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Detailed Review of Existing Empirical and Analytical Estimation Models for Multiphase Flow

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1 INTRODUCTION

Multiphase flow is one of the most challenging problems in the area of flow metering. On one hand, the underlying physics is extremely complex, the understanding and modelling of which is a daunting task. On the other hand, accurate measurement of such complex and dynamic flow conditions non-invasively is undoubtedly complicated.

There are two important factors that make multiphase flow particularly hard to characterize. One is the complex interaction between the different phases and components. The other is the dependence of this interaction on several aspects of the flow itself such as component flow rates, component densities and viscosities, temperature and pressure. Based on the characteristics of this interaction, the flow is categorized into what are referred to as flow regimes. In gas-liquid vertical flows, the flow regimes most commonly encountered are bubbly, slug, churn and annular. Early attempts at modelling the flow physics tended to lump all flow regimes into one, which while greatly simplifying the analysis, were inadequate in describing the intricacies of each regime [2]. Thus, generic models typically fall short in terms of accuracy and reliability. Yet another challenging aspect of modelling multiphase flow is its non-uniform and time varying nature. Research efforts tend to address this issue in one of two ways - make the model less complicated by assuming temporal or spatial uniformity or account for non-uniformities through correction terms determined empirically or analytically.

Of particular interest are models that predict the component phase fractions from the ideal/reference flow rates for each component. The obvious consequence of such a prediction model is the increased ability to understand and interpret complex flow phenomena. However, the model outputs are equally critical for use in the area of flow metering to relate sensor measurements to flow parameters. For instance, flow models that can predict the slip in velocity between two phases can come in very handy when there is insufficient velocity information about individual phases. Over the years, several attempts have been made to develop analytical, empirical and semi-empirical models to predict the phase fraction of individual components in a multiphase flow. Given the crucial role of a flow model, it is imperative to understand and compare the performance of these models and to determine their suitability under different operating conditions, which in essence is the aim of this paper.

Godbole et al [16] and Woldesemayat and Ghajar [17] have compiled a list of potential two-phase flow models and analysed their performance against measurements collected across different loops, flow conditions and fluids. The two phase models studied predicted the component fractions and velocities and needed one or more of the following inputs

1. Component mass/volume flow rates,
2. Fluid properties including density and viscosity
3. Pressure, temperature
4. Regime

From the point of view of drawing general conclusions about the suitability of different models for different conditions, these are the most comprehensive studies of this nature to the best of our knowledge. Typically, in order to quantify the performance of a model, its prediction is compared with sensors capable of measuring phase fraction such as gamma ray, quick closing valve, capacitance sensor etc. [16] and [17] for instance estimate model error using a consolidated database with measurements drawn from different experimental setups with different measurement techniques. The drawback of such a comparison is that the performance metric includes the unknown estimation error of the sensors themselves. This

effect is exacerbated when different types of sensors are used for validation. While qualitative conclusions can be drawn from such a comparison, it is hard to quantify the performance of the models. Moreover, the extension of the two-phase phase prediction models to a liquid-liquid-gas three phase flow has not been documented. The experimental database used by [16] and [17] only deal with liquid-gas two phase flows with water-air flows being the most common condition tested. For these reasons, finding the right model/correlation for a particular application or under specific operating conditions is a cumbersome exercise.

This paper aims to address these deficiencies and hopes to answer the following questions:

1. What kinds of correlations are applicable for three phase liquid-liquid-gas flows? What are their assumptions and under what conditions are they valid?
2. Under varying operating conditions (liquid and gas flow rates), from among the available models, which ones are the most accurate in their predictions?
3. What is the performance of the best of these models? Are there conditions under which even the best models fall short?
4. Once, the best models have been selected, how can they help in further analysis?

The rest of this paper is organized thus. Section 2 briefly explains the basic physics governing a multiphase flow and flow parameters of interest. Section 3 describes the different types of models for predicting phase fractions and their underlying assumptions, along with a list of potential models with proven accuracy. Section 4 introduces a method to evaluate model performance with experimental data. Section 5 describes the experimental setup and section 6 tabulates the results and includes a detailed analysis of model performance under different operating conditions. Section 7 concludes this paper with a summary and suggestions for future work. It is pertinent to mention at this juncture that this work focuses only on liquid-liquid-gas vertically upward flows.

2 MULTIPHASE FLOW FUNDAMENTALS

The two basic flow parameters needed to characterize a multiphase flow are component phase fraction and component velocity. Once these two parameters are known, the volume flow rates can be easily calculated using the following equations

$$Q_g = \alpha V_g A \quad (1)$$
$$Q_l = (1-\alpha) V_l A \quad (2)$$

In essence, the three quantities α, V_g, V_l completely characterize the system. However, given the time and space varying nature of the quantities in question, it needs to be kept in mind that the equations 1 and 2 need to be integrated over time and space to recover actual flow rate information.

The definition and interpretation of phase fraction or void fraction is varied, but we have adopted the most commonly used one which defines the phase fraction in terms of the cross sectional area occupied by each component of the flow. This measurement may be local i.e at a point in space at a particular time instant or an averaged measure over time and space.

Equations 1 and 2 make it evident that the problem of flow rate estimation is rather straightforward once the three parameters are known, but what we are concerned with is the inverse of this problem. In other words, given the volume flow rates of the individual components, how can we predict the component phase fractions and velocities? Complications arise due to the fact that the multiple phases may not be traveling with the same velocity. Hence, several models are focused on defining the “slip” velocity between the different phases. Unfortunately, the physics of the flow dictates that the inter-relationship between the velocities is governed by several factors other than the volume flow rates including but not restricted to the component densities, viscosities, temperature, pressure and the flow structure information characterized by the “flow regime”.
While, physical measurements related to densities, viscosities, temperature and pressure are usually readily available, information related to the flow regime is often inadequate.

2.1 Flow Regime

In multiphase flows, the interaction between the phases and their relative distribution in time and space allows their classification into what are commonly referred to as flow regimes or flow patterns. Each flow regime is characterised by readily identifiable flow structures that are usually repetitive in nature. Flow structures encountered typically in multiphase flow are shown in figure 1.

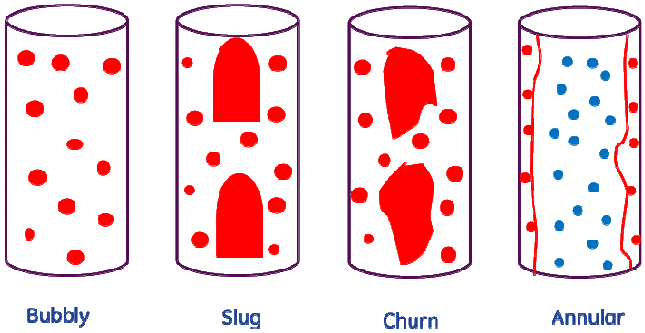

Figure 1 – Two phase flow regimes in vertical upwards flow

While the existence of a finite number of flow regimes is widely acknowledged, classification of the flow based on flow regime is not straightforward. Part of the problem is that regime identification is subjective and usually based on visual observation of the flow. Hence, regime information is ambiguous at best, with the ambiguity being especially pronounced at regime transition boundaries. This fact needs to be borne in mind when discussing flow correlations that have been defined for specific regimes.

3 MULTIPHASE FLOW MODELLING

There are several types of models that have been proposed for the prediction of component phase fraction and velocities. The following are some typical models encountered in literature:

- Homogeneous models that assume homogeneously mixed distribution and no slip.
- One dimensional models that account for the difference in velocity between the two phases.
- Models developed for specific flow regimes based on the physics of that regime.
- Empirical and semi-empirical models that are derived from experimental data.

The performance of these models is closely related to the assumptions that went into deriving them. For example, certain one dimensional models are only valid when the gas density is much smaller than the liquid density. Moreover, some models, especially the ones with empirical terms may have been over tuned to the experimental condition under which the data was collected to validate the model.

We will now offer a brief summary of existing correlations and the assumptions behind them.

3.1 Homogeneous model [2]

The homogeneous model assumes that all the components of the flow are travelling at the same velocity and that there is no slip. This provides a rather simplified framework for deriving the parameters of interest. For a gas-liquid two phase flow, the gas and liquid phases can be expressed as a function of the flow quality x.

$$V_g = \frac{Q_g}{A\alpha} = \frac{\dot{m}x}{\rho_g \alpha} \tag{3}$$

$$V_l = \frac{Q_l}{A(1-\alpha)} = \frac{\dot{m}(1-x)}{\rho_l(1-\alpha)} \tag{4}$$

Equating the two velocities, an expression for the phase fraction can be obtained in terms of the flow quality x.

$$\alpha = \frac{1}{1+\frac{1-x}{x}\left(\frac{\rho g}{\rho_l}\right)} \tag{5}$$

3.2 Slip Model [2]

The slip model provides an improvement over the homogeneous model by assuming a slip S between the two phases

$$S = \frac{V_g}{V_l} \tag{6}$$

With this expression, equation 5 can be suitably modified to account for the slip factor S.

$$\alpha = \frac{1}{1+\frac{1-x}{x}\left(\frac{\rho g}{\rho_l}\right)S} \tag{7}$$

Various expressions have been proposed for modelling the slip between the phases. Here are a few of them.

3.2.1 Momentum Flux Model [2]

The momentum flux model derives an expression for the phase fraction by using the following expression for the momentum flux for a separated flow.

$$Momentum\ Flux = \dot{m}^2\left[\frac{x^2 V_g}{\alpha} + \frac{(1-x)^2 V_l}{1-\alpha}\right] \tag{8}$$

The resultant phase fraction maximizes the momentum flux and can be obtained by differentiating equation 8 by α and equating it to zero. This results in the following expression for the slip.

$$S = \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \tag{9}$$

3.2.2 Smith Model [13]

Most correlations that predict the slip velocity in annular flows do not account for the liquid bubbles entrained in the gas. Smith's semi-empirical model provides an expression with an explicit mention of the entrainment fraction ϵ.

$$S = \epsilon + (1-\epsilon)\left[\frac{\left(\frac{\rho_l}{\rho_g}\right)+\epsilon\left(\frac{1-x}{x}\right)}{1+\epsilon\left(\frac{1-x}{x}\right)}\right]^{0.5} \tag{10}$$

Smith proposed an empirical value for the entrainment 0.4 and asserted that the model is valid for a wide range of flow rates and operating conditions. The final expression for the phase fraction is given by equation 11

$$\alpha = \frac{1}{1+0.79\left(\frac{1-x}{x}\right)^{0.78}\left(\frac{\rho g}{\rho_l}\right)^{0.58}} \tag{11}$$

3.2.2 Chisholm Correlation [15]

Chisholm proposed a modified correlation for the slip as given by equation 12.

$$S = \left[1 - x\left(1 - \frac{\rho_g}{\rho_l}\right)\right]^{0.5} \tag{12}$$

3.2 Drift Flux Model [3]

The drift flux model is perhaps the most commonly used flow correlation as is evident from the fact that ever since its inception in 1965, several researchers have contributed to its improvement and expansion by proposing suitable modifications.

The derivation of this model begins with the assumption that the gas velocity has two components to it

- Bulk velocity contributed by both liquid and gas superficial velocities. This is the velocity that each phase would have travelled with if the flow was homogeneous.
- Drift velocity due to the property difference between liquid and gas

A detailed description of the drift flux model is available in [3]. The final expression for the gas velocity is given by

$$V_g = C_0 V_h + v_{gj} \tag{13}$$

Where, V_h is defined as

$$V_h = \frac{Q_g + Q_l}{A} \tag{14}$$

C_0 is referred to as the distribution parameter which captures the non-uniform spatial profiles of velocity and phase fraction. In most drift flux models, either empirical or semi-empirical expressions have been derived for both the distribution parameter and the drift velocity.

Thus an expression for the phase fraction can be derived as given by equation 15.

$$\alpha = \frac{\frac{Q_g}{A}}{C_0 V_h + v_{jg}} \tag{15}$$

In order to predict the phase fraction, the two parameters C_0, v_{gj} need to be defined which forms the crux of the several different variations of the drift flux model, a few of which have been detailed in table 1.

Table 1 – Drift flux Models

Name	Distribution parameter	Drift Velocity
Bonnecaze et al [6]	$C_0 = 1.2$	$v_{gj} = 0.35\sqrt{gD}(1 - \frac{\rho_g}{\rho_l})$
Dix [7]	$C_0 = \frac{Q_g}{Q_g + Q_l}\left(1 + \left(\frac{Q_l}{Q_g}\right)^b\right)$ $b = \left(\frac{\rho_g}{\rho_l}\right)^{0.1}$	$v_{gj} = 2.9\left(\frac{g\sigma(\rho_l - \rho_g)}{\rho_l^2}\right)^{0.25}$
Gomez et al [8]	$C_0 = 1.15$	$v_{gj} = 1.53\left(\frac{g\sigma(\rho_l - \rho_g)}{\rho_l^2}\right)^{0.25}(1-\alpha)^{0.5} sin\theta$

Greskovich & Cooper [9]	$C_0 = 1.0$	$v_{gj} = 0.671\sqrt{gD}(sin\theta)^{0.263}$
Ishii [3]	$C_0 = 1.2 - 0.2\sqrt{\frac{\rho_g}{\rho_l}}$	$v_{gj} = \left(\frac{4g\sigma(\rho_l - \rho_g)}{\rho_l^2}\right)^{0.25}$
Kokal & Stanislav [11]	$C_0 = 1.0$	$v_{gj} = 0.345\left(gD\left(1 - \frac{\rho_g}{\rho_l}\right)\right)^{0.5}$
Morooka et al [18]	$C_0 = 1.08$	$v_{gj} = 0.45$
Nicklin et al [19]	$C_0 = 1.2$	$v_{gj} = 0.35\sqrt{gD}$
Rouhani & Axelsson [14]	$C_0 = 1 - 0.2(1-x)\left(\frac{gD\rho_l^2}{\dot{m}^2}\right)^{0.25}$ $C_0 = 1 - 0.2(1-x), for\ \alpha < 0.25$	$v_{gj} = 1.18\left(\frac{g\sigma(\rho_l - \rho_g)}{\rho_l^2}\right)^{0.25}$

3.3 Other Models

Several other empirical and semi-empirical models have been proposed for estimating the phase fraction, the most promising of which are mentioned below

3.3.1 Armand [4]-Massena [5]

$$\alpha = (0.833 + 0.167x)\left(1 + \left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)\right)^{-1} \tag{16}$$

3.3.2 Guzhov et al [10]

$$\alpha = 0.81\left(\frac{Q_g}{Q_g+Q_l}\right)(1 - e^{-2.2\sqrt{Fr}}) \tag{17}$$

$$Fr = \frac{Q_g+Q_l}{AgD} \tag{18}$$

3.3.3 Lockhart and Martinelli [12]

$$\alpha = \left[1 + 0.28\left(\frac{1-x}{x}\right)^{0.64}\left(\frac{\rho_g}{\rho_l}\right)^{0.36}\left(\frac{\mu_l}{\mu_g}\right)^{0.07}\right]^{-1} \tag{19}$$

4 MODEL EVALUATION AND VALIDATION

In the recent past, phase fraction correlations have garnered some interest, with several researchers attempting to quantify their performance using experimental data or data collected from databases. These studies compare the predicted phase fraction with a measured phase fraction obtained from a physical sensor. Typical sensor measurements used for comparison are capacitance based, gamma ray, quick closing valves etc. Experimentally determined phase fraction estimates have an inevitable measurement error associated with them. Hence, any comparison with a measured quantity is bound to be influenced by the uncertainty associated with the measurement system and measurement model.

Almost all of the phase fraction correlation comparisons stem from a need to understand and define the physics of the flow. While that is an important factor in multiphase flow analysis, the conclusions are not easily interpretable for a metering application. For example, it is hard to say what the effect of choosing a particular model will be on the accuracy of flow rate prediction. Though the drift flux and slip models have been proposed to define the component velocities, their validity for velocity prediction has not been verified in a comprehensive manner.

For these reasons, there is a need for a more in-depth analysis of the validity of the said correlations not just from the point of view of the accuracy of the predicted phase fraction, but from a flow measurement perspective as well. Hence, the following validation procedure has been proposed.

Differential pressure measurement is a very commonly used technique in multiphase flow estimation. Differential pressure measured across a venturi is proven to be an accurate indicator of the total mass flow rate of the contents of the pipe. The basic equation governing a venturi differential pressure system is given in equation 20.

$$\dot{m} = \frac{C_d}{\sqrt{\beta^4 - 1}} \sqrt{\rho_m (dP - \rho_m g h)} \tag{20}$$

Here, C_d is computed through a correlation using the Lockhart-Martinelli parameter [21] X that accounts for the irreversible loss across a venturi as well as variations due to the presence of more than one phase. This is given by equation 21, where C_{ds} is the discharge coefficient for single phase fluids.

$$C_d = C_{ds} / \sqrt{1 + \frac{20}{X} + \frac{1}{X^2}} \tag{21}$$

In order to compute the mass flow rate, a measurement of the mixture density ρ_m is needed. The mixture density can be estimated from the individual component densities if the phase fraction is available.

$$\rho_m = \alpha \rho_g + (1 - \alpha) \rho_l \tag{22}$$

Hence, in order to obtain an accurate estimate of the mass flow rate from the venturi differential pressure measurement, an accurate estimate of the mixture density is needed which in turn is dependent on the accuracy of the phase fraction.

This measurement setup can then be used to evaluate all existing phase fraction correlations by comparing the reference total mass flow rate with the mass flow rate estimated using the predicted phase fraction. The more accurate the phase fraction, the better the mass flow rate estimate. Additionally, the resultant mass flow rate error using the best prediction of the phase fraction is a strong indicator of the entitlement of the differential pressure measurement system.

5 EXPERIMENTAL SETUP

The experimental data for implementing the above proposed method of validation was collected at the National Engineering Lab (NEL) at the Scottish Technology Park, East Kilbride, Glasgow, UK. The facility's multiphase loop uses a combination of crude oil, water and nitrogen gas. The setup is shown in figure 2. The flow rates of all three components could be controlled as needed. The location marked as "Test meter" in the vertical section was used to obtain the measurements used in this analysis. The measurement system consisted of a differential pressure sensor across a venturi and an impedance sensor. The impedance sensor was used to measure the impedance of the contents of the pipe and in turn measure the phase fraction across the pipe cross section. Impedance sensors are widely used for component phase fractions in multiphase flows. The differential pressure and impedance sensors are placed at concurrent locations, which ensured that the differences in the flow conditions seen by the two sensors are minimized. These measurements, in conjunction with equations (20) and (21), were used to obtain an estimate of the mass-flow rate.

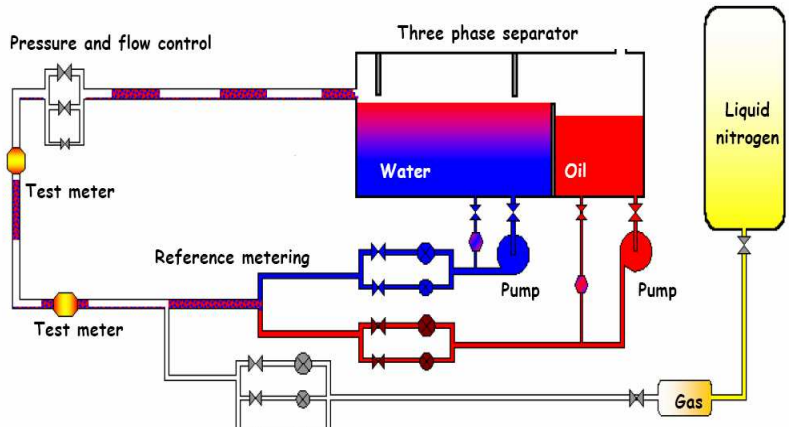

Figure 2 – Multiphase loop at NEL

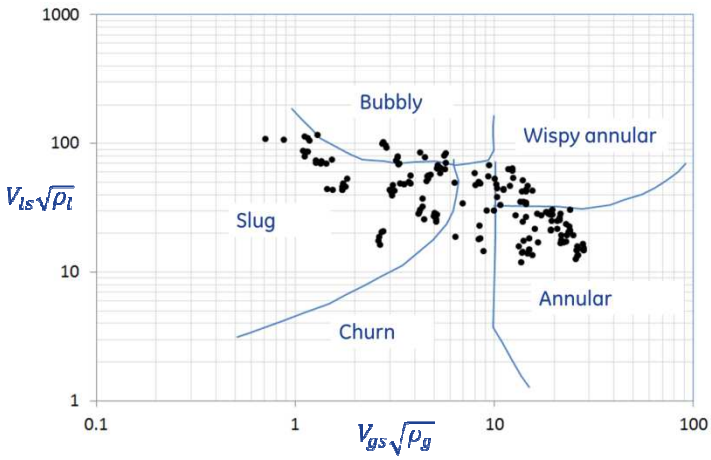

Figure 3a – Experimental Data from NEL's multiphase loop superimposed with Hewitt-Roberts flow regime map [20])

A total of around 165 data points were collected over a period of 10 days under varying flow conditions covering the entire operating region of the NEL multiphase loop. The sample points are shown in figure 3a superimposed on a traditional two-phase regime map. The flow regime map chosen, which was proposed by Hewitt-Roberts [20], accounts for variations in fluid properties and their effects on the flow regime. As is evident from figure 3a, the sample points cover most of the known flow regimes in vertical two/three phase flow. Figure 3b illustrates the fact that the flow conditions chosen were

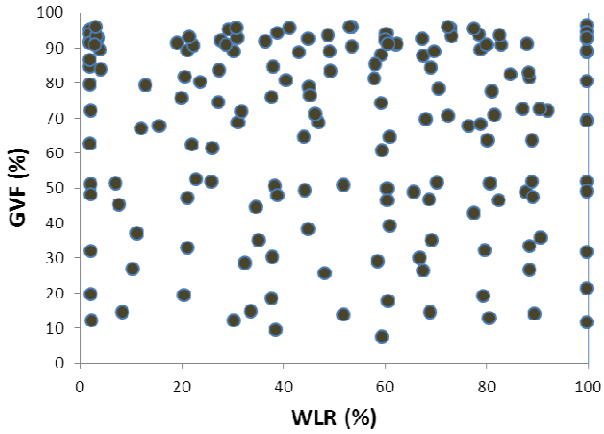

Figure 3b – Experimental Data from NEL's multiphase loop

sampled across the entire range of GVF and WLR (Water-Liquid Ratio). For every experiment, data was acquired only after allowing time for the flow to stabilize, with the duration of acquisition being 5 minutes. All measurements are averaged over the duration of acquisition to ensure that flow variations within that time did not adversely affect the analysis.

6 RESULTS AND ANALYSIS

In order to evaluate the performance of the different models, the accuracy of the mass flow rate estimated using the above mentioned method was chosen as the primary metric. The relative error in estimation was computed using the reference mass flow rate provided by NEL's single phase meters. The single phase meter measurements were suitably transformed to equivalent flow rates at the meter conditions using appropriately positioned absolute pressure measurements. It is worth mentioning at this juncture that the single flow meters were documented to have an uncertainty <1.5% over the flow conditions considered. The loop variability is not accounted for in this measure, but it is assumed that the effect of variability is minimized when the data is averaged over the 5 min of acquisition time. Gas and liquid property measurements were also available from characterization experiments carried out on the experimental fluids at NEL. The error in the mass flow rate was estimated as

$$\dot{m}_{err} = 100(1 - \frac{\dot{m}_{est}}{\dot{m}_{ref}}) \tag{23}$$

It needs to be pointed out that a relative error is used instead of absolute error since it is easier to analyse and relate to. While the error in itself is an important criterion, what is most commonly considered a performance metric is the "standard error" or the standard deviation of the error. The standard error provides bounds on the accuracy or uncertainty in estimation.

Additionally, the predicted phase fraction was also compared with the measured phase fraction using electrical impedance sensors. The metric for comparison is the correlation coefficient between the measured and predicted phase fraction. This ensures that errors due to scale or offset in the fraction measurement do not interfere with the analysis. The correlation coefficient is computed as per equation 24. The correlation coefficient is always within the range of [-1 1] with 1 corresponding to maximum positive correlation, -1 corresponding to maximum negative correlation and 0 corresponding to no correlation at all. In essence, the higher the value of R_{xy}, the better is the agreement between the predicted and measured phase fractions.

$$R_{xy} = \frac{E(\alpha_{pred} - \overline{\alpha_{pred}})(\alpha_{meas} - \overline{\alpha_{meas}})}{\sigma_{\alpha_{pred}} \sigma_{\alpha_{meas}}} \tag{24}$$

A total of 13 phase fraction predictors were chosen among the ones listed in previous sections. These were chosen based on the results from previous studies and their applicability under the given test conditions. Below is a list of the models selected and the corresponding name used to refer to each model is shown in within brackets.

1. Homogeneous model (Homogeneous)
2. Momentum Flux Model (Momentum flux)
3. Smith correlation (Smith)
4. Chisholm's correlation (Chisholm)
5. Bonnecaze et al (Bonnecaze)
6. Dix (Dix)
7. Gomez et al (Gomez)
8. Kokal-Stanislav (Kokal)
9. Ishii (Ishii)
10. Rouhani – Axelsson (Rouhani)
11. Armand – Massena (Armand)
12. Guzhov et al (Guzhov)
13. Lockhart-Martinelli (Lockhart)

The conditions tested were segregated based on the Gas Volume Fraction (GVF) as well as the Water-Liquid Ratio (WLR) to scrutinize the prediction capability of the models more closely.

6.1 GVF<20%

At lower GVFs which mostly covered slug flows, the estimated mass flow rates were found to be the most accurate, irrespective of the source of phase fraction. This may be attributed in

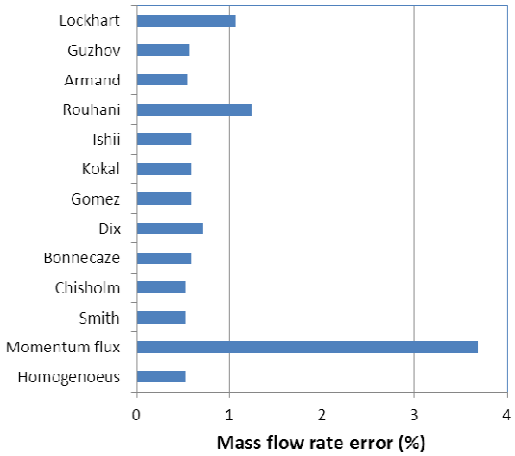

Figure 4a – Mass flow rate error for GVF<20%

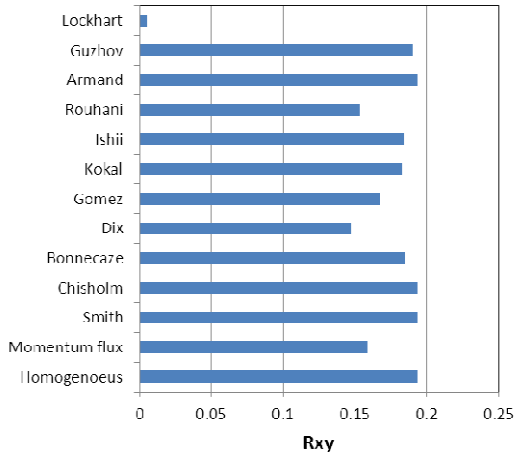

Figure 4b – Correlation with measured phase fraction for GVF<20%

part to the relatively higher mass flow rates associated with lower GVFs conditions. It is interesting to note that all models that either assume no slip or simple relationships for the slip velocity such as the Homogeneous model, Smith and Chisholm's correlation seem to perform the best. The comparison with the measured phase fraction does not give much scope for analysis, as the correlation seems to be uniformly low for this region. This may be due to the fact that for GVF<20%, the phase fractions are much lower in magnitude which in turn may result in an amplification of even small errors in the correlation coefficient. Figure 4b shows a plot of the quantity R_{xy} as defined by equation 24. The lower this term, the poorer is the prediction capability. Based on the mass flow rate estimation error, the most promising models are Homogeneous, Smith, Chisholm, Guzhov and Armand.

6.2 20%<GVF<40%

When GVF<40%, the standard error of the mass flow rate seems to be on par for most of the

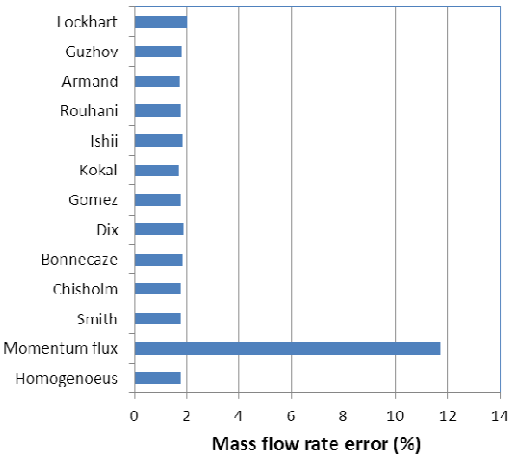

Figure 5a – Mass flow rate error for 20%<GVF<40%

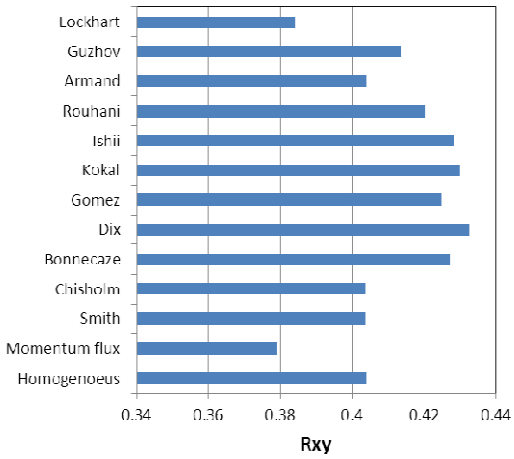

Figure 5b – Correlation with measured phase fraction for 20%<GVF<40%

models except the momentum flux and Lockhart correlations. But a quick look at the correlation coefficients in figure 5b, provides us with more discernibility for selecting the best models. Clearly, the drift flux model predictions based on Bonnecaze, Dix, Gomez, Kokal and Ishii have better correlation with the measured phase fraction. Also, the observation that the momentum flux and Lockhart models have the worst performance is corroborated by the correlation chart of figure 5b. Also, the homogeneous and slip models don't have much correlation with the measurement under these conditions. The best correlated model is the one proposed by Dix.

6.3 40%<GVF<60%

The physics of the flow dictates that as the GVF increases, and the flow transitions from bubbly-slug regimes to regimes like churn, the slip between the phases becomes more significant. This is evident from figure 6a, where the correlations with the least estimation

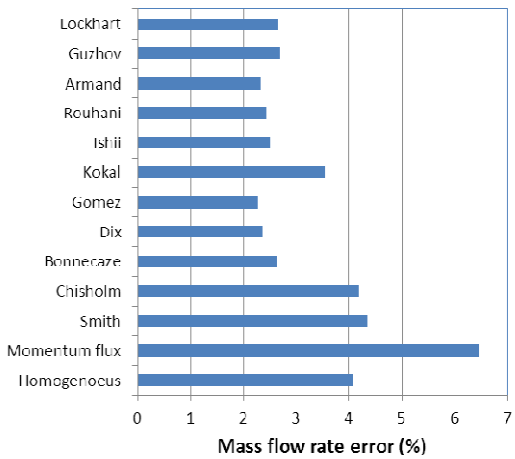

Figure 6a – Mass flow rate error for 40%<GVF<60%

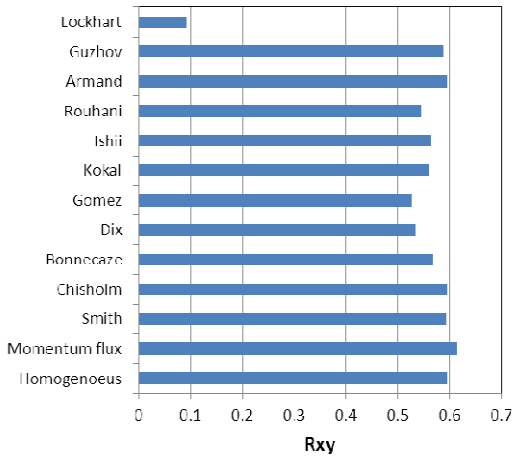

Figure 6b – Correlation with measured phase fraction for 40%<GVF<60%

error are based on the drift flux model. The best prediction models for these conditions are the ones proposed by Dix, Gomez, Ishii and Rouhani. On the other hand, the correlation plot shown in figure 6b does not indicate a clear winner, but instead seems to affirm that all models, except the one by Lockhart have a high correlation with the measured phase fraction.

6.4 60%<GVF<80%

As the GVF increases and the flow conditions are more consistent with churn-turbulent

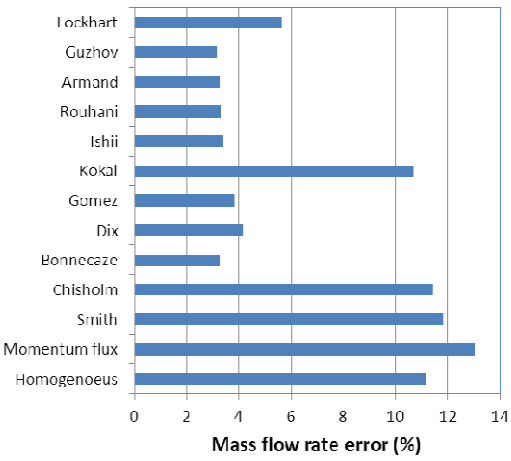

Figure 7a – Mass flow rate error for 60%<GVF<80%

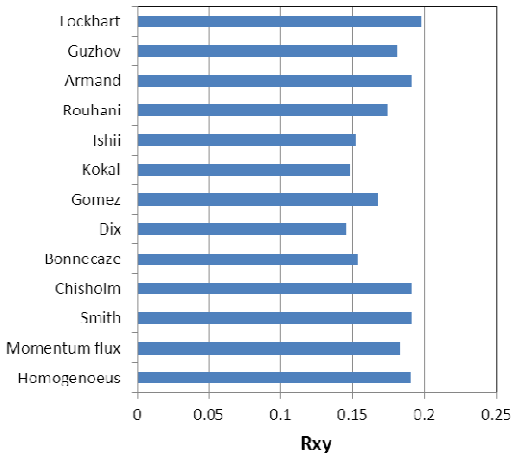

Figure 7b – Correlation with measured phase fraction for 60%<GVF<80%

regimes, two effects are clearly visible from figures 7a and 7b. The predicted phase fraction is inadequate in estimating the mass flow rate accurately as is seen in the increased standard error in figure 7a. The other observation to be made is that there is very little correlation between the predicted and measured phase fraction as is indicated by the lower numbers seen in figure 7b. These two observations together point towards a systematic error in phase fraction prediction under these conditions. Nevertheless, for 60%<GVF<80%, the best prediction models are yet again based on the drift flux assumption – Bonnecaze, Ishii, Rouhani, Armand and Guzhov. Figure 7b does not provide much scope for differentiating between the different models in terms of performance, as the correlation is uniformly poor.

6.5 80%<GVF<100%

At the highest possible GVFs, the estimation error in the mass flow rate is clearly the poorest among all the points tested. This may partly be due to the uncertainty in the differential pressure measurement itself. But two models are clearly superior to others in terms of

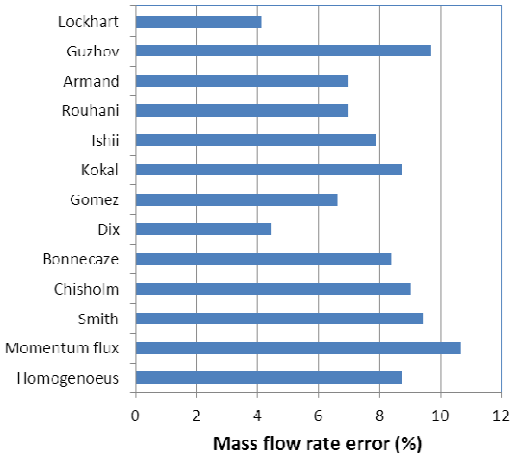

Figure 7a – Mass flow rate error for 80%<GVF<100%

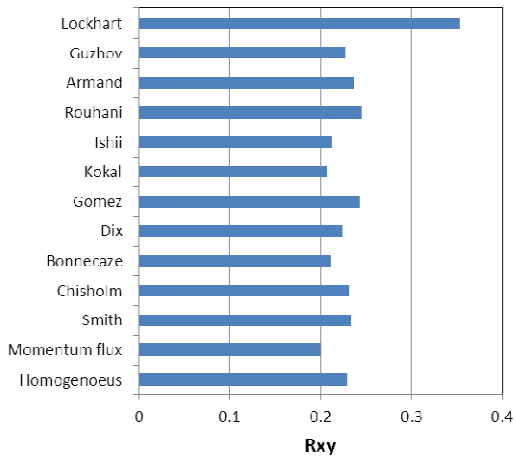

Figure 7b – Correlation with measured phase fraction for 80%<GVF<100%

estimation error and those are the correlations by Dix and Lockhart. In fact, the Lockhart correlation provides the best estimation error of around 4%. Bolstering this observation is the fact that the correlation numbers in chart 7b, where the Lockhart phase fraction prediction has significantly better correlation with the measured phase fraction when compared with all other models.

6.6 Effect of WLR

An aspect that has largely been neglected in previously analysed data is the effect of changes in the WLR in three phase flows when applying two-phase prediction models. The common practice in these situations is to treat the oil-water mixture as a single liquid, with the mixture properties calculated from component properties and the WLR using linear relationships. For example, the liquid density is given by equation 25.

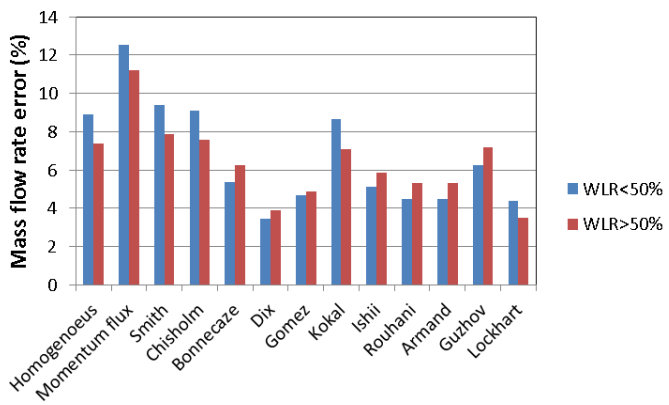

Figure 8 – Mass flow rate error for different WLR

$$\rho_l = WLR\rho_w + (1 - WLR)\rho_o \tag{25}$$

Once this assumption is made, all two-phase prediction models are deemed valid for three phase flows with appropriate corrections made to the liquid properties. But it is imperative to verify this assumption by analysing the performance of the models under varying WLR. Figure 8 shows a comparison of mass flow rate prediction for predominantly oil or predominantly water flows. While the overall performance of most of the models is consistent in both regions, some models perform better in one versus the other. The two best overall models are the ones proposed by Dix and Lockhart with the lowest errors in oil dominated flows and water dominated flows respectively.

7 CONCLUSIONS

Using experimental data collected from differential pressure and impedance measurements, a thorough analysis was conducted to determine the suitability and prediction accuracy of several types of phase fraction models for vertical three-phase flows. While several models performed better than others across the entire operating envelope, some models were found to be more accurate than others in certain regimes. This study provides recommendations on the best models to use based on the flow conditions. Unlike previous such studies, the phase fraction prediction accuracy was analysed as a function of the GVF as well as the WLR. Table 3 provides a summary with recommendations for the most effective models.

Table 3 – Model Recommendation

Operating condition	**Recommended Models**
GVF<20%	Homogeneous, Smith, Chisholm
20%<GVF<40%	Bonnecaze, Dix, Gomez, Kokal, Ishii
40%<GVF<60%	Dix, Gomez, Ishii, Rouhani
60%<GVF<80%	Bonnecaze, Ishii, Rouhani, Armand, Guzhov
80%<GVF<100%	Dix, Lockhart
WLR<50%	Dix, Lockhart, Rouhani, Armand, Gomez
WLR>50%	Lockhart, Dix, Gomez

Figure 9 shows the supporting data that corroborates the information in Table 2. Figures 10a and 10b are the error contour maps for the two best overall models as a function of GVF and WLR.

The idea of using two different metrics derived from two different types of measurement systems i.e differential pressure based and impedance based, was found to have several advantages. Not only did this make the entire analysis and subsequent conclusions more robust and trustworthy, the two measurements complemented each other in their ability to discriminate between models under different conditions. Hence, the results of this study can be considered repeatable under similar experimental conditions.

For several of these correlations, a fully-developed flow is assumed in the model formulation. Such an assumption helps simplify the analysis, as the flow is more structured. In other words, the flow profiles and other flow parameters are easier to approximate for developed flows since they are no longer a function of stream-wise position. The experimental data used to perform this study was acquired at a large enough development length to ensure developed flow at the sensor locations. This in turn meant that all the correlations could be applied as is with little modification. For flows that are still developing, it is likely that the accuracies of the models may change based on their sensitivity to the assumption of developed flow profiles.

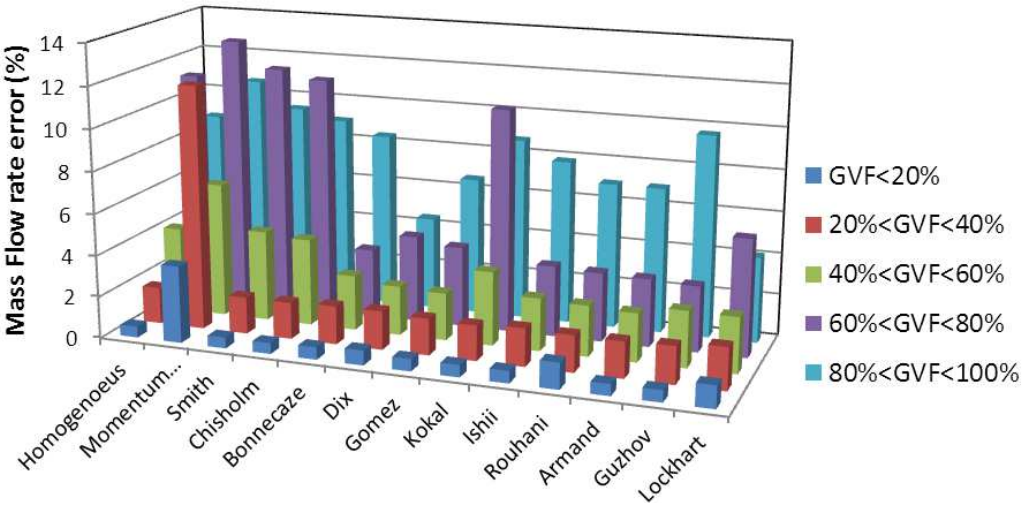

Figure 9 – Mass flow rate error as a function of GVF and model used

The models recommended by this study can be used in several different ways. A straightforward application is to use the phase fraction prediction from the models to help calibrate or correct phase fraction measurements made by different sensors. The model predictions can also be used in conjunction with sensors that measure velocity to determine their entitlement. For example, the mass flow rate error determined in this study can serve as an entitlement for the differential pressure measurements. The best performing drift-flux and slip models can be used to determine velocities of one or more components that are difficult to measure in practice. These correlations can also serve as a starting point for developing empirical models or corrections for multiphase flow measurement to improve the measurement accuracy. For example, several of the already developed semi-empirical models contain coefficients that have already been determined empirically. These coefficients

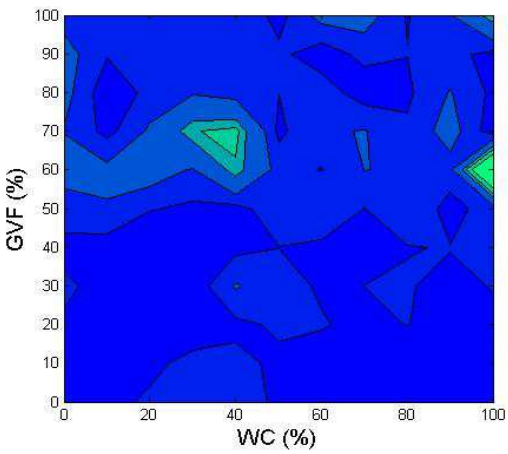

Figure 10a – Mass flow rate error(%) map for Lockhart model

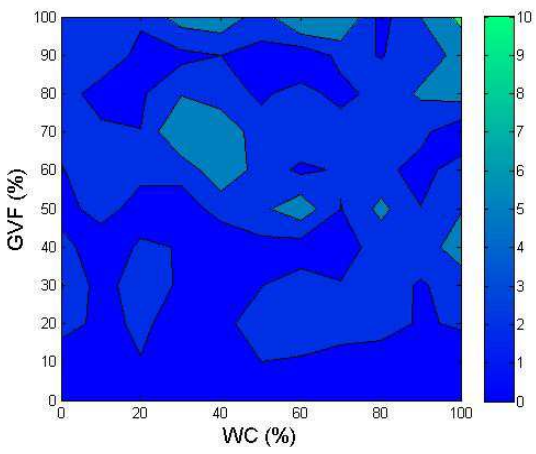

Figure 10b – Mass flow rate error(%) map for Dix's model

can be tweaked based on data acquired under specific operating conditions to further improve the accuracy of estimation.

From this study, it is also evident that the prediction capability of all available models is rather inadequate under high GVF conditions especially in churn-turbulent flows. This may be due to the fact that under churn type of regimes, the complex physics of the flow is hard to capture through simple relationships involving only a few fluid properties. Moreover, the time and space varying nature of the flow is especially pronounced under such conditions, which makes it harder to approximate quantities such as average drift velocity and average distribution parameter. There is a need for further work in building models suitable for such complex regimes.

6 NOTATION

α	Gas phase fraction
Q_g	Gas volume flow rate
Q_l	Liquid volume flow rate
V_g	Gas velocity
V_l	Liquid velocity
A	Cross sectional area
D	Pipe diameter
g	Acceleration due to gravity
ρ_g	Gas density
ρ_l	Liquid density
ρ_m	Mixture density
ρ_o	Oil density
ρ_w	Water density
μ_g	Gas viscosity
μ_l	Liquid viscosity
σ	Surface tension
x	Flow quality
$\dot{m}$	Mass flux
C_0	Distribution parameter
v_{gj}	Drift velocity
θ	Inclination angle
ϵ	Entrainment fraction
C_d	Venturi discharge coefficient
dP	Differential pressure
β	Venturi diameter ratio

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