34th International North Sea Flow Measurement Workshop
25-28 October 2016

Technical Paper

When Allocation "Back Out" Agreements can become "Lose Out" Agreements

Phil Stockton, Accord Energy Solutions Limited

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1 INTRODUCTION

1.1 Overview

When a newly developed third party field is tied back to existing infrastructure, such as a pipeline system, there is the potential for the new field's production to cause an increase in back pressure and reduce the flow of the incumbent fields already producing through the system. This is termed "back-out" and in effect defers some of the incumbent fields' production.

Back-out agreements are sometimes put in place, within the allocation system, to compensate the incumbent fields for the reduction in flow they experience. This may take the form of a transfer (or payment) from the third party of hydrocarbons to the incumbent fields so that in effect they do not experience deferment and suffer resultant economic loss.

As the production matures, the situation eventually reverses and the transfer (repayment) proceeds in the opposite direction from the incumbents to the third party. The assumption often made is that eventually the transfers will cancel one another and over their lifetime, all fields' cumulative allocation will equal their cumulative production.

This paper presents the concept of how back-out calculations typically work and presents a simplified mathematical model of a system where an incumbent field is backed out by a new field introduced into a pipeline. This simplified model is used to illustrate potential problems with the concept of back-out, particularly in the repayment period and shows that there is the very real potential for "back-out" to become "lose out" for the new field.

Finally the paper discusses methods to overcome the pitfalls exposed by the analysis.

This paper specifically addresses back-out associated with gas reservoirs producing through shared pipeline infra-structure.

1.2 Structure of the Paper

Section 2 describes a typical gas production system in which back-out occurs. This is used as an example to illustrate the issues associated with back-out. Though a fictitious system it is based on typical production rates and parameters from a real system to ensure it is reasonably representative of equivalent real systems in general. The values of the parameters used to generate the flows, pressures, etc. presented in the various charts are given in Section 6.5.

Section 3 describes a simplified mathematical model that is used to analyse the long term behaviour of the system. It thereby exposes a problem in that it appears it is impossible for the tied back field to be repaid, in any reasonable timeframe, the gas it has transferred to the incumbent field during the early back-out payment phase.

Section 4 examines options to address the problem of excessive repayment periods in an effort to ensure all fields receive the gas they have produced over the length of their lives.

Section 5 presents conclusions.

Section 6 provides a detailed description of the simplified mathematical model employed.

2 DESCRIPTION OF A SYSTEM WITH BACKOUT

2.1 Physical System

Consider the subsea gas pipeline system depicted in Figure 1:

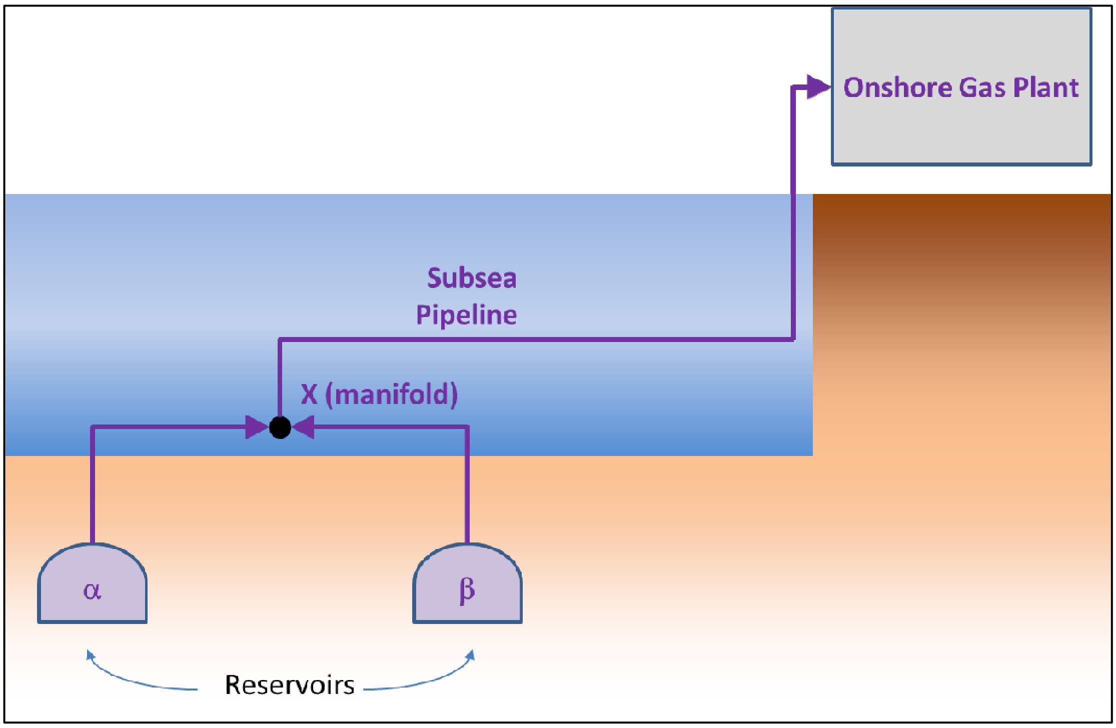


Figure 1 – System Configuration

The incumbent field has been labelled as Field Alpha (α). In order to utilise spare capacity and maximise the use of existing infrastructure it is often mutually beneficial for new developments to tie back into existing transportation and processing facilities.

A potential problem for the incumbent field is that the introduction of a new field, denoted Bravo (β), increases the total flow down the shared pipeline resulting in a higher back pressure at Field Alpha's wellhead(s).

If Alpha's wells are off plateau, the increased back pressure will reduce production from the field and this is termed back-out. (Since new field tie-backs into existing infra-structure are usually taking advantage of spare capacity, it is likely that the incumbent fields' are off-plateau).

Unless the increased back pressure has a permanently deleterious impact on the incumbent wells, back-out does not result in lost production, it merely defers it. However, due to the time value of money the incumbent field would normally wish to produce its hydrocarbons sooner rather than later and back-out is considered to have an economic cost to the incumbent.

To understand the consequences of back out, it is instructive first to consider the behaviour of the incumbent field when it is producing alone, i.e. without back-out.

2.2 Stand-Alone Production

When Field Alpha is producing alone and it is off plateau, the pressure drop across the system, from the reservoir to the delivery point at the gas plant, will determine its flow. As the reservoir pressure drops, the driving-force decreases and Alpha's flow declines. This is illustrated in Figure 2.

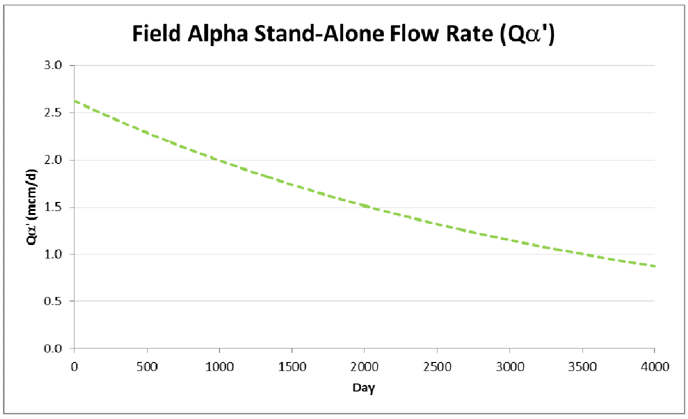


Figure 2 – Alpha Stand-Alone Flow Rate

The dash ′ in Q_α' means this is the Stand-Alone volumetric flow for Field Alpha.

The horizontal scale ranges from 0 to 4,000 days, almost 11 years of production.

The reservoir pressure also declines as shown in Figure 3.

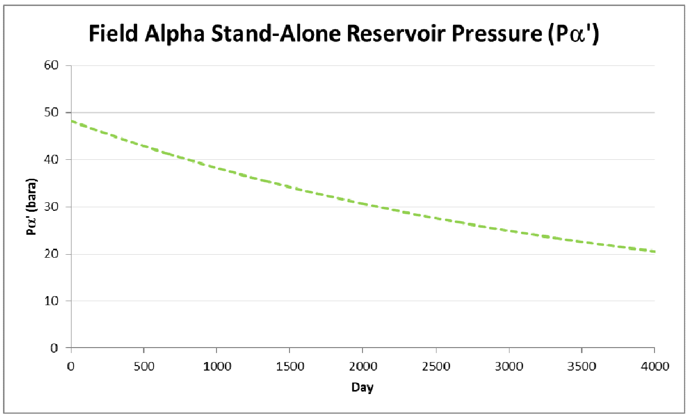
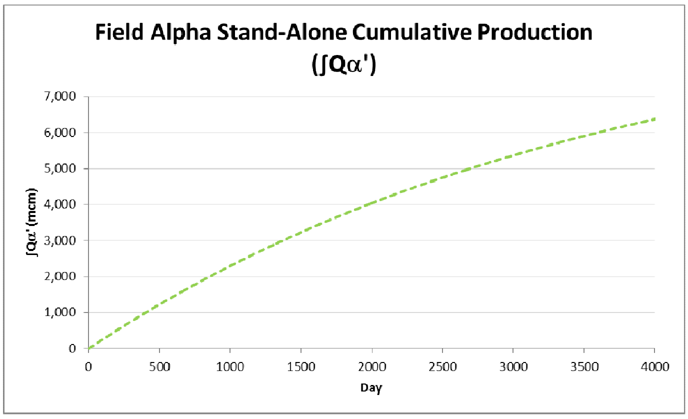


Figure 3 – Alpha Stand-Alone Reservoir Pressure

The reservoir acts like a pressurised tank. As gas is produced from the reservoir the pressure in it falls. As the pressure declines the driving force reduces, resulting in a lower flow rate.

The rate of pressure decline also falls as can be seen in the figure.



The reservoir pressure tends asymptotically to the delivery pressure at the gas plant, which in this example is set at 6.75 bara.

The fall in the reservoir pressure is directly related to the cumulative production which is plotted in Figure 4.

Figure 4 – Alpha Stand-Alone Cumulative Production

The above plots represent the evolution of Alpha's production with time in the absence of a new field being introduced. This is compared in the next section with the scenario of Shared Production with Field Bravo – a new third party tie-back.

2.3 Shared Production and Back-Out

In the Shared Production scenario, Alpha and Bravo are producing together. Alpha experiences a rise in back pressure due to the higher flow and resulting increased frictional pressure drop across the shared section of the subsea flow line. Alpha's flow rate will therefore be lower when compared with the Stand-Alone case.

The Stand-Alone and Shared Production[1] flows are compared in Figure 5:

[1] The Stand-Alone and Shared Production scenarios or cases have been capitalised for convenience of reference throughout the paper.

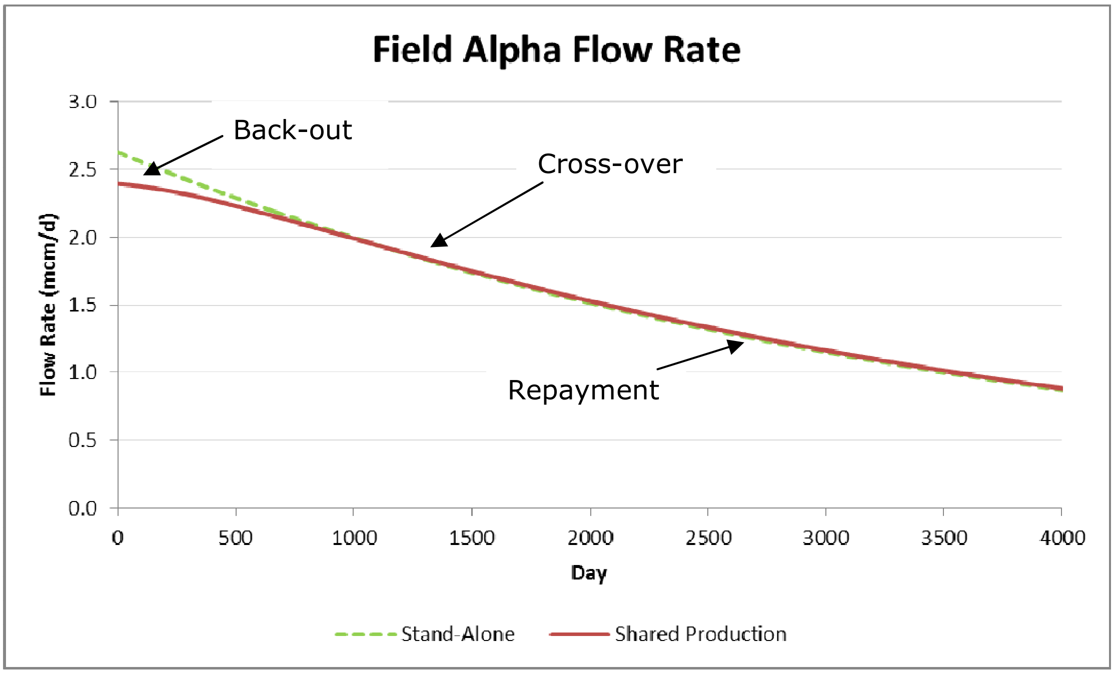


Figure 5 – Alpha Stand-Alone vs Shared Production Flow Rate

The locus of the green dashed line is identical to that in Figure 2. The red line shows the reduced flow experienced by Alpha in the early phase of the profile. The margin narrows until it becomes zero after 1,112 days, when Alpha's Shared Production exceeds its Stand-Alone level. The reason for this is revealed by comparing the difference in the reservoir pressures for the two scenarios in Figure 6:

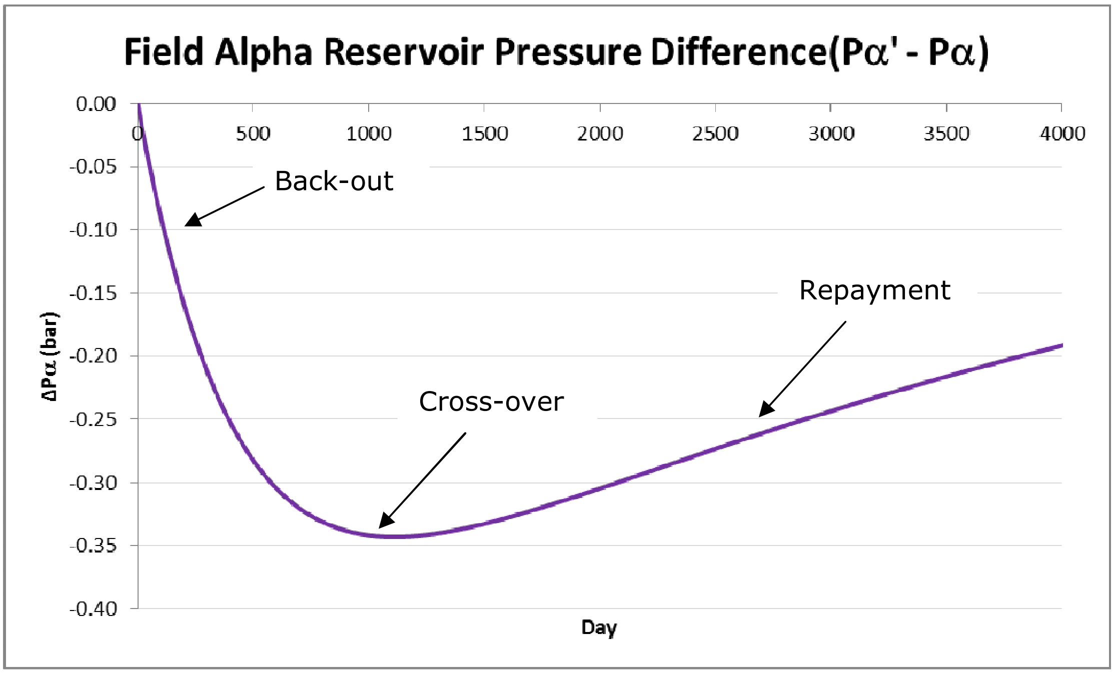


Figure 6 – Alpha Stand-Alone Minus Shared Production Reservoir Pressure

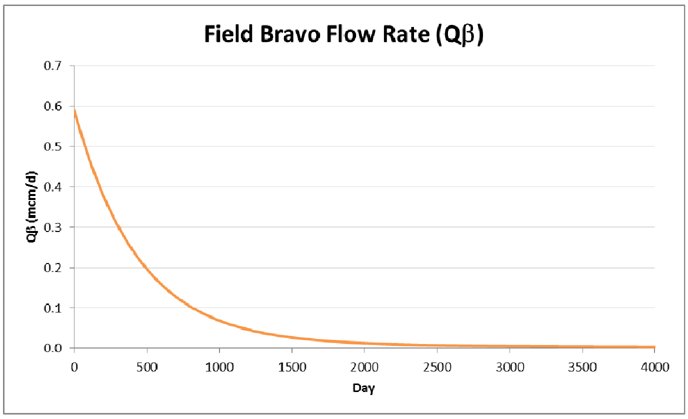
(The difference is in pressure is presented as the plots of the two reservoir pressures almost lie over one another).

The plot shows that the Stand-Alone reservoir pressure is always lower than in the Shared Production case. This is because more gas has been taken out of the reservoir in the Stand-Alone case.

Though the back pressure experienced by Alpha is greater in the Shared case, the driving force at the reservoir is also greater and this eventually results in Alpha producing more in the Shared Production scenario.

This reservoir pressure difference between the two cases reaches its maximum at the cross-over point when Alpha's flow rate is identical in both scenarios.

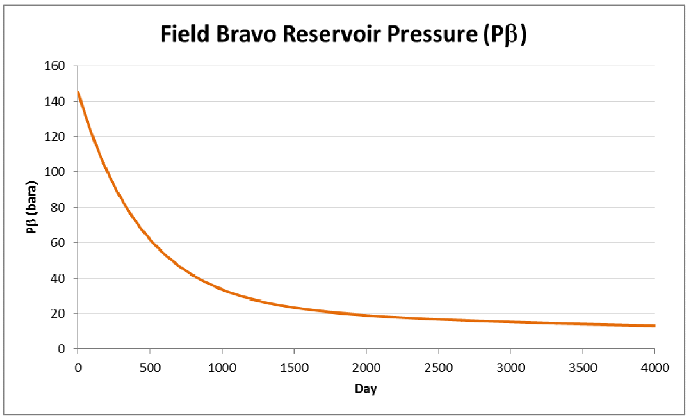
Also in the Shared case the production from Field Bravo is declining with time and therefore further reducing the back pressure.



The daily flow for Bravo is presented in Figure 7.

In the example, Bravo's production is less than Alpha's but its initial reservoir pressure is significantly greater as illustrated Figure 8.

Figure 7 – Bravo Flow Rate



Bravo's initial high reservoir pressure backs out Alpha but it declines rapidly as its reservoir is considerably smaller than Alpha's.

Figure 8 – Bravo Reservoir Pressure

Returning to Alpha, the difference in the Stand-Alone and Shared Production daily flow rates is compared in Figure 9:

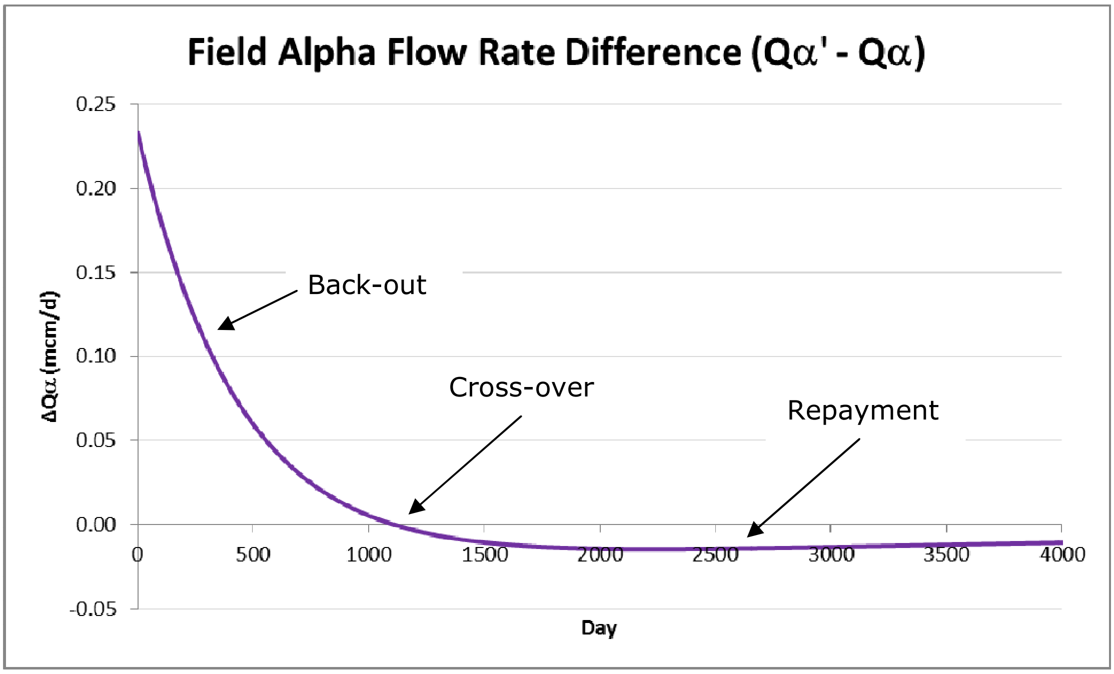


Figure 9 – Alpha Stand-Alone Production Minus Shared Production

The difference in the two daily flow rates, which is positive in the first 1,112 days, represents deferred production for Alpha and is termed the back-out.

Back-out agreements can be put in place to compensate incumbent fields (such as Alpha) for this deferred production.

2.4 Back-Out Agreement

A typical premise of back-out agreements is to allocate the incumbent field the quantity of gas it would have produced without the new field being present.

In the example, this entails calculating Alpha's deferred production as the difference between Alpha's measured (or allocated) production when co-producing compared with its estimated Stand-Alone production.

Gas is then transferred from Bravo to Alpha to compensate Alpha for the deferment, or back-out, and ensure it does not suffer economic loss.

An issue that immediately arises is the Stand-Alone Alpha production. This cannot be directly measured as it did not actually occur and is therefore hypothetical in nature. In order to overcome this, simulation models are used to calculate the hypothetical Stand-Alone production.

2.5 Back-Out Modelling

Back-out agreements usually stipulate the simulation modelling to be under-taken in order to calculate the back-out.

These simulations can vary in type and complexity but they usually involve modelling the reservoirs, wellbores and pipeline infrastructure to the point of delivery, e.g. at the inlet to a gas plant where the pressure may be considered fixed.

Commercial software programs [1], [2] such as MBAL and Eclipse model reservoirs, PROPSER and OFM model wells and GAP and PipeSim model pipeline networks. These may be integrated under a wider software framework. Such models are relatively sophisticated and require considerable expertise and effort to run.

In order to predict the level of back-out normally two models are run:

- The Shared Production case with both fields producing, i.e. mimicking reality and;
- The Stand-Alone hypothetical case with only the incumbent field producing.

The difference between the two then provides the predicted back-out experienced by the incumbent field.

At first sight, the model simulating reality appears superfluous. However, it serves two purposes:

- It allows the modelling environment to be calibrated against real production, i.e. by history matching the model against real data. The parameters calibrated in the Shared model can then be used in the Stand-Alone model and thereby improve estimates of back-out;
- It allows calculated daily back-out quantities to be projected into the future, the forecast back-out being used in the daily allocation in the forthcoming period until a new calibration exercise can be undertaken.

There are variations to the above approach but they all tend to include models that simulate both scenarios.

2.6 Deferment Back-Out is different to Hydrodynamic Back-Out

It is worth noting that back-out, in the deferred production sense and addressed in back-out Agreements, cannot be measured directly.

At first sight it may appear that the back-out experienced by the incumbent field Alpha could be determined by physically shutting in Bravo and recording the change in Alpha's production as the back-out. But this is a different kind of back-out, termed here as hydrodynamic back-out.

Hydro-dynamically, at any instant in time, Bravo is backing out Alpha in the sense that if Bravo is shut in Alpha's flow will always increase.

However, this is not how deferred production back-out is defined in back-out agreements as Alpha's flow has to be compared with the hypothetical case of Alpha having produced up to this point in time without Bravo producing at all. In such a scenario, Alpha's reservoir pressure would be lower and it would be producing less on a Stand-Alone basis.

When determining back-out in the deferred production sense, the Shared Production case, however it is determined, has to be compared against a parallel, hypothetical scenario in which only Alpha has ever produced.

2.7 Back-Out Balances

There are three phases associated with back-out which are indicated on Figure 5, Figure 6 and Figure 9:

- Back-out payment
- Cross-over
- Repayment.

During the initial back-out payment phase the transfer of gas from Field Bravo to Alpha is normally recorded and accumulated as a back-out balance.

Once the cross-over point is reached the daily back-out payments will fall to zero and the back-out balance reaches its maximum.

The repayment phase ensues and Alpha commences transfer of gas back to Bravo. The back-out balance is decremented accordingly.

The back-out balance associated with the fictitious example introduced above is presented in Figure 10:

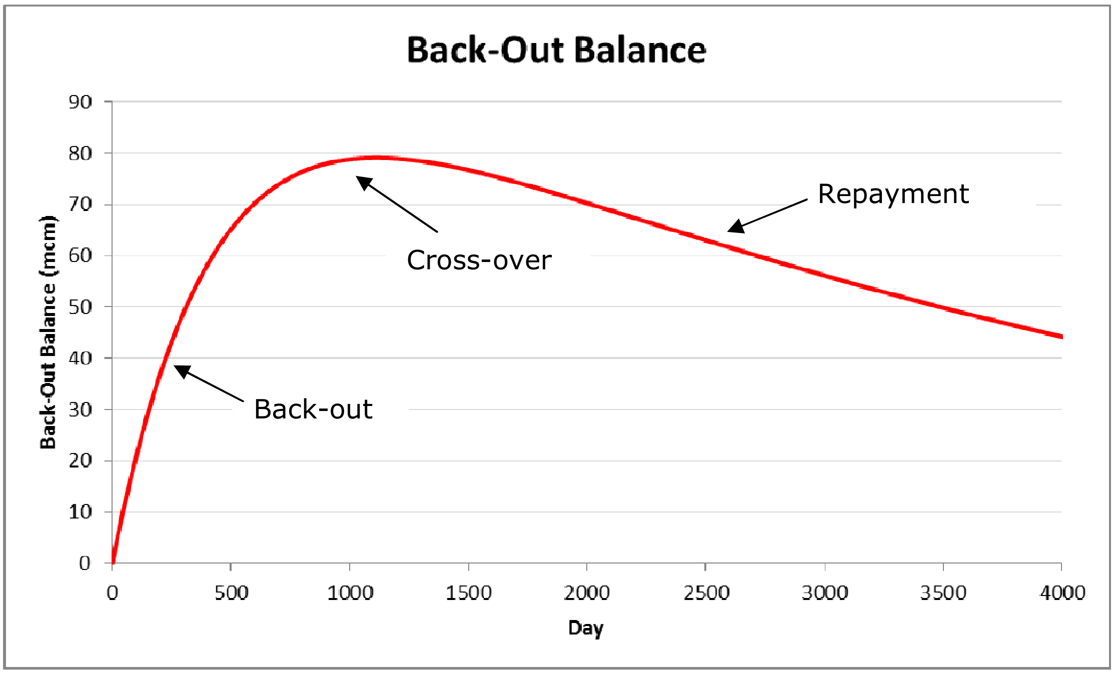


Figure 10 – Back-Out Balance

The back-out balance reaches its maximum value of 79.1 mcm at the cross-over point. This is the cumulative quantity of gas Bravo has transferred to Alpha to compensate for deferred production.

After the cross-over point Alpha Field is producing more gas than it would have been if it had produced on its own and the back-out becomes negative and repayment in the opposite direction from Alpha to Bravo takes place.

The back-out balance is then decremented by the repayment quantities.

When the back-out balance reaches zero the back-out payments will all have been repaid and the back-out calculations cease. However, how long does it take for the balance to return to zero?

Examination of Figure 10 shows that it took slightly over 3 years to build up the balance to its maximum value, but after almost a further 8 years, less than half the balance has been repaid.

In order to answer the question posed above, a simplified model of the system was developed and this is described, and its behaviour analysed, in the next section.

3 SIMPLIFIED MODEL

3.1 Reasons for the Simplified Model

In order to assess the behaviour of the repayment phase and estimate the completion date, a simplified mathematical model of the system and associated back-out was developed. The derivation of the model is presented in Section 6.

This simplified model produces analytical equations that can solve the state of the system at any point in time.

Running the rigorous, complex models mentioned in Section 2.5, is considerably more time consuming and being numerical in nature, not so easily extended far into the future in order to examine the long term behaviour of the repayment phase.

The analytical solvable equations associated with the simplified model also provide a deeper understanding of the behaviour of the system.

3.2 Model Development

The model is based on the following principles:

- the fall in reservoir pressure over a period of time is directly related to the cumulative production over that period
- the cumulative production is equal to the integrated flow rate over time
- the flow rate is equal to the rate of change of reservoir pressure.

Two principal sets of equations were developed. The first describes the behaviour of Alpha's reservoir pressure in the Stand-Alone case:

$$P_{\alpha}^{'} = \left(P_{\alpha}^{\circ} - P_{D}\right)e^{\mu t} + P_{D} \qquad (1)$$

Where,

$P_{\alpha}^{'}$	Stand-alone Alpha reservoir pressure (at time t). (The dash ′ denotes the Stand-Alone case and is not the differential operator)
P_{α}°	Initial Alpha reservoir pressure
P_D	Delivery pressure (at gas plant)
t	time

μ is a constant determined by several system parameters; see Section 6 for its definition.

The second describes Alpha's reservoir pressure in the Shared Production case:

$$P_\alpha = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + P_D \tag{2}$$

Where,

P_α Shared Production Alpha reservoir pressure (at time t)

C_1, C_2, λ_1 and λ_2 are all constants determined by the system parameters; see Section 6 for their definition.

Hence, at any point in time these equations provide the reservoir pressures for the two scenarios.

Two analogous equations provide the flow rate at any point in time. Alpha's flow rate in the Stand-Alone case is given by:

$$Q_\alpha^{'} = \frac{\mu}{k_\alpha}\left(P_\alpha^\circ - P_D\right)e^{\mu t} \tag{3}$$

Where,

$Q_\alpha^{'}$ Stand-alone Alpha flow rate (at time t)
k_α Constant relating Alpha's reservoir pressure to its gas volume

And for the Shared Production case:

$$Q_\alpha = \frac{\lambda_1}{k_w} C_1 e^{\lambda_1 t} + \frac{\lambda_2}{k_w} C_2 e^{\lambda_2 t} \tag{4}$$

Where,

Q_α Shared Production Alpha flow rate (at time t)

3.3 Cross-Over Point

The cross-over point occurs when $Q_\alpha^{'}$ and Q_α are equal and hence the cross over time ($t = \tau_X$) can be determined by combining equations (3) and (4) to give:

$$\mu\left(P_\alpha^\circ - P_D\right)e^{\mu\tau_X} = \lambda_1 C_1 e^{\lambda_1\tau_X} + \lambda_2 C_2 e^{\lambda_2\tau_X} \tag{5}$$

This equation has to be solved iteratively for τ_X.

3.4 Repayment Completion Point

The repayment phase is completed when the back-out balance falls to zero.

The back-out balance (B) at any time (t) is calculated as the difference between Alpha's Stand-Alone and Shared cumulative production:

$$B = \int_0^t Q_\alpha^{'} dt - \int_0^t Q_\alpha dt \quad (6)$$

By substituting for the flow rates from equations (3) and (4) and integrating with respect to time:

$$B = \frac{1}{k_w}\left(\left(P_\alpha^\circ - P_D\right)e^{\mu t} - C_1 e^{\lambda_1 t} - C_2 e^{\lambda_2 t}\right) \quad (7)$$

Hence by equating B to zero, the right hand side can be solved for t = τ_0, the repayment completion time. There are two solutions:

$\tau_0 = 0$

$\tau_0 = \infty$.

The back-out starts at zero, indicated by the first solution, but only returns to zero after an infinite time.

This explains the shape of the plot of the balance in Figure 10, which starts at zero, goes through a maximum at the cross-over point but then extends out with time. The second solution of the equation completes the picture suggested by the plot in that it shows that the balance returns asymptotically towards zero over an infinite time.

To further illustrate the slow rate of decline repayment, in this example after a further 10 years of production, there would still be approximately 20% of the balance unpaid.

The analysis illustrates that using this back-out approach, the new third party field (Bravo) is not going to be repaid its gas in any reasonable timeframe. Indeed, the fields will have become uneconomic to produce long before the back-out balance is appreciably repaid.

A further problem for the new third party field is that the back-out agreement may contain a clause that states that: should the incumbent field's production cease, or become uneconomic, any liability for back-out repayments ceases. This means that the new field will almost certainly be left with unpaid back-out gas.

In effect, the back-out agreement is a "Lose Out" agreement for the new field.

3.5 Comparison with Complex Model

As noted in Section 6 the simplified model does make some simplifying assumptions in order to make the equations solvable analytically and allow the underlying physics of back-out to be analysed, in particular the long term behaviour.

However, it is important that the simplified model is reasonably representative of the more complex models (and indeed actual production) in order for the conclusions drawn from it to be credible.

Figure 11 is a plot of an incumbent field's production from a real system though the data is anonymised.

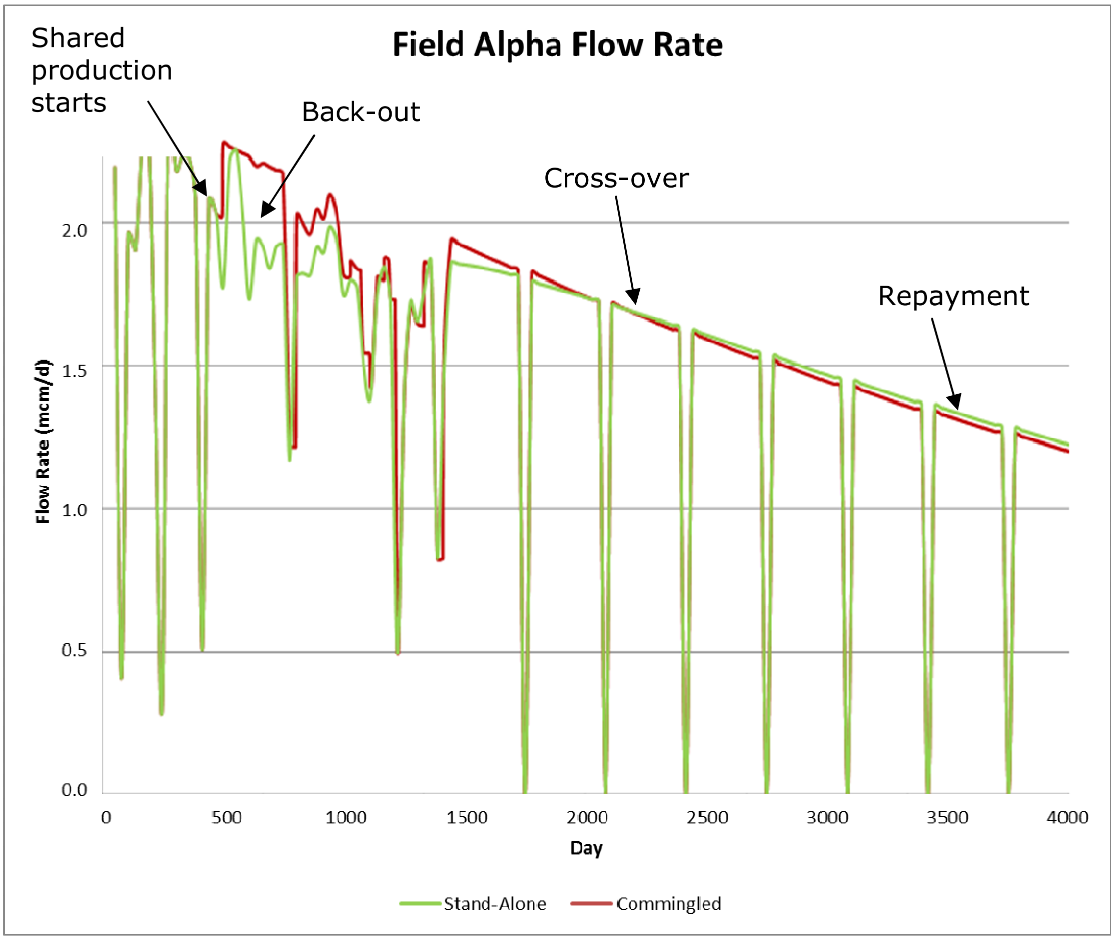


Figure 11 – Complex Model Flow Rate Results

The real system is different to that in the fictitious example used in the simplified model, which is why the timing of events is not the same when comparing the two.

The Shared Production starts after around 500 days and the difference between the forecast Stand-Alone production (red line) and the Shared Production (green line) representing back-out is immediately observed.

The data is noisy up to around day 1,300 because the models have been matched to real historical production data up to that point. Beyond that the models are predicting into the future and appear smoother.

The cross-over is observed around day 2,100 and the slow repayment behaviour observed after that.

The downward spikes to zero represent planned shutdowns which are factored into the more complex models.

Though noisier the more complex model results display all same features as the simpler model.

The same slow repayment phase can be observed with the real data but the power of the simplified model is that it shows algebraically that repayment cannot be achieved in practice and certainly not on economic timescales for such gas fields.

In the author's experience this phenomenon does not appear to have been appreciated when these back-out agreements were historically developed. It seems apparent that the focus of the agreements is on the initial back-out experienced by the host fields.

The next section discusses options to address the repayment problem.

4 ALTERNATIVE REPAYMENT OPTIONS

4.1 Contractual Agreement

Though, once realised, the unreasonably long repayment phase appears easily corrected in a technical sense. However, if the methodology is written into a contractual agreement it may be extremely difficult to obtain agreement from the incumbent field's owners to modify the repayment mechanism.

The first option presented in Section 4.2 is a relatively straight forward approach. However, it is not necessarily consistent with the premise, frequently encountered in agreements, that: the incumbent field is allocated the quantity of gas it would have produced without the new field being present.

The second option in Section Incorporation of COP into Back-Out4.3 provides a methodology which is still arguably consistent with the premise.

4.2 Fixed Period Repayment

The repayment phase could simply be based on an alternative method. For example running the balance repayments down over the same period they were built up, or some variation of this, and dispensing with the models altogether for the repayment phase.

Though pragmatic, the problem with this approach is that the incumbent field's allocated production is different to that which it would have experienced if it was produced alone as it is paying back the gas faster than the modelling predicts it should be.

4.3 Incorporation of COP into Back-Out

If the incumbent field becomes uneconomic and the repayment liability ceases before the balance is significantly repaid then in such a case the new tie-back field has effectively lost considerable volumes of gas.

However, the cessation of production (COP) does offer a route for the new field to receive repayment whilst being consistent with the agreement. The COP represents the point when the incumbent field's flow is so low that it is uneconomic to produce. So there is a definable COP flowrate. This COP flow rate would have been reached earlier in the Stand-Alone case so all the gas produced by the incumbent field between Stand-Alone COP time and Shared Production COP time should be transferred as repayment. This idea is illustrated in Figure 12:

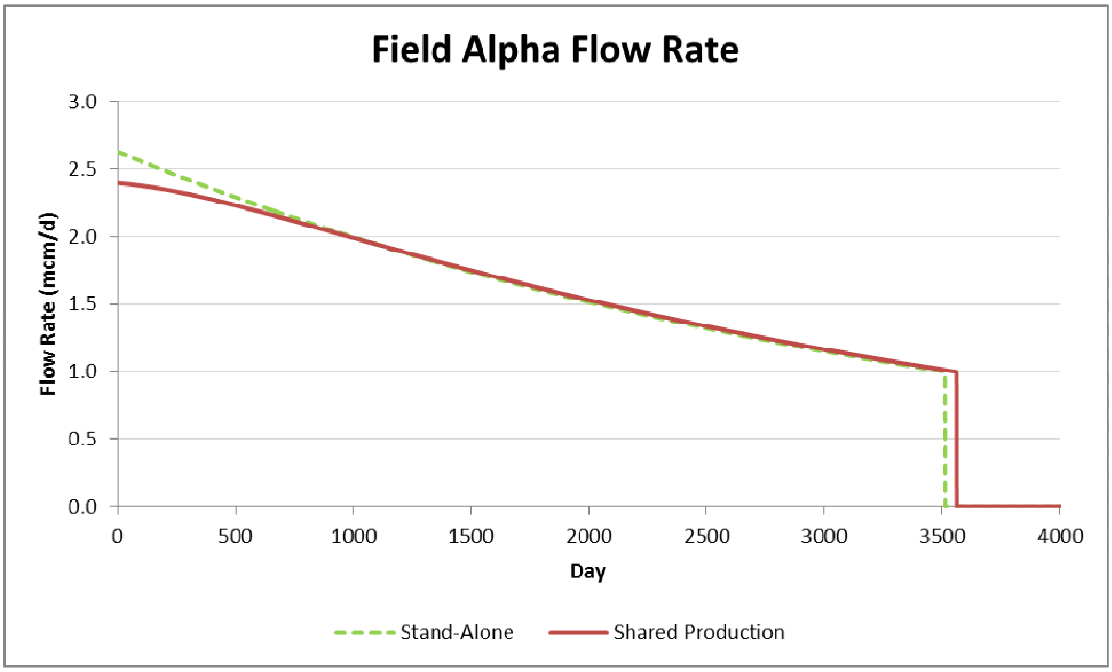


Figure 12 – Alpha Stand-Alone vs Shared Production Flow Rate with COP

Nominally, Field Alpha is considered to become uneconomic when its production falls to 1.0 mcm/d and this occurs on Day 3,562 for the Shared Production case, after which it is shut-in and its flow rate falls to zero. In the hypothetical Stand-Alone scenario this COP flow would have occurred on Day 3,513, 49 days earlier.

Hence the entire Alpha production for the last 49 days of its life should be transferred to Bravo as repayment.

The difference in the flow rates is plotted in Figure 13.

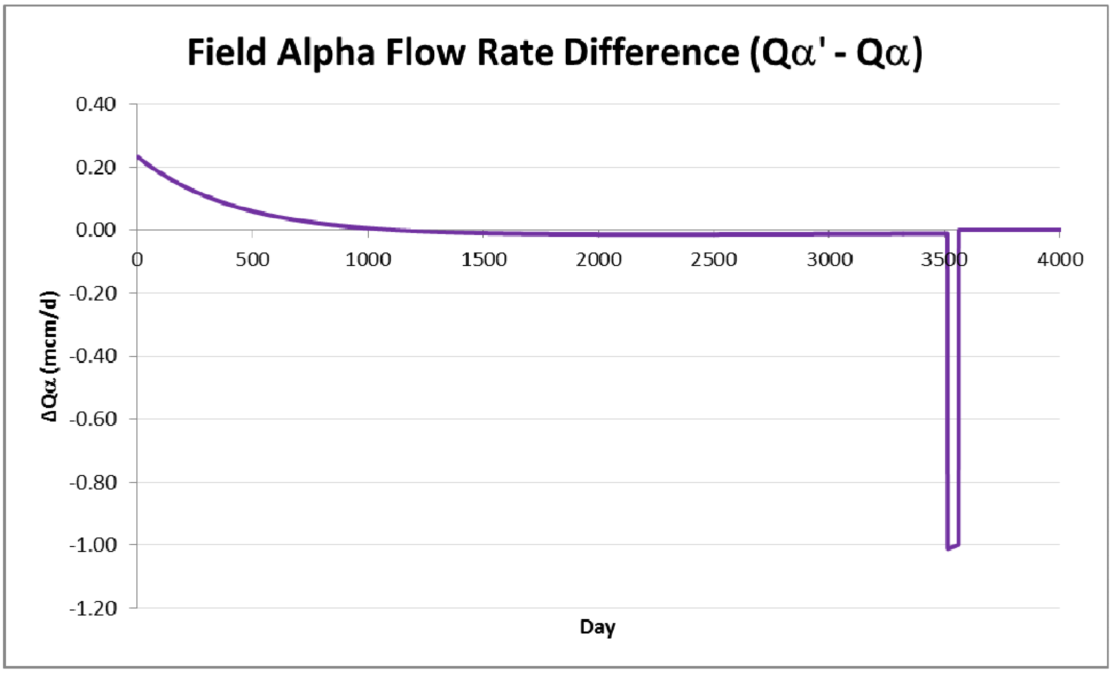


Figure 13 – Alpha Stand-Alone Production Minus Shared Production with COP

This plot graphically illustrates the slow repayment after cross over with a relatively short period of high repayment prior to Alpha's COP.

Finally Figure 14 shows the back-out balance rapidly falling to zero just prior to COP:

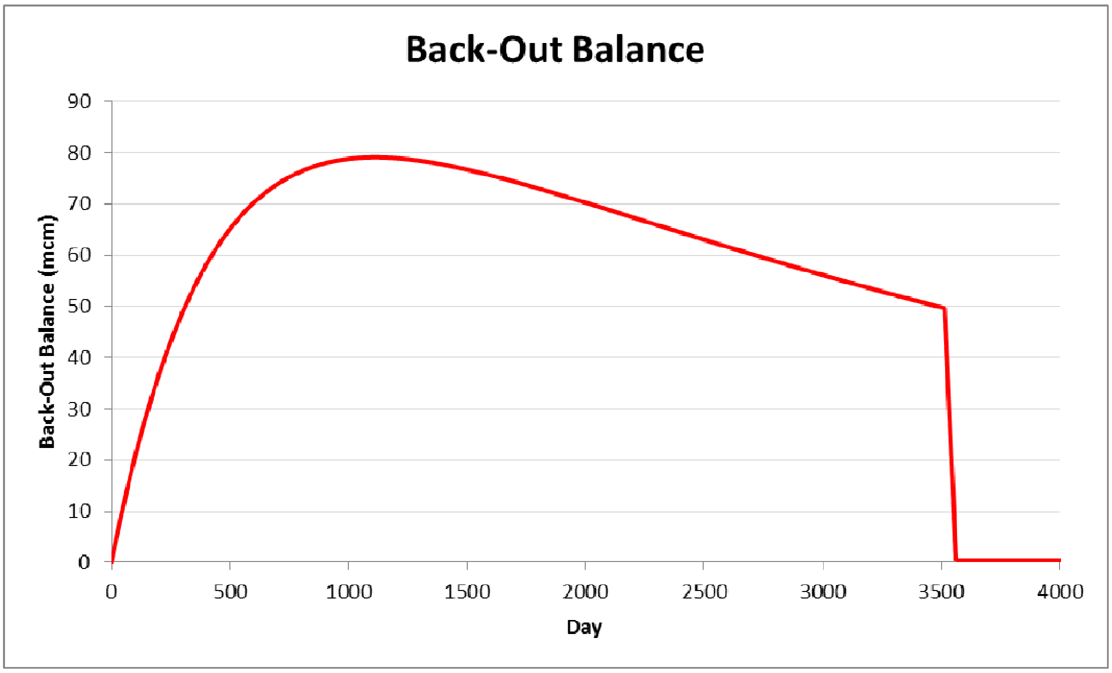


Figure 14 – Back-Out Balance with COP

As can be seen most of the repayment occurs in a single block of production just prior to COP. This could be a significant amount of time after the cross-over date but it is a mechanism by which the new field can obtain repayment, whilst still being consistent with the concept that the incumbent field's allocation is unaffected by the new field.

5 CONCLUSIONS

The paper has explored the behaviour of back-out payment and repayment mechanisms.

It has accomplished this with the use of simplified mathematical models which complement the more complex models but have the advantage that they are more tractable and readily predict the long term behaviour of such back-out systems.

The analysis has revealed that the repayment phase is so prolonged that the new third party field may not receive a significant proportion of repayment before the incumbent field's COP.

A mechanism has been suggested, that incorporates the COP, and ensures full repayment of gas. It achieves this whilst being consistent with the premise, frequently expressed in back-out agreements, that the incumbent field's allocated production is no different to that which it would have experienced if it was produced alone.

When they are written, some back-out agreements appear to focus on the initial back-out experienced by the incumbent field(s). Perhaps this is not surprising as this phase is in the near future. In contrast it appears that the consequences associated with the mechanisms for the repayment phase, are not fully realised. This can result in significant penalty of lost production for the new field.

Finally this paper has attempted to expose potential pitfalls associate with back-agreements and presented potential solutions to address them.

6 SIMPLIFIED BACK-OUT MATHEMATICAL MODEL

6.1 System

Figure 15 depicts the two gas reservoirs (α and β) producing through a shared subsea pipeline to an onshore gas plant.

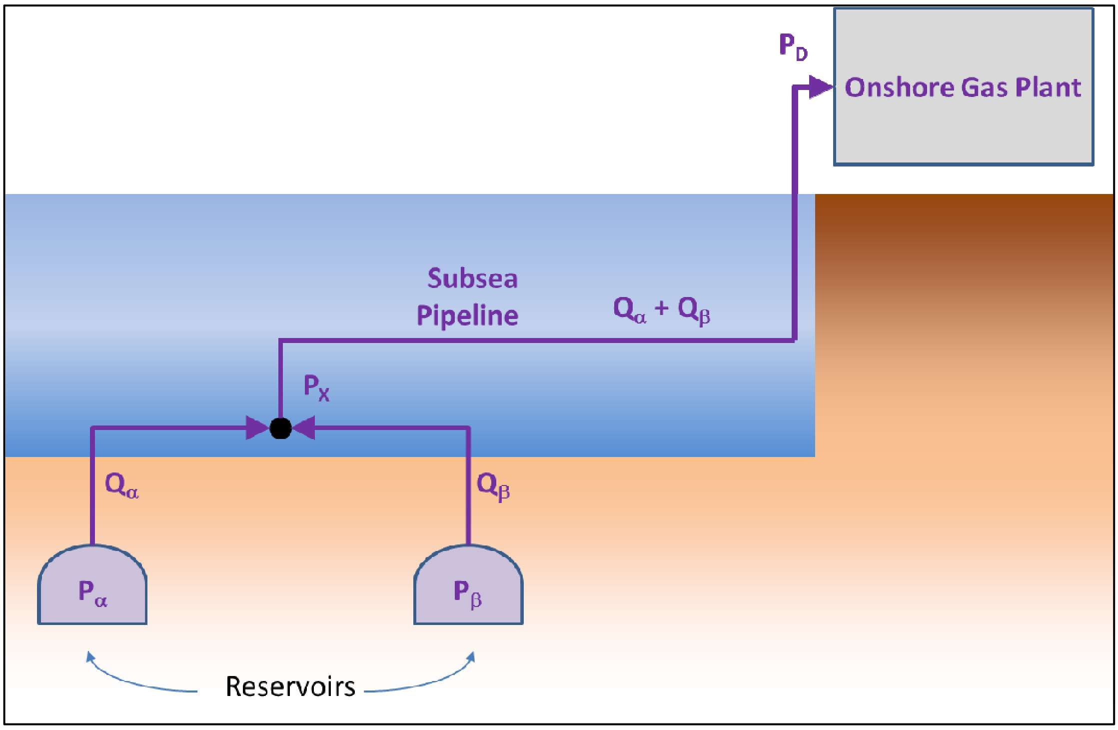


Figure 15 – System Configuration

The pressures and standard volumetric flows at various points in the system are indicated symbolically.

6.2 Simplifying Approximations

In order to render the system solvable analytically some simplifying approximations are made.

The frictional pressure loss in the pipelines is approximated to be linearly dependent on flow:

$$Q_\alpha \approx R_\alpha (P_\alpha - P_X) \tag{8}$$

$$Q_\beta \approx R_\beta (P_\beta - P_X) \tag{9}$$

$$Q_\alpha + Q_\beta \approx R_P (P_X - P_D) \tag{10}$$

The R terms represent the resistance to flow in each pipe segment and are assumed to be constants.

The typical correlation between cumulative gas produced to date (GPD) and reservoir pressure (divided by compressibility) is illustrated in Figure 16 below:

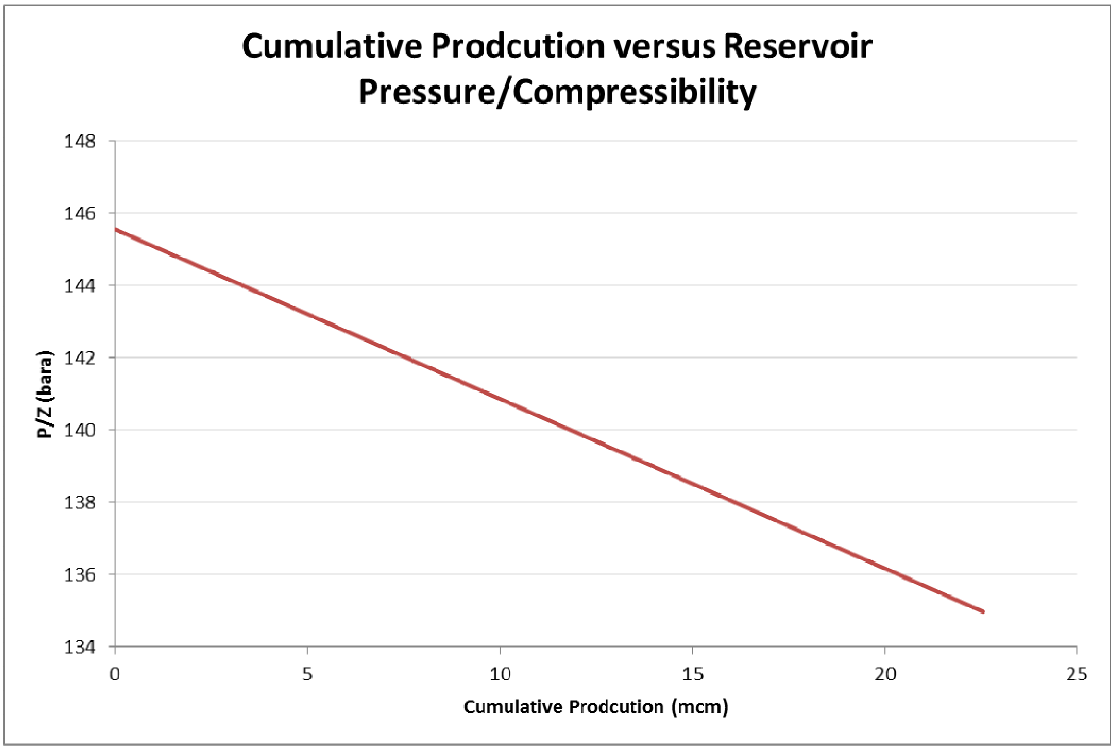


Figure 16 – Reservoir Pressure versus Cumulative Production

Again to simplify the mathematics gas compressibility (Z) is assumed to be constant over the pressure range and reservoir pressure can be expressed as a linear function of cumulative production according to:

$$P_\alpha \approx k_\alpha GPD_\alpha \tag{11}$$

6.3 Shared Production Scenario

Cumulative production is given by the integral of the flow rate over time:

$$GPD_\alpha = \int_0^t Q_\alpha dt \tag{12}$$

Rearranging (11) and substituting for GDP in (12), and rearranging in terms of reservoir pressure gives:

$$P_\alpha = k_\alpha \int_0^t Q_\alpha dt \tag{13}$$

Differentiating with respect to time:

$$\frac{dP_\alpha}{dt} = k_\alpha Q_\alpha \tag{14}$$

A similar relationship can be obtained for Bravo:

$$\frac{dP_\beta}{dt} = k_\beta Q_\beta \tag{15}$$

Substituting for flow Q_α from (8) in (14):

$$\frac{dP_\alpha}{dt} = k_\alpha R_\alpha \left(P_\alpha - P_X\right) \tag{16}$$

Similarly for Bravo:

$$\frac{dP_\beta}{dt} = k_\beta R_\beta \left(P_\beta - P_X\right) \tag{17}$$

Rearranging equation (10) in terms of P_X and substituting for Q_α and Q_β from (8) and (9) results in:

$$P_X = \frac{R_\alpha P_\alpha + R_\beta P_\beta + R_P P_D}{R_\alpha + R_\beta + R_P} \tag{18}$$

This can be substituted in (16) and (17) to obtain:

$$\frac{dP_\alpha}{dt} = \left(\frac{k_\alpha R_\alpha}{R_\alpha + R_\beta + R_P}\right)\left(P_\alpha R_P + P_\alpha R_\beta - P_\beta R_\beta - P_D R_P\right) \tag{19}$$

And:

$$\frac{dP_\beta}{dt} = \left(\frac{k_\beta R_\beta}{R_\alpha + R_\beta + R_P}\right)\left(P_\beta R_P + P_\beta R_\alpha - P_\alpha R_\alpha - P_D R_P\right) \tag{20}$$

Differentiating (19) with respect to time:

$$\frac{d^2 P_\alpha}{dt^2} = \left(\frac{k_\alpha R_\alpha}{R_\alpha + R_\beta + R_P}\right)\left(\frac{dP_\alpha}{dt}\left(R_P + R_\beta\right) - \frac{dP_\beta}{dt} R_\beta\right) \tag{21}$$

Rearranging (21):

$$\frac{dP_\beta}{dt} = \frac{1}{R_\beta}\left(\frac{dP_\alpha}{dt}\left(R_P + R_\beta\right) - \frac{d^2 P_\alpha}{dt^2}\left(\frac{R_\alpha + R_\beta + R_P}{k_\alpha R_\alpha}\right)\right) \tag{22}$$

Rearranging (19) in terms of P_β:

$$P_\beta = \frac{1}{R_\beta}\left(P_\alpha R_P + P_\alpha R_\beta - P_D R_P - \frac{dP_\alpha}{dt}\left(\frac{R_\alpha + R_\beta + R_P}{k_\alpha R_\alpha}\right)\right) \tag{23}$$

Substituting (22) and (23) in (20) and rearranging produces:

$$\frac{d^2 P_\alpha}{dt^2} - \left(\frac{k_\alpha R_\alpha (R_P + R_\beta) + k_\beta R_\beta (R_P + R_\alpha)}{R_\alpha + R_\beta + R_P}\right)\frac{dP_\alpha}{dt} + \left(\frac{k_\alpha k_\beta R_\alpha R_\beta R_P}{R_\alpha + R_\beta + R_P}\right)P_\alpha = \left(\frac{k_\alpha k_\beta R_\alpha R_\beta R_P}{R_\alpha + R_\beta + R_P}\right)P_D \tag{24}$$

This is a second order linear differential equation in P_α, which has the solution:

$$P_\alpha = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_1 t} + P_D \tag{25}$$

This is an initial value problem, with initial conditions defined when t= 0:

$$\text{At } t = 0, P_\alpha = P_\alpha^\circ, P_\beta = P_\beta^\circ \tag{26}$$

The starting reservoir pressures P_α° and P_β° are assumed to be known.

The various λ and C parameters in (25) are:

$$\lambda_1 = \frac{k_\alpha R_\alpha (R_P + R_\beta) + k_\beta R_\beta (R_P + R_\alpha) + \sqrt{\left(k_\alpha R_\alpha (R_P + R_\beta) - k_\beta R_\beta (R_P + R_\alpha)\right)^2 + 4k_\alpha^2 R_\alpha^2 k_\beta^2 R_\beta^2}}{2(R_\alpha + R_\beta + R_P)} \tag{27}$$

$$\lambda_2 = \frac{k_\alpha R_\alpha (R_P + R_\beta) + k_\beta R_\beta (R_P + R_\alpha) - \sqrt{\left(k_\alpha R_\alpha (R_P + R_\beta) - k_\beta R_\beta (R_P + R_\alpha)\right)^2 + 4k_\alpha^2 R_\alpha^2 k_\beta^2 R_\beta^2}}{2(R_\alpha + R_\beta + R_P)} \tag{28}$$

$$C_1 = \frac{1}{(\lambda_2 - \lambda_1)}\left(\frac{k_\alpha R_\alpha \left(P_\beta^\circ R_\beta - P_\alpha^\circ (R_P + R_\beta) + P_D R_P\right)}{(R_\alpha + R_\beta + R_P)} + \lambda_2 \left(P_\alpha^\circ - P_D\right)\right) \tag{29}$$

$$C_2 = \frac{-1}{(\lambda_2 - \lambda_1)}\left(\frac{k_\alpha R_\alpha \left(P_\beta^\circ R_\beta - P_\alpha^\circ (R_P + R_\beta) + P_D R_P\right)}{(R_\alpha + R_\beta + R_P)} + \lambda_1 \left(P_\alpha^\circ - P_D\right)\right) \tag{30}$$

Now P_α is obtained all other variables: P_β, Q_α and Q_β can be determined.

6.4 Stand Alone Production Scenario

For the Alpha Stand-Alone case, Q_β is zero. Equation (10) now becomes:

$$Q_\alpha^{'} \approx R_P\left(P_X - P_D\right) \tag{31}$$

Combining equation (8) and (31) to eliminate P_X:

$$Q_\alpha^{'} = \frac{R_\alpha\left(P_\alpha^{'} - P_D\right)}{\left(1 + \dfrac{R_\alpha}{R_P}\right)} \tag{32}$$

This is then substituted in (14) to obtain the first order differential equation in P_α':

$$\frac{dP_\alpha^{'}}{dt} = \frac{k_\alpha R_\alpha\left(P_\alpha^{'} - P_D\right)}{\left(1 + \dfrac{R_\alpha}{R_P}\right)} \tag{33}$$

With initial conditions defined when t= 0:

$$\text{At } t = 0, P_\alpha^{'} = P_\alpha^{\circ}, \tag{34}$$

This has the solution:

$$P_\alpha^{'} = \left(P_\alpha^{\circ} - P_D\right)e^{\mu t} + P_D \tag{35}$$

Where,

$$\mu = \frac{k_\alpha R_\alpha R_P}{\left(R_P + R_\alpha\right)} \tag{36}$$

Now P_α' is obtained, Q_α' can be determined.

6.5 Values of Parameters Used in Example System

k_α	-0.004344 bar/mcm
k_β	-0.4683577 bar/mcm
P_α°	48.3 bara
P_β°	145.3 bara
P_D	6.75 bara
R_α	0.1047 mcm/bar
R_β	0.004911 mcm/bar
R_P	0.160105 mcm/bar.

7 NOTATION

B	Back-out balance
C_1	System parameter defined in (29)
C_2	System parameter defined in (30)
GDP_α	Field Alpha cumulative gas production to date
k_α	Constant relating Alpha's reservoir pressure to its gas volume
k_β	Constant relating Bravo's reservoir pressure to its gas volume
P_α	Field Alpha Shared Production reservoir pressure
P_α'	Field Alpha Stand-Alone reservoir pressure
P_α°	Initial Alpha reservoir pressure
P_β	Field Bravo Shared Production reservoir pressure
P_β'	Field Bravo Stand-Alone reservoir pressure
P_β°	Initial Bravo reservoir pressure
P_D	Delivery pressure
P_X	Manifold pressure
Q_α	Field Alpha Shared Production volumetric flow rate
Q_α'	Field Alpha Stand-Alone volumetric flow rate
Q_β	Field Bravo Shared Production volumetric flow rate
R_α	Flow resistance factor Alpha to X (manifold)
R_β	Flow resistance factor Bravo to X (manifold)
R_P	Flow resistance factor X (manifold) to delivery point
t	Time
Z	Gas compressibility
λ_1	System parameter defined in (27)
λ_2	System parameter defined in (28)
τ_0	Repayment completion time
τ_X	Cross-over time
μ	System parameter defined in (36)

8 REFERENCES

[1] Petroleum Experts supply MBAL, PROPSER and GAP, http://www.petex.com/

[2] Schlumberger supply Eclipse, OFM and Pipesim, https://www.software.slb.com/products