procedure for evaluating and expressing uncertainties recommended in [2]⁶. The various terms involved in Eqs. (6) are further described in the following.

⁴ The digital-to-frequency converter factors (DFC) K_{DFC}^{USM} and K_{DFC}^{UMM} are sometimes referred to as the K-factors of the USM_{DM} and the USM_{MM}, respectively, during proving and calibration of the USM_{DM} and the USM_{MM}.

⁵ Single-factor correction methods for calculation of the "proving correction factor", C_{prov} , and the "calibration correction factor", C_{cal} , include e.g. (a) the flow-weighted mean error (FWME) [24], (b) the weighted mean error (WME) [25], and average meter factor methods. Multi-factor correction methods include e.g. (a) piecewise linear interpolation [24], (b) multi-point (higher order) polynomial algorithms [24], and (c) regression analysis methods.

⁶ With respect to symbol notation, the *Handbook* deviates in a few cases from the recommendations given in [2], mainly for practical reasons. E.g., for relative standard uncertainties and rel. sensitivity coefficients, the symbols " E_x " and " s_x^* " are used in [1], whereas the recommended symbols in [2] are " $u(x)/x$ " and " c_x^* ", respectively.

3.1 Pressure Transmitter Uncertainty

In the uncertainty model, Eqs. (6), the uncertainties of the pressure measurements, \hat{P}_m , \hat{P}_M , \hat{P}_d and \hat{P}_ρ , are involved in the evaluation of $E_{liq}^{\Delta c}$, $E_{liq}^{m\Delta c}$, $E_{liq}^{\Delta m\Delta c}$ and E_{USM} , to be discussed below.

The relative combined standard uncertainty of a pressure measurement, \hat{P} , is given as $E_p \equiv u_c(\hat{P})/\hat{P}$, where [1,11]

$$u_c^2(\hat{P}) = u^2(\hat{P}_{transmitter}) + u^2(\hat{P}_{stability}) + u^2(\hat{P}_{RFI}) + u^2(\hat{P}_{temp}) + u^2(\hat{P}_{atm}) + u^2(\hat{P}_{vibration}) + u^2(\hat{P}_{power}) + u^2(\hat{P}_{misc}) \quad (7)$$

gives the combined standard uncertainty of the pressure measurement⁷. The eight terms at the right hand side of Eq. (7) account, respectively, for uncertainties related to (a) the pressure transmitter (hysteresis, terminal-based linearity, repeatability, pressure calibration laboratory), (b) stability of the pressure transmitter, (c) radio-frequency interference (RFI) effects, (d) temperature effects, (e) atmospheric pressure, (f) vibration effects, (g) power supply effects, and (h) miscellaneous effects (mounting, etc.). With exception for (c) and (e), information on these input uncertainties may be provided by the instrument manufacturer or calibration laboratory, cf. e.g. [21].

3.2 Temperature Element / Transmitter Uncertainty

In the uncertainty model, Eqs. (6), the uncertainties of the temperature measurements \hat{T}_m , \hat{T}_M , \hat{T}_d and \hat{T}_ρ , are involved in the evaluation of $E_{liq}^{\Delta c}$, $E_{liq}^{m\Delta c}$, $E_{liq}^{\Delta m\Delta c}$ and E_{USM} , to be discussed below.

The relative combined standard uncertainty of a temperature measurement, \hat{T} , is given as $E_T \equiv u_c(\hat{T})/\hat{T}$, where [1,11]

$$u_c^2(\hat{T}) = u^2(\hat{T}_{elem,transm}) + u^2(\hat{T}_{stab,transm}) + u^2(\hat{T}_{RFI}) + u^2(\hat{T}_{temp}) + u^2(\hat{T}_{stab,elem}) + u^2(\hat{T}_{vibration}) + u^2(\hat{T}_{power}) + u^2(\hat{T}_{cable}) + u^2(\hat{T}_{misc}) \quad (8)$$

gives the combined standard uncertainty of the temperature measurement. The nine terms at the right hand side of Eq. (8) account, respectively, for uncertainties related to (a) the temperature element and transmitter calibrated as a unit, (b) stability of the temperature transmitter, (c) RFI effects, (d) temperature effects, (e) stability of the Pt100 element, (f) vibration effects, (g) power supply effects, (h) lead resistance effects, and (i) miscellaneous effects. With exception for (c), information on these input uncertainties may be provided by the instrument manufacturer or calibration laboratory, cf. e.g. [22].

3.3 Densitometer Uncertainty

In the uncertainty model, Eqs. (6), the uncertainty of the liquid (oil) density at line conditions, ρ_d , is involved in the evaluation of E_{qm} and E_{ρ_0} , see below.

The relative combined standard uncertainty of the liquid (oil) density measurement, $\hat{\rho}_d$, is given as $E_{\rho_d} \equiv u_c(\hat{\rho}_d)/\hat{\rho}_d$, where [1,11]

⁷ To distinguish between a quantity and an estimated value of the quantity, the symbol “ \hat{x} ” (the “hat notation”) is used here to denote the estimated value of the quantity “ x ”.

$$u_c^2(\hat{\rho}_d) = s_{\rho_u}^2 u^2(\hat{\rho}_u) + u^2(\hat{\rho}_{stab}) + u^2(\hat{\rho}_{rept}) + s_{\rho,T_d}^2 u_c^2(\hat{T}_d) + s_{\rho,P_d}^2 u_c^2(\hat{P}_d) + u^2(\hat{\rho}_{temp}) + u^2(\hat{\rho}_{pres}) + u_c^2(\hat{\rho}_{inst}) + u^2(\hat{\rho}_{misc}) \quad (9)$$

gives the combined standard uncertainty of the density measurement. In addition to the usual frequency relationship regression curve, the functional relationship of the vibrating-element liquid densitometer used here accounts for temperature correction and pressure correction [1,11].

The nine terms at the right hand side of Eq. (9) account, respectively, for uncertainties related to (a) the indicated (uncorrected) density (calibration laboratory instruments, reading error during calibration, hysteresis, etc.) (also referred to as the “densitometer accuracy” [23]), (b) stability, (c) repeatability, (d) temperature measurement in the densitometer, T_d , (e) pressure measurement in the densitometer, P_d , (f) the temperature correction model, (g) the pressure correction model, (h) temperature and pressure effect of an on-line installation (by-pass) of the densitometer, and (i) miscellaneous effects (reading error, deposits, corrosion, vibrations, power supply variations, self-induced heat, flow in by-pass line, etc.). The various $s_{x,y}$ terms are sensitivity coefficients given in [1].

Data for the input uncertainties $u(\hat{\rho}_u)$, $u(\hat{\rho}_{stab})$, $u(\hat{\rho}_{rept})$, $u(\hat{\rho}_{temp})$, $u(\hat{\rho}_{pres})$ and $u_c(\hat{\rho}_{inst})$, as well as contributions to $u(\hat{\rho}_{misc})$, should normally be provided by the instrument manufacturer or the calibration laboratory, cf. e.g. [23]. $u_c(\hat{P}_d)$ and $u_c(\hat{T}_d)$ are calculated according to Eqs. (7)-(8).

3.4 Reference Density Uncertainty

The uncertainty of the liquid (oil) density at standard reference conditions (1 atm., 15 °C), ρ_0 , is involved in the evaluation of $E_{liq}^{\Delta c}$, $E_{liq}^{m\Delta c}$ and $E_{liq}^{\Delta m\Delta c}$.

The relative combined standard uncertainty of the reference density estimate of the liquid (oil), $\hat{\rho}_0$, is given as $E_{\rho_0} \equiv u_c(\hat{\rho}_0)/\hat{\rho}_0$, where

$$E_{\rho_0}^2 = (s_{\rho_0\rho_d}^* E_{\rho_d})^2 + (s_{\rho_0T_d}^* E_{T_d})^2 + (s_{\rho_0P_d}^* E_{P_d})^2 + (s_{\rho_0,ild}^* E_{ild}^{mod})^2 + (s_{\rho_0,pld}^* E_{pld}^{mod})^2 + (s_{\rho_0,P_e}^* E_{P_e})^2 \quad (10)$$

The six terms at the right hand side of Eq. (10) account, respectively, for uncertainties related to (a) the density measurement, ρ_d , (b) the temperature measurement in the densitometer, T_d , (c) the pressure measurement in the densitometer, P_d , (d) the model for the densitometer liquid volume temperature correction factor, C_{ild} , (e) the model for the densitometer liquid volume pressure correction factor, C_{pld} , and (f) the equilibrium pressure, P_e . The various $s_{x,y}^*$ terms are relative sensitivity coefficients given in [1].

3.5 Liquid Volume Expansion Uncertainties

The relative combined standard uncertainties related to the liquid volume expansion involved in the evaluation of E_{qv} , E_{Qv} and E_{qm} of Eqs. (6), are given as

$$(E_{liq}^{\Delta c})^2 = (s_{liq,\Delta c,P_p}^* E_{P_p})^2 + (s_{liq,\Delta c,P_M}^* E_{P_M})^2 + (s_{liq,\Delta c,T_p}^* E_{T_p})^2 + (s_{liq,\Delta c,T_M}^* E_{T_M})^2 + (s_{liq,\Delta c,P_e}^* E_{P_e})^2 + (s_{liq,\Delta c,plp}^* E_{plp}^{mod})^2 + (s_{liq,\Delta c,tlp}^* E_{tlp}^{mod})^2 + (s_{liq,\Delta c,\rho_0}^* E_{\rho_0})^2 \quad (11a)$$

$$\begin{aligned} (E_{liq}^{m\Delta c})^2 &= (s_{liq,m\Delta c,P_p}^* E_{P_p})^2 + (s_{liq,m\Delta c,P_M}^* E_{P_M})^2 + (s_{liq,m\Delta c,P_m}^* E_{P_m})^2 \\ &+ (s_{liq,m\Delta c,T_p}^* E_{T_p})^2 + (s_{liq,m\Delta c,T_M}^* E_{T_M})^2 + (s_{liq,m\Delta c,T_m}^* E_{T_m})^2 \\ &+ (s_{liq,m\Delta c,P_e}^* E_{P_e})^2 + (s_{liq,m\Delta c,tlp}^* E_{tlp}^{mod})^2 + (s_{liq,m\Delta c,plp}^* E_{plp}^{mod})^2 + (s_{liq,m\Delta c,\rho_0}^* E_{\rho_0})^2 \end{aligned} \quad (11b)$$

$$\begin{aligned} (E_{liq}^{\Delta m\Delta c})^2 &= (s_{liq,\Delta m\Delta c,P_p}^* E_{P_p})^2 + (s_{liq,\Delta m\Delta c,P_M}^* E_{P_M})^2 + (s_{liq,\Delta m\Delta c,P_m}^* E_{P_m})^2 + (s_{liq,\Delta m\Delta c,P_d}^* E_{P_d})^2 \\ &+ (s_{liq,\Delta m\Delta c,T_p}^* E_{T_p})^2 + (s_{liq,\Delta m\Delta c,T_M}^* E_{T_M})^2 + (s_{liq,\Delta m\Delta c,T_m}^* E_{T_m})^2 + (s_{liq,\Delta m\Delta c,T_d}^* E_{T_d})^2 \\ &+ (s_{liq,\Delta m\Delta c,P_e}^* E_{P_e})^2 + (s_{liq,\Delta m\Delta c,tlp}^* E_{tlp}^{mod})^2 + (s_{liq,\Delta m\Delta c,plp}^* E_{plp}^{mod})^2 + (s_{liq,\Delta m\Delta c,\rho_0}^* E_{\rho_0})^2 \end{aligned} \quad (11c)$$

respectively. The terms at the right hand side of Eqs. (11) account for uncertainties related to the pressure measurements P_p , P_M , P_m and P_d , the temperature measurements T_p , T_M , T_m and T_d , the model for the densitometer liquid volume temperature correction factor, C_{tlc} , the model for the densitometer liquid volume pressure correction factor, C_{plc} , and the equilibrium pressure, P_e . The various $s_{x,y}^*$ terms are relative sensitivity coefficients given in [1].

3.6 Calibration Uncertainty

Let the index j represent the calibration flow rate which is closest to the measured flow rate. In Eq. (6a), the relative combined standard uncertainty related to calibration of the USM_{MM} is given by [1]

$$E_{cal}^2 \equiv (E_{dev}^{cal})^2 + (E_{UMM, rept, j}^{cal})^2 + E_{SVP, j}^2 \quad (12)$$

The three terms at the right hand side of Eq. (12) account, respectively, for uncertainties related to (a) the deviation factor of the K-factor calibration curve resulting from calibration of the USM_{MM} vs. the SVP, C_{dev}^{cal} , (b) the repeatability of the USM_{MM} and the SVP at calibration flow rate no. j , and (c) $q_{SVP, j}$, the volumetric flow rate of the liquid measured by the SVP at calibration flow rate no. j .

$E_{UMM, rept, j}^{cal}$ represents the relative standard deviation of the spread of measured flow rates, at calibration flow rate no. j , due to random effects related to the USM_{MM} and the SVP measurements during calibration. It thus represents a "calibration repeatability", like e.g. the 0.027 % limit at 95 % conf. lev. (with coverage factor $k = 2$) found in API specifications [7].

The calibration curve deviation factor C_{dev}^{cal} represents the uncorrected deviation between the USM_{MM} measurement and the "deviation curve" resulting from calibrating the USM_{MM} at the M calibration points, after correction of the USM_{MM} measurement by the correction factor C_{cal} has been made. It can be shown [1] that the relative standard uncertainty of the calibration curve deviation factor may be expressed as

$$E_{dev}^{cal} = \frac{1}{\sqrt{3}} \left| \frac{Dev_{C, j}^{cal}}{\hat{C}_{dev, j}^{cal}} \right| \approx \frac{Dev_{C, j}^{cal}}{\sqrt{3}}, \quad Dev_{C, j}^{cal} = \frac{C_{cal} \cdot Q_{UMM, j}^{cal} - Q_{SVP, j}}{Q_{SVP, j}}, \quad (13)$$

where, at calibration flow rate no. j , $Q_{UMM, j}^{cal}$ is the standard volumetric flow rate measured by the USM_{MM} during calibration, $Q_{SVP, j}$ is the standard volumetric flow rate as measured by the small-volume prover during calibration, $Dev_{C, j}^{cal}$ is the corrected relative deviation (i.e. the relative deviation after multiplication with the calibration correction factor, C_{cal}), and $C_{dev, j}^{cal} = 1 + Dev_{C, j}^{cal}$ is the calibration deviation factor. The deviation data $Dev_{C, j}^{cal}$, $j = 1, \dots, M$,

are available from proving at the M proving flow rates. Details are given in [1], including the method used to calculate E_{dev}^{cal} at flow rates in-between the calibration flow rates.

The relative standard uncertainty of $\hat{q}_{SVP,j}$, the volumetric flow rate of the liquid measured by the SVP at calibration flow rate no. j , is given as [1]

$$E_{SVP,j} \equiv \frac{u(\hat{q}_{SVP,j})}{\hat{q}_{SVP,j}}, \quad (14a)$$

where $u(\hat{q}_{SVP,j})$ is the standard uncertainty of $\hat{q}_{SVP,j}$. In practice, from the calibration certificate of the USM_{MM}, this figure may be taken as

$$E_{SVP,j} \equiv \frac{u(\hat{K}_{cal,j}^{UMM})}{\hat{K}_{cal,j}^{UMM}}, \quad (14b)$$

where $u(\hat{K}_{cal,j}^{UMM})$ is the standard uncertainty of the calibration K-factor estimate of the USM_{MM}, $\hat{K}_{cal,j}^{UMM}$, $j = 1, \dots, M$. $u(\hat{K}_{cal,j}^{UMM})$ and $\hat{K}_{cal,j}^{UMM}$ are given in the calibration certificate of the USM_{MM}.

3.7 Proving Uncertainty

Let the index ℓ represent the proving flow rate which is closest to the measured flow rate. In Eq. (6a), the relative combined standard uncertainty related to proving of the USM_{DM} is given by [1]

$$E_{prov}^2 \equiv (E_{dev}^{prov})^2 + (E_{USM,repr,\ell}^{prov})^2 + (E_{UMM,repr,\ell}^{prov})^2. \quad (15)$$

The three terms at the right hand side of Eq. (15) account, respectively, for uncertainties related to (a) the deviation factor of the proving curve, C_{dev}^{prov} , for proving of the USM_{DM} vs. the USM_{MM}, (b) the repeatability of the USM_{DM} during proving at proving flow rate no. ℓ , and (c) the repeatability of the USM_{MM} during proving at proving flow rate no. ℓ .

In Eq. (15), $E_{USM,repr,\ell}^{prov}$ and $E_{UMM,repr,\ell}^{prov}$ represent the relative standard deviations of the spread of measured flow rates by the USM_{DM} and USM_{MM}, respectively, at proving flow rate no. ℓ , due to random effects in the USM_{DM} and USM_{MM}, respectively. These “USM_{DM} proving repeatability” and “USM_{MM} proving repeatability” terms are given by [1]

$$(E_{USM,repr,\ell}^{prov})^2 \equiv 2 \sum_{i=1}^N (s_{t_{li}}^* E_{t_{li,U}^{USM,prov}})^2, \quad (E_{UMM,repr,\ell}^{prov})^2 \equiv 2 \sum_{i=1}^N (s_{t_{li}}^* E_{t_{li,U}^{UMM,prov}})^2. \quad (16)$$

The relative sensitivity coefficient $s_{t_{li}}^*$ is given in [1]. $E_{t_{li,U}^{USM,prov}}$ and $E_{t_{li,U}^{UMM,prov}}$ are the relative standard uncertainties of those contributions to the transit times of path no. i which are uncorrelated with respect to upstream and downstream propagation, for the USM_{DM} and USM_{MM} during proving, respectively. They are given as

$$E_{t_{li,U}^{USM,prov}} = u(\hat{t}_{li}^{USM,prov,random}) / \hat{t}_{li}^{USM}, \quad E_{t_{li,U}^{UMM,prov}} = u(\hat{t}_{li}^{UMM,prov,random}) / \hat{t}_{li}^{UMM}, \quad (17)$$

where \hat{t}_{li}^{USM} and \hat{t}_{li}^{UMM} are the upstream transit times of path no. i in the USM_{DM} and USM_{MM}, respectively, and $u(\hat{t}_{li}^{USM,prov,random})$ and $u(\hat{t}_{li}^{UMM,prov,random})$ are the standard uncertainties due to random effects on transit times (after possible signal averaging). In practice, $u(\hat{t}_{li}^{USM,prov,random})$ and $u(\hat{t}_{li}^{UMM,prov,random})$ represent the standard deviations of the spread of measured transit times in path no. i , for the two meters during proving, at the proving flow rate in question.

The proving curve deviation factor C_{dev}^{prov} represents the uncorrected deviation between the USM_{DM} measurement and the "deviation curve" resulting from proving the USM_{DM} at the L proving points, after correction of the USM_{DM} measurement by the correction factor C_{prov} has been made. It can be shown [1] that the relative standard uncertainty of the deviation factor may be expressed as

$$E_{dev}^{prov} = \frac{1}{\sqrt{3}} \left| \frac{Dev_{C,\ell}^{prov}}{\hat{C}_{dev,\ell}^{prov}} \right| \approx \frac{Dev_{C,\ell}^{prov}}{\sqrt{3}}, \quad Dev_{C,\ell}^{prov} = \frac{C_{prov} \cdot q_{USM,\ell}^{prov} - q_{UMM,\ell}^{prov}}{q_{UMM,\ell}^{prov}}, \quad (18)$$

where, at proving flow rate no. ℓ , $q_{USM,\ell}^{prov}$ is the volumetric flow rate measured by the USM_{DM} during proving, $q_{UMM,\ell}^{prov}$ is the volumetric flow rate measured by the USM_{MM} during proving, $Dev_{C,\ell}^{prov}$ is the corrected relative deviation (i.e. the relative deviation after multiplication with the proving correction factor, C_{prov}), and $C_{dev,\ell}^{prov} = 1 + Dev_{C,\ell}^{prov}$ is the proving deviation factor. The deviation data $Dev_{C,\ell}^{prov}$, $\ell = 1, \dots, L$, are available from proving at the L proving flow rates.

In the theory above, multipoint proving ($L \geq 1$) is used for the description, as a convenient way to account for the uncertainty of the proving K-factor when the metering flow rate is different from the proving flow rate, accounted for by the term E_{dev}^{prov} . In practice, proving is made at a single flow rate, so that $L = 1$. E_{dev}^{prov} is zero when the metering flow rate equals the proving flow rate. For metering flow rates different from the proving flow rate, such as in an interval around the proving flow rate, E_{dev}^{prov} is - as a practical approach - taken to increase linearly with flow rate in both directions away from the proving flow rate. Due to similar ultrasonic technology in the USM_{DM} and the USM_{MM} instruments, it is here assumed that for this interval around the proving flow rate, the slope of E_{dev}^{prov} (for the USM_{DM}) with changing flow rate is the same as the slope of E_{dev}^{cal} (for the USM_{MM}) with changing flow rate. Details are given in [1].

3.8 USM Duty Operation Uncertainty

In Eq. (6d), the relative combined standard uncertainty of the USM in duty operation (i.e., during metering) is given by [1]

$$E_{USM}^2 \equiv (E_{USM, rept}^{met})^2 + (E_{USM, \Delta mpc})^2 + E_{misc}^2. \quad (19)$$

The three terms at the right hand side of Eq. (19) account, respectively, for uncertainties related to (a) repeatability of the USM_{DM} measurement in duty operation, at the flow rate in question (due to random transit time effects), (b) systematic effects related to the USM_{DM} and the USM_{MM}, due to change of conditions from calibration to proving (for the USM_{MM}), and from proving to metering (for the USM_{DM}), and (c) miscellaneous systematic effects in the USM_{DM} and/or USM_{MM} measurements which are not eliminated by proving and calibration, and which are not covered by other uncertainty terms accounted for here (e.g. inaccuracy of the USM functional relationship (the underlying mathematical model), etc.).

In Eq. (19), $E_{USM, rept}^{met}$ represents the relative standard deviation of the spread of measured flow rates by USM_{DM} during metering, due to random effects in the USM_{DM}. This "USM_{DM} metering repeatability" term is given by [1]

$$(E_{USM, rept}^{met})^2 \equiv 2 \sum_{i=1}^N (s_{tli}^* E_{tli,U}^{USM, met})^2. \quad (20)$$

The relative sensitivity coefficient $s_{t_{li}}^*$ is given in [1]. $E_{t_{li,U}}^{USM,met}$ is the relative standard uncertainty of those contributions to the transit times of path no. i which are uncorrelated with respect to upstream and downstream propagation. It is given as

$$E_{t_{li,U}}^{USM,met} = u(\hat{t}_{li}^{USM,met,random}) / \hat{t}_{li}^{USM}, \quad (21)$$

where \hat{t}_{li}^{USM} is the upstream transit time of path no. i in the USM_{DM}, and $u(\hat{t}_{li}^{USM,met,random})$ is the standard uncertainty due to in-duty random effects on transit times (after possible signal averaging), such as (a) turbulence, (b) incoherent noise (due to pressure reduction valves, RFI, vibrations, etc.), (c) coherent noise (acoustical and electrical cross-talk, acoustic reverberation, other signal interference), (d) finite clock resolution, (e) electronics stability (possible random effects), (f) possible random effects in signal detection/processing (e.g. erroneous signal period identification), and (g) power supply variations. $u(\hat{t}_{li}^{USM,met,random})$ represents the standard deviation of the spread of measured transit times in path no. i , at the actual flow rate.

In Eq. (19), the “systematic USM_{DM} effects” term can be expressed by⁸

$$E_{USM,\Delta mpc}^2 \equiv E_{body,\Delta mpc}^2 + E_{time,\Delta mpc}^2 + E_{I,\Delta mpc}^2, \quad (22)$$

where the three terms at the right hand side of Eq. (22) account, respectively, for uncertainties related to (a) possible uncorrected change of the USM meter body dimensions (radius, lateral chord positions, inclination angles) from calibration to proving (for the USM_{MM}), and from proving to metering (for the USM_{DM}), caused by possible deviation in P and T between calibration, proving and metering conditions, (b) possible uncorrected systematic effects on the transit times caused e.g. by deviation in conditions from calibration to proving (for the USM_{MM}), and from proving to metering (for the USM_{DM}) (P , T , transducer deposits, transducer ageing, etc), and (c) possible change of installation conditions from calibration to proving (for the USM_{MM}), and from proving to metering (for the USM_{DM}) (related to the USM integration method).

In Eq. (22), the “meter body uncertainty” term can be shown [1] to be given as

$$E_{body,\Delta mpc} \equiv E_{rad,\Delta mpc} + E_{chord,\Delta mpc} + E_{angle,\Delta mpc}, \quad (23)$$

where

$$E_{rad,\Delta mpc} \equiv s_R^* \cdot (E_{R,\Delta mpc} + E_{R,\Delta pc}), \quad (24a)$$

$$E_{chord,\Delta mpc} \equiv \sum_{i=1}^N \text{sign}(\hat{y}_i) s_{y_i}^* \cdot (E_{y_i,\Delta mpc} + E_{y_i,\Delta pc}), \quad (24b)$$

$$E_{angle,\Delta mpc} \equiv \sum_{i=1}^N \text{sign}(\hat{\phi}_i) s_{\phi_i}^* \cdot (E_{\phi_i,\Delta mpc} + E_{\phi_i,\Delta pc}), \quad (24c)$$

and

$$E_{R,\Delta mpc} + E_{R,\Delta pc} = \sqrt{(E_{KP,\Delta mpc})^2 + (E_{KT,\Delta mpc})^2}, \quad (24d)$$

$$E_{y_i,\Delta mpc} + E_{y_i,\Delta pc} = \sqrt{(E_{KP,\Delta mpc})^2 + (E_{KT,\Delta mpc})^2}, \quad (24e)$$

$$E_{\phi_i,\Delta mpc} + E_{\phi_i,\Delta pc} = \frac{B \sin 2\hat{\phi}_{i0}}{2\hat{\phi}_{i0}} E_{KP,\Delta mpc}. \quad (24f)$$

⁸ The subscript “ Δmpc ” relates to changes from calibration to proving conditions (for the USM_{MM}) and changes from proving to metering conditions (for the USM_{DM}). That means, uncertainty contributions which are practically eliminated by proving and calibration, are *not* to be included in these expressions.

Here, s_R^* , $s_{y_i}^*$ and $s_{\phi_i}^*$ are relative sensitivity coefficients given in [1], ϕ_{i0} is the inclination angle of path no. i at “dry calibration” conditions, N is the number of acoustic paths in the USM, and B is a constant defined in [1]. $E_{KP,\Delta mpc}$ and $E_{KT,\Delta mpc}$ are the relative combined standard uncertainties of the radial pressure and temperature correction factors for the two USM meter bodies, $K_p = 1 + \beta \cdot \Delta \hat{P}$ and $K_T = 1 + \alpha \cdot \Delta \hat{T}$, respectively, accounting for changes from calibration to proving conditions (for the USM_{MM}) and proving to metering conditions (for the USM_{DM}). These are given as

$$E_{KP,\Delta mpc} = \sqrt{(\Delta \hat{P}_m^{\Delta mp} + \Delta \hat{P}_M^{\Delta pc})^2 \cdot u^2(\hat{\beta}) + \hat{\beta}^2 \cdot u_c^2(\Delta \hat{P}_m^{\Delta mp} + \Delta \hat{P}_M^{\Delta pc})}, \quad (25a)$$

$$E_{KT,\Delta mpc} = \sqrt{(\Delta \hat{T}_m^{\Delta mp} + \Delta \hat{T}_M^{\Delta pc})^2 \cdot u^2(\hat{\alpha}) + \hat{\alpha}^2 \cdot u_c^2(\Delta \hat{T}_m^{\Delta mp} + \Delta \hat{T}_M^{\Delta pc})}, \quad (25b)$$

where $u(\hat{\alpha})$ and $u(\hat{\beta})$ are the standard uncertainties of the coefficients of linear temperature and pressure expansion of the meter body material (usually steel), α and β , respectively. $\Delta \hat{P}_m^{\Delta mp}$ and $\Delta \hat{T}_m^{\Delta mp}$ are the difference in pressure and temperature, respectively, from proving to metering conditions, for the USM_{DM}. $\Delta \hat{P}_M^{\Delta pc}$ and $\Delta \hat{T}_M^{\Delta pc}$ are the difference in pressure and temperature, respectively, from calibration to proving conditions, for the USM_{MM}.

For calculation of Eqs. (25), two cases are addressed here. In cases for which P and T corrections of the meter body are *not* used, these P and T uncertainties are determined by the *span* of the pressure and temperature measurements, so that

$$u_c^2(\Delta \hat{P}_m^{\Delta mp} + \Delta \hat{P}_M^{\Delta pc}) = \frac{1}{\sqrt{3}} (\Delta \hat{P}_m^{\Delta mp} + \Delta \hat{P}_M^{\Delta pc}) = \frac{1}{\sqrt{3}} (\hat{P}_m^{met} - \hat{P}_m^{prov} + \hat{P}_M^{prov} - \hat{P}_M^{cal}), \quad (26a)$$

$$u_c^2(\Delta \hat{T}_m^{\Delta mp} + \Delta \hat{T}_M^{\Delta pc}) = \frac{1}{\sqrt{3}} (\Delta \hat{T}_m^{\Delta mp} + \Delta \hat{T}_M^{\Delta pc}) = \frac{1}{\sqrt{3}} (\hat{T}_m^{met} - \hat{T}_m^{prov} + \hat{T}_M^{prov} - \hat{T}_M^{cal}), \quad (26b)$$

where subscript “m” and “M” refer to the USM_{DM} and the USM_{MM}, respectively, and superscript “met” and “prov” refer to “at metering conditions” and “at proving conditions”, respectively.

In cases where P and T corrections of the meter body *are* used, these uncertainties are determined by the pressure and temperature *measurement uncertainties*, so that

$$u_c(\Delta \hat{P}_m^{\Delta mp} + \Delta \hat{P}_M^{\Delta pc}) = \sqrt{(\sqrt{2} \cdot u_c(\hat{P}_m))^2 + (\sqrt{2} \cdot u_c(\hat{P}_M))^2} \quad (27a)$$

$$u_c(\Delta \hat{T}_m^{\Delta mp} + \Delta \hat{T}_M^{\Delta pc}) = \sqrt{(\sqrt{2} \cdot u_c(\hat{T}_m))^2 + (\sqrt{2} \cdot u_c(\hat{T}_M))^2} \quad (27a)$$

where $u_c(\hat{P}_m)$, $u_c(\hat{T}_m)$, $u_c(\hat{P}_M)$ and $u_c(\hat{T}_M)$ are calculated according to Eqs. (7)-(8). Details are given in [1].

In Eq. (22), the “systematic transit time effects” term is calculated as [1]

$$E_{time,\Delta mpc} \equiv \sum_{i=1}^N [(s_{t1i}^* E_{t1i,C}^{\Delta mp} - s_{t2i}^* E_{t2i,C}^{\Delta mp})^2 + (s_{t1i}^* E_{t1i,C}^{\Delta pc} - s_{t2i}^* E_{t2i,C}^{\Delta pc})^2]. \quad (28)$$

$E_{t1i,C}^{\Delta mp}$ and $E_{t2i,C}^{\Delta mp}$ are the relative standard uncertainties of uncorrected systematic transit time effects on upstream and downstream propagation in acoustic path no. i of the USM_{DM}, due to possible deviation in pressure and/or temperature from proving to metering, defined as

$$E_{t_{1i,C}}^{\Delta mp} = u(\hat{t}_{1i}^{USM, \Delta mp, systematic}) / \hat{t}_{1i}^{USM} , \quad E_{t_{2i,C}}^{\Delta mp} = u(\hat{t}_{2i}^{USM, \Delta mp, systematic}) / \hat{t}_{2i}^{USM} . \quad (29)$$

Similarly, $E_{t_{1i,C}}^{\Delta cp}$ and $E_{t_{2i,C}}^{\Delta cp}$ are the relative standard uncertainties of uncorrected systematic transit time effects on upstream and downstream propagation in acoustic path no. i of the USM_{MM}, due to possible deviation in pressure and/or temperature from calibration to proving, defined as

$$E_{t_{1i,C}}^{\Delta pc} = u(\hat{t}_{1i}^{UMM, \Delta pc, systematic}) / \hat{t}_{1i}^{UMM} , \quad E_{t_{2i,C}}^{\Delta pc} = u(\hat{t}_{2i}^{UMM, \Delta pc, systematic}) / \hat{t}_{2i}^{UMM} . \quad (30)$$

In Eqs. (29)-(30), \hat{t}_{1i}^{USM} and \hat{t}_{2i}^{USM} are the upstream and downstream transit times of path no. i in the USM_{DM}, from proving to metering conditions, and $u(\hat{t}_{1i}^{USM, \Delta mp, systematic})$ and $u(\hat{t}_{2i}^{USM, \Delta mp, systematic})$ are the standard uncertainties of uncorrected systematic effects in these transit times. Similarly, \hat{t}_{1i}^{UMM} and \hat{t}_{2i}^{UMM} are the upstream and downstream transit times of path no. i in the USM_{MM}, from calibration to proving conditions, and $u(\hat{t}_{1i}^{UMM, \Delta pc, systematic})$ and $u(\hat{t}_{2i}^{UMM, \Delta pc, systematic})$ are the standard uncertainties of uncorrected systematic effects in these transit times. $s_{t_{1i}}^*$ and $s_{t_{2i}}^*$ are relative sensitivity coefficients given in [1].

Such systematic transit time effects may be due to (a) cable/electronics/transducer/diffraction time delay (due to line pressure and temperature effects, ambient temperature effects, drift, effects of possible transducer exchange), (b) possible Δt -correction (line pressure and temperature effects, ambient temperature effects, drift, reciprocity effects, effects of possible transducer exchange), (c) possible systematic effects in signal detection/processing, (d) possible cavity time delay correction effects, (e) possible transducer deposits (wax, scaling, etc.), and (f) sound refraction (flow profile effects (“ray bending”).

In Eq. (22), the “installation effects” term,

$$E_{I, \Delta mpc} \equiv \sqrt{E_{I, \Delta mp}^2 + E_{I, \Delta pc}^2} , \quad (31)$$

is closely related to the integration methods used in the USM_{DM} and the USM_{MM}. $E_{I, \Delta mp}$ and $E_{I, \Delta pc}$ serve as input uncertainties to the EMU program. $E_{I, \Delta mp}$ represents installation effects for the USM_{DM} from proving to metering conditions, and $E_{I, \Delta pc}$ represents installation effects for the USM_{MM} from calibration to proving conditions. Such installation effects on the USM integration uncertainty may be due to change of *axial flow velocity profile* and/or change of *transversal flow velocity profiles*. Such changes may be due to e.g. (i) line pressure and temperature effects, (ii) ambient temperature effects, (iii) possible changed wall roughness over time (corrosion, wear, pitting, etc.), in the pipe and meter body, and (iv) possible wall deposits / contamination in the pipe and meter body (wax, scaling), etc.

3.9 Signal Communication And Flow Computer Uncertainties

In Eqs. (6d), the relative uncertainty term E_{comm} accounts for the uncertainties due to the signal communication between the USM field electronics and the flow computer (e.g. the flow computer calculation of frequency (i.e., pulses/s) in case of frequency output). E_{flocm} accounts for the uncertainty of the flow computer calculations. Both should be specified by the USM manufacturer, and are normally relatively small.

4 MICROSOFT EXCEL PROGRAM “EMU - USM FISCAL OIL METERING STATION”

A PC program has been implemented based on the uncertainty model for the Scenario B metering station described in Section 3. The program *EMU - USM Fiscal Oil Metering Stations* is implemented as a Microsoft Excel 2003 spreadsheet.

The program calculates the expanded and relative expanded uncertainties of an oil metering station operated according to Scenario B (cf. Tables 1 and 2, and Fig. 1), for the three measurands in question, q_v , Q_v and q_m .

In addition to calculation/plotting/reporting of the expanded uncertainty of the oil metering station and the individual instruments of the station, the Excel program can be used to calculate, plot and analyse the relative importance of the various contributions to the uncertainty budget for the various instruments of the metering station (e.g. using bar-charts).

For several of the instruments and procedures involved in the metering station, an implementation strategy has been chosen which enables the user to give uncertainty input at two levels: (1) an “overall level”, and (2) a more “detailed level”, cf. Table 3. This provides a useful flexibility in operation of the program.

At the *overall level*, the user specifies the combined standard uncertainty of the instrument in question directly as input to the program. It is left to the user to calculate and document this uncertainty figure. This covers most methods of obtaining the uncertainty figure⁹.

Table 3. Uncertainty model contributions, and optional levels for specification of input uncertainties to the program *EMU - USM Fiscal Oil Metering Stations*, for Scenario B.

Uncertainty contribution	Overall level	Detailed level
Pressure measurement uncertainty (for each of: USM _{DM} , USM _{MM} , densitometer and SVP)	✓	✓
Temperature measurement uncertainty (for each of: USM _{DM} , USM _{MM} , densitometer and SVP)	✓	✓
Density measurement uncertainty	✓	✓
Reference density calculation	✓	✓
Liquid volume expansion uncertainties	✓	✓
Small-volume prover (SVP) uncertainty	✓	
Calibration uncertainty		✓
Proving uncertainty		✓
USM duty operation uncertainty	✓	✓
Signal communication and flow computer calculations	✓	

At the *detailed level*, the combined standard uncertainty of the instrument in question is calculated by the *EMU* program, from more basic input for the instrument provided e.g. by the instrument manufacturer and calibration laboratory/certificates, as outlined in Section 3.

With respect to USM_{MM} calibration, USM_{DM} proving and USM_{DM} duty operation (metering), the level for specification of input uncertainties at the detailed level is adapted to data available from calibration, proving and duty operation (metering), from the USM manufacturer. In particular this concerns:

(1) Calibration of the USM_{MM}:

- **Calibration deviation factor.** The user specifies the M calibration flow rates and the corresponding calibration K-factors of the USM_{MM}, $K_{cal,j}^{UMM}$, $j = 1, \dots, M$, as given in the calibration certificate of the USM_{MM}. On this basis, the EMU computer program calculates the relative uncertainty E_{dev}^{cal} at and in-between the calibration flow rates according to a method described in [1], cf. also Eq. (13).
- **SVP uncertainty (reference).** The user specifies the expanded uncertainty of the calibration K-factors of the USM_{MM}, at each calibration flow rate, as given in the calibration certificate of the USM_{MM}. The relative uncertainty of the SVP, $E_{SVP,j}$, is calculated according to Eq. (14b), at each calibration flow rate.

⁹ The “overall level” option may be of interest in several cases, such as e.g.: (a) if the user wants a “simple” and quick evaluation of the influence of an instrument uncertainty on the expanded (overall) uncertainty of the oil metering station, (b) in case of a different installation of the oil densitometer (e.g. in-line), (c) in case of a different oil densitometer functional relationship, or (d) in case the input used at the “detailed level” does not fit sufficiently well to the type of input data / uncertainties which are relevant for e.g. the pressure transmitter or temperature element/transmitter at hand.

- **Calibration repeatability.** The user specifies the repeatability (relative standard deviation) of the indicated USM_{MM} flow rate measurement during calibration, $E_{UMM, rept, j}^{cal}$, cf. Eq. (12). That is, the combined repeatability of the USM_{MM} and the small-volume prover (SVP). It can be given in the EMU program to be flow rate dependent.

(2) Proving of the USM_{DM}:

- **USM_{MM} repeatability during proving.** The user specifies the repeatability (relative standard deviation) of the indicated USM_{MM} flow rate measurement during proving, $E_{UMM, rept, \ell}^{prov}$, cf. Eq. (15). It can be given in the program to be flow rate dependent.
- **USM_{DM} repeatability during proving.** The user specifies the repeatability (relative standard deviation) of the indicated USM_{DM} flow rate measurement during proving, $E_{USM, rept, \ell}^{prov}$, cf. Eq. (15). It can be given in the program to be flow rate dependent.
- **Proving deviation factor.** The user specifies the L proving flow rates and the proving K-factors of the USM_{DM}, $K_{prov, \ell}^{USM}$, $\ell = 1, \dots, L$, as determined from proving of the USM_{DM} vs. the USM_{MM}. Normally, $L = 1$. On this basis, the EMU computer program calculates the relative uncertainty E_{dev}^{prov} in an interval of possible metering flow rates around the proving flow rate, according to a method described in [1], cf. also Eq. (18).

(3) USM_{DM} in duty operation (metering):

- **USM_{DM} repeatability during metering.** The user specifies either (a) the repeatability (relative standard deviation) of the indicated USM flow rate measurement in duty operation, $E_{USM, rept}^{met}$, or (optionally) (b) the repeatability (standard deviation) of the measured transit times, $u(\hat{t}_{1i}^{random})$, cf. Eqs. (17) and (18), respectively. Both can be given in the program to be flow rate dependent.
- **USM_{DM} and USM_{MM} body expansions.** The user specifies whether correction for pressure and temperature effects is used or not for the USM_{DM} and USM_{MM} meter bodies, and the uncertainties of the temperature and pressure expansion coefficients, $u(\hat{\alpha})$ and $u(\hat{\beta})$. Cf. Eqs. (25)-(27).
- **Systematic transit time effects.** The user specifies the uncertainty of uncorrected systematic effects on the measured upstream and downstream transit times, $u(\hat{t}_{1i}^{USM, \Delta pc, systematic})$, $u(\hat{t}_{2i}^{USM, \Delta pc, systematic})$, $u(\hat{t}_{1i}^{USM, \Delta mp, systematic})$ and $u(\hat{t}_{2i}^{USM, \Delta mp, systematic})$, cf. Eqs. (28)-(30). These relate to changes from calibration to proving (USM_{MM}), and proving to metering (USM_{DM}).
- **Integration method (installation effects).** The user specifies the uncertainties related to the integration method / installation effects, $E_{I, \Delta pc}$ and $E_{I, \Delta mp}$, cf. Eq. (31). These relate to changes from calibration to proving (for the USM_{MM}), and proving to metering (for the USM_{DM}), respectively.

With respect to the USM technology, the program can thus be run in two modes:

- (A) Completely meter independent, and
- (B) Weakly meter dependent.

Mode (A) corresponds to choosing the overall level for the USM duty operation uncertainty (both for the repeatability and the systematic deviation re. proving). Mode (B) corresponds to choosing the “detailed level”.

By “weakly meter dependent” is here meant that the inner diameter, number of paths and the number of reflections for each path are to be known. However, actual values for the inclination angles, lateral chord positions and integration weights do not need to be known. Only very approximate values for these quantities are needed (used for calculation of certain sensitivity coefficients).

5 UNCERTAINTY EVALUATION EXAMPLE

As an example of evaluation of the expanded uncertainty of a Scenario B USM fiscal oil metering station using the EMU program described in Section 4, consider the metering station instrumentation example given in Table 2: 6" pipeline, a 6" USM duty meter (USM_{DM}) with flow computer, a 6" USM master meter (USM_{MM}), a small-volume prover system (SVP), four Rosemount 3051P Reference Class Pressure Transmitters [21], four Pt 100 4-wire RTD elements and Rosemount 3144 Smart Temperature Transmitters [22], and an on-line (bypass) installed Solartron 7835 Liquid Density Transducer [23]. The type of flow computer and small-volume prover (SVP) are arbitrary (unspecified), but in the example used here, the SVP consists of a compact prover and a turbine transfer meter (TM), treated here as a unit. The 6" USM is a 0.13 m diam. 4-path meter with parallel chords, non-reflecting paths and Gauss-Jacobi integration. A typical North Sea oil is considered [11], with density 776 kg/m³ at densitometer conditions. Line conditions (in the meter run) are taken to be 18 barg and 65 °C (P_m^{met}, T_m^{met}). Densitometer conditions are 19 barg and 64 °C (P_d, T_d). Proving conditions are taken to be 18 barg and 65 °C (for the USM_{DM} and the USM_{MM}, P_m^{prov}, T_m^{prov} and P_M^{prov}, T_M^{prov} , respectively). Calibration conditions are 19 barg and 64 °C (P_M^{cal}, T_M^{cal}).

Evaluation of e.g. the relative expanded uncertainty of the axial volumetric flow rate, kE_{qv} , according to Eq. (6a), involves the USM_{DM}, the USM_{MM}, the small-volume prover (SVP), the pressure and temperature measurements, the densitometer, the signal communication / flow computer, and the oil parameters. The "detailed level" of input is used here. $M = 6$ calibration flow rates are considered, in the range 25 – 280 m³/h (corresponding to a flow velocity range of about 0.5 – 6 m/s). A single proving flow rate is considered (i.e. $L = 1$), at 120 m³/h. Metering flow rates in an interval up to ± 20 m³/h relative to the proving flow rate are considered, i.e. in the interval 100 – 140 m³/h.

The 6 calibration flow rates and a set of example calibration K-factors of the USM_{MM}, $\hat{K}_{cal,j}^{UMM}$, $j = 1, \dots, M$, are given in Table 4, together with an example set of expanded uncertainties of the calibration K-factors $k \cdot u(\hat{K}_{cal,j}^{UMM})$ (given at 95 % conf. lev., with coverage factor $k = 2$).

Other information given in Table 4 (columns 3 - 7) includes the K-factor of the turbine transfer meter (TM) used as part of the SVP calibration system, the uncertainty of this turbine meter K-factor, the accumulated volume measured by the USM_{MM} and TM over the calibration period, and the difference between these two accumulated volumes (the "deviation" resulting from the calibration). This "other information" is included here for convenience, and is not used as input to the uncertainty calculations. That is, only columns 2, 8 and 9 are used as input to the uncertainty calculations presented here.

Table 4. Values from a constructed and tentative example of a calibration certificate for the USM_{MM}, based on calibration of the USM_{MM} using a SVP (Scenario B).

$K_{DFC}^{USM} =$		6000							
$M =$		6							
Calibr. flow rate	Calibration flow rate	K-factor turbine meter,	Unc. (k = 2) K-factor	Accumulated calibr. volume,	Accumulated calibr. volume,	Difference in acc. volume	K-factor USM _{MM}	Unc. (k = 2) K-factor	
rate no.	(m ³ /h)	TM (p/l)	TM (p/l)	TM (m ³)	USM _{MM} (m ³)	USM _{MM} - TM (%)	$K_{cal,j}^{UMM}$ (p/m ³)	USM _{MM} (p/m ³)	
1	25	6.65449	0.00239	2.5146	2.5102	-0.175	5894.3	3.8	
2	58	6.64842	0.00239	4.9422	4.9370	-0.105	5895.7	3.8	
3	95	6.64189	0.00239	8.1954	8.1827	-0.155	5893.6	3.8	
4	155	6.64783	0.00239	13.0666	13.0382	-0.217	5886.3	3.8	
5	210	6.63342	0.00239	15.8647	15.8380	-0.168	5885.4	3.8	
6	280	6.62891	0.00239	20.9535	20.9197	-0.161	5890.6	3.8	

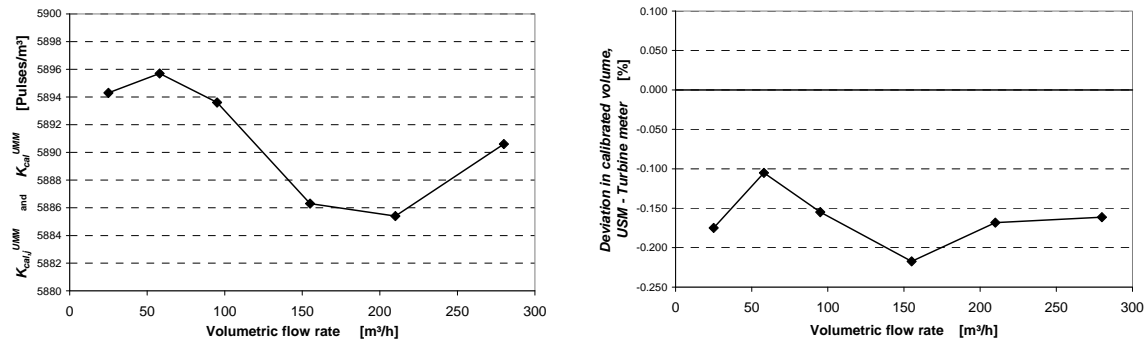


Fig. 2. Example of calibration K-factors of the USM_{MM} (left), and the deviation of the accumulated calibrated volume of the USM_{MM} relative to the SPV (right), as a result of the USM_{MM} calibration, cf. Table 4. Markers indicate the values at the 6 calibration flow rates. In this example, linear interpolation is used for the curves, indicating K-factor and deviation values for flow rates in-between the calibration flow rates.

For calculation of E_{liq}^{Ac} , a number of input uncertainties are to be specified with respect to the liquid (oil) parameters. The relative standard uncertainties of all oil temperature and pressure measurements (four of each see above), E_T and E_p , are taken to be 0.023 % and 0.044 %, respectively. Due to space limitations, it is referred to [1] for details on other liquid (oil) parameter input uncertainties.

For calibration of the USM_{MM} vs. the SVP, a number of input uncertainties are in question, cf. Eqs. (12)-(14). The relative uncertainty of the SVP, $E_{SVP,j}$, is calculated according to Eq. (14b), at each calibration flow rate. The repeatability (relative standard deviation) of the indicated USM_{MM} flow rate measurement during calibration, $E_{UMM,repr,j}^{cal}$, is taken to be 0.0135 %, in agreement with API specifications [7]. This figure represents the combined repeatability of the USM_{MM} and the small-volume prover (SVP). The calibration deviation factor uncertainty, kE_{dev}^{cal} , is calculated to be 0.026 % (100 % conf. lev., rect. distrib.)¹⁰.

For proving of the USM_{DM} vs. the USM_{MM}, several input uncertainties are in question as well, cf. Eqs. (15)-(18). The proving deviation factor uncertainty, kE_{dev}^{prov} , is calculated to be 0.010 % (100 % conf. lev., rect. distrib.). The repeatabilities (i.e. standard deviations) of the USM_{DM} and USM_{MM} proving measurements, $E_{USM,repr,\ell}^{prov}$ and $E_{UMM,repr,\ell}^{prov}$, are both taken to be 0.02 % [12-15].

For the USM in duty operation, the uncertainties in question are given by Eqs. (19)-(31). The USM repeatability in duty operation is tentatively taken to be $E_{USM,repr}^{met} = 0.02$ % (rel. standard deviation) [12-15].

The relative standard uncertainty of the USM_{DM} integration method (accounting for possible changes in installation effects from proving to metering) is taken to be $E_{I,\Delta mp} = 0.01$ %, as a tentative and possibly large example value. Similarly, the relative standard uncertainty of the USM_{MM} integration method (accounting for possible changes in installation effects from calibration to proving) is taken to be $E_{I,\Delta pc} = 0.01$ %, as an example value.

The uncertainties of uncorrected systematic effects on the measured upstream and downstream transit times in USM_{DM} (accounting for possible changes from proving to

¹⁰In the present calculation example given here, all input uncertainties given in the text are taken to correspond to 67 %, 95 % or 100 % confidence levels (depending on type of uncertainty), with normal or rectangular probability distribution, and coverage factor $k = 1, 2$ or $\sqrt{3}$, respectively.

metering), $k \cdot u(\hat{t}_{1i}^{USM, \Delta pc, systematic})$ and $k \cdot u(\hat{t}_{2i}^{USM, \Delta pc, systematic})$, are given as 10 ns with 40 ps time difference (100 % conf. lev., rect. distrib.), respectively, as a tentative example.

The case is considered where pressure and temperature correction is used for the meter body. The uncertainties of the temperature and pressure expansion coefficients $k \cdot u(\hat{\alpha})$ and $k \cdot u(\hat{\beta})$ are set to 20 % (100 % conf. lev., rect. distrib.), as example values. Details are given in [1].

The relative standard uncertainties related to the signal conversion/communication and flow computer calculations, E_{comm} and E_{flocom} , are set to zero in this example.

Fig. 3 shows the relative expanded uncertainty of the volumetric flow rate measurement for this illustration example, plotted over the volumetric flow rate range 100 to 140 m³/h. In this example, calibration of the USM_{MM} is made at 6 calibration points (where the closest calibration flow rates to the measured flow rate are 95 and 155 m³/h, cf. Table 4), proving is made at 120 m³/h, and metering is assumed made in the interval 100 – 140 m³/h.

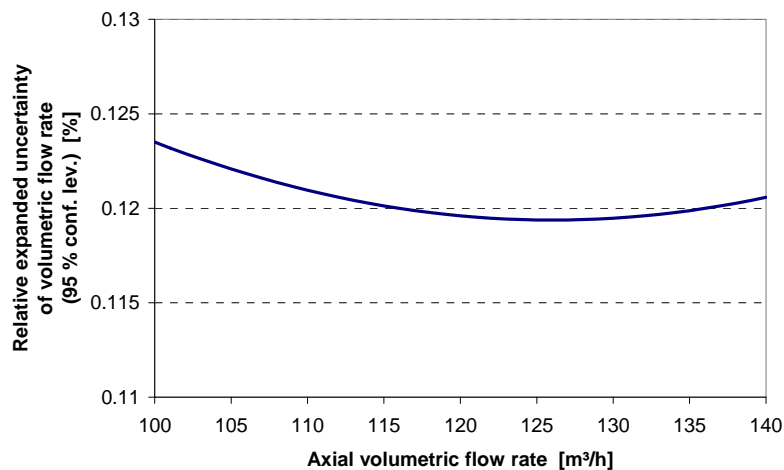


Fig. 3. Illustration example of the relative expanded uncertainty of the volumetric flow rate measurement, kE_{ν} . In this example, calibration of the USM_{MM} is (among other calibration points, cf. Table 4) made at 95 and 155 m³/h, proving is made at 120 m³/h, and metering is assumed made in the interval 100 – 140 m³/h.

The various contributions to the relative expanded uncertainty of the volumetric flow rate measurement may be investigated in further detail at each of the metering flow rates shown in Fig. 3. For example, at a metering flow rate of 110 m³/h, Fig. 3 indicates a relative expanded measurement uncertainty of 0.121 %, at 95 c.l. The bar-chart shown in Fig. 4 gives the relative importance of the various uncertainty contributions at this metering flow rate. The figure indicates that three uncertainty contributions essentially dominate the uncertainty budget in this tentative example; the calibration uncertainty, the proving uncertainty and the uncertainty of the USM in duty operation.

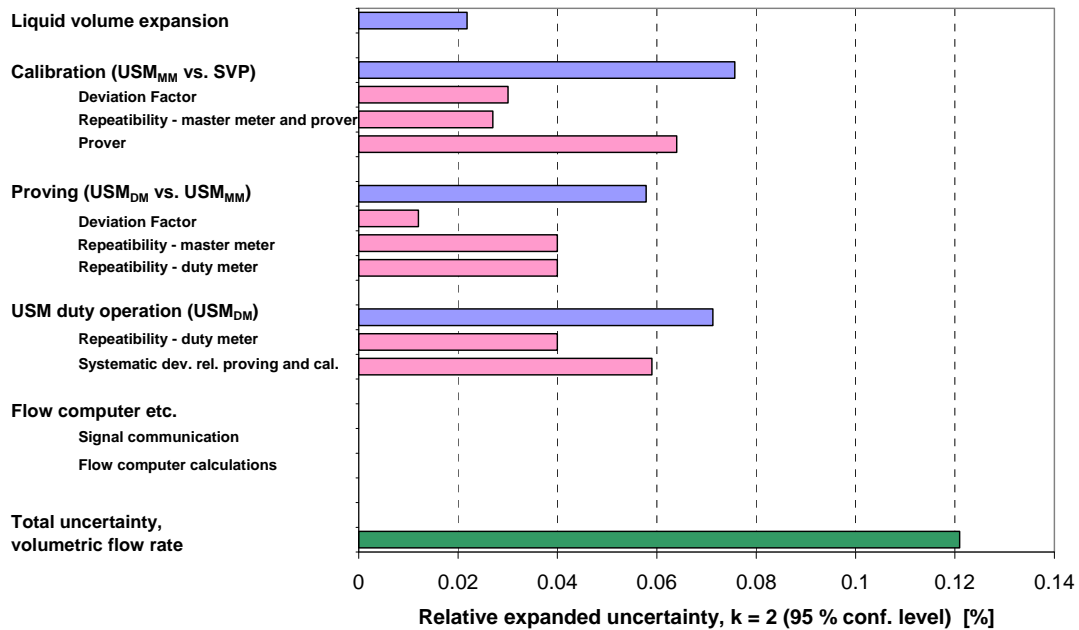


Fig. 4. Bar-chart showing the various contributions to the calculated relative expanded uncertainty of the volumetric flow rate measurement shown in Fig. 3, at a metering flow rate of 110 m³/h. See the text for further information. Each cluster of red bars represent "detail" results which accumulate up to the "sub-main" result given by the blue bar above the red bar cluster. The green bar gives the calculated total relative expanded uncertainty at this metering flow rate.

In Fig. 4, the relative expanded uncertainty of the USM duty measurement is calculated to about 0.071 %. It may be of interest to investigate the relative importance of the various contributions to this uncertainty figure. Fig. 5 shows the contributions to the expanded uncertainty of the USM_{DM}, calculated according to Eqs. (19)-(31). In addition to the miscellaneous effects (which are set to zero here), these are organized in four groups: USM repeatability in duty operation, meter body uncertainty, uncertainty of systematic transit time effects, and the integration method uncertainty (installation effects). In general the latter two groups are the most difficult to specify (only the USM repeatability is normally available from current USM manufacturer data sheets), and only tentative uncertainty figures have been used in the present calculation example, to illustrate use of the program, and demonstrate the sensitivity to these uncertainty contributions. It appears that in this illustration example the uncertainty of the USM_{DM} in duty operation is dominated by the systematic transit time effects and the USM_{DM} repeatability in operation. Other uncertainty contributions are relatively smaller.

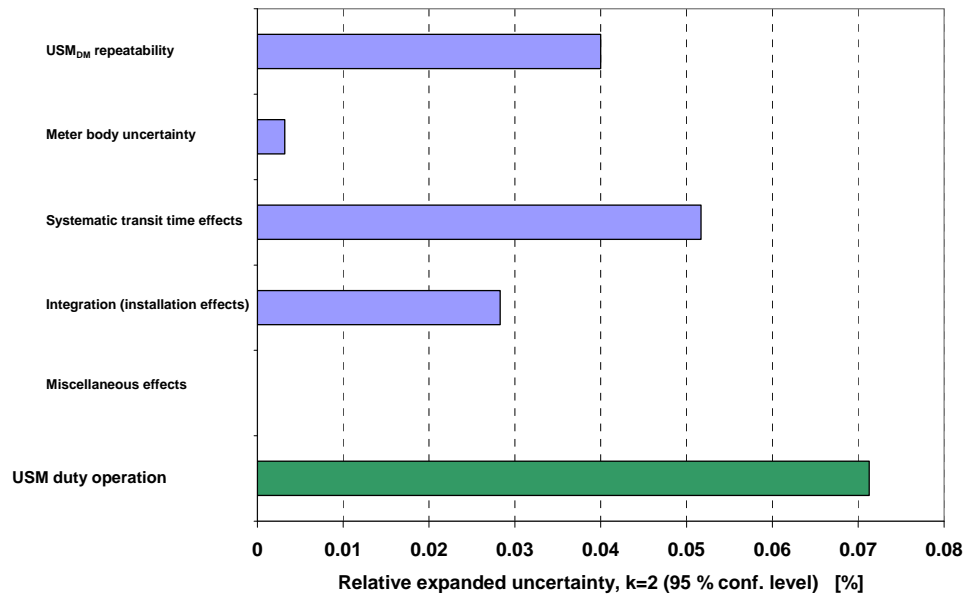


Fig. 5. Bar-chart showing the various contributions to the expanded uncertainty of the USM duty measurement given (by blue colour) in Fig. 4, at the 110 m³/h metering flow rate, calculated according to Eqs. (19)-(31).

Table 5 summarizes the calculation results of this example in an overall uncertainty budget for the volumetric flow rate measurement, \hat{q}_v , for a metering flow rate of 110 m³/h. It appears that the calibration and USM duty uncertainties contribute almost equally to the relative expanded uncertainty of the volumetric flow rate measurement, at this flow rate, followed by the proving uncertainty. All three contribute significantly in this illustration example.

Table 5. Illustration example of an overall uncertainty budget for the USM fiscal oil metering station, for the volumetric flow rate, q_v , and the example input data given in the text, cf. also Figs. 4-5. The calibration flow rates are given in Table 4, proving is made at 120 m³/h, and the metering flow rate is 110 m³/h.

Source	Uncertainty contribution				Combined uncertainty	
	Relative expanded uncertainty	Confidence level & probab. distribution	Cover. factor, k	Relative standard uncertainty	Rel. sens. coeff.	Relative Variance
Liquid volume expansion	0.0218 %	95 % (normal)	2	0.0109 %	1	1.188·10 ⁻⁸
Calibration	0.0757 %	95 % (normal)	2	0.0378 %	1	1.432·10 ⁻⁷
Proving	0.0578 %	95 % (normal)	2	0.0289 %	1	8.362·10 ⁻⁸
USM duty operation	0.0713 %	95 % (normal)	2	0.0356 %	1	1.270·10 ⁻⁷
Signal comm. / flow comp.	Neglected	-	-	-	1	0
Sum of relative variances				$E_{q_v}^2$		3.658·10 ⁻⁷
Relative combined standard uncertainty				E_{q_v}		0.0605 %
Relative expanded uncertainty (95 % confidence level, $k = 2$)				$k \cdot E_{q_v}$		0.1210 %

The uncertainty evaluation example discussed above for illustration purposes, addresses the volumetric flow rate, q_v . Using the *EMU* program, similar analyses can be made for the standard volumetric flow rate, Q_v , and the mass flow rate, q_m [1].

Calculations of this type may be made for the pressure transmitter and the temperature element/transmitters (in the USM_{DM}, the USM_{MM}, the densitometer and the SVP), the densitometer, the reference density, the calibration of the the USM_{MM}, and the proving of the USM_{DM}. For documentation purposes, necessary reporting of input data and calculated results are available, for the three measurands given by Eqs. (1).

6 CONCLUSIONS

The NPD regulations states that “it shall be possible to document the total uncertainty of the measurement system. An uncertainty analysis shall be prepared for the measurement system within a 95 % confidence level” [5]. The *GUM* [2] put requirements to such documentation. To meet such requirements and harmonize procedures and industry practice in the documentation of fiscal measurements, a series of handbooks on uncertainty calculation of fiscal oil and gas metering stations is developed in a cooperation between NFOGM, NPD, Tekna and CMR [9-11,1,4].

In the present paper, the ongoing development of a handbook of uncertainty calculations of ultrasonic fiscal oil metering stations [1] is described. Three different metering station scenarios are being addressed in [1], Scenarios A, B and C, cf. Table 1 and Fig. 1.

The present paper gives a description of Scenario B (cf. Table 2), including an uncertainty calculation example. The example is meant essentially to illustrate and demonstrate the possibilities of the *Handbook* and the accompanying *EMU* program, and does not serve as an uncertainty analysis of a USM fiscal oil metering station.

The expanded uncertainties calculated by the program *EMU - USM Fiscal Oil Metering Stations* may be used in documentation of the metering station uncertainty. That means, provided the user of the program (on basis of manufacturer information or other sources) can document the numbers used for the input uncertainties to the program, the *Handbook* and the program gives procedures for propagation of these input uncertainties.

For traceability purposes the input data to the program (quantities and uncertainties) must be documented by the user. The user must also document that the calculation procedures and functional relationships implemented in the program are in conformity with the ones actually applied in the fiscal oil metering station¹¹.

The uncertainty model for USM fiscal oil metering stations presented in the *Handbook* is based on present-day “state of the art of knowledge” for stations of this type, and is not expected to be complete with respect to description of effects influencing on such metering stations. In spite of that, the uncertainty model does account for a large number of the important factors that influence on the expanded uncertainty of metering stations of this type. It is expected that the most important uncertainty contributions have been accounted for. Evaluation of the effects of these factors on the uncertainty of the metering station should be possible with the uncertainty model and the program developed here.

It is the intention and hope of the partners presenting this *Handbook* that - after a period of practical use of the *Handbook* and the program - the uncertainty model presented here will be subject to necessary comments and viewpoints from users and developers of USMs, and others with interest in this field, as a basis for a possible later revision of the *Handbook*. The overall objective of such a process would of course be that - in the end - a useful and accepted method for calculation of the uncertainty of USM fiscal oil metering stations can be agreed on, in the Norwegian metering society as well as internationally.

¹¹ Especially if the “overall level” options of the program are used, the program should cover a wide range of situations met in practice.

SYMBOL NOTATION

USM_{DM} :	ultrasonic duty meter,
USM_{MM} :	ultrasonic master meter, used for proving of the USM_{DM} (Scenario B),
SVP:	small-volume prover system, used for calibration of the USM_{MM} (Scenario B),
q_v :	axial volumetric flow rate at line (metering) conditions [m^3/h],
Q_v :	axial volumetric flow rate at standard reference conditions (1 atm. and 15 °C) [Sm^3/h],
q_m :	axial mass flow rate [kg/h],
q_{USM} :	axial volumetric flow rate indicated by the USM_{DM} under duty operation (line conditions), before the "proving correction factor", C_{prov} , and the "calibration correction factor", C_{cal} , have been applied [m^3/s],
ρ :	liquid (oil) density in the meter run (at line conditions) [kg/m^3],
ρ_d :	liquid (oil) density in the densitometer [kg/m^3],
ρ_0 :	liquid (oil) density at standard reference conditions (1 atm. and 15 °C) [kg/m^3],
C_{prov} :	correction factor accounting for proving of the USM_{DM} vs. the USM_{MM} in Scenario B (here referred to as the "proving correction factor") [-],
C_{cal} :	correction factor accounting for calibration of the USM_{MM} vs. the SVP in Scenario B (here referred to as the "calibration correction factor") [-],
C_{ℓ}^{prov} :	meter factor determined in proving of the USM_{DM} vs. the USM_{MM} in Scenario B, $\ell = 1, \dots, L$ (here referred to as a "proving meter factor") [-],
C_j^{cal} :	meter factor determined in calibration of the USM_{MM} vs. the SVP in Scenario B, $j = 1, \dots, M$ (here referred to as a "calibration meter factor") [-],
$K_{prov,\ell}^{USM}$:	K-factor of the USM_{DM} at proving flow rate no. ℓ , $\ell = 1, \dots, L$, determined in proving of the USM_{DM} vs. the USM_{MM} in Scenario B [$pulses/m^3$],
$K_{cal,j}^{UMM}$:	K-factor of the USM_{MM} at calibration flow rate no. j , $j = 1, \dots, M$, determined in calibration of the USM_{DM} vs. the SVP in Scenario B, and given in the USM_{MM} calibration certificate [$pulses/m^3$],
K_{DFC}^{USM} :	DFC (digital-to-frequency converter) factor of the USM_{DM} [$pulses/m^3$],
K_{DFC}^{UMM} :	DFC (digital-to-frequency converter) factor of the USM_{MM} [$pulses/m^3$],
L :	number of proving flow rates ("proving points"),
M :	number of calibration flow rates ("calibration points"),
C_{itm} :	volume correction factor (VCF) for the effect of temperature on the liquid (oil) in the meter, re. standard reference conditions [-],
C_{plm} :	volume correction factor (VCF) for the effect of pressure on the liquid (oil) in the meter, re. standard reference conditions [-],
C_{ild} :	volume correction factor (VCF) for the effect of temperature on the liquid (oil) in the densitometer, re. standard reference conditions [-],
C_{pld} :	volume correction factor (VCF) for the effect of temperature on the liquid (oil) in the densitometer, re. standard reference conditions [-].
E_p :	relative combined standard uncertainty of the liquid pressure measurement [-],
E_T :	relative combined standard uncertainty of the liquid temperature measurement [-],
E_{ρ_d} :	relative combined standard uncertainty of the liquid density at line (metering) conditions [-],
E_{ρ_0} :	relative combined standard uncertainty of the liquid density at standard reference conditions (1 atm. and 15 °C) [-],
E_{q_v} :	relative combined standard uncertainty of the axial volumetric flow rate estimate, \hat{q}_v [-],
E_{Q_v} :	relative combined standard uncertainty of the standard axial volumetric flow rate estimate, \hat{Q}_v [-],
E_{q_m} :	relative combined standard uncertainty of the axial mass flow rate estimate, \hat{q}_m [-],

- $E_{liq}^{\Delta c}$: relative combined standard uncertainty of the liquid volume expansion at calibration conditions, involved in the evaluation of E_{qv} [-],
- $E_{liq}^{m\Delta c}$: relative combined standard uncertainty of the liquid volume expansion at metering and calibration conditions, involved in the evaluation of E_{Q_v} [-],
- $E_{liq}^{\Delta m\Delta c}$: relative combined standard uncertainty of the liquid volume expansion at metering and calibration conditions, involved in the evaluation of E_{q_m} [-],
- E_{qv}^B : relative combined standard uncertainty, used for some of the uncertainty contributions to E_{qv} , E_{Q_v} and E_{q_m} [-],
- E_{prov} : relative combined standard uncertainty of the estimate \hat{q}_v , related to proving of the USM_{DM} vs. the USM_{MM} in Scenario B [-],
- E_{cal} : relative combined standard uncertainty of the estimate \hat{q}_v , related to calibration of the USM_{MM} vs. the SVP in Scenario B [-],
- E_{dev}^{cal} : relative standard uncertainty of the calibration curve deviation factor estimate, \hat{C}_{dev}^{cal} [-],
- $E_{UMM, rept, j}^{cal}$: relative combined standard uncertainty representing the repeatability of the USM_{MM} and SVP measurements during calibration, at calibration flow rate no. j , $j=1, \dots, M$ [-],
- $E_{SVP, j}$: relative combined standard uncertainty of, $\hat{q}_{SVP, j}$, the volumetric flow rate of the liquid measured by the SVP at calibration flow rate no. j , $j=1, \dots, M$ [-],
- $C_{dev, j}^{cal}$: calibration deviation factor at calibration flow rate no. j , $j=1, \dots, M$ [-],
- $Dev_{C, j}^{cal}$: corrected relative calibration deviation (i.e. the relative deviation after multiplication with the calibration correction factor, C_{cal}), at calibration flow rate no. j , $j=1, \dots, M$ [-],
- $Q_{UMM, j}^{cal}$: standard volumetric flow rate measured by the USM_{MM} during calibration, at calibration flow rate no. j , $j=1, \dots, M$ [m³/s],
- $Q_{SVP, j}$: standard volumetric flow rate as measured by the small-volume prover during calibration, at calibration flow rate no. j , $j=1, \dots, M$ [m³/s],
- E_{dev}^{prov} : relative standard uncertainty of the proving curve deviation factor estimate, \hat{C}_{dev}^{prov} [-],
- $E_{USM, rept, \ell}^{prov}$: relative combined standard uncertainty representing the repeatability of the USM_{DM} measurement during proving, at proving flow rate no. ℓ , $\ell=1, \dots, L$ [-],
- $E_{UMM, rept, \ell}^{prov}$: relative combined standard uncertainty representing the repeatability of the USM_{MM} measurement during proving, at proving flow rate no. ℓ , $\ell=1, \dots, L$ [-],
- $C_{dev, \ell}^{prov}$: proving deviation factor at proving flow rate no. ℓ , $\ell=1, \dots, L$ [-],
- $Dev_{C, \ell}^{prov}$: corrected relative proving deviation (i.e. the relative deviation after multiplication with the proving correction factor, C_{prov}), at proving flow rate no. ℓ , $\ell=1, \dots, L$ [-],
- $q_{USM, \ell}^{prov}$: volumetric flow rate measured by the USM_{DM} during proving, at proving flow rate no. ℓ , $\ell=1, \dots, L$ [m³/s],
- $q_{UMM, \ell}^{prov}$: volumetric flow rate measured by the USM_{MM} during proving, at proving flow rate no. ℓ , $\ell=1, \dots, L$ [m³/s],
- E_{USM} : relative combined standard uncertainty of the estimate \hat{q}_v , related to duty operation of the USM_{DM} [-],
- $E_{USM, rept}^{met}$: relative combined standard uncertainty (i.e. relative standard deviation) representing the repeatability of the USM_{DM} measurement during metering, at the metering flow rate in question [-],
- $E_{USM, \Delta mpc}$: relative combined standard uncertainty of the estimate \hat{q}_v , due to systematic effects in the USM_{DM} and USM_{MM} measurements, caused by changes in conditions from calibration to proving (USM_{MM}), and from proving to metering (USM_{DM}) [-],

- E_{misc} : relative combined standard uncertainty of the estimate \hat{q}_v , due to miscellaneous systematic effects in the USM_{DM} and/or USM_{MM} measurements which are not eliminated by calibration and proving, and which are not covered by other uncertainty terms accounted for here (e.g. inaccuracy of the USM functional relationship (the underlying mathematical model), etc.) [-],
- $E_{body,\Delta mpc}$: relative combined standard uncertainty of the estimate \hat{q}_v , due to changes of the USM_{DM} and USM_{MM} meter bodies from calibration to proving (USM_{MM}) and from proving to metering (USM_{DM}). That is, uncertainty of the meter body inner radius, \hat{R} , the lateral chord positions of the N acoustic paths, \hat{y}_i , and the inclination angles of the N acoustic paths, $\hat{\phi}_i$, $i = 1, \dots, N$, caused by possible deviation in pressure and/or temperature between calibration, proving and metering [-],
- $E_{time,\Delta mpc}$: relative combined standard uncertainty of the estimate \hat{q}_v , due to systematic effects on the transit times of the N acoustic paths, \hat{t}_{1i} and \hat{t}_{2i} , $i = 1, \dots, N$, caused by possible deviation in pressure and/or temperature from calibration to proving (USM_{MM}), and from proving to metering (USM_{DM}) [-],
- $E_{I,\Delta mpc}$: relative standard uncertainty of the estimate \hat{q}_v related to the USM integration method, due to changes of installation conditions from calibration to proving (USM_{MM}), and from proving to metering (USM_{DM}) [-],
- $E_{rad,\Delta mpc}$: relative combined standard uncertainty of the estimate \hat{q}_v , due to uncertainty of the meter body inner radius, \hat{R} , caused by possible deviation in pressure and/or temperature from calibration to proving (USM_{MM}), and from proving to metering (USM_{DM}) [-],
- $E_{chord,\Delta mpc}$: relative combined standard uncertainty of the estimate \hat{q}_v , due to uncertainty of the lateral chord positions of the N acoustic paths, \hat{y}_i , $i = 1, \dots, N$, caused by possible deviation in pressure and/or temperature from calibration to proving (USM_{MM}), and from proving to metering (USM_{DM}) [-],
- $E_{angle,\Delta mpc}$: relative combined standard uncertainty of the estimate \hat{q}_v , due to uncertainty of the inclination angles of the N acoustic paths, $\hat{\phi}_i$, $i = 1, \dots, N$, caused by possible deviation in pressure and/or temperature from calibration to proving (USM_{MM}), and from proving to metering (USM_{DM}) [-],
- $E_{KP,\Delta mpc}$: relative combined standard uncertainty of the radial pressure correction factor for the two USM meter bodies, K_P , accounting for changes in conditions from calibration to proving (USM_{MM}) and proving to metering (USM_{DM}) [-],
- $E_{KT,\Delta mpc}$: relative combined standard uncertainty of the radial temperature correction factor for the two USM meter bodies, K_T , accounting for changes in conditions from calibration to proving (USM_{MM}) and proving to metering (USM_{DM}) [-],
- E_{comm} : relative standard uncertainty of the estimate \hat{q}_v , due to signal communication between the USM field electronics and the flow computer (e.g. the flow computer calculation of frequency (pulses/s) in case of frequency output) [-],
- E_{flocom} : relative standard uncertainty of the estimate \hat{q}_v , due to uncertainties in the flow computer calculations [-],
- k : coverage factor [-],
- \hat{x} : estimated value of a quantity, x ,
- $u(\hat{x})$: standard uncertainty of an estimated quantity, \hat{x} ,
- $u_c(\hat{x})$: combined standard uncertainty of an estimated quantity, \hat{x} ,
- $s_{x,y}^*$: relative sensitivity coefficient.

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