

Limits on Achieving Improved Performance from Gas Ultrasonic Meters and Possible Solutions

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1 INTRODUCTION

At the 2010 NSF MW, we presented a paper [Ref 1] covering the last 25 years of USM development. The paper chronicled the progression and adoption of gas custody transfer ultrasonic meters from the perspective of technology development, application experience, standards development and path configuration. During session 6 of the same conference, a paper [Ref 2] was presented on a 32-path USM, which had a similar un-calibrated uncertainty (0.2 to 0.3%) as conventional multipath meter. Surprised by the fact that introducing significantly more paths did not improve the meter uncertainty, Klaus asked if 100-paths would help improve the un-calibrated uncertainty. The surprising answer from the paper's author was **NO**. This was surprising in the sense that more is ordinarily assumed to mean better.

The 32-path meter has 16 paths at $z/R = 0.25$ and 16 paths at $z/R = 0.75$, which presents a detailed velocity profile. With fully developed flow the profile was two perfectly concentric rings. With disturbed flow several points were thrown off the two perfectly concentric rings. This certainly detected the presence of a disturbance, but it was not at all obvious how to improve the uncertainty. Some point shifts were positive and some negative, so to a first order the integration technique gave a reasonably accurate average flow. It was neither easy to estimate the direction nor the magnitude of any error.

This interaction at the 2010 workshop inspired the motivation for this present paper. In particular it begged the question, what are the limitations on achieving improved performance from gas ultrasonic meters and what are the possible solutions to these limitations.

2 SOME BASIC LIMITATIONS

Limitations in accuracy can range from the practical reality of using this technology in industrial applications to the highly complex, multi-physics nature of the device and its interaction with flowing gas.

The first and most obvious limitation relates to the high pressure, high flow, natural gas calibration facilities. These have typical uncertainties of 0.2% to 0.3%, so it is difficult to achieve higher absolute accuracies. However, the repeatability of these calibration facilities is better, making it possible to detect smaller differences in meter response. This fact alone serves to limit the meter uncertainty since the product cannot have a stated uncertainty that is better than the laboratory. Moreover, what happens to this characterised uncertainty when the meter arrives at site? Is it possible to truly quantify the change in uncertainty due to installation conditions?

A more complex limitation comes from the fact that the fundamental physical interaction between the 3-D acoustic beam and the 3-D turbulent flow is extremely complex and not well understood. The process of launching the acoustic beam from the transmitting transducer into the flow, the acoustic beam travelling through the turbulent flow and the generation of a signal in the receiving transducer are difficult to analyse. At present simple 1-D ray theory is the most predominant approach to predict meter behaviour. We are not aware of any multi-physics tools that are presently available to tackle these fundamental problems.

3. OTHER LIMITATIONS

The USM is not a bulk flow meter (like the orifice or turbine), where **all** the flow passes through the meter to generate a signal related to **all** the flow, but rather it is a **sampling** device. It samples along multiple path (line average) velocities, uses an integration technique to calculate the (area average) flow velocity and the meter area to calculate the flow.

Most flow meters perform best when presented with fully developed turbulent flow: a time averaged velocity and turbulence profile that does not change with increasing pipe length. Disturbed flow (swirl, asymmetry, cross flow, etc) presents a non-steady measurand to the flow meter that gradually changes towards fully developed flow as it moves downstream.

A turbine meter that measures just one angular velocity or an orifice plate that measures just one differential pressure cannot detect or compensate for disturbed flow. The multipath USM produces much more information and diagnostics that can potentially detect and compensate for disturbed flow, because it is a sampling system. This is a major advantage of a sampling system over a bulk meter,

To investigate these ideas we will use a Power Law to describe the velocity profile and the Westinghouse [Ref 3] path numbers, locations and integration techniques.

3.1 The Power Law Profile

$$\frac{v}{V_{MAX}} = \left(\frac{y}{R}\right)^{1/n} \quad (1)$$

$$y = R - r \quad (2)$$

v = the velocity at **y**, the distance from the pipe wall
r = is the distance from the centre of the pipe
R = the radius of the pipe
V_{MAX} = the velocity at centre of the pipe
n = the **Power**

$$\text{The Flow } Q = V_{AVG} \pi R^2 = \int_0^R 2 \pi r v dr \quad (3)$$

From this integration of the power law we can get

$$\frac{V_{MAX}}{V_{AVG}} = \frac{(n+1)(2n+1)}{2n^2} \quad (4)$$

This allow us to show the profile as **v / V_{AVG}**

Fig 1 does so for **n = 8**, which corresponds to Reynolds number of about 10⁶.

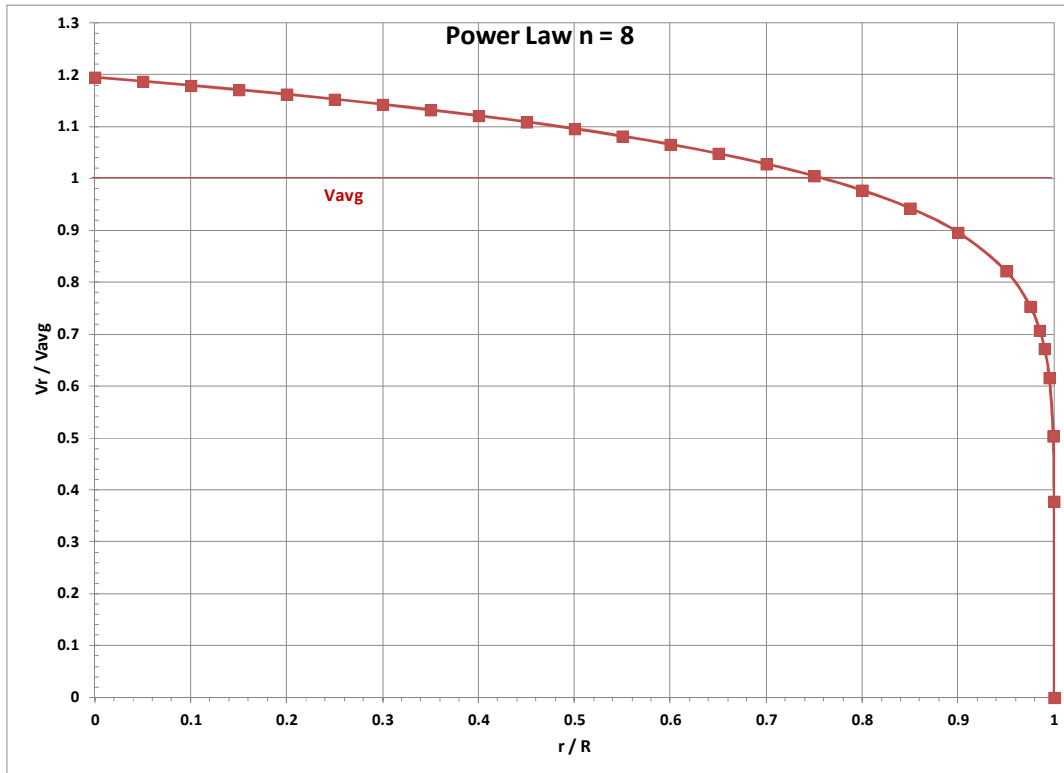


Fig 1. Point velocity profile in terms of v / V_{AVG} as a function of r/R

Note that V_{AVG} occurs at $r/R = 0.75$, which is the bases of a single point velocity measurement placed at the three-quarter radius, or $R/4$ from the wall [Ref 4]. Typically, V_{MAX} is about $= 1.20 V_{AVG}$ and to sample lower velocities one needs to get closer to the wall.

The power law is an approximation with obvious limitations:

- There is a discontinuity at the centreline; however there is no flow at the centreline (no area).
- The flow at the wall is far more complex than the simple power law; there is a wall layer, core region and a matching zone between them.

The power law is used because it is simple to integrate and differentiate analytically.

We can look at the distribution of flow by showing the integral $Q = \int_0^R 2 \pi r v dr$ in Fig 2.

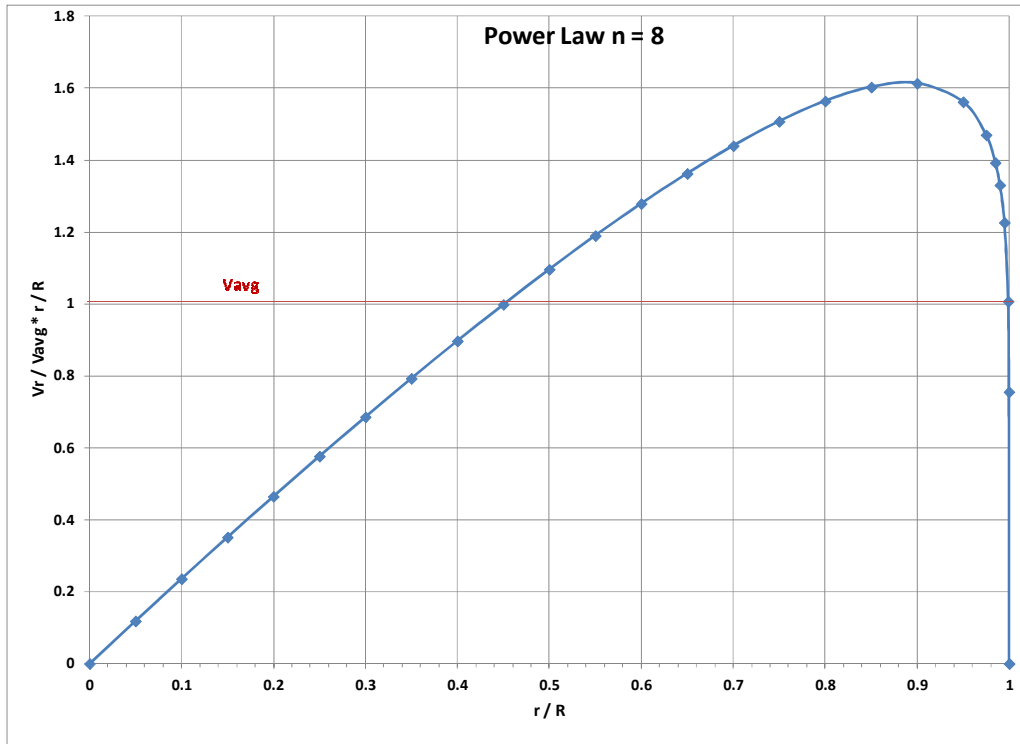


Fig 2. Flow profile in terms of \mathbf{v}^*r as a function of r/R

The area under the curve is the volume flow rate and is the same as the rectangular area under V_{AVG} . The maximum increment in volume flow is associated with $r/R = 0.9$, which is close to the wall.

We can integrate this curve to show the cumulative volume flow in Fig 3.

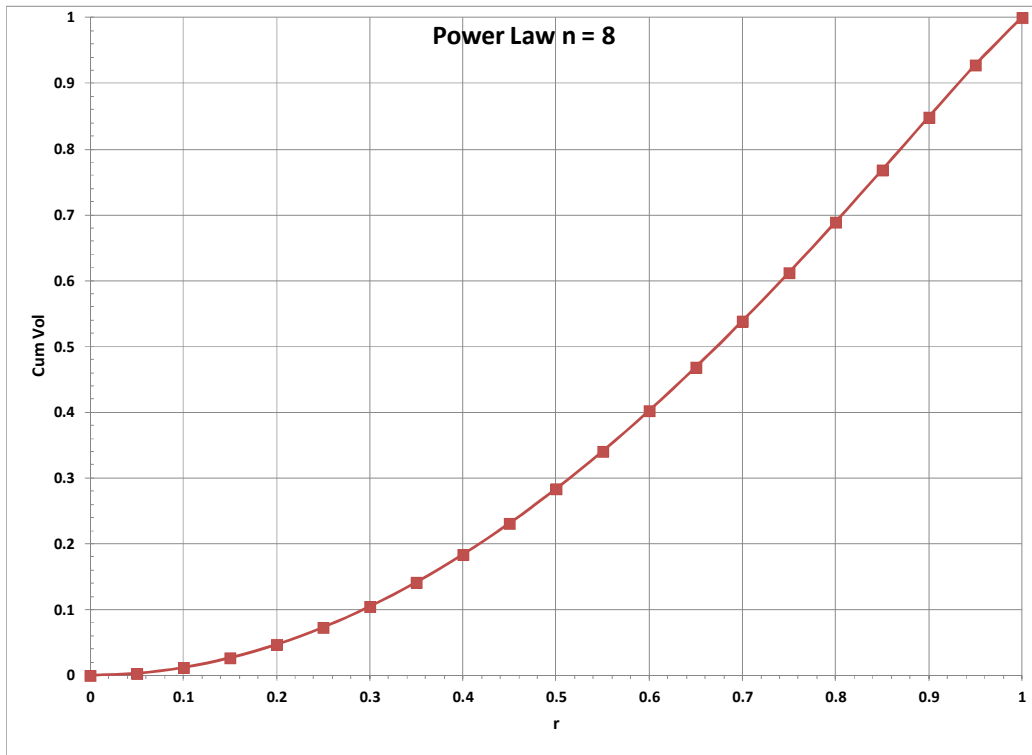


Fig 3. Cumulative volume flow as a function of r/R

The 10% of the pipe radius closest to the centre (r/R from 0 to 0.1) accounts for 1.2% of the volume flow. The 10% of the pipe radius closest to the wall (r/R from 0.90 to 1.0) accounts for 15.1% of the volume flow, while the 5% of the pipe radius closest to the wall (r/R from 0.95 to 1.0) accounts for 7.2% of the volume flow. Note that the area increases as r^2 , but the velocity decreases to zero at the wall ($r/R = 1$).

We see from the power law that the wall zone contains much of the volume flow and the greatest velocity gradient. This is important as it suggests that one of the paths (samples) should be taken close to the pipe wall to account for these effects. By sampling closer to the wall will the integration technique yield a better result? Challenges associated with doing that will be discussed later in the paper.

3.2 The Westinghouse Integration Technique

This is part of the general numerical integration process known as Gaussian-Quadrature. It is based on the integration of polynomials and many schemes exist depending on the type of polynomial e.g. Legendre, Chebycheff, Jacobi, Labatto, etc. They all use the values of the integrand at a few specific points to estimate the total integral.

This is specifically useful for the ultrasonic meter, because it samples the velocity on a few chords and we wish to know the average velocity over the whole pipe area. Westinghouse uses Chebycheff polynomials of the second type to decide path locations and weighting factors.

Note that the different integration schemes give different answers for the same profile, with typically a $\pm 1\%$ spread. Giving rise to a further need for flow calibration and the possible use of additional corrections.

Westinghouse gives the location of the paths as \mathbf{z} , the distance from the centre normal to the path as a proportion of the pipe radius \mathbf{R} . It also gives a weighting factor \mathbf{W} associated with \mathbf{z} that is used in the numerical integration:

$$V_{AVG} = \sum_1^n V_i W_i \quad (5)$$

\mathbf{n} = number of paths, \mathbf{V}_i = path velocity from 1 to \mathbf{n} and \mathbf{W}_i = path weight from 1 to \mathbf{n} .
Once \mathbf{n} is chosen \mathbf{z}_i and \mathbf{W}_i are fixed constants.

Westinghouse values of \mathbf{z} and \mathbf{W} are given for \mathbf{n} from 2 to 7 in Table 1.

	Radial	Position		Center	Radial	Position		Sum W
z2			0.5000		-0.5000			
W2			0.5000		0.5000			1.0000
z3			0.7071	0.0000	-0.7071			
W3			0.2500	0.5000	0.2500			1.0000
z4		0.8090	0.3090		-0.3090	-0.8090		
W4		0.1382	0.3618		0.3618	0.1382		1.0000
z5		0.8660	0.5000	0.0000	-0.5000	-0.8660		
W5		0.0833	0.2500	0.3333	0.2500	0.0833		1.0000
z6	0.9010	0.6235	0.2225		-0.2225	-0.6235	-0.9010	
W6	0.0538	0.1746	0.2716		0.2716	0.1746	0.0538	1.0000
z7	0.9239	0.7071	0.3827	0.0000	-0.3827	-0.7071	-0.9239	
W7	0.0366	0.1250	0.2134	0.2500	0.2134	0.1250	0.0366	1.0000

Table 1. Westinghouse values of **z** and **W**

Note that all paths are symmetrical about the centre, when **n** is odd, one path is on the centre and ΣW is always one. This is important for two reasons:

1. If the flow is uniform: $V_1 = V_2 \dots = V_n = V_{AVG}$, then the integration gives the correct answer.
2. We can interpret **W** as the fractional pipe area associated with the path velocity and the total area is 1 (Fig 4).

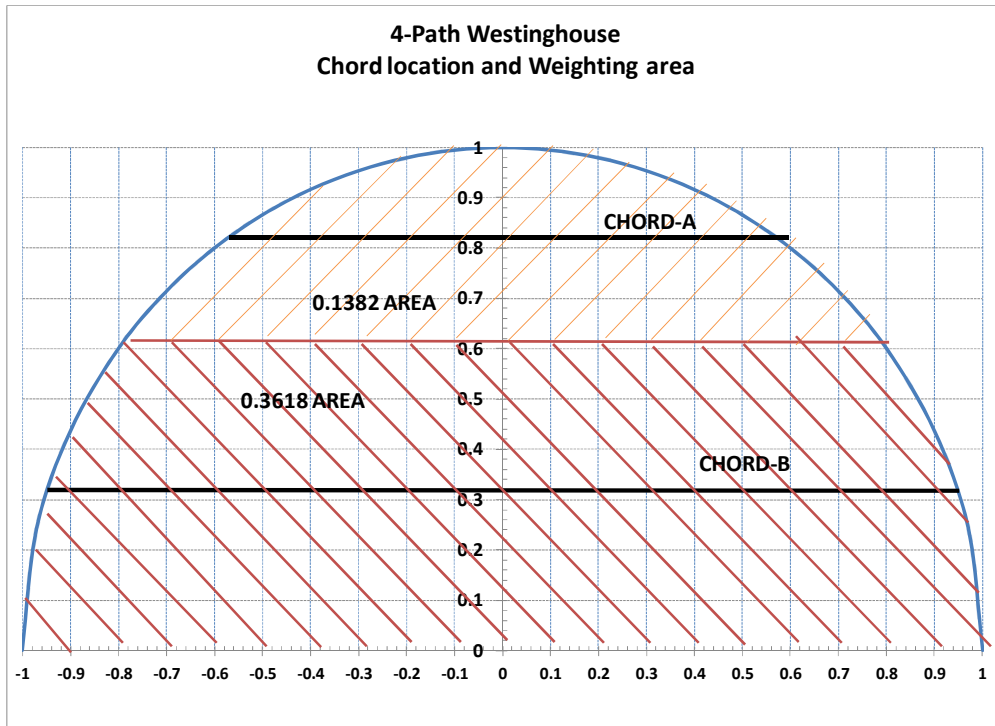


Fig 4. Velocity-Area representation of Westinghouse integration for **n** = 4

Chord-A at $z/R = 0.809$ covers 13.82% of the pipe area. Chord-B at $z/R = 0.309$ and covers 36.18% of the pipe area. Together this is (13.82+36.18) 50% of the pipe area, cords C & D do the same for the other half.

We can see that as the number of paths increase the outer path get closer to the pipe wall.

To associate a path velocity with the location we can use the Power Law directly for the point velocity and a line integral for the path velocity (Fig 5).

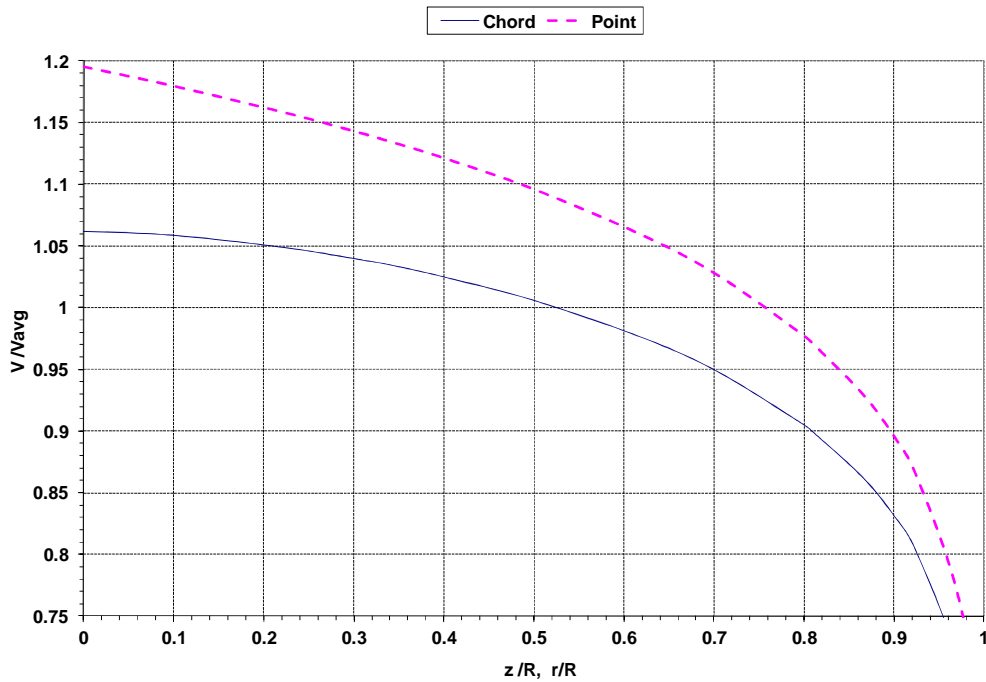


Fig 5. Point and Chord Velocities for Power Law $n = 8$

We can now associate the outer path average line velocity V to the area average V_{AVG} with the number of paths n (Table 2).

n	2	3	4	5	6	7
z/R	0.5000	0.7071	0.8090	0.8660	0.9010	0.9239
V/Vavg	1.0058	0.9470	0.9001	0.8622	0.8308	0.8043

Table 2. Outer path velocity with n

Note that it is difficult to sample lower velocities.

We can now see if increasing the number of paths improves the Westinghouse integration by using the $n = 8$ Power Law [Fig 6]. Here we will test the argument that sampling near the wall will yield a better integration result.

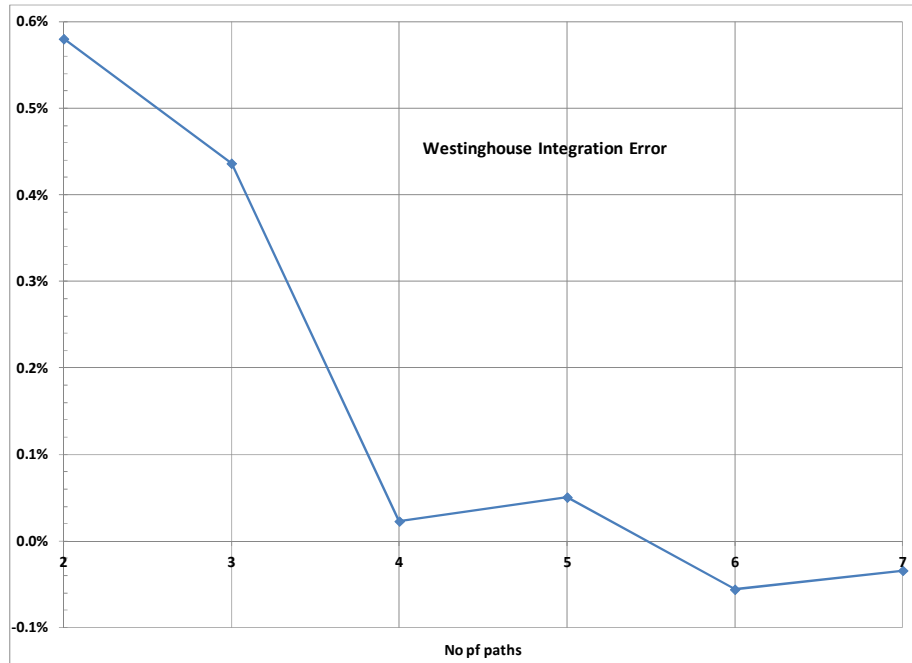


Fig 6. The Westinghouse integration improvement with number of paths

The results shown in Fig 6 suggest that with more than four paths there is little improvement. The idea proposed earlier was that paths closer to the wall should improve the integration; however this has been shown not to be the case once the number of paths reaches four. However this simple theoretical line integral has ignored many practical problems that occur near the walls:

- Steep velocity gradient
- High shear stresses
- High turbulence
- Acoustic refraction
- Acoustic reflection
- Disturbance from transducer port geometry
- The Power Law approximation

To mitigate some of the limitations of the power law, we answered the question using CFD. This yielded an almost identical result, see figure 7. Here the situation was replicated by using a fully developed flow with $Re = 1 \times 10^6$ and performing CFD integrations for 2 to 7 paths. The results were remarkably similar to the simple power law approach.

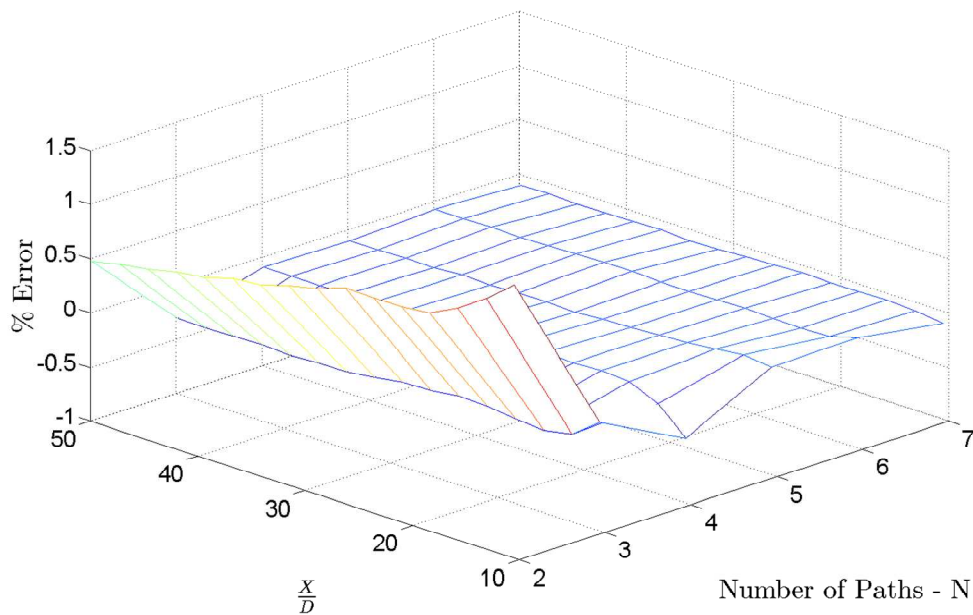


Fig 7. Simulated error versus number of Westinghouse paths using CFD

We postulate that this problem with sensing what is close to the wall is a potential limit on the USM accuracy. However our simple power law analysis plus a more rigorous CFD analysis showed this not to be the case.

Note that for the Instronet meter bounce paths [Ref 5]:

- Single bounce has $z/R = 0$
- Double bounce, inscribed triangle, has $z/R = 0.500$
- Triple bounce, inscribed square, has $z/R = 0.707$

The 32-path meter [Ref 1] has:

- 16 paths at $z/R = 0.25$
- 16 paths at $z/R = 0.75$

Neither of these meters have paths close to the wall. This is also why increasing the 32-path meter from 32 to 100 at the same **locations** would not help, as it would add no additional information about the velocity profile.

• 3.3 Manufacturing tolerances

We have assumed that the paths are perfectly placed at the correct locations and that the acoustic signal is a simple ray travelling from one transducer to the other. The real situation is more complex:

- Mechanical tolerances on the body, ports, transducers and mounts
- The acoustic signal is a three-dimensional wave
- The physical and acoustic axis of the transducer might not be the same
- The transducer port geometry is difficult to define

This all leads to errors in the precise location and geometry of the acoustic paths.

The ultrasonic meter measures transit times and calculates velocity by distance / time. Two distances are important; the distance between the transducers and the axial distance travelled in the flow. It is not easy to measure these distances directly, leading to potential errors.

Fundamentally length and time can be measured with great precision, but again these practical problems make calibration necessary. It is not always obvious if the calibration is correcting a time or length error.

We can estimate errors associated with path location by using the information in Fig 5 to calculate the slopes of the curve at the path locations. Again we do this for a 4-path Westinghouse meter with the $n = 8$ power law profile.

			For $dz = 0.01 = 1\%$ of R					
$z (r/R)$	$V (Vch/Vav)$	dV/dz	dV	% dV	Wt	% Flow	n Cords	$n \cdot U^2$
0.309	1.0385	-0.1241	-0.0012	-0.1195	0.3618	-0.0432	2	0.0037
0.809	0.9000	-0.5905	-0.0059	-0.6561	0.1382	-0.0907	2	0.0164
							Sum	0.0202
							Sq Rt	0.1421

dV/dz is negative because as z increases towards the wall V decreases. We look at a change of $dz = 0.01$ or 1% of R , the pipe radius. The slope (dV/dz) and velocity change (dV) at $z = 0.809$ is about five times that at $z = 0.309$, while with the weighting, the flow change is double. Treating the 4 paths as random, the probable flow error would be $\pm 0.142\%$, which is not very large, but could be significant as a bias.

4. POSSIBLE SOLUTIONS:

The solutions are based on trying to mitigate the problems identified above

4.1 Steep velocity gradient

The nature of real viscous fluids (however small the viscosity) will always produce a steep velocity gradient close to the wall, which can affect the integration accuracy. A better fundamental understanding of fluid mechanics / boundary layer theory could help understand this process.

4.2 High turbulence

A contraction into the meter can make a more uniform profile and less turbulent flow [Ref 6]. However it will have a steeper velocity gradient at the wall, not be stable (since the flow has been pushed from equilibrium) and probably affect the integration. A second consequence is that a more forced profile also reduces the sensitivity of the diagnostics.

It is possible to reduce turbulence with a flow conditioner having many small holes, which reduce the scale and increase the frequency of the turbulence [Ref 7]. However it too is not stable and will tend towards fully developed flow. It is also prone to blockage by trash, scale or hydrate.

A better model of acoustic interaction with turbulence is needed to help understand the process [Ref 8].

4.3 Acoustic refraction

Differences in temperature, density, speed of sound and velocity can all cause refraction. It will be different when the signal is propagated upstream or downstream. A better model of acoustic interaction with flow is needed to help understand the process.

4.4 Acoustic reflection

As a path gets closer to the wall the difference in transit time between a direct path and a reflected path from the wall become smaller. This does not affect the leading edge of the signal, but could cause interference if the rest of the signal is used to as part of the process to detect the signal arrival time.

4.5 Disturbance from transducer port

Most meters place the transducers half in half out of the port to put the transducer centreline at the pipe wall. The protrusion produces a flow disturbance. The British Gas USM has the transducer recessed in the port (to allow pigging).

4.6 Flow disturbances can reduce the integration accuracy [Ref 9]

Use a flow conditioner to establish a fixed stable profile. Although conditioners claim to be **isolating** they may not be able to eliminate all Re and disturbance effects. Any disturbance caused by the flow conditioner will be changing towards fully developed flow. Note that a flow conditioner is an obstruction that causes a pressure loss. It can also catch debris, which would change the velocity profile and defeat the object.

4.7 Use a Reynolds Number correction on all of the individual paths

It is a common misconception that Reynolds number corrections are valid for industrial equipment in field applications. Reynolds number corrections are only valid for fully developed smooth pipe flow. The velocity profile depends on both Re and pipe roughness since the single factor that defines velocity profile is friction factor, which itself depends on Re and pipe roughness.

A potential alternative is to use a **profile factor** correction, however this assumes it is unique and that the relationship between the profile factor and error is known.

4.8 Integration accuracy

In numerical analysis, a **quadrature rule** is an approximation of the definite integral of a function, usually stated as a weighted sum of function values at specified points within the domain of integration. An n -point **Gaussian quadrature rule**, named after Carl Friedrich Gauss, is a quadrature rule constructed to yield an exact result for polynomials of degree $2n - 1$ or less by a suitable choice of the points x_i and weights w_i for $i = 1, \dots, n$. The domain of integration for such a rule is conventionally taken as $[-1, 1]$, so the rule is stated as

$$\int_{-1}^1 f(x) dx \approx \sum_{i=1}^n w_i f(x_i).$$

Gaussian quadrature will only produce accurate results if the function $f(x)$ is well approximated by a polynomial function within the range $[-1, 1]$. Perhaps a typical velocity profile is not well represented by a polynomial.

Our discussion of flow distribution in a circular pipe (Fig 3) shows that most of the flow (70%) is in the outer half of the pipe, but the weighting is low there.

5. PROFILE FACTOR CORRECTIONS

To examine how this correction might work we will look at two cases: an asymmetric profile and a symmetric profile. Again we will use the Power Law and the 4-path Westinghouse integration.

5.1 Asymmetric Velocity Profile

The asymmetry is produced by moving the cords of a 4-path meter off centre on a Power Law ($n = 8$) profile, with the results shown in (Fig 8).

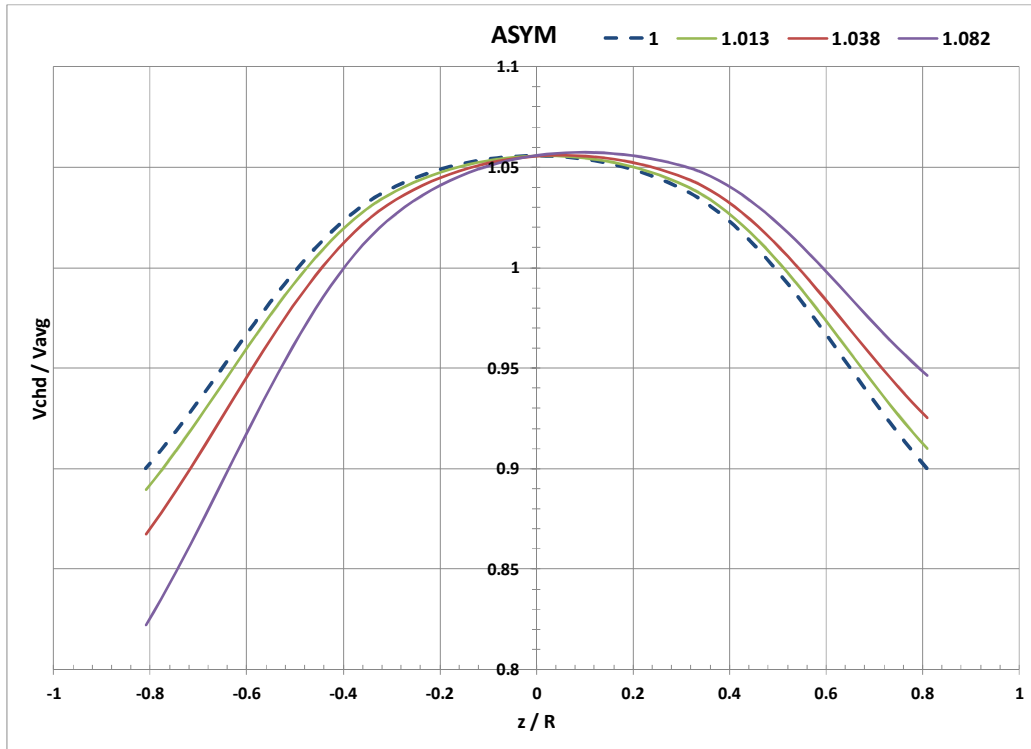


Fig 8. Asymmetric velocity profiles

We can calculate the flow error by using the Westinghouse integration as the actual profile has not changed, just where it is sampled (Table 3).

	Error	Asym	ProFct	Cross
	0.000%	1.0000	1.1538	1.0000
	-0.017%	1.0131	1.1543	1.0081
	-0.131%	1.0375	1.1579	1.0233
	-0.570%	1.0817	1.1723	1.0520
Change	-0.57%	8.17%	1.85%	5.20%

Table 3. Error due to asymmetry

Where for the 4-path Westinghouse (BG crossed paths) meter

$$\text{Asym} = \text{Asymmetry} = (V_a + V_b) / (V_c + V_d) \quad (6)$$

$$\text{ProFct} = \text{Profile Factor} = (V_b + V_c) / (V_a + V_d); \quad (7)$$

$$\text{Cross} = \text{Cross Flow} = (V_a + V_c) / (V_b + V_d) \quad (8)$$

See the results in (Fig 9)

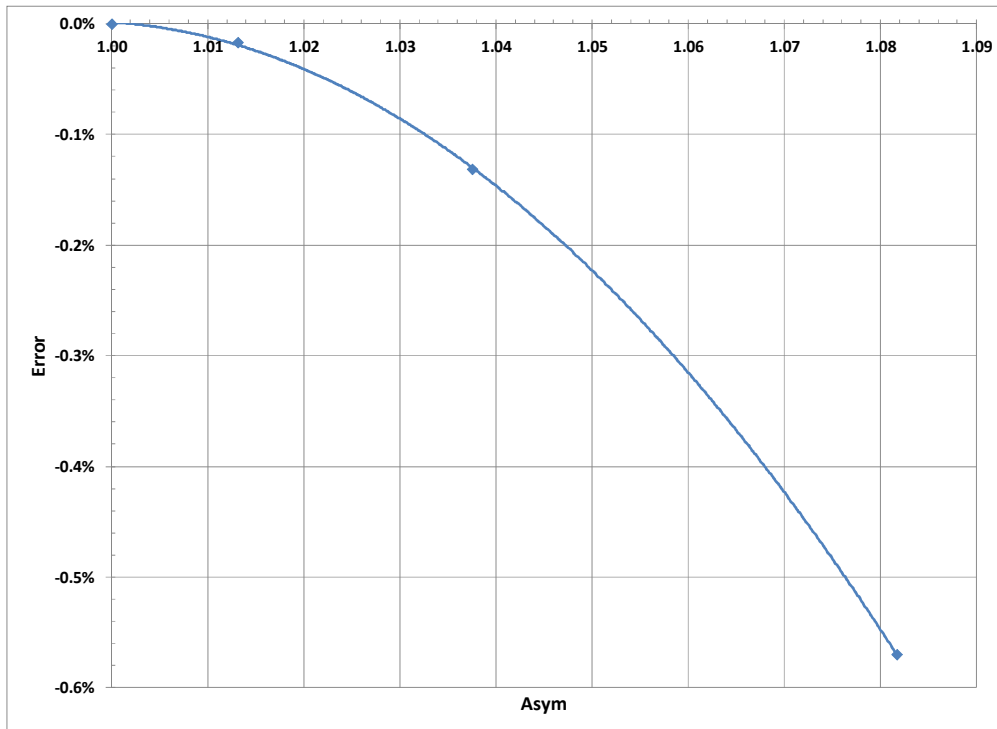


Fig 9. Flow error with asymmetry

The change in ProFct is negligible and it takes a change in Asym > 3% to give an error > 0.1%. The integration technique is robust and can cope with modest profile disturbances. Asymmetry causes the velocity on one side of the centreline to increase and on the other side to decrease. As the velocity decreases the slope gets steeper and the magnitude decreases more than the increase on the other side. Thus the net effect is a reduction in flow and a negative error. The cross flow is an artefact of the way asymmetry was produced without changing the actual flow profile.

This gives an insight into possible profile corrections.

5.2 Symmetric Velocity Profile

Symmetrical profiles are produced by changing the Power Law n values from 5 to 12 for the 4-path Westinghouse meter, with the results shown in (Fig 9).

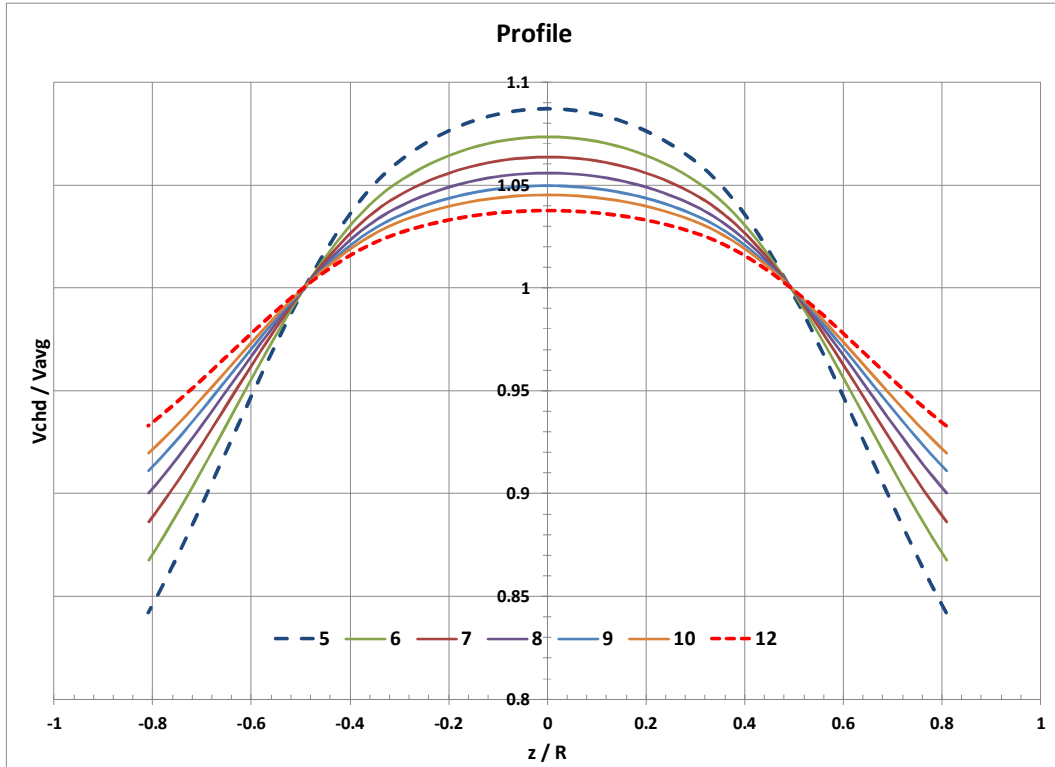


Fig 10. Symmetrical profiles for Power Law n values

It is striking that all profiles pass through $V_{chd}/V_{avg} = 1.00$ at $z/R = 0.500$

We can calculate the flow error by using the Westinghouse integration, the ProFct from equation (7) and show the result in (Fig 11)

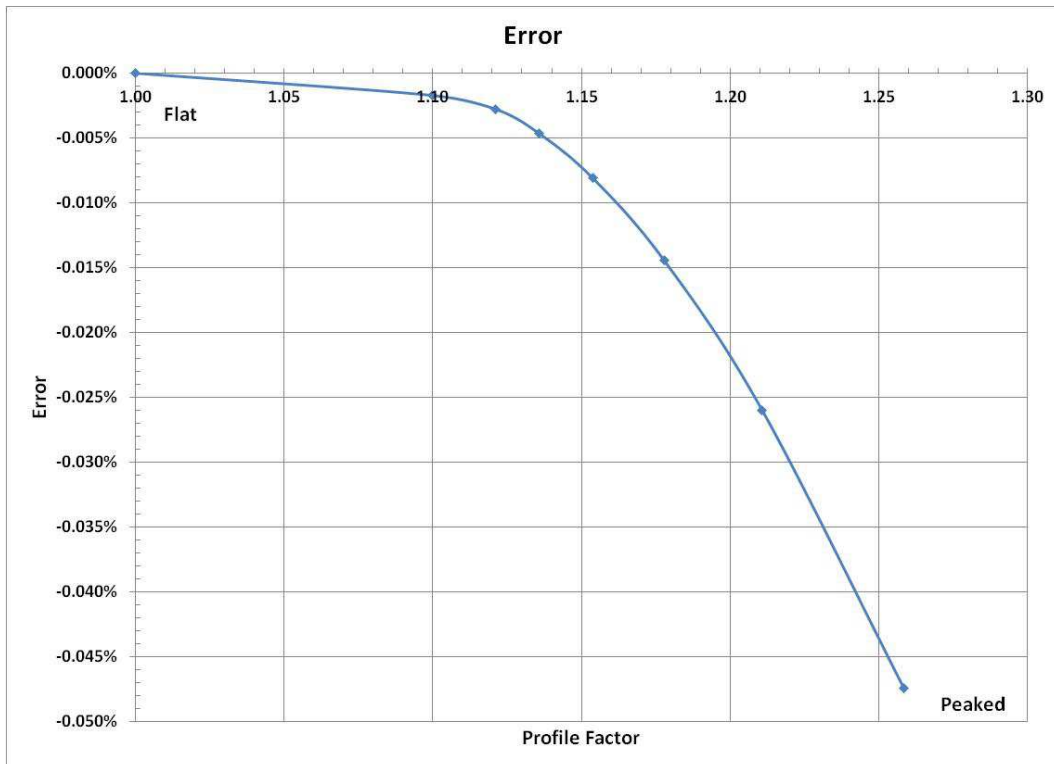


Fig 11. Flow error with profile factor

The change in ProFct is 30% and the error is only 0.05%, which is negligible. The integration technique is robust and can cope with this disturbance. A peaked velocity profile has the central velocity increased and the wall velocity decreased. As the wall velocity slope gets steeper the magnitude decreases more than the centre increases. Thus the net effect is a reduction in flow and a negative error. The cross flow and asymmetry are both one, so this is a more realistic disturbance. This profile factor change is equivalent to Reynolds numbers change from about 10^3 to $2 \cdot 10^9$. Thus this meter does not need a Re No correction, within the limits of the power law to represent the flow profile.

6. ULTRASONIC DESIGN

There is a large variation in meter design:

- Number of paths
- Location of paths
- Direct or reflecting paths
- Paths in one of more planes
- Integration schemes
- Corrections
- Diagnostics
- Transit time detection
- Discarding bad data
- Compensating for failed paths
- Cast or fabricated bodies
- Manufacturing tolerances
- Transducer size and operating frequency
- Electronics and software

The design can aim at: high accuracy, high tolerance to flow disturbances, the production of useful diagnostic data and a self checking ability. The ideal would be to meet them all, but this may not be possible and probably explains the large variety of meter designs on the market.

Two different approaches seem to have evolved:

1. Install a flow conditioner to establish a good flow profile and use the diagnostics to confirm the good profile to validate the meter accuracy.
2. Design the meter to detect disturbances and be immune to them to confirm the meter accuracy.

The price for 1 is the cost of obstruction and pressure loss caused by the flow conditioner. The advantage of 2 would be the elimination of the flow conditioner, if it proves possible.

7. CONCLUSIONS

Ultrasonic meters are highly accurate volumetric measurement devices, however manufacturers and operators are continually looking for ways to further improve performance. In this paper, we have looked at several potential limitations for improved accuracy of ultrasonic meters including calibration accuracy, sampling system versus bulk flow meter, meter design, manufacturing tolerance, flow disturbances, integration schemes, flow conditioners, basic physical interaction of acoustic waves with turbulent flow, transducer port disturbances and correction factors.

As an example, we have postulated that obtaining velocity information closer to the wall should help improve performance, but have subsequently shown using the power law and also CFD that this may not be the case. One could almost equally well argue that it is better to stay away from the pipe wall, to avoid all the associated practical problems associated with flow and acoustic interactions.

In examining these many possible limitations to improving USM performance, from an accuracy perspective, it became clear to us that the vast majority of currently outstanding technical challenges are very subtle. We were looking for a 0.3% improvement in meter uncertainty and it is clear there is no single contributor that can account for that level of improvement. The fact, there will be multiple effects. It is possible that we are looking to find nine 0.1% effects to account for a 0.3% ($0.1\% * \sqrt{9}$) improvement in meter uncertainty,

With these effects being so subtle and complex any corrections for them or design solutions to them will also be subtle. This ‘subtleness’ may be such that the reality of an industrial setting, (the changing measurand and non steady nature of metering station) will swamp it.

While we continue to study these subtle effects, it behoves us to continually focus on overall station design best practice to ensure the flow meter and associated equipment are given the best possible chance to deliver highly accurate measurement

9 NOTATION

USM	Ultrasonic meter	W_i	path weight
v	point velocity at y	i	path a, b, c, d
y	distance from pipe wall	z	path location
r	distance from pipe centre	dV/dz	slope
R	pipe radius	Σ	Sum
n	Power Law power	U	uncertainty
Q	flow rate	CFD	computational fluid dynamics
V_{AVG}	average velocity = $Q/\pi R^2$	NMi	Netherlands measurement institute
V_{MAX}	velocity at pipe centre V_i path velocity		

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