

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

### Simplified Equations for Some Thermophysical Properties Used in High Pressure Metering of Natural Gas

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## 1 INTRODUCTION

This paper specifies a method to calculate viscosity and other properties, excluding density, for use in the metering of natural gas flow.

This paper gives simplified methods for the calculation of (dynamic) viscosity, Joule-Thomson coefficient, and isentropic exponent for use in natural gas calculations in the temperature range -10 to 40 °C and pressures up to 100 bar(abs) in the gas phase.

For Joule-Thomson and isentropic exponent, the uncertainty of the equations provided is greater than that obtained from a complete equation of state such as ISO-20765:2 or GERG-2008 (reference [1]), but is considered to be fit for purpose. The equations are much simpler.

The motivation for providing simplified methods is mainly for the calculations required, according to ISO-5167, to measure flow of high pressure natural gas with an orifice plate meter (references [2], [3], [4] and [5])

The basic mass flowrate equation is :-

$$q = \frac{C}{\sqrt{1-\beta^4}} \varepsilon \frac{\pi}{4} d^2 \sqrt{2 \cdot \Delta p \cdot \rho} \quad (1)$$

where C is function of  $\beta$  and Re, and of the type of orifice pressure tappings, and  $\varepsilon$  is a function of  $\beta$ , P,  $\Delta p$ , and  $\kappa$ . The above standards differ in the functions for C and  $\varepsilon$ . Although q is given by equation (1), iteration is required since C is a function of Re and Re is a function of q. Similarly, given q, equation (1) does not directly give  $\Delta p$  since  $\varepsilon$  is a function of  $\Delta p$ .

The use of the equations in ISO-5167 (2003) [5] for calculating flow (q) for an orifice plate meter, over a typical input range of temperature, pressure, differential pressure, and gas composition, gives the following uncertainty equation (when the only source of uncertainties is considered to be in the calculation of the required gas thermophysical properties):

$$\begin{aligned} [u(q)/q]^2 &= [0.5 \pm 0.0002]^2 \cdot [u(\rho)/\rho]^2 && \text{mass density} \\ &+ [0.0006 \pm 0.0002]^2 \cdot [u(\eta)/\eta]^2 && \text{viscosity} \\ &+ [0.002 \pm 0.0012]^2 \cdot [u(\kappa)/\kappa]^2 && \text{isentropic exponent} \\ &+ [-0.0004 \pm 0.0002]^2 \cdot [u(\mu)/\mu]^2 && \text{Joule-Thomson coefficient} \end{aligned} \quad (2)$$

This equation can be used to estimate the required uncertainty for the calculation of the properties.

For the contribution to the expanded uncertainty (U) (coverage factor k=2, 95% confidence interval) in the flow to be less than 0.1 %, then

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

$$U(\rho)/\rho < 0.1 \% \quad U(\eta)/\eta < 85 \% \quad U(\kappa)/\kappa < 25 \% \quad U(\mu)/\mu < 125 \%$$

For the uncertainty contribution to be less than 0.02 %, then

$$U(\rho)/\rho < 0.02 \% \quad U(\eta)/\eta < 17 \% \quad U(\kappa)/\kappa < 5 \% \quad U(\mu)/\mu < 25 \%$$

Thus, density needs to be calculated as accurately as possible, while the calculation does not need to be very accurate for the other properties. Their target uncertainty is no better than about 25%.

## 2 VISCOSITY ( $\eta$ )

There are many methods for the calculation of gas phase (dynamic) viscosity. Some, based in theory, are quite complicated. Of all the methods, the Lohrenz-Bray-Clark method is relatively simple, requires minimal component data, and is a method that is widely implemented (reference [6]). It is the method recommended here. One disadvantage is that it is sensitive to the input density; but for the application considered here, accurate densities will be available, so this is not a problem.

Below outlines the required parameters and equations to implement this method.

**Table 1 – Component Parameters**

	MW	Tc (K)	Pc (bar)	Zc
N <sub>2</sub>	28.0134	126.21	33.9	.2909877
CO <sub>2</sub>	44.0095	304.13	73.75	.2742627
CH <sub>4</sub>	16.0425	190.564	45.992	.2862833
C <sub>2</sub> H <sub>6</sub>	30.0690	305.33	48.714	.2788728
C <sub>3</sub> H <sub>8</sub>	44.0956	369.85	42.47	.2764399
iC <sub>4</sub>	58.1222	407.85	36.34	.2743873
nC <sub>4</sub>	58.1222	425.25	37.92	.2722036
neoC <sub>5</sub>	72.1488	433.75	31.94	.2730688
iC <sub>5</sub>	72.1488	460.45	33.77	.2708153
nC <sub>5</sub>	72.1488	469.80	33.75	.2686985
nC <sub>6</sub>	86.1754	507.90	30.35	.2646740

$$\text{Mixture parameters:} \quad MW_{\text{mix}} = \sum_{i=1}^N X_i MW_i \quad (3)$$

$X_i$  is component mole fraction

$$Vc_{\text{mix}} = \sum_{i=1}^N X_i \frac{Zc_i Tc_i}{Pc_i} \quad Tc_{\text{mix}} = \sum_{i=1}^N X_i Tc_i \quad Pc_{\text{mix}} = \sum_{i=1}^N X_i \frac{Pc_i}{1.01325} \quad (4)$$

$$\text{Viscosity at low pressure:} \quad T_r = T / Tc_i \quad (5)$$

$$T_r \leq 1.5 \quad \alpha = 0.00034 T_r^{0.94} \quad (6)$$

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

$$T_r > 1.5 \quad \alpha = 0.0001778 \times (4.58 T_r - 1.67)^{0.625} \quad (7)$$

Component viscosity: 
$$\eta_i = \frac{MW_i^{1/2}}{Tc_i^{1/6}} \left( \frac{Pc_i}{1.01325} \right)^{2/3} \alpha \quad (8)$$

Mixture viscosity: 
$$\eta_{mix} = \frac{\sum_{i=1}^N X_i \eta_i \sqrt{MW_i}}{\sum_{i=1}^N X_i \sqrt{MW_i}} \quad (9)$$

Viscosity at high density: 
$$\xi = \frac{MW_{mix}^{1/2} Pc_{mix}^{2/3}}{Tc_{mix}^{1/6}} \quad (10)$$

where the units are :  $MW_{mix}$  [g/mol],  $Pc_{mix}$  [atm],  $Tc_{mix}$  [K]

$\rho_r = V_{cmix} \times P / ( Z \times T )$   $P$  [bar],  $T$  [K],  $Z$  calculated by ISO-20765:2

$$\alpha = 0.1023 + 0.023364 \rho_r + 0.058533 \rho_r^2 - 0.040758 \rho_r^3 + 0.0093324 \rho_r^4 \quad (11)$$

Viscosity: 
$$\eta = \eta_{mix} + \xi (\alpha^4 - 0.0001) \quad (12)$$

This is the viscosity of the natural gas mixture (units mPa.s = cP)

From the following experimental data, the estimated uncertainty of this method is about 2%.

Total number of points	627
Temperature range	260 to 344 K (-13 to 71 °C)
Pressure range	1 to 122 bar

Experimental data is in references [7], [8], [9], [10] and [11].

The following figures show the distribution of the errors.

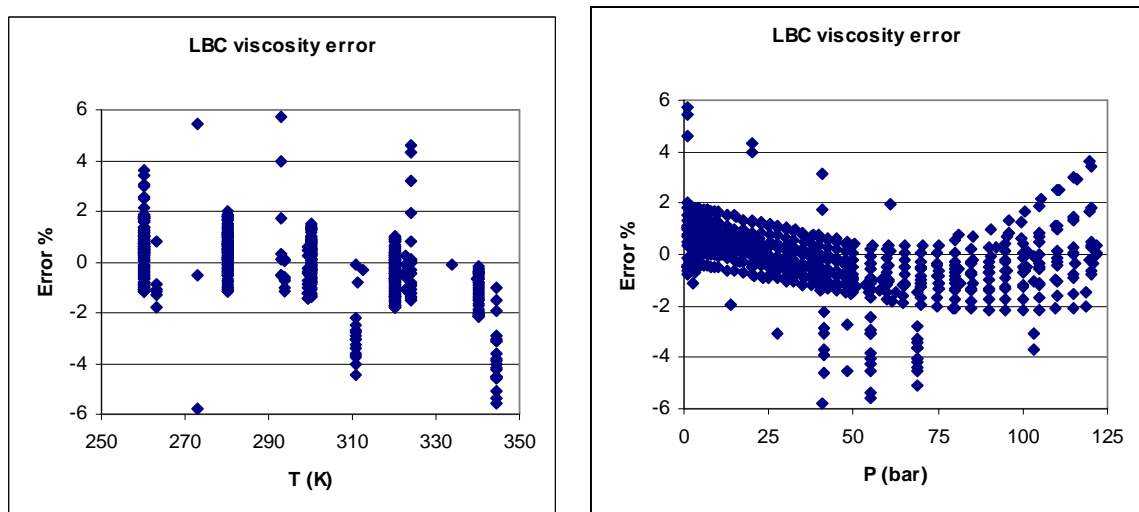


Fig. 1 – errors in viscosity as a function of temperature and of pressure

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

If a detailed composition is not available, but only bulk properties, e.g., calorific value (CV), relative density (RD), and CO<sub>2</sub> mol%, then this can be converted to an equivalent N<sub>2</sub>/CO<sub>2</sub>/CH<sub>4</sub>/C<sub>3</sub>H<sub>8</sub> mixture, and the above equation for viscosity can be used for this equivalent 4 component mixture.

This 4 component mixture has 2 unknown mol% (N<sub>2</sub>-mol% and C<sub>3</sub>H<sub>8</sub>-mol%) (CO<sub>2</sub>-mol% is given, and CH<sub>4</sub>-mol%=100-N<sub>2</sub>-CO<sub>2</sub>-C<sub>3</sub>H<sub>8</sub>). These 2 unknowns are determined from the CV and RD. The procedure is to assume Z (e.g. 0.9975), solve the linearized CV and RD equations, update Z, and repeat until converged. Convergence is rapid since Z does not change much with natural gas composition.

### 3 OTHER PROPERTIES

Other properties can be accurately calculated using the GERG-2008 equation of state (as detailed in ISO-20765:2), and implemented in the DNV-GL program GasVLe.

There are no existing widely-used simple methods for these properties (unlike the case above for viscosity), so new equations were derived.

To determine the optimal equations, a range of simulated natural gases was generated based on the following rules:

Table 2 – Natural Gas rules

mol %	Lower limit	Upper limit
N <sub>2</sub>	0.05	10
CO <sub>2</sub>	0.01	4
CH <sub>4</sub>	80	98
C <sub>2</sub> H <sub>6</sub>	0.25	9
C <sub>3</sub> H <sub>8</sub>	0.01	3.5
nC <sub>4</sub>	0.001	1
nC <sub>5</sub>	0.001	0.2
nC <sub>6</sub>	0.001	0.1
iC <sub>4</sub> /nC <sub>4</sub>	0.45	0.83
iC <sub>5</sub> /nC <sub>5</sub>	0.83	1.33
neoC <sub>5</sub> /nC <sub>5</sub>	0.01	0.015
C <sub>n</sub> /C <sub>n-1</sub>	0.2	0.4
CV (MJ/sm <sup>3</sup> )	35	45

The procedure was to generate N<sub>2</sub>, CO<sub>2</sub>, and C<sub>2</sub>H<sub>6</sub> composition values uniformly within this range; C<sub>3</sub>H<sub>8</sub> values were generated from ratio limits (C<sub>n</sub>/C<sub>n-1</sub>) using the C<sub>2</sub>H<sub>6</sub> value, and similarly for nC<sub>4</sub>, nC<sub>5</sub>, and nC<sub>6</sub> (using ratio limits with C<sub>3</sub>H<sub>8</sub>, nC<sub>4</sub>, or nC<sub>5</sub>, respectively); generate iC<sub>4</sub>, iC<sub>5</sub>, and neoC<sub>5</sub> from ratio limits. The CH<sub>4</sub> composition is the remainder. The CH<sub>4</sub> value and CV ranges were checked (as well as C<sub>3</sub>H<sub>8</sub>, nC<sub>4</sub>, nC<sub>5</sub>, and nC<sub>6</sub> values to be within their ranges). The composition was accepted if all the above limits were satisfied.

The GERG-2008 equation of state was then used to calculate the properties for all the mixtures in a grid of temperatures and pressures over the range of interest. From these calculations it was observed that the compositional variation was not significant, compared to the temperature and pressure variation, and was within the target uncertainty outlined in the introduction. Thus, equations as a function of T and P only were sought. (The compositional variation is accounted for in the final overall uncertainty.)

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

To determine the optimal equation, a bank of terms with powers of T and P (including fractional powers, and positive and negative values) was used with the SuperFit routine of the DNV GL Excel Add-In GasTools. In the end, the equations chosen as achieving the requirements were very straightforward.

The recommended equations are given below, with a table of values (from the equation), a table of bias errors, and a table of RMS errors (as absolute values and as percent). The RMS can be interpreted as a standard uncertainty (coverage factor k=1).

The bias and RMS (root-mean-squared) errors with respect to the calculation of ISO-20765:2 (GERG-2008 equation of state) are (for property X):

$$\text{bias} = \frac{1}{N} \sum_{i=1}^N (X_i^{\text{eqn}} - X_i^{\text{gerg}}) \quad \text{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i^{\text{eqn}} - X_i^{\text{gerg}})^2} \quad (13)$$

The value N is the number of test points.

The major contribution to the RMS comes from the compositional variation in the property, rather than from the inadequacy of the simple equation.

### 3.1 Joule-Thomson coefficient ( $\mu$ )

Definition:  $\mu = \left( \frac{\partial T}{\partial P} \right)_H$

Equation:  $\mu = (0.594 - 0.0042 \times t) + (-0.177 + 0.0021 \times t) \times (P/100)^2 \quad (14)$

where t is °C and P is bar(abs)

The data were fitted in the range 0-30 °C, 10-100 bar(abs)

**Table 3 – Joule-Thomson coefficient equation value and bias**

P/bar	Joule-Thomson coefficient value						P/bar	Bias					
	100	80	60	40	20	10		100	80	60	40	20	10
100	0.438	0.417	0.396	0.375	0.354	0.333	100	0.019	0.008	0.004	0.003	0.003	0.002
80	0.509	0.481	0.452	0.424	0.395	0.366	80	0.004	0.005	0.007	0.007	0.006	0.003
60	0.565	0.530	0.496	0.461	0.427	0.393	60	-0.005	0.003	0.008	0.009	0.007	0.002
40	0.604	0.566	0.527	0.488	0.450	0.411	40	-0.004	0.006	0.010	0.010	0.006	-0.001
20	0.628	0.587	0.546	0.505	0.463	0.422	20	0.001	0.009	0.011	0.009	0.003	-0.005
t/°C	-10	0	10	20	30	40	t/°C	-10	0	10	20	30	40

**Table 4 – Joule-Thomson coefficient uncertainty (RMS and RMS %)**

P/bar	RMS						P/bar	RMS %					
	100	80	60	40	20	10		100	80	60	40	20	10
100	0.021	0.018	0.019	0.019	0.019	0.019	100	4.8	4.3	4.8	5.1	5.4	5.7
80	0.028	0.028	0.028	0.027	0.025	0.023	80	5.5	5.8	6.2	6.4	6.3	6.3
60	0.040	0.037	0.034	0.031	0.028	0.025	60	7.1	7.0	6.9	6.7	6.6	6.4
40	0.045	0.040	0.037	0.034	0.030	0.026	40	7.5	7.1	7.0	7.0	6.7	6.3
20	0.044	0.040	0.037	0.033	0.029	0.027	20	7.0	6.8	6.8	6.5	6.3	6.4
t/°C	-10	0	10	20	30	40	t/°C	-10	0	10	20	30	40

### 3.2 Isentropic Exponent ( $\kappa$ )

Definition:  $\kappa = -\frac{V}{P} \left( \frac{\partial P}{\partial V} \right)_S$

Equation:  $\kappa = (1.3028 - 0.0005794 \times t) + (-0.08437 + 0.002658 \times t) \times (P/100) + (0.3267 - 0.005517 \times t) \times (P/100)^2 \quad (15)$

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

where t is °C and P is bar(abs)

The data were fitted in the range 0-20 °C, 5-75 bar(abs)

**Table 5 – Isentropic exponent equation values and bias**

P/bar	Isentropic exponent value						P/bar	Bias					
<b>100</b>	1.580	1.545	1.511	1.476	1.442	1.408	<b>100</b>	-0.107	-0.055	-0.033	-0.027	-0.032	-0.042
<b>80</b>	1.464	1.444	1.425	1.405	1.385	1.365	<b>80</b>	-0.024	-0.010	-0.005	-0.007	-0.011	-0.018
<b>60</b>	1.379	1.370	1.360	1.350	1.341	1.331	<b>60</b>	0.001	0.002	0.002	0.000	-0.003	-0.005
<b>40</b>	1.325	1.321	1.317	1.313	1.309	1.305	<b>40</b>	0.001	0.000	-0.001	-0.001	-0.001	0.000
<b>20</b>	1.302	1.299	1.296	1.294	1.291	1.288	<b>20</b>	-0.002	-0.002	-0.002	-0.002	-0.001	0.000
t/°C	<b>-10</b>	<b>0</b>	<b>10</b>	<b>20</b>	<b>30</b>	<b>40</b>	t/°C	<b>-10</b>	<b>0</b>	<b>10</b>	<b>20</b>	<b>30</b>	<b>40</b>

**Table 6 – Isentropic exponent uncertainty (RMS and RMS %)**

P/bar	RMS						P/bar	RMS %					
<b>100</b>	0.119	0.060	0.034	0.027	0.032	0.043	<b>100</b>	7.5	3.9	2.3	1.8	2.2	3.1
<b>80</b>	0.025	0.010	0.009	0.012	0.016	0.022	<b>80</b>	1.7	0.7	0.6	0.9	1.2	1.6
<b>60</b>	0.012	0.013	0.014	0.014	0.014	0.015	<b>60</b>	0.9	0.9	1.0	1.0	1.0	1.1
<b>40</b>	0.015	0.015	0.015	0.014	0.014	0.014	<b>40</b>	1.1	1.1	1.1	1.1	1.1	1.1
<b>20</b>	0.013	0.013	0.013	0.013	0.013	0.013	<b>20</b>	1.0	1.0	1.0	1.0	1.0	1.0
t/°C	<b>-10</b>	<b>0</b>	<b>10</b>	<b>20</b>	<b>30</b>	<b>40</b>	t/°C	<b>-10</b>	<b>0</b>	<b>10</b>	<b>20</b>	<b>30</b>	<b>40</b>

### 3.3 Speed of Sound (W)

Definition:  $W = \sqrt{[\kappa \times P \times 10^5 / \rho]}$  m/s, P bar(abs),  $\rho$  kg/m<sup>3</sup>

Equation:  $\kappa$  from equation (15) above,  $\rho$  from accurate equation of state

Because W varies with  $\rho$ , it is only useful to provide a table of RMS % uncertainty.

**Table 7 – Speed of sound uncertainty (RMS %)**

P/bar	RMS %					
<b>100</b>	3.51	1.86	1.11	0.91	1.09	1.48
<b>80</b>	0.84	0.36	0.32	0.42	0.57	0.79
<b>60</b>	0.43	0.48	0.51	0.52	0.54	0.57
<b>40</b>	0.56	0.56	0.55	0.55	0.54	0.54
<b>20</b>	0.50	0.50	0.50	0.49	0.49	0.49
t/°C	<b>-10</b>	<b>0</b>	<b>10</b>	<b>20</b>	<b>30</b>	<b>40</b>

# 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

## Technical Paper

### 4 USAGE

In order to calculate flow (according to equation (1)), the required properties ( $\rho$ ,  $\eta$  and  $\kappa$ ) are calculated at the upstream temperature and pressure. However, to calculate the upstream temperature from the measured downstream temperature using  $\kappa$  (references [2] and [3]) or  $\mu$  (references [4] and [5]), there are a number of feasible conditions to use. There is also the option of using an averaged value (rather than a value from the equation at a specified T & P). Using the property equation at the measured downstream T and upstream P has the merit of not requiring recalculation of the property. In general, the best property to use is calculated at the average upstream & downstream temperature & pressure (compared with an exact isentropic or isenthalpic calculation). The difference between the various options is general insignificant. The variation in upstream temperature is usually less than 0.01 K (whilst the difference with a proper thermodynamic calculation is more like 0.05 K).

### 5 CONCLUSION

Equations have been given that are simple to implement, but that are accurate enough to be useful (especially for high pressure orifice plate metering) for natural gas in the temperature range -10 to 40 °C and pressure range up to 100 bar.

Some of the equations have already been implemented by some manufacturers in their flow computer. The methods and equations are proposed to be ISO 20765 part 4.

### 6 NOTATION

Symbol	Meaning	Units
C	discharge coefficient	[-]
d	orifice diameter	[m]
D	pipe (inside) diameter	[m]
P	pressure (absolute) (upstream)	[Pa]
T	temperature (upstream)	[K]
t	temperature	[°C]
q	mass flowrate	[kg/s]
Re	Reynolds number, $(4 \cdot q) / (\pi \cdot \eta \cdot D)$	[-]
W	speed of sound	[m/s]
X <sub>i</sub>	mole fraction of component i	[mol/mol]
$\beta$	diameter ratio, $d/D$	[-]
$\Delta p$	differential pressure	[Pa]
$\Delta \omega$	permanent pressure loss	[Pa]
$\epsilon$	expansibility factor	[-]
$\eta$	fluid dynamic viscosity (at T,P)	[Pa.s]
$\kappa$	isentropic exponent	[-]
$\mu$	Joule-Thomson coefficient	[K/bar]

## 32<sup>nd</sup> International North Sea Flow Measurement Workshop 21-24 October 2014

### Technical Paper

$\pi$  = 3.141592654...

$\rho$  fluid density (at T,P) [kg/m<sup>3</sup>]

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