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Could Allocation be Rocket Science?
Using the Kalman Filter to Optimise Well Allocation Accuracy

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1 INTRODUCTION

In the 1960s, the Kalman filter was applied to navigation for the Apollo Project, which required estimates of the trajectories of manned spacecraft going to the Moon and back. The Kalman filter is an optimal estimation technique that uses dynamic mathematical models and physical measurements to obtain the most probable estimates of underlying variables as they vary with time. It incorporates both the measurement and also model uncertainties into its highly efficient estimation algorithm. Today, the use of the Kalman filter is extremely widespread throughout many science and engineering applications.

This paper describes the application of the Kalman filter to the problem of estimating (and hence allocating) well production based on intermittent and possibly poor quality measurement data from well tests. The technique recognises the increasing uncertainty in the estimated well flows as the time since the last well test elapses.

It also utilises the additional measurement of the wells’ daily, aggregate, commingled production as it exits a process. It can take advantage of process upsets in that when wells shut in, the commingled production of the remaining wells is measured and the drop in production is an estimate of the shut in wells. This information is smoothly incorporated into the well estimates and propagates forward in time.

The efficacy of the Kalman filter is demonstrated using simplified examples and then applied to typical real world, anonymised data.

The industry has called for better reservoir management techniques (Wood Review [1]). As part of the response, adoption of the Kalman filter can provide a mechanism to improve well production estimates.

Section 2 introduces the main concepts of the Kalman filter and illustrates its widespread practical use in numerous applications. Section 3 describes the potential uses of the filter in terms of oil and gas allocation, specifically focussing on its application to improve well production estimates. Section 4 explores the suitability of the filter using theoretical models and Section 5 applies the filter to real data. Section 6 provides some conclusions. Sections 7, 8, 9 and 10 present: Mathematical Analyses, Notation, References and an Appendix of Figures, respectively.

2 INTRODUCTION TO THE KALMAN FILTER

2.1 What is the Kalman Filter?

The Kalman filter is an algorithm that uses a series of measurements observed over time, containing statistical noise (i.e. uncertainty), and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone.

The algorithm works in a two-step process. In the **prediction step**, the Kalman filter produces estimates of the current state variables[1], along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of uncertainty, including random noise) is observed, these estimates are updated (in the **update step**) using a weighted average, with more weight being given to estimates with lower certainty. The algorithm is recursive in that it uses only the present input measurements and the previously calculated state and its uncertainty matrix; no additional past information is required.

2.2 History and Applications

The filter is named after Rudolf E. Kálmán, who wrote the original paper describing the filter [2] in 1960. (It is referred to as a filter as it filters the noise (or uncertainties) in the system and measurements). It was subsequently developed at NASA [3] and applied to the problem of trajectory estimation for the Apollo program, leading to its incorporation in the Apollo navigation computer. Apollo 8 (December 1968), the first human spaceflight from the Earth to an orbit around the Moon, would certainly not have been possible without the Kalman filter.

Since then, Kalman filters have been employed in a wide variety of practical applications, which include:

- Aircraft autopilots
- Navigation
- Dynamic positioning
- Computer vision
- Economic modelling
- Weather forecasting
- Climate modelling
- Missile guidance
- Speech enhancement.

[1] In this paper, these state variables are the well flow estimates or potentials.

3 APPLICATION TO WELL ESTIMATION AND ALLOCATION

3.1 Concept

The Kalman filter is applied to dynamic systems and uses process models, along with noisy measurements, to provide best estimates of variables in the system.

The mathematics of steady state data reconciliation has previously been used to develop an approach to uncertainty-based allocation (UBA) [5]. UBA reconciles noisy measurements to perform a daily allocation. An extension of these ideas to dynamic systems involves the use of the Kalman filter to perform the data reconciliation and is described further in [10]. Hence, the use of the Kalman filter appeared a natural extension of the ideas used to develop UBA.

There are potentially a number of applications of the Kalman filter in allocation systems. Though daily hydrocarbon allocation is essentially steady state, there can be a number of dynamic variables that feature in allocation systems. For example:

- Storage tank volumes, which fill over several days then are emptied periodically;
- Oil and gas pipelines, in which hydrocarbons of different compositions are commingled and through which they flow, possibly being accelerated and decelerated, over a number of days;
- Well production estimation.

It is the final example which has been used in this study to explore the feasibility and effectiveness of the Kalman filter for allocation purposes.

3.2 Application to Well Production Estimation for Allocation

The system envisaged in which the Kalman filter could be applied is the case where individual well production is measured intermittently using well tests. In the interval between well tests the well oil production can vary due to a number of factors, for example: rising water cut, change in downhole pressures, changes in lift flow or choke settings, etc. Ideally, a well should be retested if a significant adjustment takes place. However, in practice this is not always possible.

So, in the interval between tests it is possible that the uncertainty in the estimate of the well production can become significant. Various methods may be used to estimate the interim production: well decline equations, productivity indices, bottom hole and well head pressures, gas lift curves, etc. However, these estimates are in effect based on simplistic models of the well production, which cannot capture the complexity of the noisy real well production, and hence these estimates may still exhibit significant uncertainty.

A typical allocation approach in such a system is to allocate produced oil on a day in proportion to the estimated well production, based on well tests, after accounting for any processing effects (e.g. shrinkage) and well uptime (hours on production):

$$a_{w,d} = m_d * \left(\frac{x_{w,d} q_{w,d}}{\sum_w x_{w,d} q_{w,d}} \right) \qquad (1)$$

Where,

$a_{w,d}$	Oil allocation for well, w, on day, d
$a_{w,d}$	Oil allocation for well, w, on day, d
m_d	Produced oil measurement, on day, d
$q_{w,d}$	Daily flow potential estimate for well, for well, w, on day, d
$x_{w,d}$	Fractional uptime (i.e. hours on production÷24), for well, w, on day, d.

In this equation, the shrinkage is ignored (assumed to be 1) and the well flow estimate is in terms of a daily (24 hour) potential production $\mathbf{q_w}$, i.e. what the well would produce if it was operating steadily over a day. A well therefore still has a potential even if it is shut in. The well potential is the key variable that is estimated throughout this paper.

On a day when a well is tested $\mathbf{q_{w,d}}$ can be equated to the test rate $\mathbf{t_{w,d}}$ (adjusted to a 24 hour rate). However, it should be borne in mind that the well test measurement is noisy and hence exhibits a degree of uncertainty. In addition, as time elapses since a well was last tested, the uncertainty in $\mathbf{q_{w,d}}$ will rise.

There is additional information provided by Equation (1), in that the sum of all wells' production is measured by $\mathbf{m_d}$, which though it also exhibits uncertainty, is normally a fiscal meter and therefore more accurate than the well test measurement. Changes in $\mathbf{q_{w,d}}$ will be reflected in $\mathbf{m_d}$, although without further information it is not possible to determine confidently which wells' $\mathbf{q_{w,d}}$ to assign a change in $\mathbf{m_d}$ to. However, in real systems, wells are frequently shut in and started up (due to operational issues). So, for a specific well (w=A), $\mathbf{x_{A,d}}$ reduces from 1 if it is shut in (possibly falling to zero if shut in for the whole day), during which time the the drop in $\mathbf{m_d}$ may to some extent be attributed to $\mathbf{q_{A,d}}$. However, it must also be borne in mind that the remaining wells which have continued producing may also have changed their collective potential to some extent. What is evident from these considerations though is that the total measured flow, $\mathbf{m_d}$, does provide some information about individual well production, but knowledge of how $\mathbf{m_d}$ varies from day to day is required, which means the system has a temporal dependency, i.e. it is dynamic.

If changes in $\mathbf{m_d}$ are providing updates in $\mathbf{q_{w,d}}$, it also means that previous values of $\mathbf{q_{w,d}}$, will have an impact on the current day; again the temporal dependency and dynamic nature of the problem arises.

In summary, an approach is required that produces better estimates of wells' daily oil potentials ($\mathbf{q_{w,d}}$) that incorporates the following features:

- Uses well test measurements;
- Accounts for the uncertainties in the well test measurements;
- Recognises the rising relative uncertainty in the potential as time elapses since the last well test measurement was obtained;
- Utilises the daily product oil flow measurement;

- Accounts for the uncertainty in the product oil flow measurement;
- Infers information about well potentials from changes in the product oil flow, from one day to the next;
- Infers information about well potentials from a knowledge of their uptimes;
- Carries the most recent well potential estimates into the next day's allocation and hence updated well potential estimates.

The answer to this problem is provided by the Kalman filter.

3.3 Specific Implementation of the Kalman Filter for Well Estimation

A number of extensions and generalised methods have been developed but the Kalman filter proposed here is a linear discrete filter. The proposed implementation is relatively simple, in that the full Kalman filter includes terms for control variables which are not required in the well estimation system. A more complete description of the Kalman filter and its applications is provided by [4].

A mathematical presentation of the Kalman filter equations, as developed for this well test based allocation system, is provided in Section 7; the various steps are described briefly below.

Prediction Step

The first equation in the predict phase of the Kalman filter employs the transition matrix, which predicts how the well flow potential from the previous day propagates to the current day. This normally involves some multiplier, to account for well decline, etc. and could simply be 1 if the well flow is assumed constant. In effect, this is our model of how the well potentials evolve over time. In the real system described in Section 5, the transition matrix accounts for changes in well choke position.

The uncertainty in the previous day's potentials (this is an output from the Update Step of the Kalman filter run on the previous day) is used - with the addition of uncertainty due to process noise - to obtain the uncertainty in the predicted well flow potentials. This process noise reflects the uncertainty in the model that we have assumed.

Update Step

This step incorporates any measurements that are made. If any wells are tested the well test along with its measurement uncertainty are used to update an individual well's potential. Similarly the total metered product oil is a measurement of the sum of the production of all wells flowing on that day. This is used to update the potential of all wells flowing on the day.

These measurements are necessarily uncertain and hence the well flow potentials are updated using a weighted average of all estimates and measurements, with more weight being given to those with lower uncertainty.

Because the uncertainty of the measurements and the process model uncertainty (process noise) may be difficult to determine precisely, it is common to discuss the filter's behaviour in terms of gain. The Kalman gain is a function of the relative uncertainty of the measurements and current estimate of the predicted well potentials, and can be tuned to achieve particular performance. With a high gain, the filter places more confidence on the measurements, and thus follows them more closely. With a low gain, the filter follows the model predictions more closely, smoothing out noise but decreasing the responsiveness.

The uncertainties of the updated well potentials are also output by the filter. This is in the form of the covariance matrix which is a familiar feature in data reconciliation techniques.

Recursion

The calculated well potentials and their associated uncertainties form the input to the calculations the following day. This is the recursive nature of the Kalman filter and all the information required to perform the calculations the next day is contained within the estimates and uncertainties from the current day.

This feature makes the Kalman filter computationally very efficient, which was one of its attractions in the days of limited computing power available during the Apollo missions. Though computing power is not such an issue now, the computational efficiency is attractive from an allocation viewpoint because the algorithm is concise and only requires input from the previous day. This means the filter does not increase effort required to perform allocation re-runs, etc. It is analogous to any other balance that is carried forward from one day to the next in allocation systems, e.g. pipeline stocks, gas substitution accounts, etc.

4 FEASIBILITY WITH SIMPLIFIED EXAMPLES

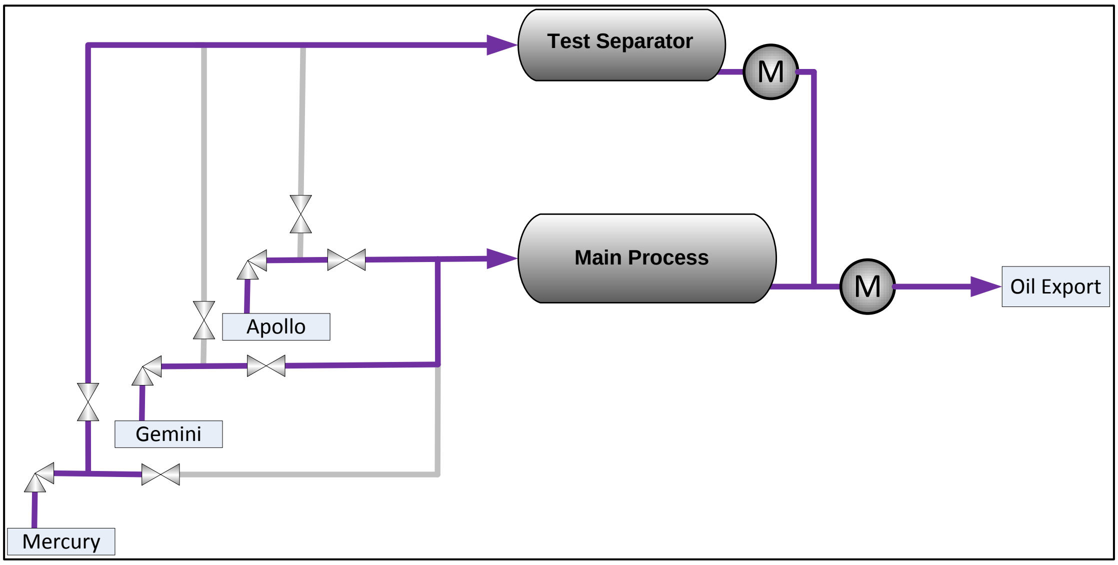
4.1 Introduction

A simplified theoretical process model is utilised in this section, in which the true well production can be modelled. Random process noise is introduced to reflect a more realistic process. Meter uncertainties are then applied to mimic the real world measurements and well estimates generated based on the Kalman filter equations. The results of the Kalman filter will be affected by the measurement noise but its performance can be determined by comparing the results with the known true underlying well production. In particular, the Kalman filter estimates can be compared against more conventional well test allocation approaches and hence an assessment of the filter’s efficacy assessed.

4.2 Description of System

There are assumed to be three wells, labelled Apollo (α), Gemini (γ) and Mercury (μ) that are normally commingled in a main process but individual wells can be tested in a dedicated test separator. The process is illustrated schematically in Figure 1:

Figure 1 – Simplified Model Process Schematic



In this idealised theoretical model it is possible to know the true production potential of each well as it declines with time. This is modelled assuming a simple exponential decline curve and the potential from one day to the next changes according to:

$$q_{\alpha,d} = e^{-b_\alpha \Delta} q_{\alpha,d-1} \qquad (2)$$

Where,

b_α Well Apollo's exponential decline constant
Δ Day step interval (=1 day).

In order to introduce more realistic noise into the system the decline well flow is randomly varied in accordance with a Gaussian random variable with an uncertainty equal to ±1% of the flow. This is to reflect other unknown process effects that affect the flow but which cannot be modelled.

The well uptime is also modelled simplistically in that a well can be shut in or producing at maximum rate, i.e. the fractional daily uptime can only be 0 or 1. It was assumed that there was a 10% chance that an individual well would be shut in on any day.

It is acknowledged that when a well is producing for less than 24 hours but not shut in for the whole day, the fractional daily uptime introduces an extra level of uncertainty since production hours themselves will be estimates. The uncertainty in the flow is exacerbated because wells do not attain full production immediately on start up. This issue is discussed further in Section 5 when the Kalman filter is applied to real data.

Hence, the true production from each well can be modelled on a day by day basis and the combined flow calculated as the sum of daily well production. Ignoring processing effects (shrinkage) this summated production represents the true measured daily oil production from the process.

In addition each well is tested at fixed intervals, although the interval is different for each well.

In reality, the measured total production and well test measurements would include measurement uncertainties and these were assumed to be ±1% and ±20% respectively.

4.3 Simplified Model Results

The calculations were run for a 50 day period. The results from the simplified model are presented in Figure 2, Figure 3 and Figure 4 for Apollo, Gemini and Mercury, respectively:

Figure 2 – Simplified Model Apollo Well

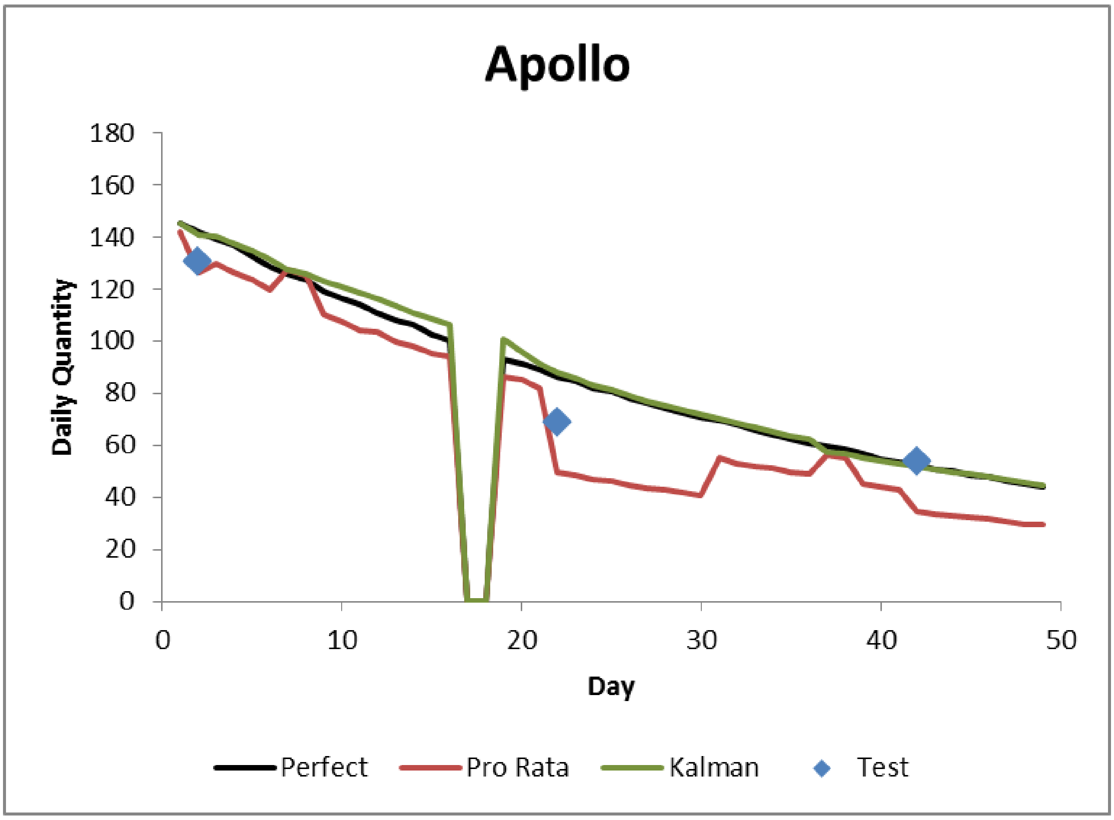


Figure 3 – Simplified Model Gemini Well

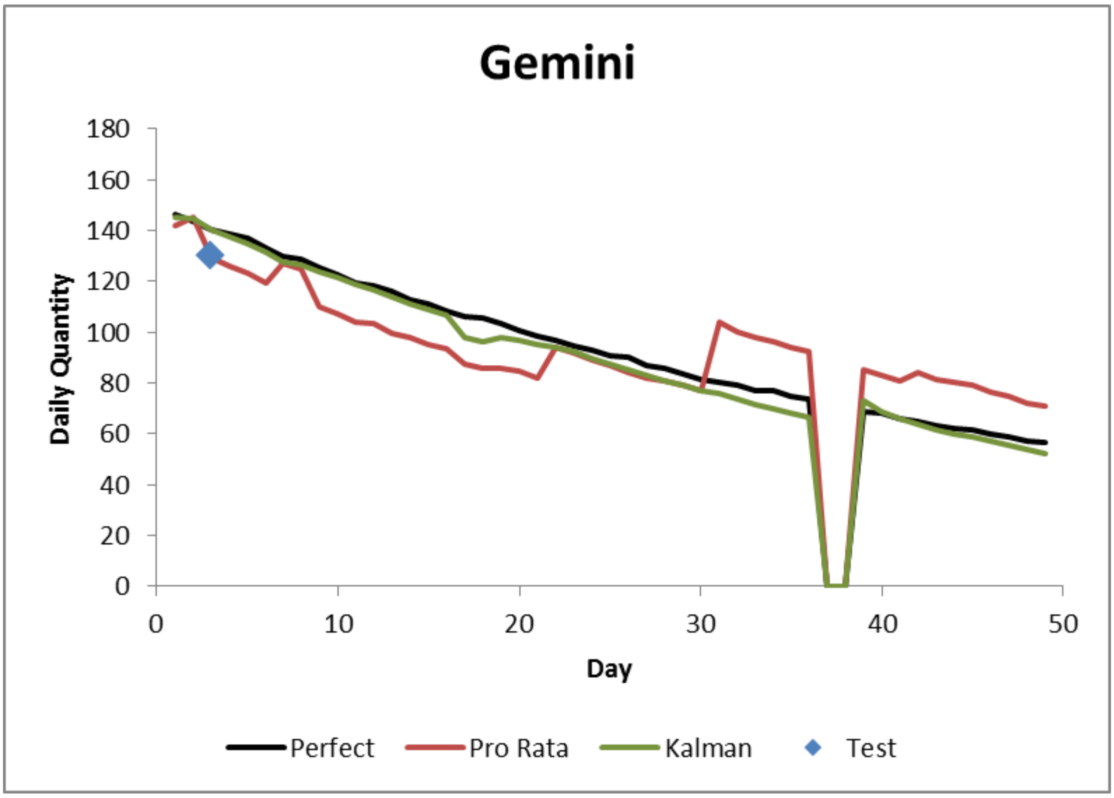
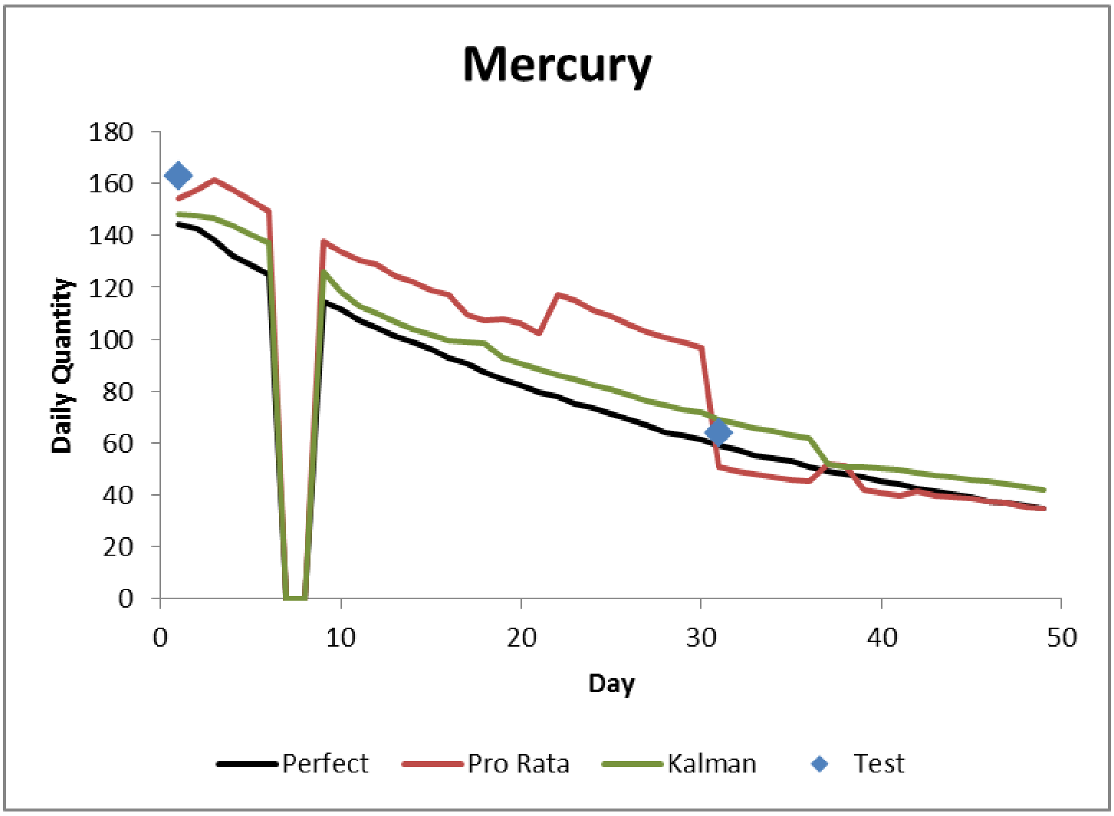


Figure 4 – Simplified Model Mercury Well



The black line is the true modelled flow from each well which declines exponentially with some daily variation due to other random process noise.

The blue diamonds are the well tests which include measurement uncertainty.

The red line is the amount that would be allocated to wells based on the latest well tests allocated in a pro rata fashion – which is a typical approach. The green line shows the results obtained from the Kalman filter.

These plots are intended to illustrate some of the features that were anticipated in Section 3.2. As can be observed, the Kalman filter predicts the well flows and associated potentials better than the simple pro rata approach. Use of the commingled meter production to inform the individual well rates in the Kalman filter mitigates the impact of relatively high well test uncertainties which distort the pro rata results.

With the well test based pro rata approach, a poor well test for one well will have a deleterious impact on the other wells, as can be observed on Day 22 when the Apollo well test significantly under-measured the flow and the Gemini and Mercury wells saw their allocated oil increase in a step wise fashion. These particular relative distortions persist until further well tests are conducted.

Also, the days when wells shut in reveal further information about the individual well rates and this is most noticeably observed on Day 37 when Gemini shuts in. In the Kalman filter this results in the other two wells' oil flows being corrected even after Gemini restarts. This is not the case for the pro rata approach which continues to estimate the Apollo and Gemini rates poorly.

These plots were generated randomly but a specific example was selected to illustrate the features discussed. A more meaningful analysis is to perform a whole series of 50 day allocation runs allowing the various input parameters to vary randomly in each Monte Carlo trial and continuing with the random daily fluctuations also within the trial. This analysis is presented in the next section.

4.4 Monte Carlo Analysis

In the analysis, each well's initial flow was independently, randomly varied between 50 and 200 tonnes per day[2]. The well decline factor (b) was similarly randomly varied between 0.00 and 0.03 for each well independently. Both the initial production and well decline factor were held constant for the duration of each Monte Carlo trial of 50 days.

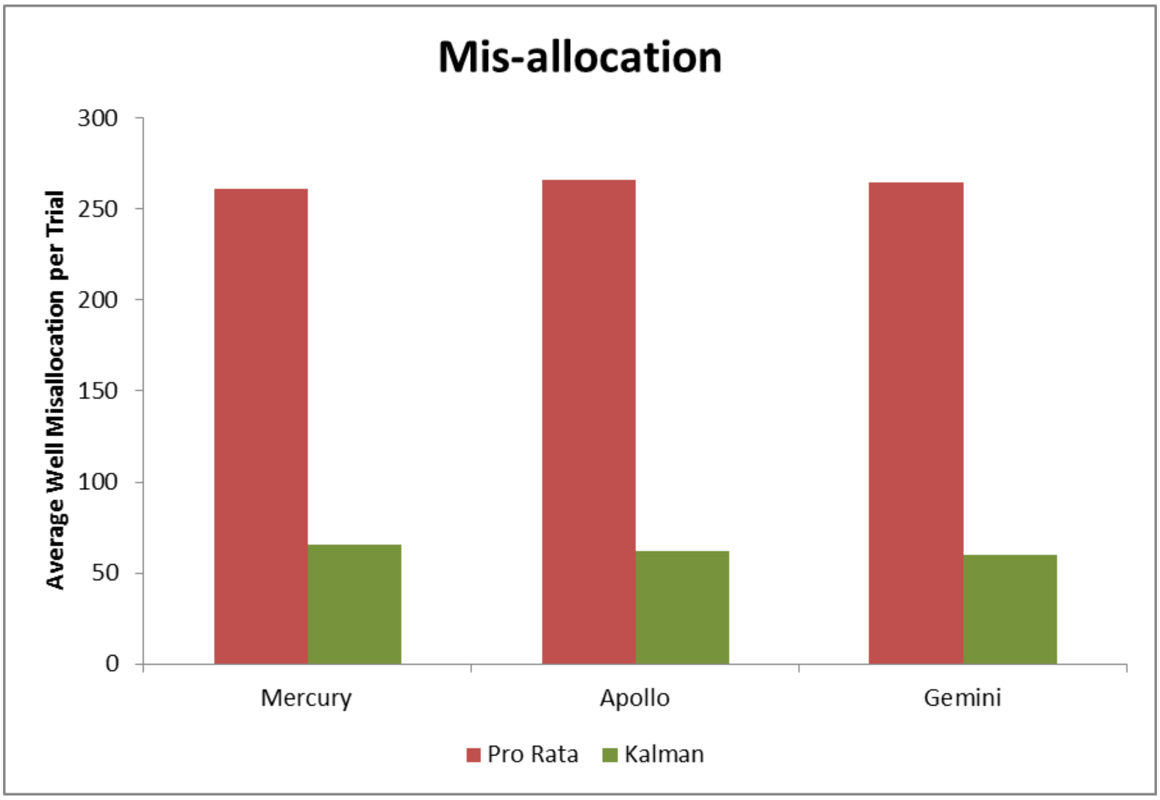
Similarly, the assumed process noise for the Kalman filter was varied randomly between 0% and 10% (i.e. normally significantly over estimating the true noise value of 1%) between trials.

[2] This is an arbitrary range and the designation of tonnes per day is not of special significance.

The test interval was varied randomly between 10 and 30 days, independently for each well but the intervals were held constant for the duration of a single Monte Carlo trial.

1,000 trials were run in the Monte Carlo simulation and the absolute difference, or mis-allocation, between the estimated and true production from each well was recorded each day for each method. These daily absolute differences were then summed over the 50 days for each trial. The averages of these 50 day mis-allocations from the 1,000 trials for each method are compared in Figure 5 for all three wells.

Figure 5 – Average 50 Day Total Well Mis-allocation from 1,000 Monte Carlo trials



This illustrates the Kalman filter estimates to be significantly more accurate than the simple well test based approach: the average mis-allocation is reduced by approximately 75%.

The above analysis illustrates the potential benefits of the application of the Kalman filter to well flow estimation.

The theoretical environment allows us to assess the efficacy of the Kalman filter in a unique way in that we do know the true values of the variables we are estimating. Significant effort has been taken to try and account for all foreseeable issues that may confound the estimation, for example - accounting for the fact that the Kalman model does not assume the correct initial well flow, the decline factor or process noise. This has been accounted for by randomly varying these assumed values (around the true values) between the Monte Carlo trials. However, this is still a relatively idealised environment and though many variables have been accounted for and their influence tested, the theoretical model may not necessarily include the possibilities of unknown process noise, gross errors, human error, etc. The next section addresses this by applying the Kalman filter to real data.

5 APPLICATION TO REAL DATA

5.1 Introduction

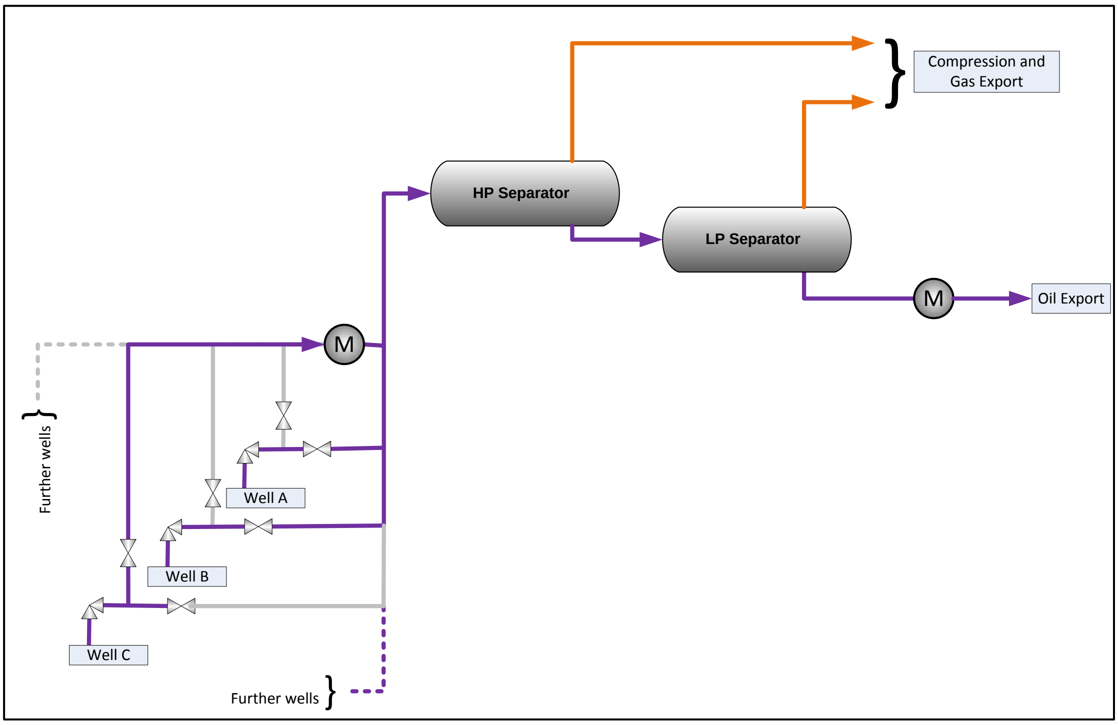
The chaos and noise of real world data presents a more formidable test of the Kalman filter than any theoretical data in assessing the feasibility and robustness of the method.

The problem with real data is that we can never know the true values of well flows we are estimating and the performance of the filter has to be assessed by indirect means. This has been accomplished by: some direct numerical estimations of performance, by comparison with the simple well based pro rata approach and the use of engineering judgement.

5.2 Description of System

A schematic of the process and well configuration is presented in Figure 6:

Figure 6 – Simplified Schematic of Process



The process handles production from five wells (labelled A to E, though only the first three are shown in the schematic). The oil and gas products are fiscally metered. There is a single subsea multiphase flow meter (MPFM), through which the wells can be individually routed to test their performance.

As shown in Figure 6, the well stream fluids are commingled upstream of the 1st stage separator. Oil separation is achieved using two-stage separation and gas from separation is sent via compression trains to fuel and export.

Though real, the data is anonymised and several years old. The period examined covers just over 150 days (5 months) of production. Only the oil is considered in the following analysis.

5.3 Kalman Filter Process Model

The assumed model for the real system is similar in many respects to that described for the theoretical model presented in Section 4. However, the exponential decline was not applied to the real wells as they were choked back during the period examined. In fact the choke positions were varied daily and significantly so between well tests.

It was found by an analysis of the well test data that the well flows varied roughly linearly with the choke position. The choke position was the dominant variable in determining flow. The well rates appeared much less well correlated with other variables such as bottom-hole pressure, etc.

The effect of the choke changes was therefore incorporated into the process model and the change in potential from one day to the next was calculated according to:

$$q_{w,d} = \frac{ch_{w,d}}{ch_{w,d-1}} q_{w,d-1} \tag{3}$$

Where,

$ch_{w,d}$ Choke opening position, for well, w, on day, d.

w represents any of the wells, A to E. In effect, the well potential, (i.e. the state variable), is determined as the flow at the new choke position.

5.4 Estimated Process Noise

The process noise has to be estimated using engineering judgement to some extent, though it is adjusted as part of the tuning process, discussed in Section 5.7. A value of ±10% was assumed as the baseline uncertainty in the model. This was further increased in proportion to the choke change (up to maximum of ±50%), to reflect the additional uncertainty introduced by the choke changes and the assumption of a linear dependence on choke position.

These values were determined after some trial and error. In fact, the performance of the filter was relatively insensitive to the process noise uncertainties unless they were extremely low (<±1%) or high (>±100%).

5.5 Measurement Model

Well Tests

The well test oil represents a direct measurement of the oil potential.

$$q_{w,d} = t_{w,d} \tag{4}$$

Where,

$t_{w,d}$ Well test rate (24 hour basis) for well, w, on day, d.

Product Oil Measurement

The product oil is a measurement of the sum of the producing wells' flow, after allowing for shrinkage:

$$\sum_{w} q_{w,d} * x_{w,d} * sh_{w,d} = m_d \tag{5}$$

Where,

$sh_{w,d}$ Oil shrinkage from well test to export for well, w, on day, d.

That is, the sum of each well's potential from the update step in Equation (3), multiplied by their fractional up time for the day (i.e. hours producing/24), multiplied by the shrinkage from well test to export product oil conditions, should be equal to the total measured product oil on the day.

5.6 Measurement Uncertainties

The oil product and sub-sea multiphase well test flow meter measurement uncertainties are presented in Table 1:

Table 1 –Well MPFM and Oil Product Uncertainties

Meter	Uncertainty type	Uncertainty (±%)
Oil Export Meter	relative	1%
MPFM Water Liquid Ratio (WLR)	absolute	3%
MPFM Liquid Flow	relative	3%

The relative uncertainty in the oil export meter (from Table 1) was assumed to be ±1%. This was based on a nominal uncertainty of ±0.5% for the fiscal meter but degraded to reflect the presence of a low, but still significant, water content.

The quoted MPFM uncertainties are typical nominal values appropriate for the type of meter installed. For an MPFM, the dry oil flow uncertainty is a function of the measured liquid and WLR and their associated uncertainties and the relative uncertainty in the oil flow is given by:

$$e_{t,o} = \frac{\sqrt{\left(e_{t,l}(1-WLR)\right)^2 + (\varepsilon_{WLR})^2}}{(1-WLR)} \tag{6}$$

Where,

$e_{t,l}$	Relative uncertainty in measured liquid flow rate
$e_{t,o}$	Relative uncertainty in oil flow rate
WLR	Water liquid ratio
ε_{WLR}	Absolute uncertainty in measured WLR.

The above equation was presented in [7] and derived using the approach described in the GUM [8], termed Taylor Series Method (TSM), which is used to model the propagation of uncertainties.

Though the meter was used to measure the flow from a number of different wells, they were all produced from the same reservoir which was at a pressure above the bubble point. Hence, the wellstream composition for all wells should remain ostensibly constant. Similarly, the produced water composition should also remain stable.

The measured oil in the MPFM also undergoes some shrinkage from the point of measurement to export. Monte Carlo simulation (described in a Supplement to the GUM [9]) of the process found the shrinkage to be 0.928 on average with an uncertainty of ±3%. The approach used in [6] was used to obtain the average shrinkage and associated uncertainty.

5.7 Kalman Gain Tuning

It is unlikely that the assumed uncertainty for the process noise is entirely accurate. It is also unlikely that the product oil and MPFM measurement uncertainties are wholly precise. This issue is well recognised in the implementation of Kalman filters in real world applications and such filters are termed sub-optimal.

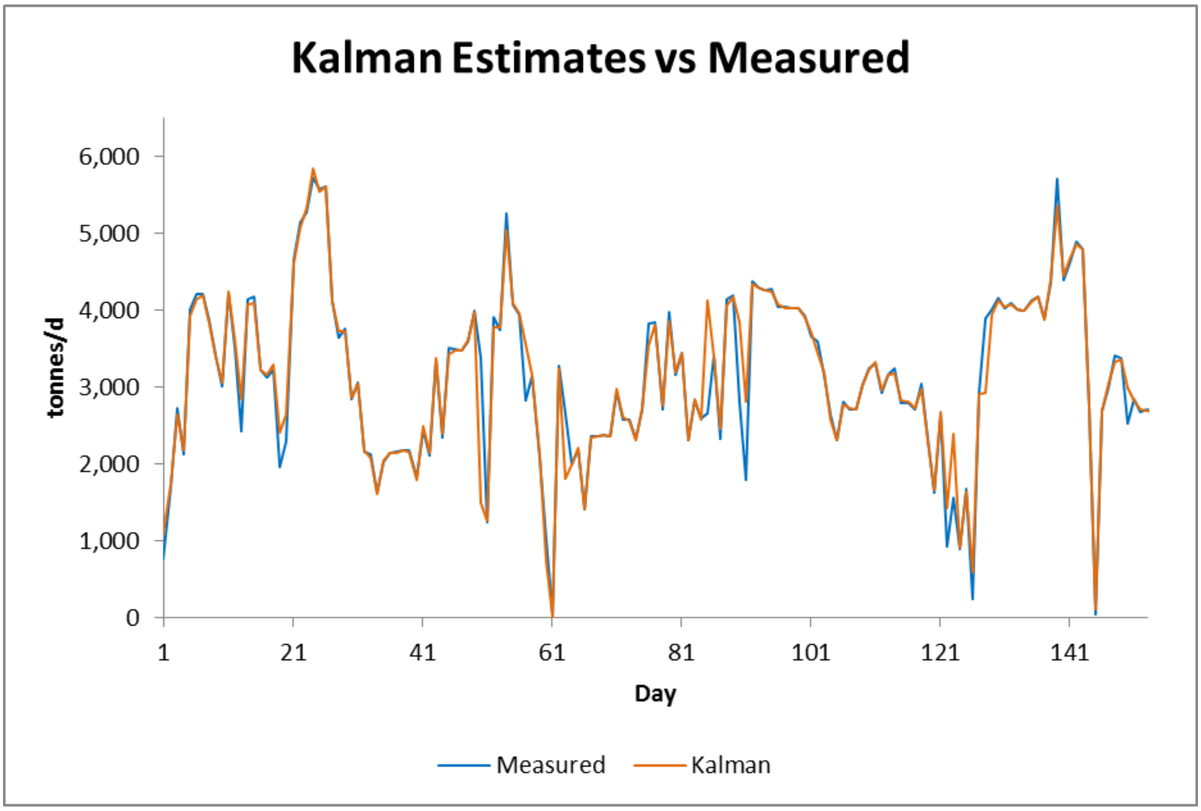
The behaviour of the filter is determined by the ratio of the process to measurement uncertainties in the form of the Kalman gain. Hence, the filter is less sensitive to the accuracy of the uncertainties so long as their values relative to each other are representative. This was accomplished in this application by adjusting the process noise variance, in effect tuning the Kalman gain. This tuning was optimised by observing the behaviour of the filter in estimating the wells' future test rates and the daily produced oil.

The tuning process is an attempt to optimise the sub-optimal filter to render it as close as possible to a truly optimal filter. Sub-optimal filters, Kalman gain tuning and practical implementation considerations have been thoroughly analysed in the literature and are discussed for example in [4].

5.8 Results

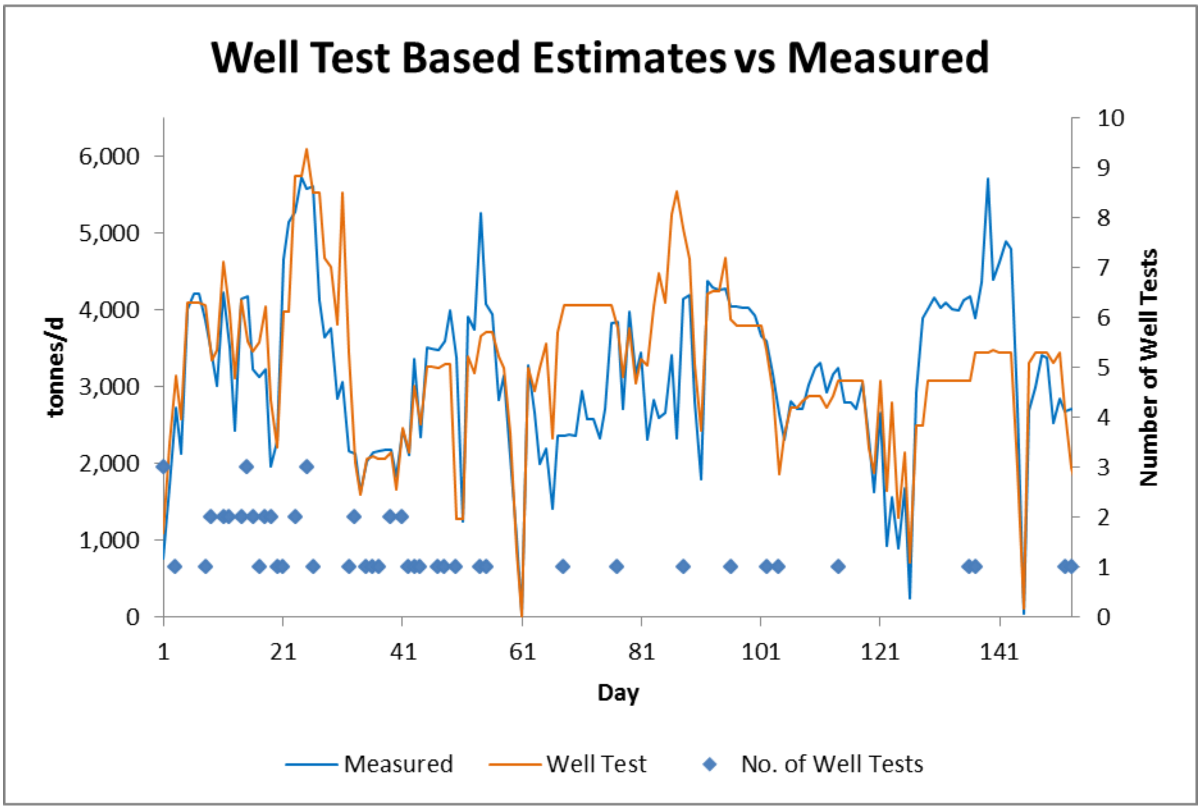
Figure 7 is a plot of the predicted total oil from the **Predict Step** in the Kalman filter ($Q_{d|d-1}$) versus the actual measured rate, i.e. prior to the reconciliation of the potentials against the oil export meter on that day.

Figure 7 – Kalman Predicted versus Measured Oil



As can be observed the Kalman totals are a close match with the metered oil. To put this in context, compare the total predicted oil from the well test based pro rata approach in Figure 8:

Figure 8 – Well Test Based Predicted Total Oil versus Measured Oil

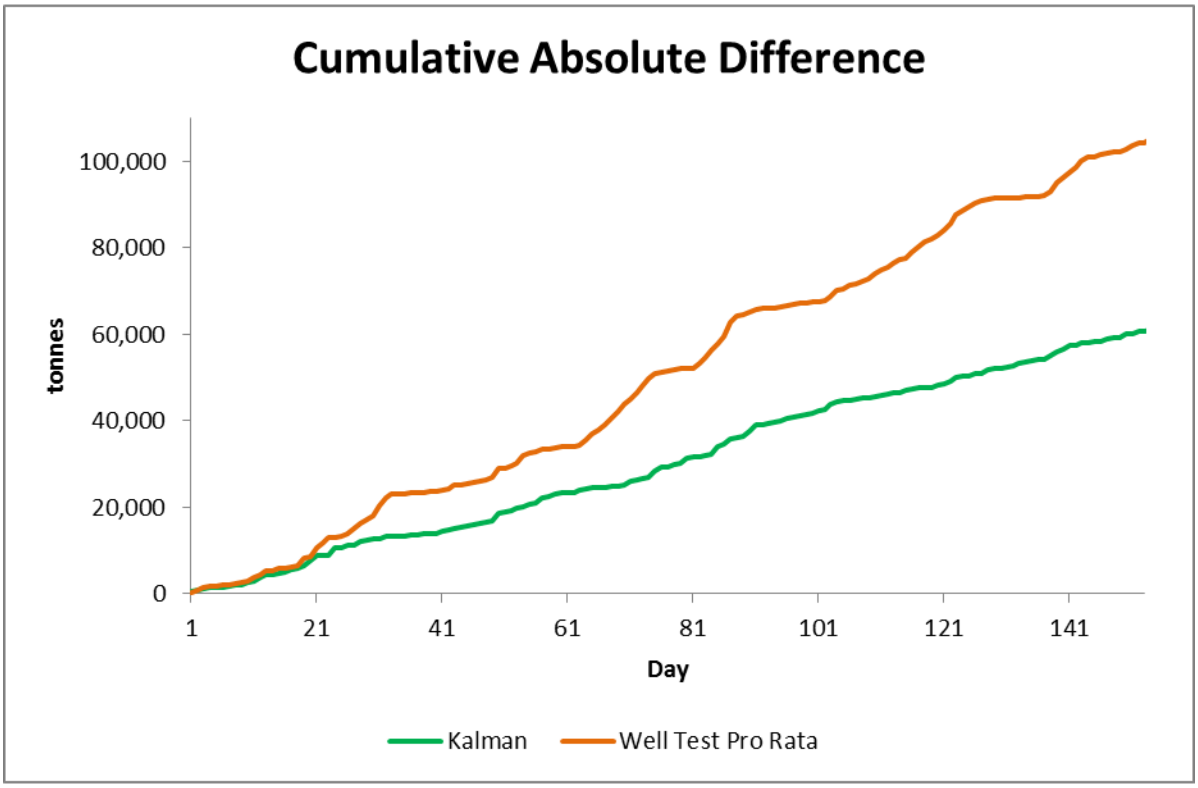


Looking at the well test based estimates, the match is not as good and there are clear periods when the well tests appear to be significantly under- or over-estimating production. The diamonds (plotted against the right hand axis) indicate the number of well tests conducted each day. The periods of drift in the estimated oil production do appear to coincide roughly with periods when the well test frequency is low.

The advantage the Kalman filter offers is its ability to exploit the additional information provided by each day's measured product oil.

This difference in predictive ability was calculated from the absolute difference in the predicted and the measured oil for the two methods over the 153 day period. The cumulative absolute differences are plotted in Figure 9:

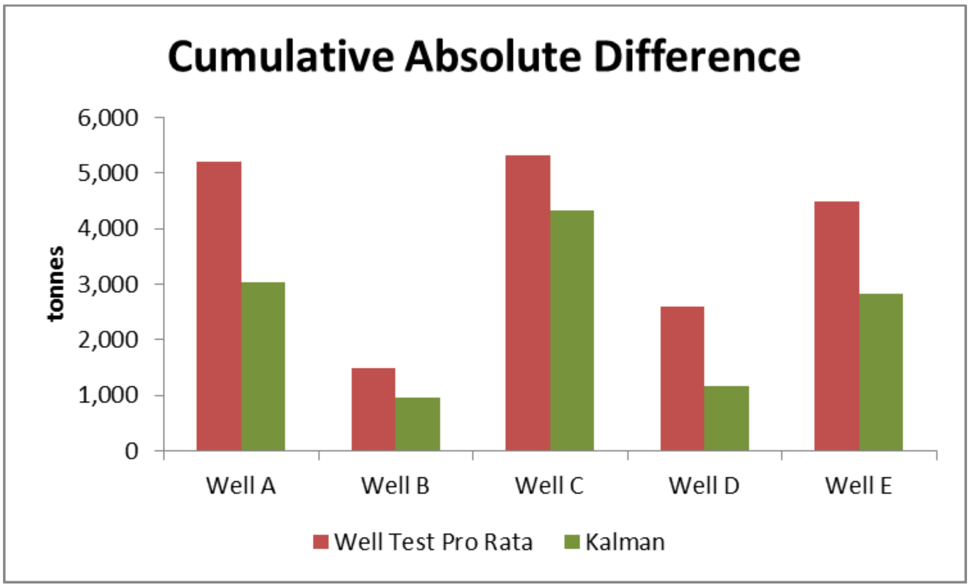
Figure 9 – Cumulative Absolute Difference Predicted versus Measured Oil



At the end of the period the Kalman filter cumulative difference is 42% lower than the well test pro rata value.

A second metric with which to compare the methods is to determine the absolute difference between each well's predicted flow just prior to a well test and the measured well test flow. This is plotted in Figure 10:

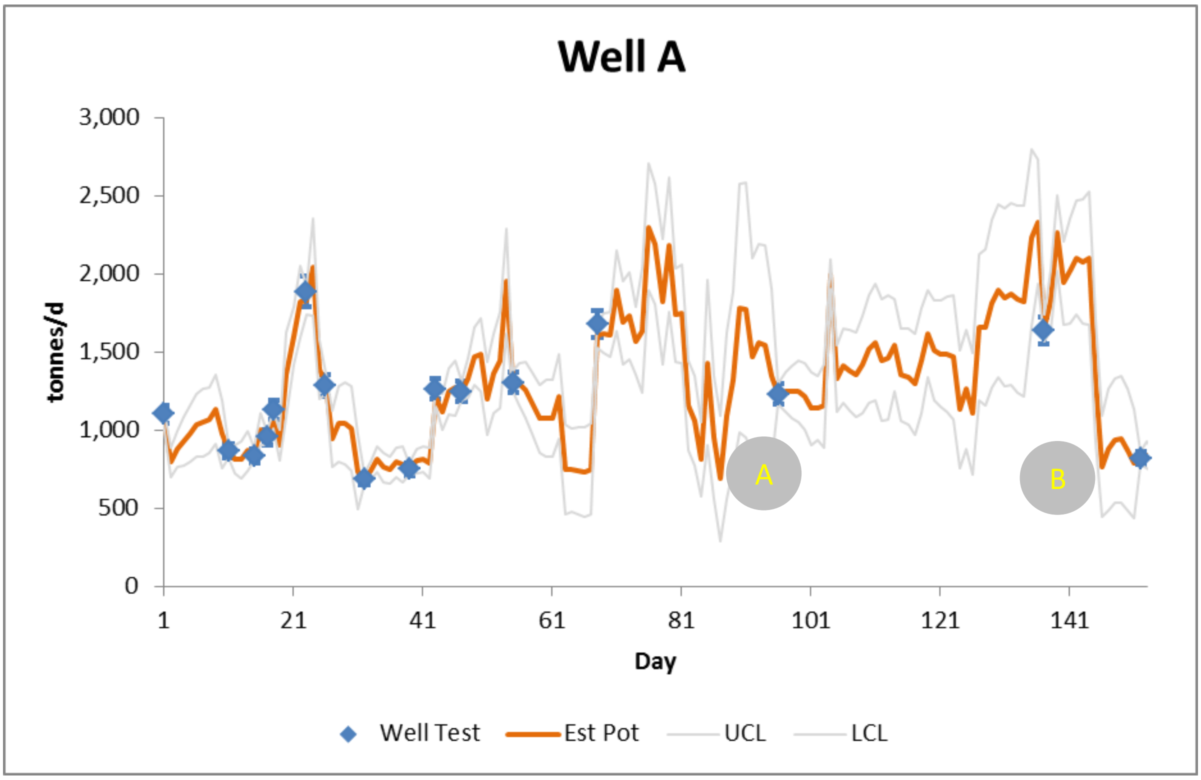
Figure 10 – Cumulative Absolute Difference Predicted versus Measured Well Test Oil



At the end of the period the Kalman filter cumulative difference is 36% lower than the well test pro rata value for the well test data.

The following figures examine the results of the Kalman filter for individual wells. Figure 11 is a plot of the Kalman estimated potentials for Well A:

Figure 11 – Well A: Kalman Oil Potentials

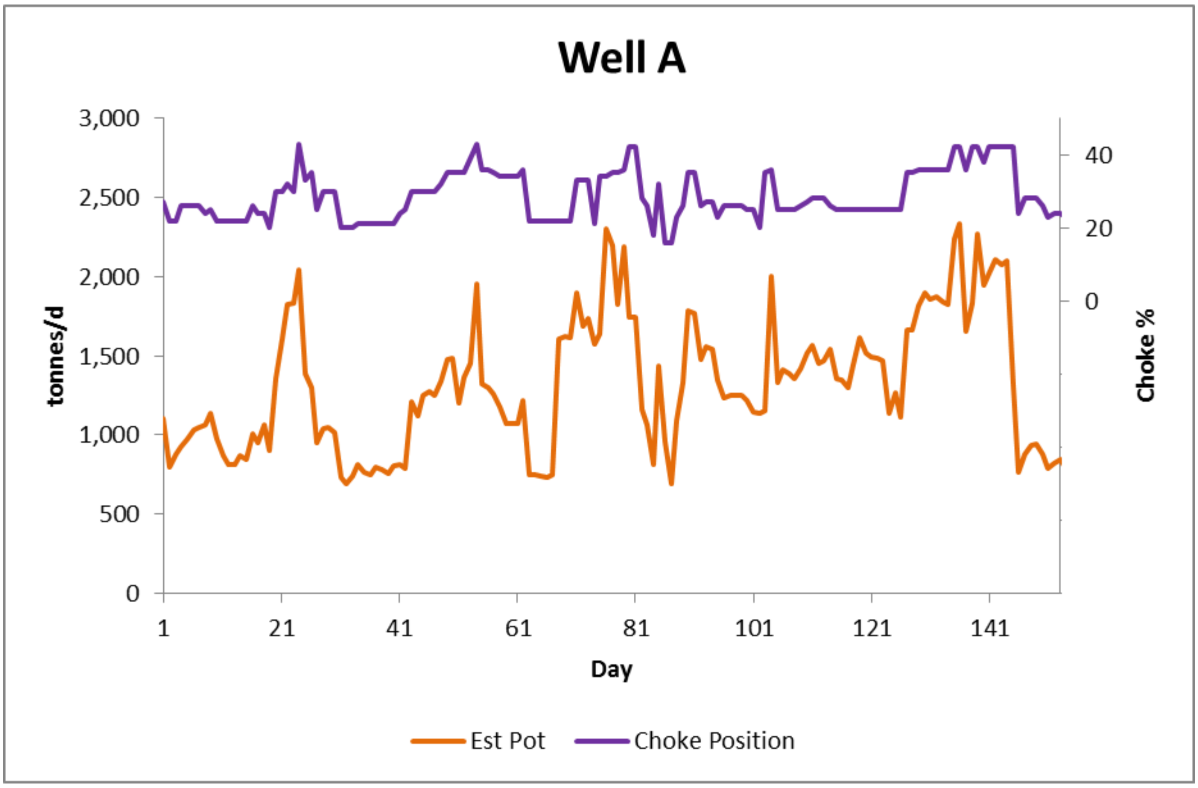


The orange line plots the Kalman filter estimate of the well's potential. The blue diamonds are well test rates which have error bars to indicate their measurement uncertainty. The grey lines, labelled UCL (upper confidence limit) and LCL (lower confidence limit), represent the ± uncertainties in the Kalman potentials, which are generated from the covariance matrix ($\mathbf{P}_{\mathbf{d|d}}$). As can be observed, the uncertainty rises as the time elapsed since the last well test increases, which was one of the objectives presented in Section 3.2. It also reduces when a well test is conducted as the confidence in the potential increases when a direct measurement is made.

An attractive feature of the Kalman filter is along the evolution of the potential, it predicts changes in potential prior to a well test taking place. This is most obviously illustrated at the points labelled A and B. Around point A, the predicted potential is varying but falls smoothly to agree with the well test on Day 96. Even more markedly at point B, Well A's predicted potential falls from around 2,000 tonnes/d to 800 tonnes/d on Day 146 (due to a choke change – see Figure 12), the new reduced rate is then confirmed by the well test on Day 152.

There is considerable variability in the potential and this is largely driven by changes in the well choke position. The changes in flow roughly correspond with the changes in potential as illustrated in Figure 12:

Figure 12 – Well A: Potential and Choke Position

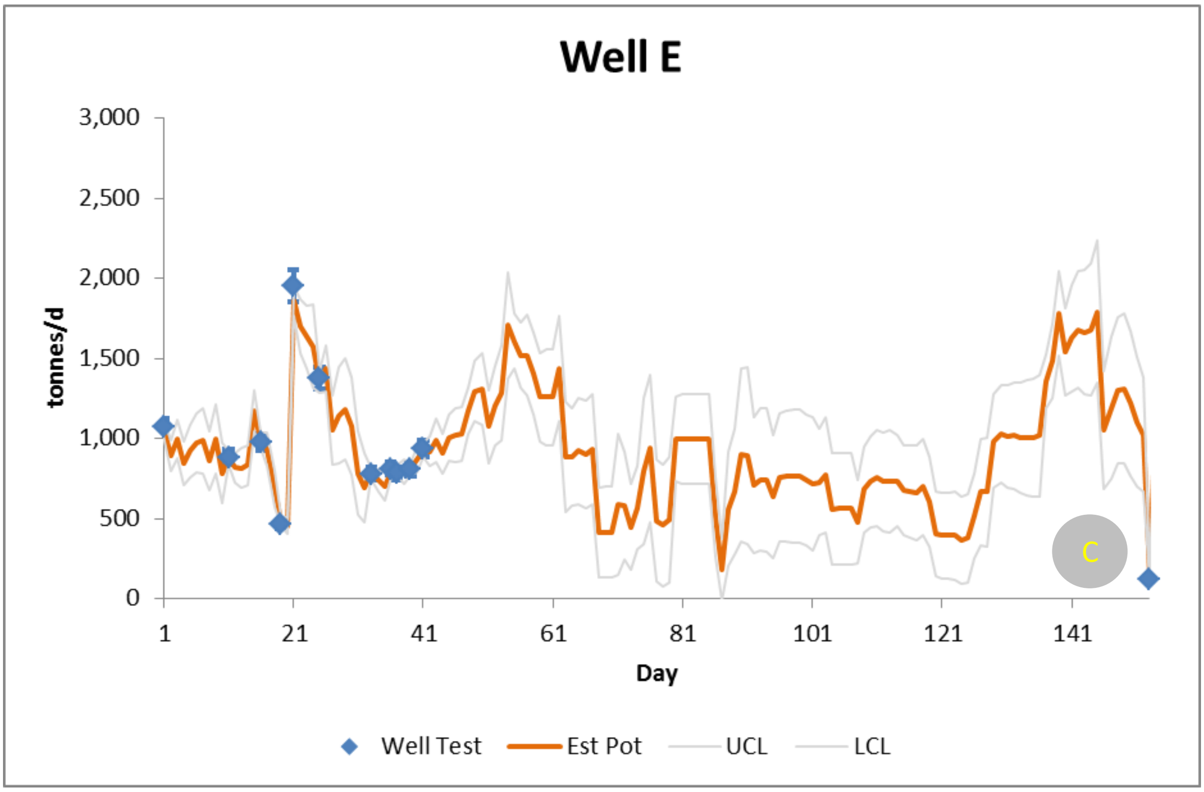


The potential is plotted in orange, identical to that in Figure 11, but the choke % opening is also plotted in purple against the right hand axis.

Similar charts for all the wells are presented in the Section 10 Appendix of Figures, where similar features for the other wells can be observed.

The Kalman potential for Well E, as plotted in Figure 13, is worthy of further comment however:

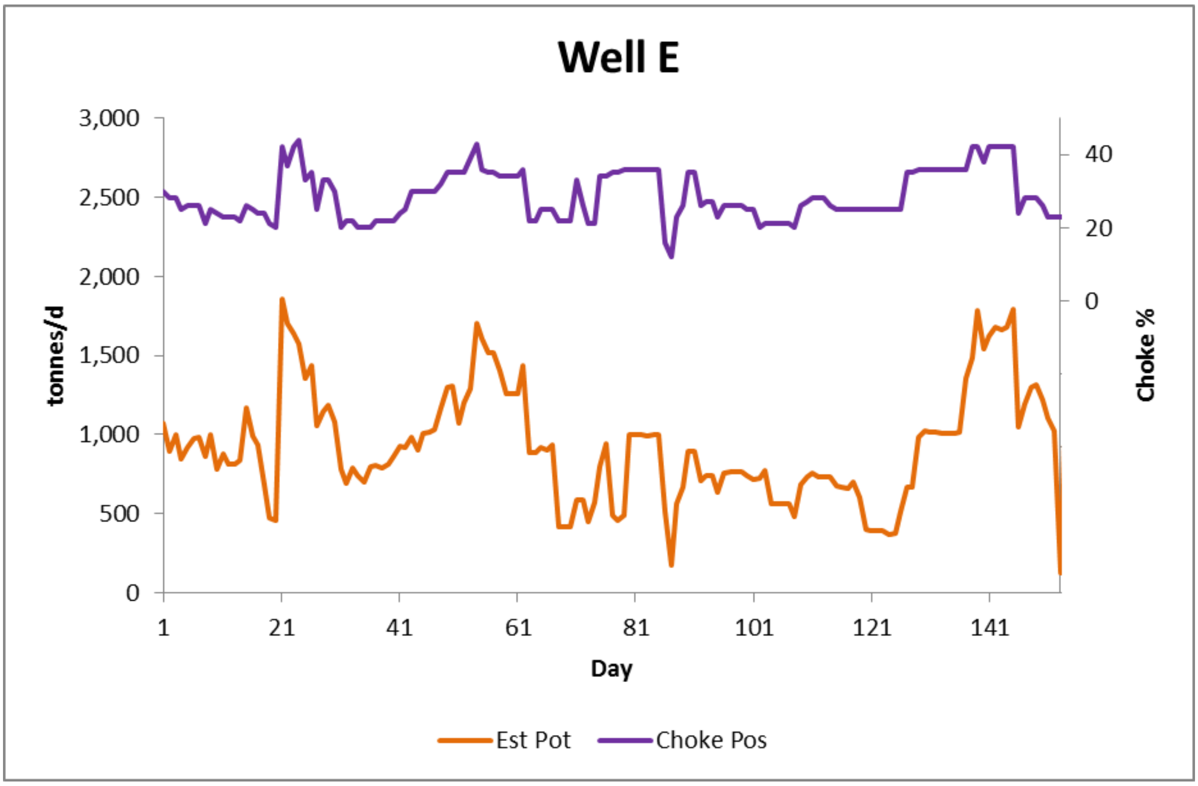
Figure 13 – Well E: Kalman Oil Potentials



An interesting feature of Well E is that it is not tested after Day 41 until Day 153, when there is an extremely significant drop in its potential from 1,024 tonnes/d to 123 tonnes/d as indicated by C.

Examination of its choke position in Figure 14 shows that the variation in its flow generally reflects its choke position:

Figure 14 – Well E: Kalman Oil Potentials

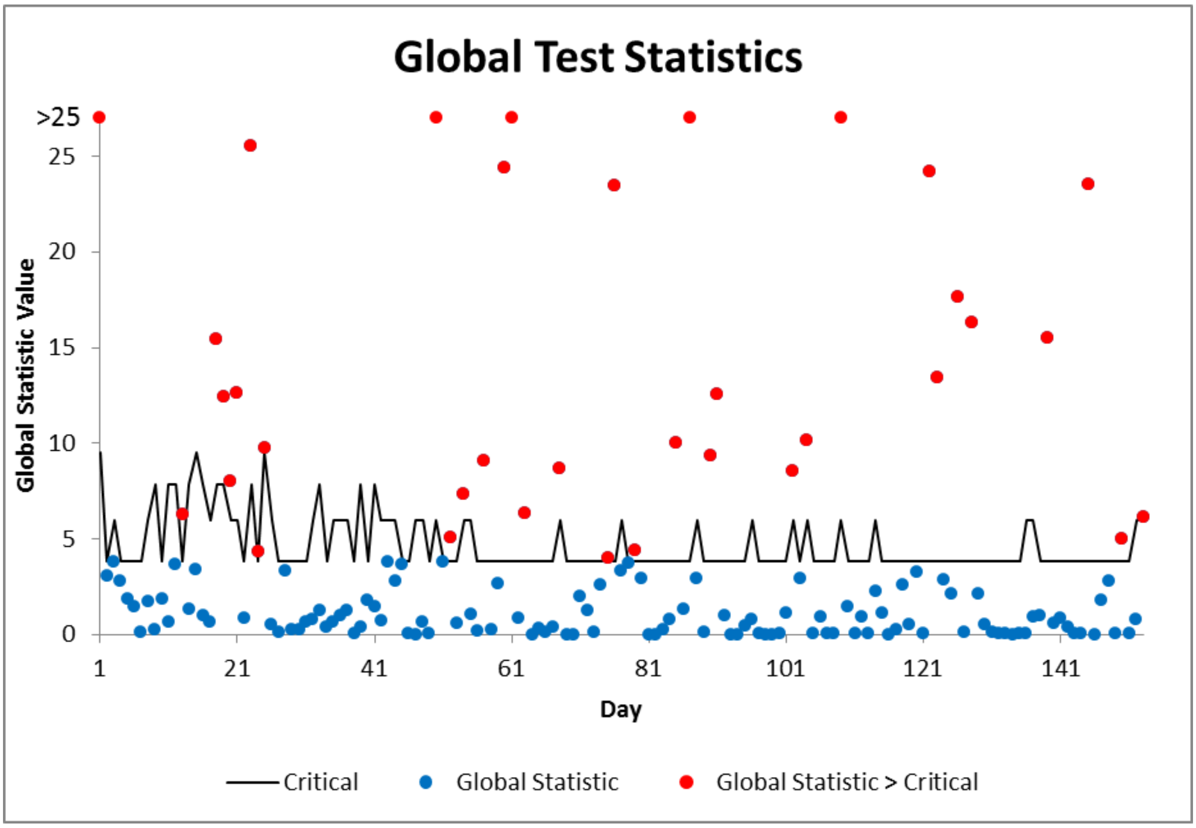


However, this is not borne out for the day (Day 153) of the well test when the choke position remained the same as the previous day and the flow of all producing wells had been steady. In fact this drop leads us to suspect the well test itself, especially when the next test on well E, performed less than two weeks later, measured an oil rate of 1,174 tonnes/d.

The Kalman filter provides information on the uncertainty in the estimates which, when coupled with the measurement data, allows statistical tests to be performed on the data. One such test is the Global Test which considers the mass imbalances (residuals in the constraint equations) in the system and determines statistically whether these are within the range expected given the measurement and well potential uncertainties. The mathematics of the test are presented in Section 7.4. The Global Test identifies the potential presence of gross errors but does not identify the source of the errors.

The statistic is plotted in Figure 15:

Figure 15 – Global Test Statistic



The black line is a critical value of the statistic, which is dependent on the number of measurements made each day. The circles indicate the calculated daily Global statistic: blue are below the critical value and red are above it indicating the potential presence of gross errors in the data.

The Global test statistic is above the critical value on Day 153, indicating there is potentially a problem and perhaps confirming our suspicion of the test on Well E.

In fact, there are a number of other days where the critical test statistic is exceeded and these days have been examined on a day by day basis and ignored if they are deemed to corrupt the estimates. These days when gross errors occur are generally when wells restart after being shut in, as was alluded to previously in Section 4.2).

6 CONCLUSIONS

The Kalman filter brings new dimensions to estimation techniques. It utilises not only all the available measurement data, but also employs a process model to enable estimates to be propagated from one day to the next – hence prior information gained is retained. It performs this in a statistically near optimal manner by utilising the uncertainty in the measurements and the process model.

The Kalman filter has been demonstrated to be a feasible tool to optimise the estimation of well potentials using theoretical models. In addition, Kalman filter has been successfully applied to a real system and produces improved estimates of oil potentials compared with simply using well test data.

7 MATHEMATICAL DEVELOPMENT OF THE USE OF A KALMAN FILTER APPROACH TO OPTIMAL WELL FLOW ESTIMATES

7.1 Introduction

The mathematical development presented in this section is based on the simplified theoretical system discussed in Section 4. A similar development is applicable to the real world data in Section 5, the principal difference being the elements of the phase transition matrix which are based on change in choke position rather than well decline.

Notation is presented in Section 8.

7.2 Predict Phase of Filter

In first phase of the filter, the state transition matrix $\mathbf{F_d}$ is used to estimate the updated well flow **q** (on day **d**) from the previously estimated flow (on day **d-1**) in Equation (7). This is in effect the model estimated update and, for the case of the three wells, this is given by the exponential decline equation.

$$Q_{d|d-1} = F_d Q_{d-1|d-1} \tag{7}$$

More explicitly,

$$\begin{bmatrix} q_{\alpha,d|d-1} \\ q_{\beta,d|d-1} \\ q_{\gamma,d|d-1} \end{bmatrix} = \begin{bmatrix} e^{-b_\alpha \Delta} & 0 & 0 \\ 0 & e^{-b_\beta \Delta} & 0 \\ 0 & 0 & e^{-b_\gamma \Delta} \end{bmatrix} \begin{bmatrix} q_{\alpha,d-1|d-1} \\ q_{\beta,d-1|d-1} \\ q_{\gamma,d-1|d-1} \end{bmatrix} \tag{8}$$

Similarly the predicted variance (directly related to the uncertainty) of the well flows is calculated from:

$$P_{d|d-1} = F_d P_{d-1|d-1} F_d^T + N_d \tag{9}$$

F_t remains fixed and initially **P** has estimates of the uncertainty in the well flows, which can be set somewhat arbitrarily (safer to assume a high value).

$$P_{d=0} = \begin{bmatrix} \left(\varepsilon_{\alpha,0}/2\right)^2 & 0 & 0 \\ 0 & \left(\varepsilon_{\beta,0}/2\right)^2 & 0 \\ 0 & 0 & \left(\varepsilon_{\gamma,0}/2\right)^2 \end{bmatrix} \tag{10}$$

Initially, the **P** matrix only has diagonal elements, but as it is updated, the off-diagonal co-variance terms become populated. $\mathbf{N_d}$ is the process noise matrix.

7.3 Update Phase of Filter

In the second phase of the filter any measurements are taken into account. The residual vector $\mathbf{Z_d}$ (in effect the mass balances) is calculated from:

$$Z_d = Y_d - H_d Q_{d|d-1} \tag{11}$$

The observation matrix $\mathbf{H_d}$, relates the measurements to the well rates. There are two sets of measurements, the total oil production measured daily (i.e. every update) and individual well tests which occur at intervals (possibly irregular). On a day without any well tests and all wells flowing:

$$H_d = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{12}$$

The top row relates the wells to measured well tests. On this day there aren't any, so all zeroes are entered. The second row relates the wells to the total measured flow; on this day all wells are flowing and are added to produce the total flow. If a well isn't flowing then a zero would be correspondingly entered in the second row. For example, in (13), α well is being tested and γ is not producing:

$$H_d = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \tag{13}$$

The vector $\mathbf{Y_d}$ has two elements, the top corresponding to the measured well test rate and the bottom the total produced oil measured rate:

$$Y_d = \begin{bmatrix} t_d \\ m_d \end{bmatrix} \tag{14}$$

With this arrangement it is assumed that only one well is tested on any day. If there are multiple tests on the same day then $\mathbf{H_d}$ and $\mathbf{Y_d}$ have additional rows.

The residual matrix variance, $\mathbf{S_d}$, is calculated using the variances in the well rate estimates from the predict phase and the variance in the measurements (obtained from the well test and produced oil measurement uncertainties):

$$S_d = H_d P_{d|d-1} H_d^T + \Sigma_d \tag{15}$$

This is equivalent to the calculation of the propagation of uncertainties described in the GUM [8] but expressed in matrix notation. $\mathbf{\Sigma_d}$ is given by:

$$\Sigma_d = \begin{bmatrix} \left({\varepsilon_{t,d}}/{2} \right)^2 \\ \left({\varepsilon_{m,d}}/{2} \right)^2 \end{bmatrix} \quad (16)$$

The updated state estimation vector of well flows (i.e., the solution for day **d**) is calculated in the final innovation step by the filter:

$$Q_{d|d} = Q_{d|d-1} + K_d Z_d \quad (17)$$

Where $\mathbf{K_d}$ is the Kalman gain matrix and is calculated, using the results from (9) and (15), according to:

$$K_d = P_{d|d-1} H_d^T S_d^{-1} \quad (18)$$

In essence, this incorporates the uncertainties in the model estimates and any measurements in a statistically optimal way.

Finally, the updated well flow estimate variance matrix is calculated (i.e. the variance associated with $\mathbf{Q_{d|d}}$ obtained in (17)):

$$P_{d|d} = (I - K_d H_d) P_{d|d-1} \quad (19)$$

$\mathbf{Q_{d|d}}$ and $\mathbf{P_{d|d}}$ in (17) and (19) now become $\mathbf{Q_{d-1|d-1}}$ and $\mathbf{P_{d-1|d-1}}$ in equations (7) and (9) on the next day.

7.4 Global Test for Gross Errors

The Global Test, described in [10], is used to test if there are any gross errors in the data. It does not identify the source(s) of each error, simply their presence. The Global Test essentially considers whether the observed mass balance deviation is within the uncertainty that would be expected given the uncertainties in the variables and measurements used to compute it. In the context of the Kalman filter this means using the residuals matrix Z_d from (11) and its associated variance S_d calculated in (15). The Global Test statistic $\boldsymbol{\gamma}$, is computed from:

$$\gamma = Z_d^T S_d^{-1} Z_d \quad (20)$$

Under the null hypothesis, that the data does not include any gross errors, this statistic follows a χ^2 distribution with $\boldsymbol{\nu}$ degrees of freedom, where $\boldsymbol{\nu}$ is the number of measurements. Each day there is at least one measurement - specifically, the produced oil, plus any well tests on that day. Hence, if the calculated value of $\boldsymbol{\gamma}$ is greater than this figure, the hypothesis that the data do not contain gross errors is rejected.

8 NOTATION

Generally, upper case letters have been used to represent vectors and matrices, whilst lower case has been used to denote individual variables. Symbols in the general text (i.e. outside if a numbered equation) have been emboldened.

a	Oil allocation for well
b	Well exponential decline constant
ch	Choke opening position
d	Day
e	Uncertainty, relative
F	State transition matrix
H	Observation matrix
I	Identity matrix
K	Kalman gain matrix
m	Produced oil measurement
N	Process noise variance matrix
$P_{d-1\|d-1}$	Matrix of well flow estimate uncertainties on day, d-1
$P_{d\|d-1}$	Predicted matrix of well flow estimate uncertainties on day, d
$P_{d\|d}$	Final matrix of well flow estimate uncertainties on day, d
$Q_{d-1\|d-1}$	Vector of well flow estimates on day, d-1
$Q_{d\|d-1}$	Predicted vector of well flow estimates on day, d
$Q_{d\|d}$	Final vector of well flow estimates on day, d
q	Daily flow potential estimate for well
sh	Oil shrinkage from well test to export
S	Residuals variance matrix
t	Well test flow measurement
WLR	Water liquid ratio
x	Fractional uptime (i.e. hours on production÷24)
Y	Measurement vector
Z	Measurement residual vector

Greek

γ	Global Test statistic
Δ	Day step interval
ε	Uncertainty, absolute
Σ	Measurement variance matrix
ν	Degrees of freedom
χ	Chi statistic

Subscripts

$_d$	Day number
$_l$	Liquid
$_m$	Measured product oil
$_o$	Oil
$_t$	Well test
$_w$	Well
$_{WLR}$	Water liquid ratio
$_\alpha$	Well α (Apollo)
$_\gamma$	Well γ (Gemini)
$_\mu$	Well μ (Mercury)

Superscripts

^{T}	Matrix transpose
$^{-1}$	Matrix inverse

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10 APPENDIX OF FIGURES

Figure 16 – Well A: Kalman Oil Potentials

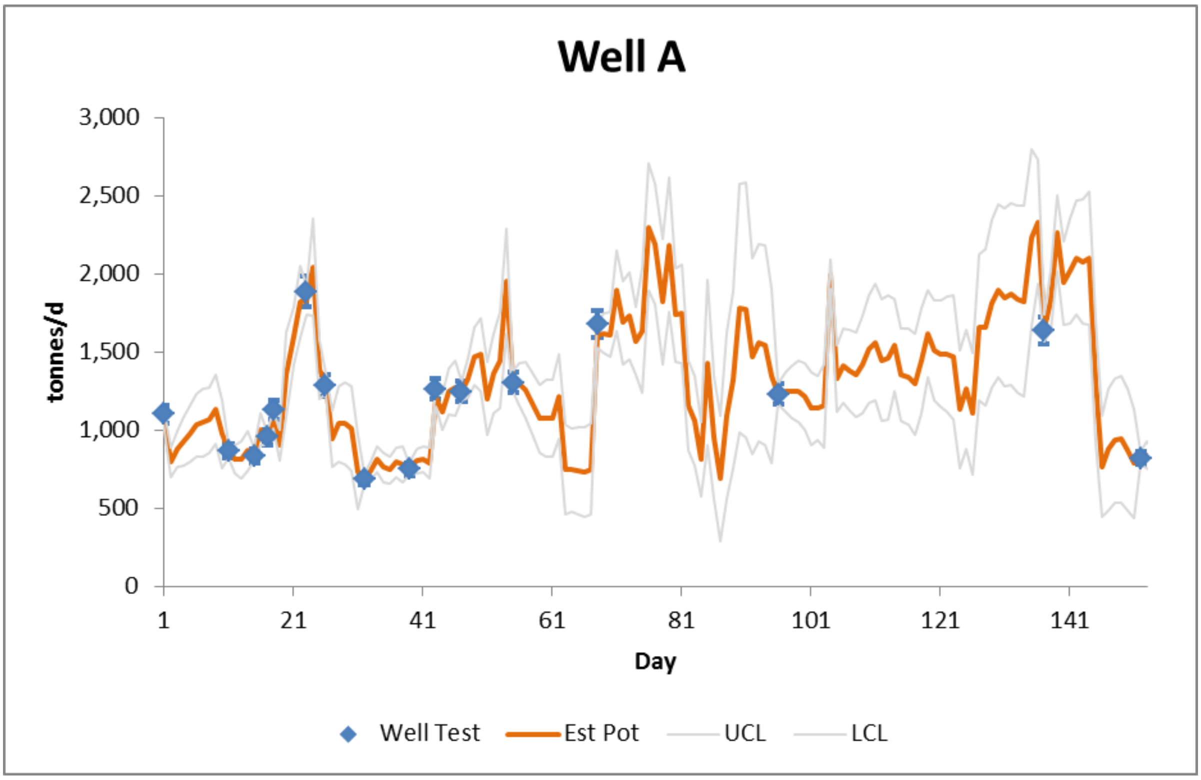

Figure 17 – Well A: Potential and Choke Position

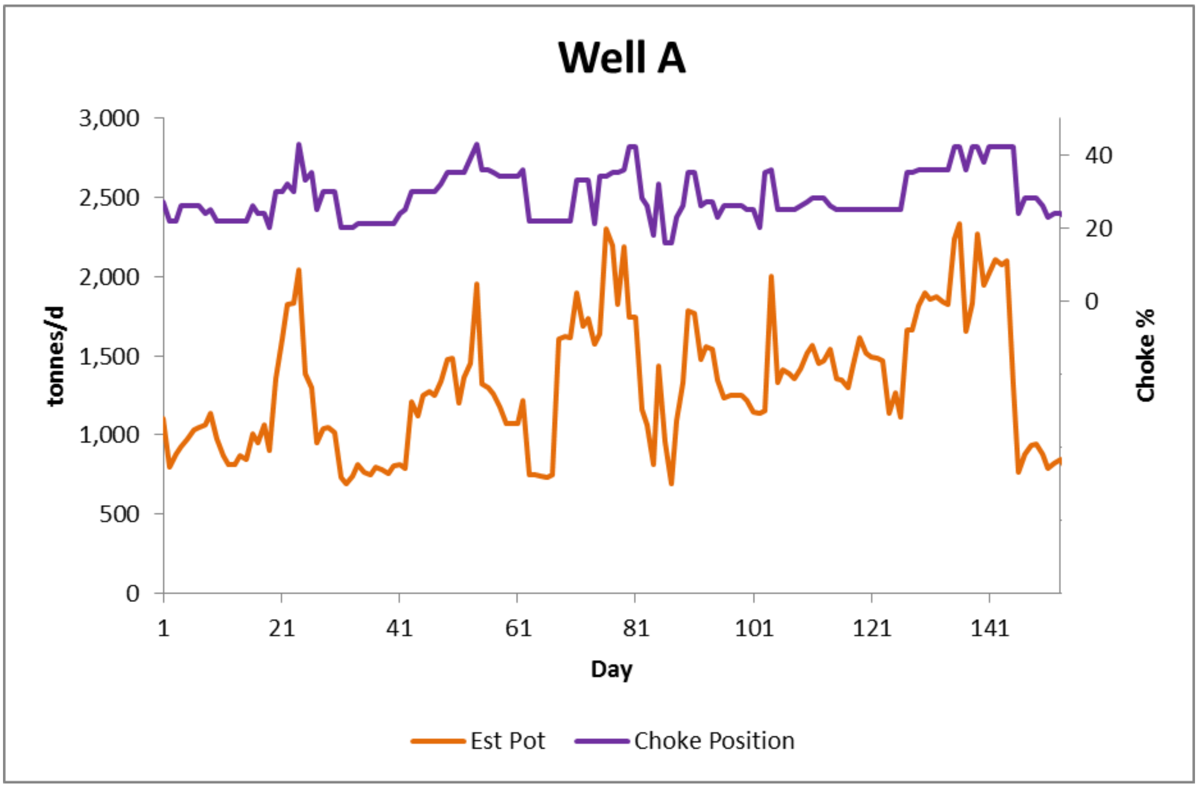

Figure 18 – Well B: Kalman Oil Potentials

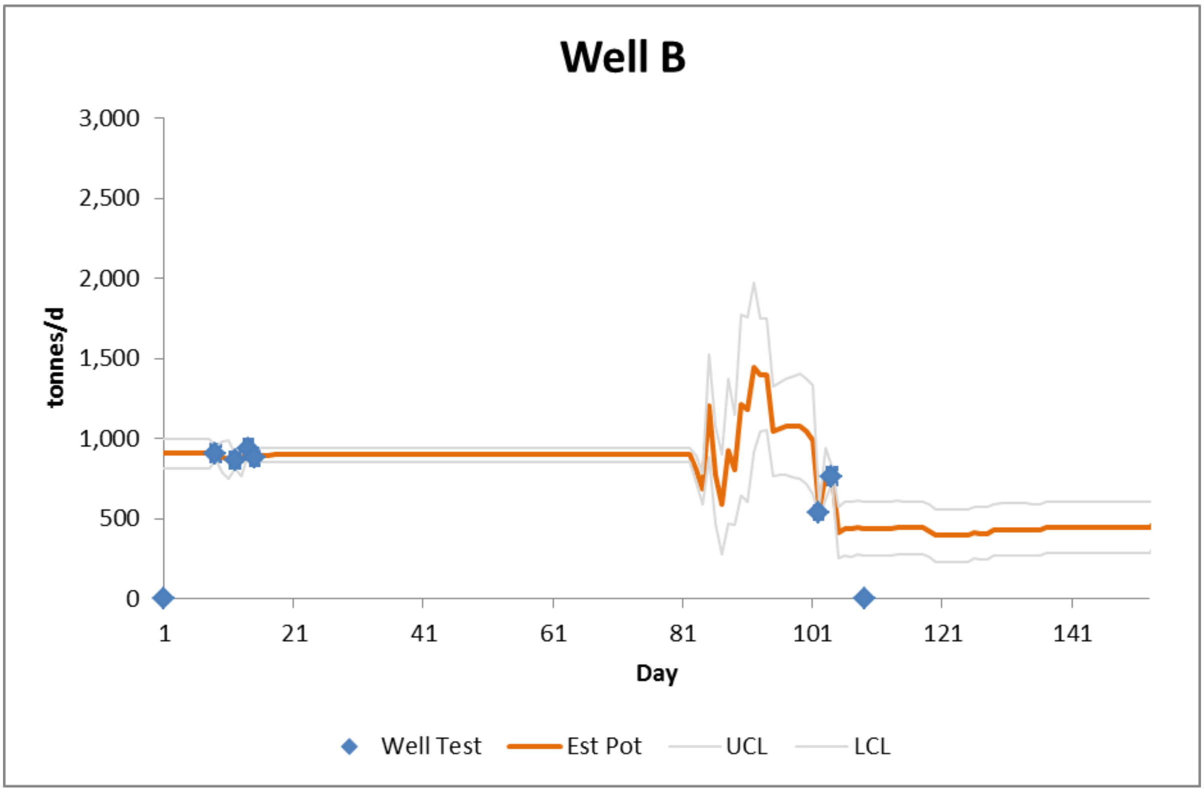

Figure 19 – Well B: Potential and Choke Position

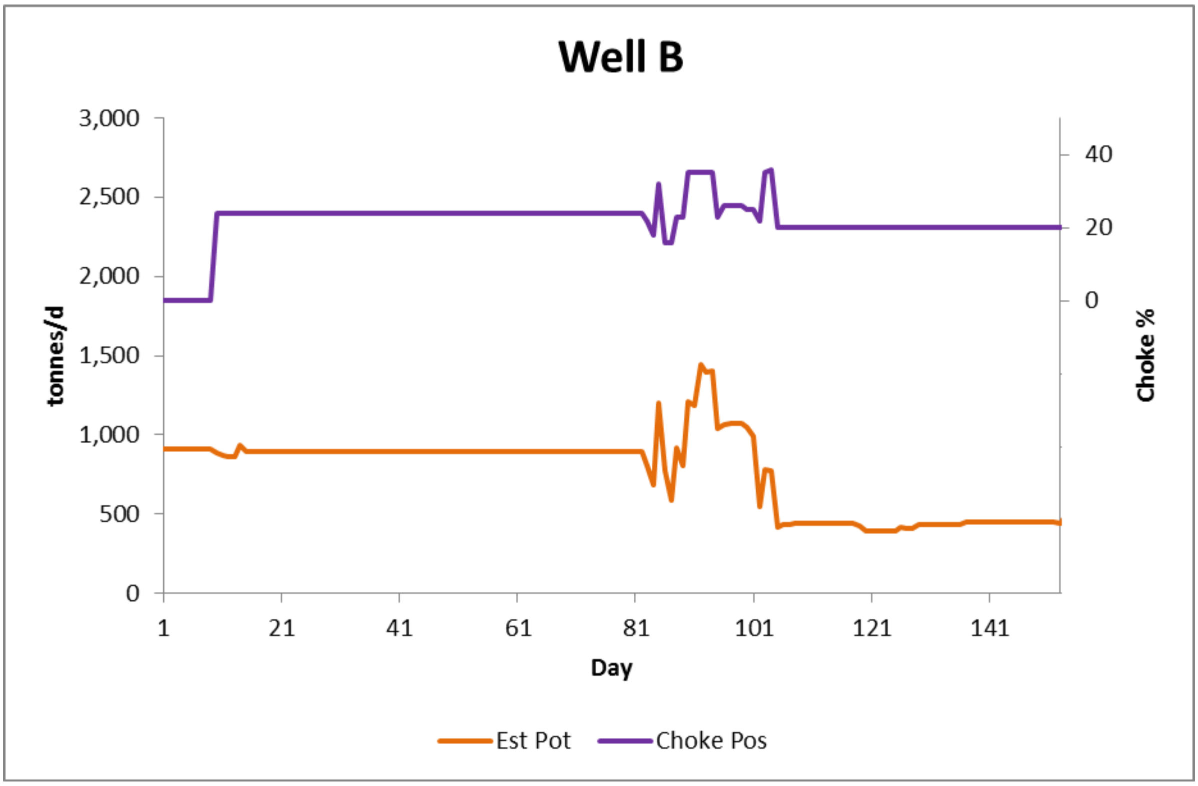

Figure 20 – Well C: Kalman Oil Potentials

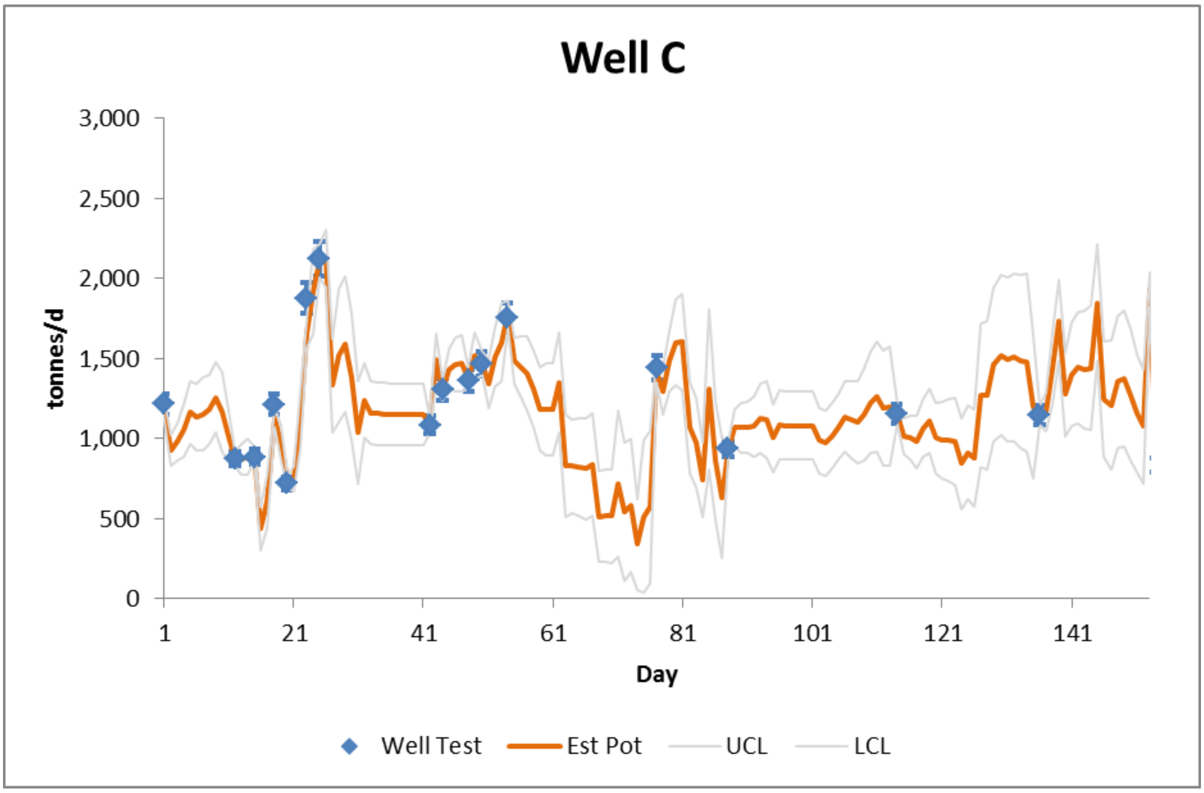

Figure 21 – Well C: Potential and Choke Position

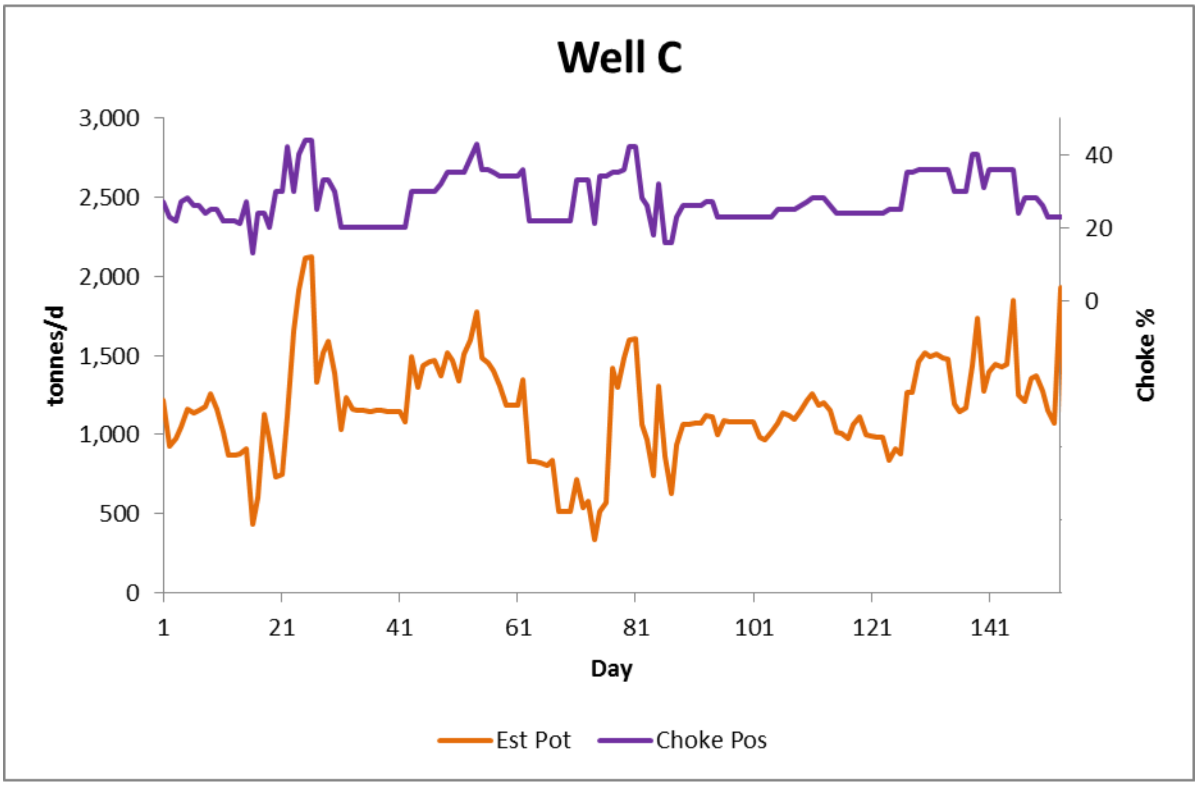

Figure 22 – Well D: Kalman Oil Potentials

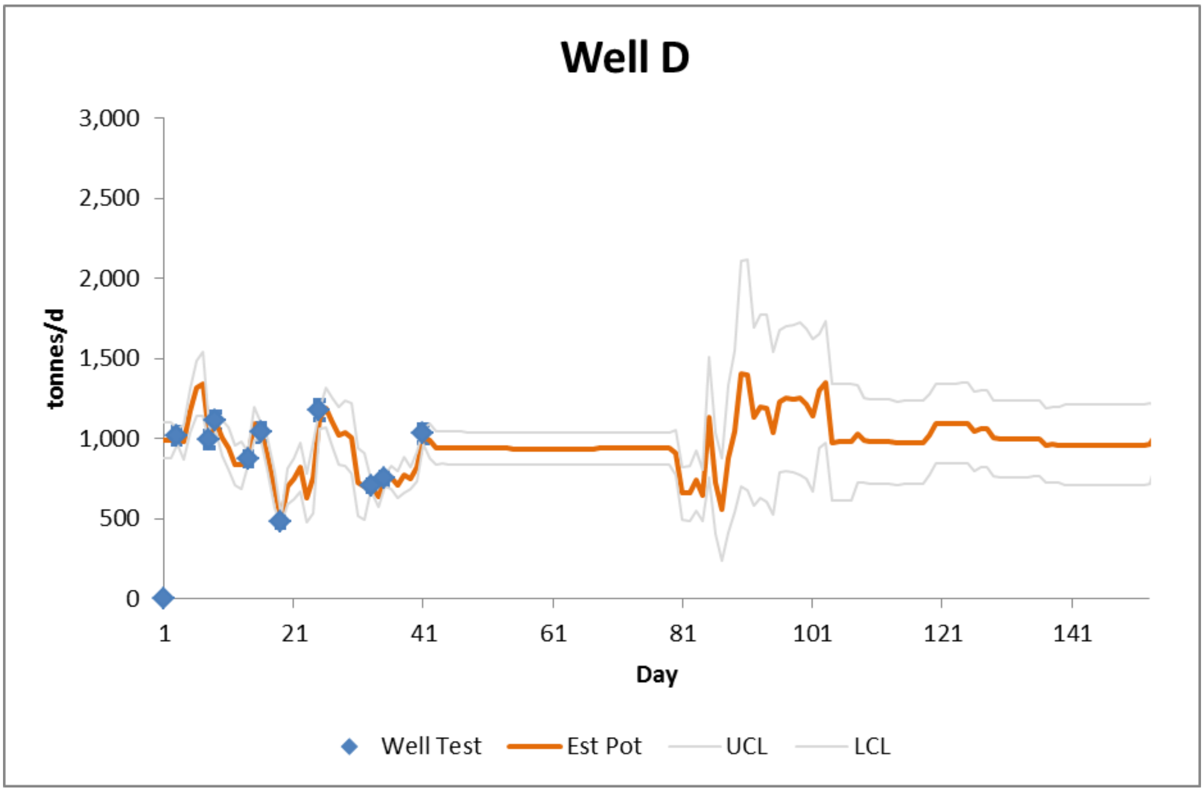

Figure 23 – Well D: Potential and Choke Position

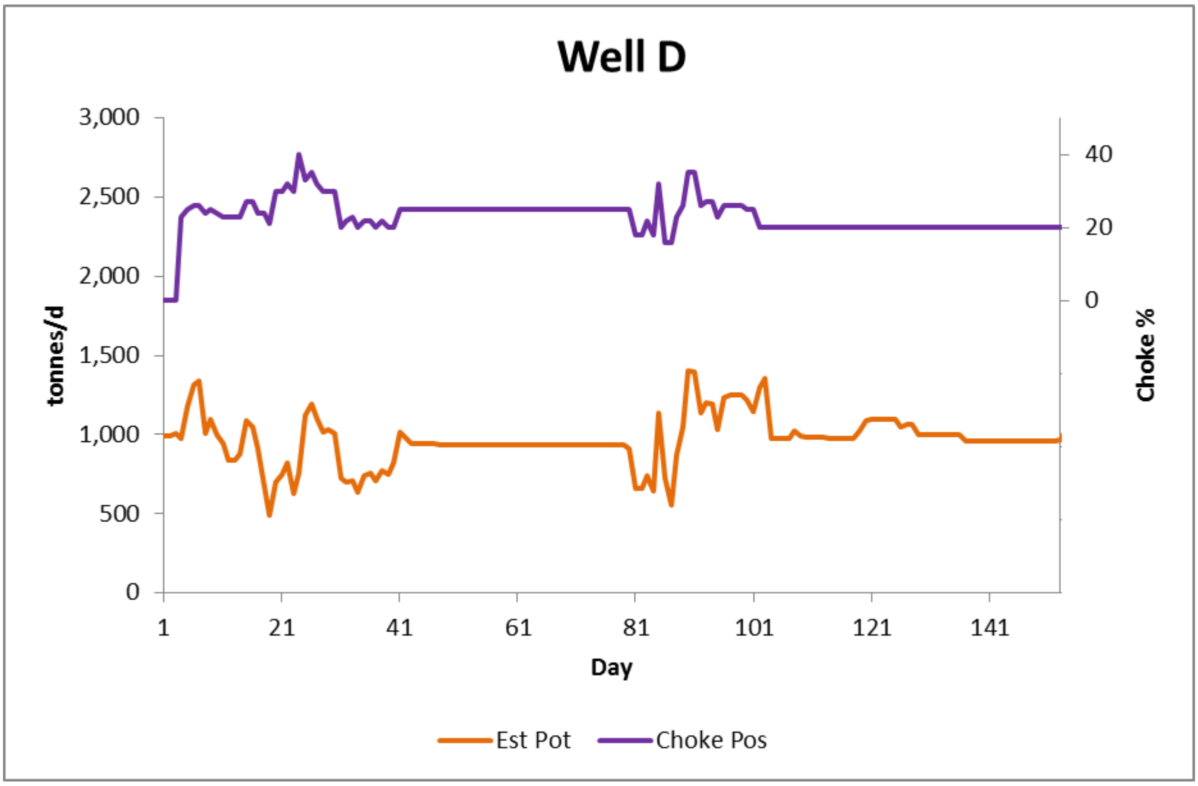

Figure 24 – Well E: Kalman Oil Potentials

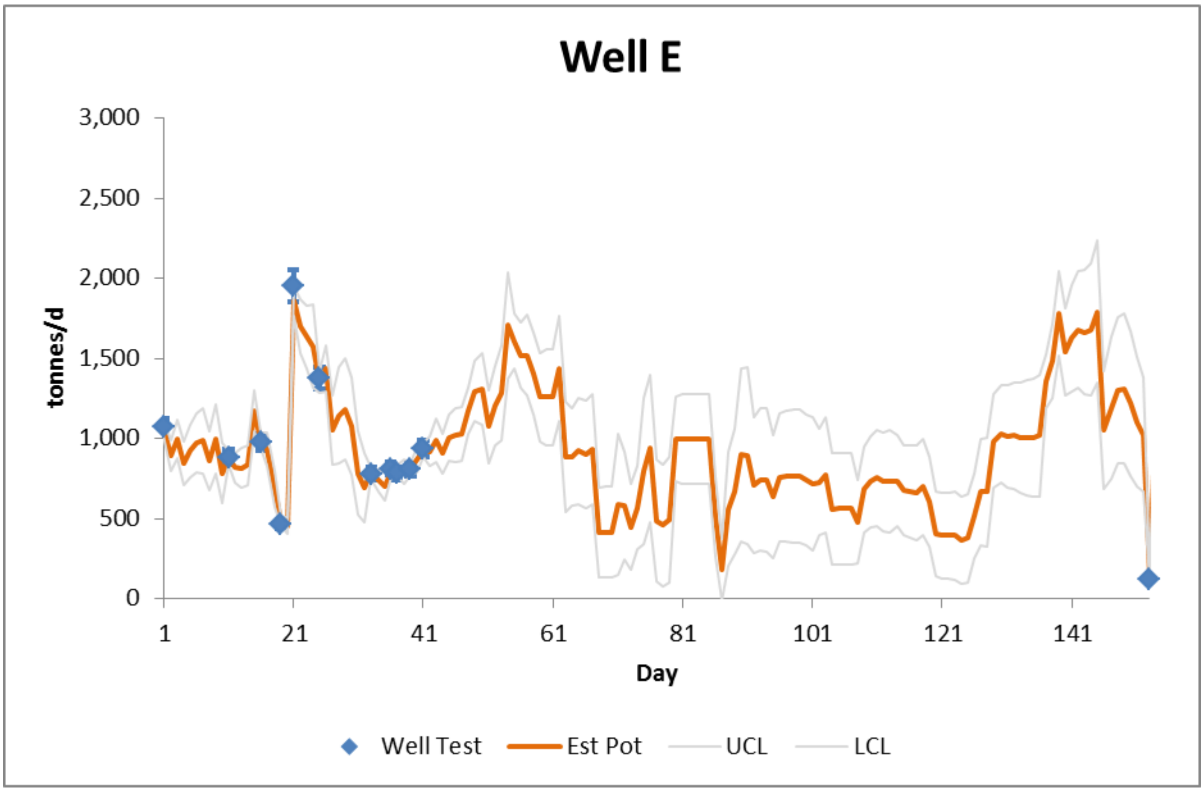

Figure 25 – Well E: Potential and Choke Position