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Determination of Optimal Calibration Intervals – A Risk Based Approach

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1 INTRODUCTION

The current economic challenges facing the upstream oil and gas sector has driven an urgent need to reduce costs and improve efficiency. A reduction of operating costs may be achieved through the extension of the calibration intervals. The maintenance strategies associated with fiscal and custody transfer measurement systems have traditionally been based on a 'time-based' approach (maintenance activities are scheduled at fixed intervals), without much consideration to past equipment performance and the introduction of ever more intelligent diagnostic capabilities of modern instrumentation. However, custody transfer, fiscal and allocation metering are essentially the cash registers of the company and their performance must be assured to minimise financial exposure. Similarly, complying with EU regulation for the monitoring, reporting and verification of greenhouse gas emissions requires the operators to have in place a measurement plan defining the calibration and maintenance regime required in order to meet the specified uncertainty levels for activity data for the applied tiers.

It is this drive, to reduce costs, whilst maintaining control of the measurement uncertainty that has resulted in the Oil & Gas Authority (UK) to strongly urge operators to consider abandoning the traditional time-based maintenance in favour of a 'risk-based' or 'condition-based' maintenance, or combination of both [3]. It is suggested that a risk-based approach to maintenance, as outlined in section 5.2.5 of the current issue of the DECC Measurement Guidelines [1], should be the default methodology.

The impact of measurement bias is proportional to the time over which it exists without correction. The effect can be eliminated by calibration and adjustment at appropriate intervals, however, the determination of calibration intervals is a complex mathematical and statistical process requiring accurate and sufficient data taken during the calibration process [7]. There is no universally applicable single best practice and thus there is a need for a better understanding of the mechanism required for the determination of an appropriate calibration interval. There are very few official documents available which provide guidance on establishing the optimal calibration interval [2], [4].

This paper details the application of a risk-based approach to determine the optimal calibration interval.

Section 2 covers the concept of the risk-based approach. Section 2.1 introduces the term 'total costs' and provides insight as to how this can be established. Section 2.2 details the mechanism of measuring instrument ageing and introduces some models which can be used to define the evolution of the measurement bias over time. Section 2.3 describes the calculation of financial exposure in terms of the 'expected loss', considering the likelihood and consequences of the measurement bias, and the 'value flow rate'. Section 2.4 defines the measurement costs calculation. The methodology is demonstrated on the example of a gas ultrasonic meter, pressure and temperature transmitters using real anonymised data in Section 3. Section 4 provides conclusions.

2 RISK-BASED APPROACH CONCEPT

For deployment of the risk-based approach [1] the following parameters have to be evaluated:

- **the value flow rate** is the product of the relevant flow rate Q and the relevant measured product value PV.

- **the estimate of a measurement error** (measurement bias) $\Delta(t)$ over a given period of time (refer to section 2.2).

- **the financial exposure** FE is the product of the expected loss and the value flow rate $(Q \cdot PV)$ accumulated over the period of time T during which the measurement error $\Delta(t)$ may be expected to occur (refer to section 2.3).

- **the measurement costs** MC take account of the costs of ownership of a measuring instrument including calibration costs, repair costs, maintenance and metering service, depreciation and other costs (refer to section 2.4).

The concept of the risk-based approach is to balance the measurement costs against the financial exposure, determined by estimating the likelihood and consequences of the measurement error over a given period of time. The balance equation can be expressed as follows:

$$FE = MC \tag{1}$$

2.1 Total Costs

For implementation of the risk-based approach the time-dependent total costs function is defined as the sum of financial exposure and the measurement costs as follows:

$$TC(t) = FE(t) + MC(t) \tag{2}$$

The optimal calibration interval will correspond to the minimum of the total costs function [6]. This can be calculated by differentiating equation (2) and setting the found derivative equal to zero, thus the optimal calibration interval is the one which satisfies the following condition:

$$\frac{d\big(FE(t) + MC(t)\big)}{dt} = 0 \tag{3}$$

To solve the differential equation (3) for the optimal calibration interval T, the financial exposure and the measurement costs have to be defined as a function of time. In many cases obtaining such an analytical solution to equation (3) is infeasible and thus the solution can be determined graphically by observing a minimum of the total costs function.

This can be demonstrated by the following example. Substituting $FE(t)$ from (11) and $MC(t)$ from (21) in (3):

$$\frac{d}{dt}\left(\int EL(t) \cdot Q \cdot PV dt\right) + \frac{d(A \cdot t^B)}{dt} = 0 \tag{4}$$

Substituting $EL(t)$ from (12) in to equation (4) differentiates to:

$$c \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot (\sigma_0 + v_\sigma \cdot t + a_\sigma \cdot t^2) + A \cdot B \cdot t^{B-1} = 0 \qquad (5)$$

The optimal calibration interval T is the one which satisfies equation (5), but it is not possible to solve this equation in closed form, but it can be solved numerically.

2.2 Measurement Bias

Following the international vocabulary of metrology [5]:

- **systematic measurement error** is the component of measurement error that in replicate measurements remains constant or varies in a predictable manner and can be known (and be corrected) or unknown (considered as random).

- **random measurement error** is the component of measurement error that in replicate measurements varies in an unpredictable manner, which can be evaluated with the help of probability theory.

- **instrumental drift** or '**drift error'** is the continuous or incremental change over time in indication, due to changes in metrological properties of a measuring instrument. Instrumental drift is related neither to a change in a quantity being measured nor to a change of any recognized influence quantity, but related to an interaction of a measuring instrument with an operating environment and this process is defined in the paper as 'ageing'. Ageing does not depend on whether a measuring instrument is in operation or in storage. The drift error can be corrected at a given time and then again starts to age going forward from this point, thus repeated corrections are required over a measuring instrument life time. The rate of ageing depends on a manufacturing process, used materials and operating temperature of a measuring instrument.

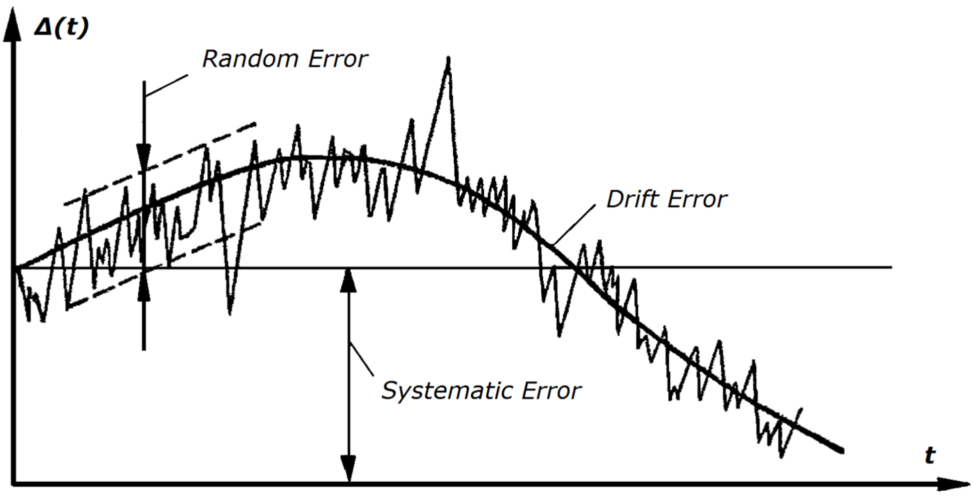

Figure 1 – Measurement Bias

The systematic and random errors are characteristics of a stationary random process, but the drift error is characteristic of a nonstationary random process and varies continuously with time. In reality all these three types of errors are combined in the form of a seamless nonstationary random process shown on Figure 1 and within this paper its estimate is defined as a function of time using the term of measurement bias $\Delta(t)$. Note that the measurement bias term is broadened in comparison with the definition given in [5].

A measuring instrument after calibration can be characterized over its measurement range by the relation between the quantity Q and the measurement bias $\Delta(t)$. Figure 2 shows a one-to-many relation and the width of the strip provides a measuring instrument uncertainty which is supposed to be within the maximum permissible limits $\pm\Delta_p$. After initial calibration the relation tends to be as graph a) shows. Over the period of time T the joint action of the above mentioned errors causes an additive and multiplicative shift in relation as graph b) shows. The additive shift can be corrected by zero adjustment as graph c) shows but it has to be kept in mind that with zero adjustment expands the width of the strip. The multiplicative shift reflects the presence of the drift error and graph c) shows the most common situation for all subsequent calibrations. Consequently, it is more likely that a measuring instrument uncertainty exceeds the maximum permissible limit towards the end of its measurement range with half of all instruments failing calibration at these points [11].

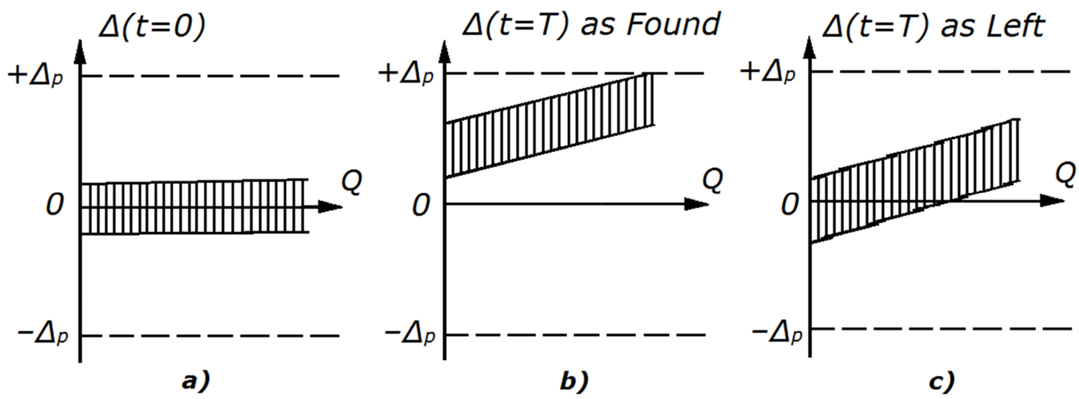

Figure 2 – Measurement Bias

Along with predictable calibration failure, that is those where the measurement bias $\Delta(t)$ exceeds the permissible limits $\pm\Delta_p$ due to drift error, unexpected calibration failure may also occur. As shown in [11] only 5 % of all calibration failures can be classified as unexpected and will not be considered in the present paper.

The measurement bias can be defined by several mathematical models based on a hypotheses of the continuous changing or ageing of a measuring instrument in the process of its operation or storage.

In general, the exponential model can be used and the measurement bias over the measuring instrument lifetime may be written in terms of time (an instrument age, years) as follows:

$$\Delta(t) = \Delta_0 + \Delta_m \cdot \frac{\omega_0}{a} \cdot (e^{a \cdot t} - 1) \tag{6}$$

where Δ_0 is the measurement bias defined at initial calibration ($t = 0$), $\Delta_m = \Delta_p - \Delta_0$ is the difference between the measurement bias defined at initial calibration and the maximum permissible limit, ω_0 is the calibration failure frequency, a is the acceleration of the measuring instrument ageing.

Substituting function $e^{a \cdot t} = 1 + a \cdot t + (a \cdot t)^2/2 + (a \cdot t)^3/3 + \cdots$ by first three terms of its expansion, equation (6) can be approximated by a quadratic equation as follows:

$$\Delta(t) = \Delta_0 + v \cdot t + a_\Delta \cdot t^2 \tag{7}$$

where $v = \Delta_m \cdot \omega_0$ is the velocity of the measuring instrument ageing (%/year), $a_\Delta = (\Delta_m \cdot \omega_0 \cdot a)/2$ is the absolute acceleration of the measuring instrument ageing (%/year^2). v and a_Δ can be defined on the basis of experimental data (calibration results over a number of years) using least squares or any other suitable technique.

In terms of the standard deviation σ of the measurement bias, equation (7) can be expressed as follows:

$$\sigma(t) = \sigma_0 + v_\sigma \cdot t + a_\sigma \cdot t^2 \tag{8}$$

where σ_0 is the standard deviation at initial calibration ($t = 0$), v_σ and a_σ are the velocity and acceleration of the standard deviation which can be defined on the basis of experimental data (calibration results over a number of years).

If the calibration failure frequency is higher at the beginning of the measuring instrument life time and lower at the end as graph a) on Figure 3 shows, then the acceleration of ageing is a negative value. And vice versa, if the calibration failure frequency is increasing towards the end of the measuring instrument life time the acceleration of ageing is a positive value as graph b) on Figure 3 shows. With every adjustment at a time T_i the measurement bias is set to the value close to initial calibration Δ_0 but it continues to deviate following its ageing curve and showing the nature of the instrumental drift.

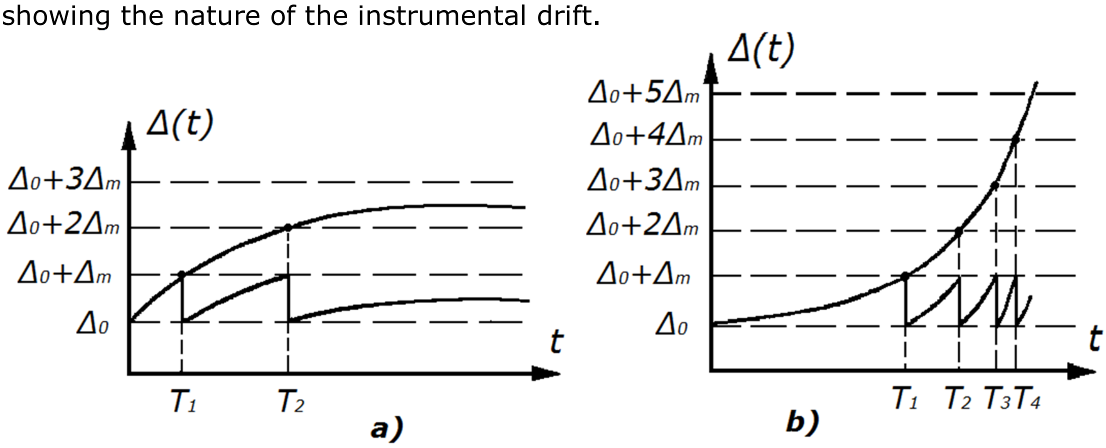

Figure 3 – Exponential Ageing Model

If the calibration failure frequency is roughly the same over a measuring instrument life time, then the linear mathematical model ($a = 0$) describing the measurement bias behaviour can be assumed and equation (7) is reduced to:

$$\Delta(t) = \Delta_0 + v \cdot t \tag{9}$$

The measurement bias behaviour defined by the linear model is shown on Figure 4. In this case the calibration interval will remain constant over the measuring instrument life time. The linear model is referred to in [8] for optimal calibration interval determination.

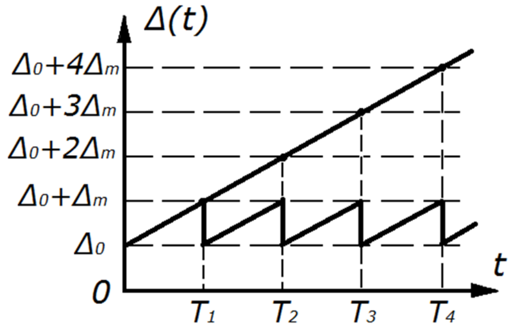

Figure 4 – Linear Ageing Model

Defining the errors on the grounds of their frequency (drift error – lower frequency, random error – higher frequency) refers them to different regions of the frequency spectrum and specifies the spectral properties of the measurement bias. It allows the prediction of the ageing peak maximum of a measuring instrument as a maximum of the measurement bias logarithmical spectrum [11] defined by the following function:

$$S(t) = \sqrt{S_i^2 + 2 \cdot v \cdot t \cdot e^{a \cdot t} \cdot \left[S_i + \frac{v}{a} \cdot (e^{a \cdot t} - 1)\right]} \tag{10}$$

where $S_i = \sigma_i^2 / ln(T_i/t)$ is the value of the logarithmical spectrum at a particular time moment T_i, σ_i is the standard deviation of the measuring instrument.

Summarising the models described in this section it should be noted that the linear model (9) facilitate the prediction of the measurement bias in the period from 1 to 5 years, the exponential model (6) in the period from 1 to 100 years and the logarithmic spectrum model may be used to predict the measurement bias behaviour in the range from 1 second to 100 years. The models can show good results if at least 5 calibration results are available, thus sufficient as-found and as-left data have to be available for a multiple of instruments of the same type.

2.3 Financial Exposure

The consequences of the measurement bias can be determined by a loss function and in conjunction with the likelihood of the measurement bias to occur, the expected loss can be calculated as detailed in Section 7.

The expected loss may include a penalty due to failure to deliver an agreed amount of product or an incorrect evaluation and forecasting of product.

The consequences of the measurement bias grow with time, the longer calibration interval the higher the expected loss. The accumulated product of the expected loss and the value flow rate over the period of time T between two subsequent calibrations defines the financial exposure as follows:

$$FE(t = T) = \int_0^T EL(t) \cdot Q \cdot PVdt \tag{11}$$

where T is the number of days between two subsequent calibrations, Q is the product flow rate per day, PV is the product value per flow rate unit. The financial exposure is expressed in currency units.

Assuming the exponential model (8) of the normally distributed measurement bias and absolute loss function (29), the expected loss defined by equation (32) is expressed in terms of t as follows:

$$EL(t) = c \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot (\sigma_0 + v_\sigma \cdot t + a_\sigma \cdot t^2) \tag{12}$$

where σ_0 is the measuring instrument standard deviation defined following the initial calibration.

Equation (12) allows the expression of the financial exposure in the following form:

$$\begin{aligned} FE(t = T) &= \int_0^T c \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot (\sigma_0 + v_\sigma \cdot t + a_\sigma \cdot t^2) \cdot Q \cdot PVdt = \\ &= Q \cdot PV \cdot T \cdot c \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \left(\sigma_0 + \frac{1}{2} \cdot v_\sigma \cdot T + \frac{1}{3} \cdot a_\sigma \cdot T^2\right) \end{aligned} \tag{13}$$

2.4 Measurement Costs

The measurement costs include capital and operating expenditures and the **annual** measurement costs MC (£/year) are calculated as follows:

$$MC = OPEX + EF \cdot CAPEX \tag{14}$$

where $OPEX$ (£/year) is the operating expenditures, EF (1/year) is the efficiency factor of capital expenditures, $CAPEX$ (£) is the capital expenditures.

The operating expenditures calculation can be formularized as follows:

$$OPEX = CalC + RepC + SalC + DepC + AsC \tag{15}$$

where $CalC$ (£/year) is the calibration costs, $RepC$ (£/year) is the minor repair or servicing costs, $SalC$ (£/year) is the costs associated with the workforce (engineers and technicians assuring trouble-free operation of measuring instruments) salary, $DepC$ (£/year) is the depreciation (allocation of an assets cost to periods in which the assets are used) costs, AsC (£/year) is the electrical energy consumption and processed materials supply costs.

The calibration costs $CalC$ may include but not limited to:

- calibration at an accredited laboratory,

- attendance to witness a calibration by manufacturer's and operator's engineers,
- expenses related with the attendance,
- transportation of the measuring instrument to a calibration laboratory and back,
- removal and replacement of the measuring instrument,
- scaffolding over meter stream to allow removal and replacement,
- lagging removal and replacement,
- updating records

The minor repair or servicing costs are calculated as follows:

$$RepC = t \cdot RPC + Prt \qquad (16)$$

where t (hours/year) is the average time spent for repair or service, RPC (£/hour) is the repair personnel costs, Prt (£/year) is the average cost of replaced parts.

The metering personnel salary costs are calculated as follows:

$$SalC = 12 \cdot \sum MPC \cdot K \qquad (17)$$

where MPC (£/month) is the metering personnel costs, K is the personnel workload factor showing a fraction of time spent for the particular measuring instrument, 12 (month/year) is the constant.

The depreciation costs are calculated as follows:

$$DepC = K_D \cdot PC \qquad (18)$$

where K_D (1/year) is the depreciation coefficient defining the period of the metering instrument life time, PC (£) is the measuring instrument purchase costs.

The electrical energy consumption EnC (£/year) and processed materials supply costs MSC (£/year) are calculated as the sum of two:

$$AsC = EnC + MSC \qquad (19)$$

The capital expenditures calculation can be formularized as follows:

$$CAPEX = K_{PC} \cdot PC + K_{ICC} \cdot ICC + K_{RDC} \cdot RDC \qquad (20)$$

Where K_{PC}, K_{ICC} and K_{RDC} are the coefficients defining the part of the costs associated with the measurement of the specified process parameter and applicable if the measuring instrument is capable to measure more than one parameter, PC (£) is the measuring instrument purchase costs, ICC (£) is the installation and commissioning costs, RDC (£) is the research and development costs which include but not limited to:

- development of measurement, calibration and maintenance procedures,
- evaluation of uncertainty and obtaining of specified approval,
- upgrade and development of data control system and related software.

The estimate of the installation and commissioning costs is assumed as 20 % of the measuring instrument purchase costs or $ICC = 0.2 \cdot PC$. The estimate of the coefficients in (20) is assumed as $1/n$, where n is the number of process parameters the measuring instrument is capable to measure and control.

Considering the nature of the measurement costs nonlinearly decreasing with the calibration interval extension it can be approximated by a power function as follows:

$$MC(t) = A \cdot t^B \tag{21}$$

where A and B are the parameters of the approximation function which can be obtained using the least squares technique or any other suitable technique.

3 OPTIMAL CALIBRATION INTERVAL CALCULATION EXAMPLE

For illustration of the aspects covered in the paper let us consider a gas metering system incorporating an ultrasonic flow meter, pressure and temperature transmitters.

The mass flow rate, Q_m (kg/h), is calculated as follows:

$$Q_m = Q_v \cdot [1 + K_T \cdot (Ta - Ta_{cal})] \cdot [1 + K_P \cdot (P - P_{cal})] \cdot \rho \tag{22}$$

where Q_v (m^3/h) is the measured gross volume flow rate, K_T is the material specific temperature coefficient of the meter body, K_P is the pressure coefficient of the meter body, Ta_{cal} and P_{cal} are the calibration absolute temperature and pressure respectively, $\rho = \frac{M \cdot P}{Z \cdot R \cdot T}$ (kg/m^3) is the flowing density of the gas, M is the molar mass, P is the absolute line pressure, Z is the compressibility factor, R is the gas constant, Ta is the absolute line temperature.

As the first step the measurement bias as a function of time is determined on the basis of available calibration data (refer to section 8) for the ultrasonic flow meter, pressure and temperature transmitters with no considerations of zero and span adjustments. The statement in section 2.2 that in 50 % of all cases the measurement bias exceeds the maximum permissible limit towards the end of the measurement range is proved by the results. Thus for the measurement bias development the following calibration points are considered: 2800 m3/h, 90 barg and 40 °C.

The exponential model as per equations (7) and (8) is used for the measurement bias determination. For the ultrasonic meter the ageing velocity is defined as positive value and the acceleration as negative value. The opposite situation was observed with pressure and temperature transmitters, but it should be noted that calibration results were only available for a period of two years, further data is required to establish a more reliable ageing model.

The second step supposes the calculation of the financial exposure using equation (13) and assuming the normally distributed measurement bias and absolute loss function. The following parameters were used for evaluation:

- natural gas is the fluid type
- 0.41 £/thm is the product value PV

- 164,038.0 thm /day is the product flow rate Q
- sensitivity coefficient on the basis of equation (22) is calculated as $c = 1$ for ultrasonic flow meter, $c = 0.99$ for pressure transmitter and $c = 0.03$ for temperature transmitter.

The third step specifies the measurement costs calculation and in the current example operating expenditures (15) are limited by calibration and depreciation costs but the capital expenditures are defined by equation (20).

The last step directs us to balance the calculated measurement bias and measurement costs by searching the minimum of the total costs function defined by equation (2). Defining the values of the total costs function by varying the variable T in the range from 0 to 100 months and mapping it on the graph allows the determination of the optimal calibration interval corresponding to the minimum of the total costs function.

The total costs function minimum was identified at the level of 14 months for the ultrasonic flow meter (the current calibration interval is 12 months) as shown on Figure 5.

The total costs function minimum was identified at the level of 5 months for the pressure transmitter (the current calibration interval is 3 months) as shown on Figure 6.

The total costs function minimum was identified at the level of 13 months for the temperature transmitter (the current calibration interval is 2 months) as shown on Figure 7.

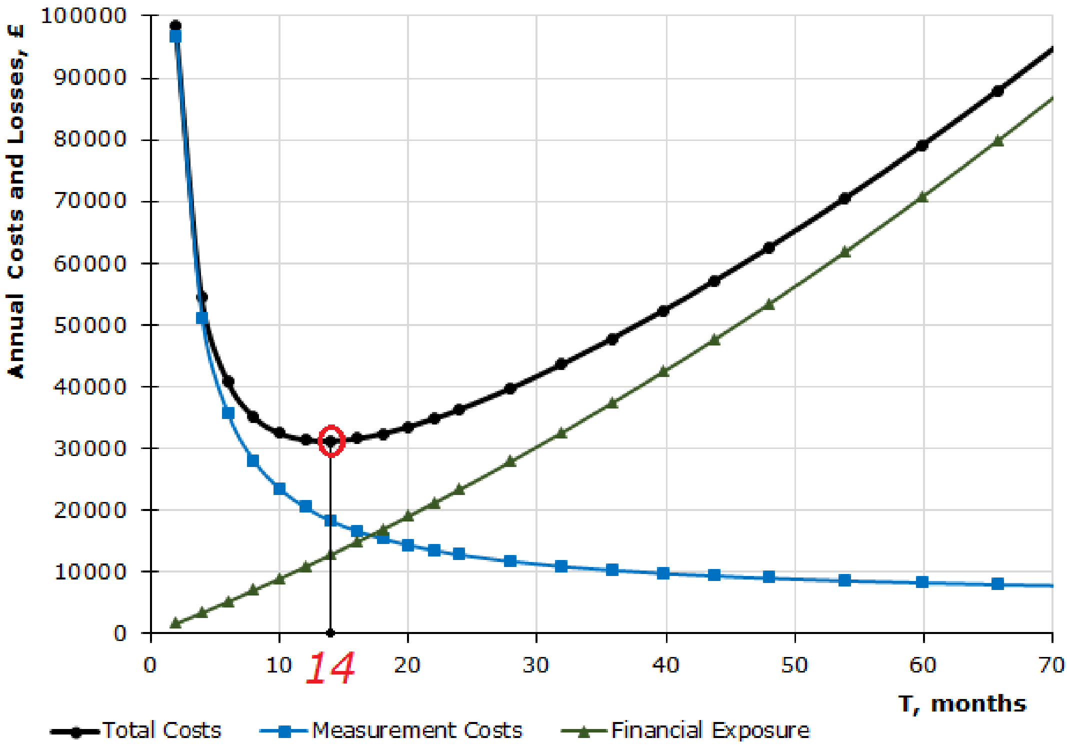

Figure 5 – Total Costs Function for Gas Ultrasonic Flow Meter

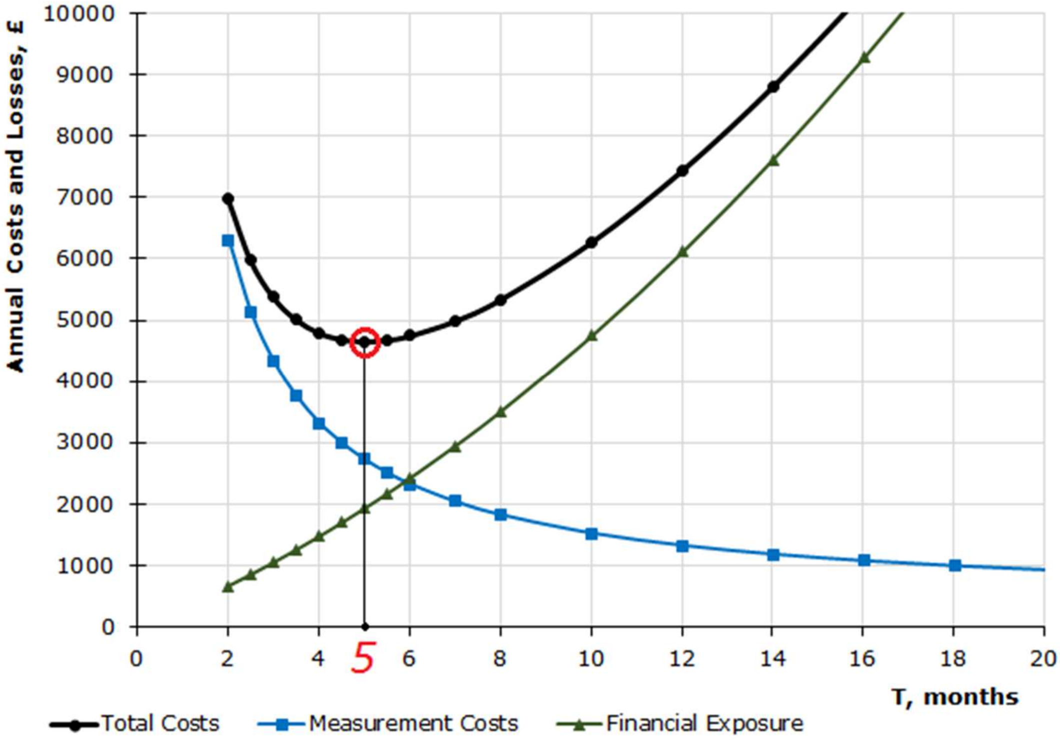

Figure 6 – Total Costs Function for Pressure Transmitter

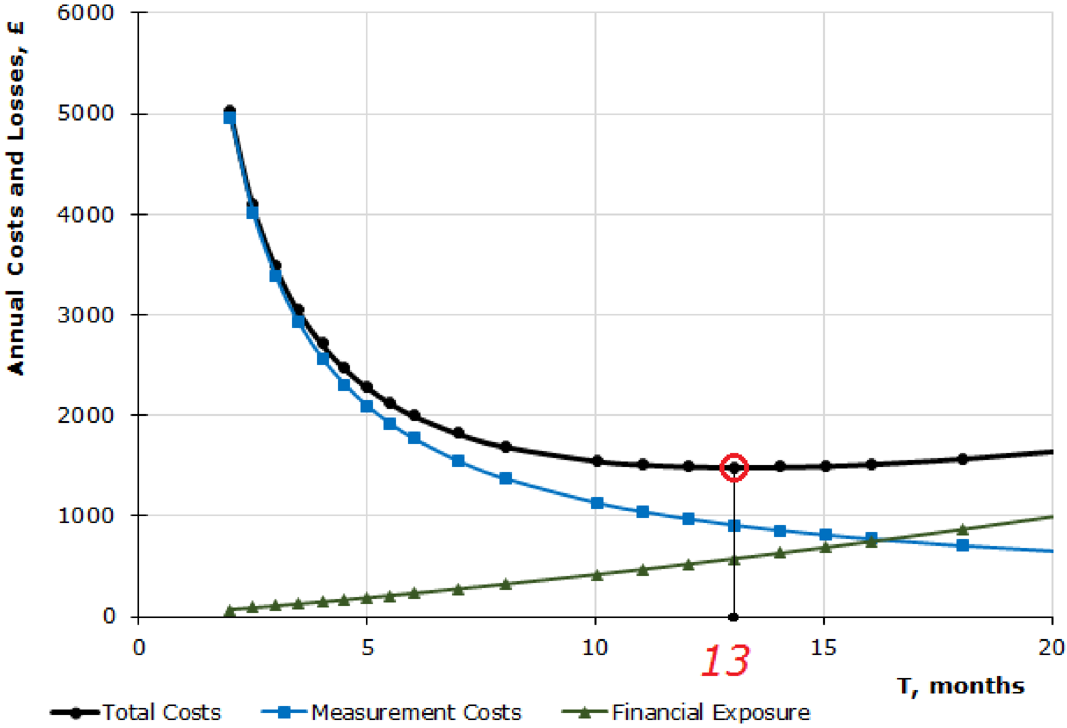

Figure 7 – Total Costs Function for Temperature Transmitter

4 CONCLUSIONS

The present paper details a risk-based approach to the maintenance of measuring instruments determining the optimal calibration interval. Each parameter of the risk-based concept is analysed separately, in particular, the models allowing the definition of the measuring instrument ageing process and the loss functions modifying the expected loss and consequently the financial exposure.

A step-by-step methodology has been provided which specifies how to implement the proposed risk-based approach. It has been shown on real data that the risk-based approach can play an important role and provide an efficient tool for the optimization of maintenance resources.

It is shown that the process of establishing an optimum calibration interval can be a complex endeavour requiring the use of mathematical and statistical processes and demanding a sufficient quantity of calibration data. The development of a standardised suite of tools to perform the process can make the process simpler but still retain the mathematical rigour to provide confidence that the resultant calibration intervals are optimal from a financial exposure perspective and the total costs are minimised.

In the current climate faced by the industry operators can obtain significant benefit by assessing the financial risk associated with an implemented maintenance methodology and therefore ensuring that the resultant financial exposure is properly understood.

5 NOTATION

The symbols defined and used within the section of the paper are not listed below.

A, B	Parameters of MC approximation	t	time
a	Ageing acceleration of measurement bias	T	Period of time between calibrations
a_σ	Ageing acceleration of standard deviation	TC	Total costs
a_Δ	Absolute acceleration	Q	Determined quantity value
c	Sensitivity coefficient	v	Ageing velocity of measurement bias
$CAPEX$	Capital expenditures	v_σ	Ageing velocity of standard deviation
EF	Efficiency factor of $CAPEX$		**Greek**
EL	Expected loss	Δ	Measurement bias
FE	Financial exposure	Δ_0	Measurement bias at initial calibration
MC	Measurement costs	Δ_m	difference between Δ_p and Δ_0
$OPEX$	Operating expenditures	Δ_p	Maximum permissible limit
PV	Product value	σ_0	Standard deviation at initial calibration
S	Measurement bias logarithmical spectrum	ω_0	Calibration failure frequency

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7 APPENDIX A

A.1 Expected Loss Calculation

There is a relationship between a measurement bias, Δ, and the associated change in the quantity value ΔQ which can be formularized as $\Delta Q = \Psi(\Delta)$, where the function Ψ is known as a loss function (also known as a risk function). A loss (or cost – "the price paid for inaccuracy") will arise if the measurement error turns to be different from agreed value and the larger the measurement error the greater the shift in the determined parameter and the greater the loss.

If the measurement bias remains constant or varies in a predictable manner, then the measurement bias is defined as systematic and the shift in the determined quantity value will have a particular sign. In practice, corrections are applied to compensate for a known systematic measurement error, but unknown systematic measurement errors are considered to be random. In this paper only unknown systematic and random measurement errors are considered and thus the shift in the determined quantity value ΔQ is treated as a random variable as well.

Ideally the loss function has to be minimised, but it is incorrect to find an extremum of the function of a random variable. Instead, considering a large number of measurements a mean or expectation of the loss function $\Psi(\Delta)$ can be minimised and an expected loss (a risk measure), EL, may be written as:

$$EL = E[\Psi(\Delta)] = \int_{-\infty}^{+\infty} \Psi(\Delta) \cdot f(\Delta)\, d\Delta \qquad (23)$$

where $f(\Delta)$ is a probability density function of the measurement bias Δ.

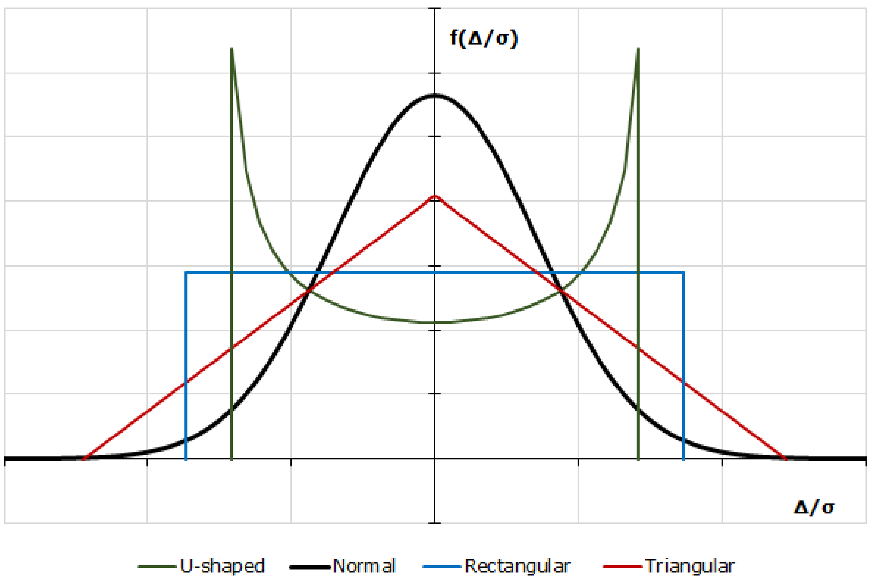

Figure 8 – Probability Density Functions

Most frequently the measurement bias Δ is normally distributed because its components have similar significance. In practice, rectangular, triangular and U-shaped distribution of measurement bias may occur. Figure 8 shows probability density function of these distributions, where σ is a standard deviation of the measurement bias.

In this paper, as the most common, the normal distribution will be considered with the probability density function written as follows:

$$f(\Delta) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} \tag{24}$$

where m is the mean and σ is the standard deviation of the distribution.

There are three commonly used loss functions: squared, absolute and '0-1'. Additionally, the hybrid absolute function is also considered in the article and all of them are shown in Figure 9. Obviously, the expected loss will be calculated differently for each function as shown in the sections below.

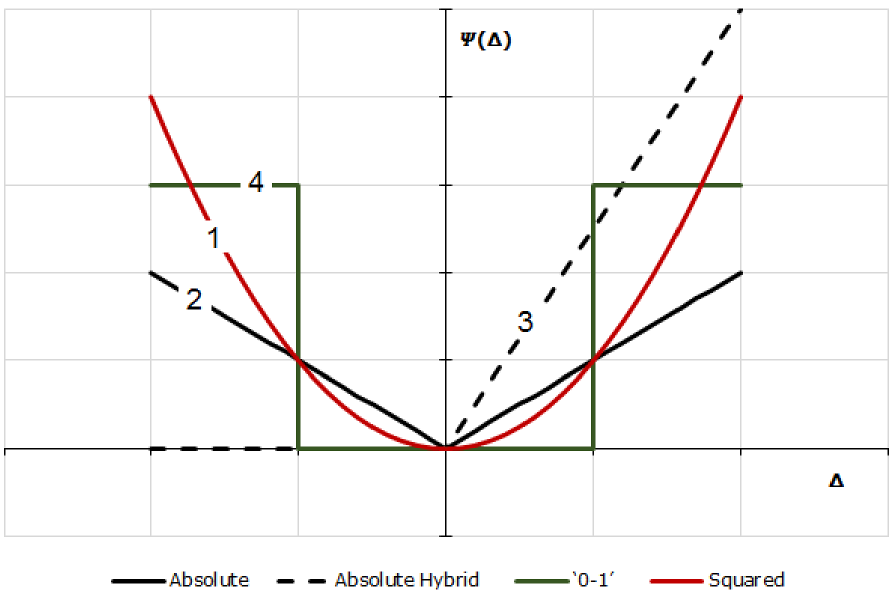

Figure 9 – Loss Functions

A.2 Squared Loss Function

The squared loss function (curve 1 on Figure 9) is commonly used in mathematical optimization when the relationship between the measurement bias and the quantity value is explicitly nonlinear. The function that represents the squared loss is given by:

$$\Psi(\Delta) = c \cdot \Delta^2 \tag{25}$$

where c hereinafter is a coefficient, representing the sensitivity coefficient of the determined quantity value to the measurement bias. This is applicable to indirect measurements and elements of secondary instrumentation having an influence on the determined quantity.

Then the expected loss is calculated by substituting from (24) and (25) to (23):

$$EL = \int_{-\infty}^{+\infty} c \cdot \Delta^2 \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} d\Delta \tag{26}$$

Making the substitution of a variable $z = (\Delta - m)/(\sigma \cdot \sqrt{2})$ and considering that $\Delta = z \cdot \sigma \cdot \sqrt{2} + m$ and $d\Delta = \sigma \cdot \sqrt{2} \cdot dz$, then equation (26) can be expressed in terms of z as follows:

$$\begin{aligned} EL = \frac{c}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \left(z \cdot \sigma \cdot \sqrt{2} + m\right)^2 \cdot e^{-z^2} dz = \\ = \frac{c \cdot \sigma^2 \cdot 2}{\sqrt{\pi}} \int_{-\infty}^{+\infty} z^2 \cdot e^{-z^2} dz + \frac{c \cdot m^2}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-z^2} dz \\ + \frac{c \cdot \sigma \cdot 2\sqrt{2} \cdot m}{\sqrt{\pi}} \int_{-\infty}^{+\infty} z \cdot e^{-z^2} dz \end{aligned} \tag{27}$$

The third integral in equation (27) is equal to zero as the integral of an odd function over symmetric limits, the second integral is the Euler-Poisson (Gaussian) integral equal to $\sqrt{\pi}$ and the first integrand can be integrated by parts with the result of $\sqrt{\pi}/2$. Hence, the expected loss is given by:

$$\boldsymbol{EL} = \frac{c \cdot \sigma^2 \cdot 2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} + \frac{c \cdot m^2}{\sqrt{\pi}} \cdot \sqrt{\pi} + 0 = \boldsymbol{c} \cdot (\boldsymbol{\sigma^2} + \boldsymbol{m^2}) \tag{28}$$

If the known systematic errors are excluded from the measurement results the number and values of positive and negative errors remaining is balanced and the mean is equal to zero. Thus, hereinafter $m = 0$ is assumed.

A.3 Absolute Loss Function

The absolute loss function (curve 2 on Figure 9) is used when the relationship between measurement bias and the quantity value can be expressed in the form of piecewise linear function. The function that represents the absolute loss is given by:

$$\Psi(\Delta) = c \cdot |\Delta| \tag{29}$$

Then the expected loss is calculated by substituting from (24) and (29) to (23):

$$EL = \int_{-\infty}^{+\infty} c \cdot |\Delta| \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} d\Delta \tag{30}$$

Making the substitution of a variable $z = (\Delta - m)/(\sigma \cdot \sqrt{2})$ and considering that $\Delta = z \cdot \sigma \cdot \sqrt{2} + m$ and $d\Delta = \sigma \cdot \sqrt{2} \cdot dz$, then equation (30) can be expressed in terms of z assuming for simplification $m = 0$ as follows:

$$EL = \frac{c}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \left|z \cdot \sigma \cdot \sqrt{2} + m\right| \cdot e^{-z^2} dz = \frac{c \cdot \sigma \cdot \sqrt{2}}{\sqrt{\pi}} \int_{-\infty}^{+\infty} |z| \cdot e^{-z^2} dz \tag{31}$$

The integral in equation (31) is equal to one as the integral of an even function with symmetric limits. Hence, the expected loss is given by:

$$\boldsymbol{EL} = \frac{c \cdot \sigma \cdot \sqrt{2}}{\sqrt{\pi}} \cdot 1 = \boldsymbol{c} \cdot \frac{\sqrt{\mathbf{2}}}{\sqrt{\boldsymbol{\pi}}} \cdot \boldsymbol{\sigma} \tag{32}$$

If the absolute hybrid loss function (curve 3 on Figure 9) is piecewise function defined by two sub-functions as follows:

$$\Psi(\Delta) = \begin{cases} 0, & \Delta < 0 \\ c \cdot \Delta, & \Delta \geq 0 \end{cases} \tag{33}$$

Then the expected loss is calculated by substituting from (24) and (33) to (23):

$$EL = \int_{0}^{+\infty} c \cdot \Delta \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} \, d\Delta \tag{34}$$

Making the substitution of variable $z = (\Delta - m)/(\sigma \cdot \sqrt{2})$ the equation (34) can be expressed in terms of z as follows:

$$\begin{aligned} EL &= \frac{c}{\sqrt{\pi}} \int_{0}^{+\infty} \left(z \cdot \sigma \cdot \sqrt{2} + m\right) \cdot e^{-z^2} \, dz = \\ &= \frac{c \cdot \sigma \cdot \sqrt{2}}{\sqrt{\pi}} \int_{0}^{+\infty} z \cdot e^{-z^2} \, dz + \frac{c \cdot m}{\sqrt{\pi}} \int_{0}^{+\infty} e^{-z^2} \, dz \end{aligned} \tag{35}$$

Hence the integration interval is half of the interval in equation (31), and the expected loss is given by:

$$\boldsymbol{EL} = \frac{c \cdot \sigma \cdot \sqrt{2}}{\sqrt{\pi}} \cdot \frac{1}{2} + \frac{c \cdot m}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = \boldsymbol{c} \cdot \left(\frac{\boldsymbol{\sigma}}{\sqrt{\mathbf{2} \cdot \boldsymbol{\pi}}} + \frac{\boldsymbol{m}}{\mathbf{2}} \right) \tag{36}$$

Obviously, the expected loss evaluated using equation (36) is only half as large as the expected loss evaluated using equation (32). In this case the loss occurs when the measurement bias deviates only in one direction and it means that the systematic measurement bias shall be greater than $0.66 \cdot \sigma$ at the 90 % confidence level [11]. Assuming freedom from the known systematic errors $m = 0$ and the loss function slope $a = 1$, equation (36) takes on the form of the "risked mis-allocation exposure" calculation equation presented in paper [9].

A.4 '0-1' Loss Function

The '0-1' loss function (curve 4 on Figure 9) is used when the relationship between measurement error and the quantity value has a form of discontinuous change if the error is found outside of its permissible limits. The function that represents the '0-1' loss is given by:

$$\Psi(\Delta) = \begin{cases} 0, & -\Delta_p \leq \Delta \leq \Delta_p \\ \Delta Q_p, & |\Delta| > \Delta_p \end{cases} \tag{37}$$

where Δ_p is a maximum permissible error, ΔQ_p is a deviation of the quantity value caused by error Δ_p.

Then the expected loss is calculated by substituting from (24) and (37) to (23) and assuming the even loss function:

$$\begin{aligned} EL &= \int_{-\infty}^{-\Delta_p} \Delta Q_p \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} d\Delta + \int_{\Delta_p}^{+\infty} \Delta Q_p \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} d\Delta \\ &= 2 \int_{\Delta_p}^{+\infty} \Delta Q_p \cdot \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\Delta - m)^2}{2 \cdot \sigma^2}} d\Delta \end{aligned} \tag{38}$$

Making the substitution of variable $z = (\Delta - m)/(\sigma \cdot \sqrt{2})$ the equation (38) can be expressed in terms of z as follows:

$$\begin{aligned} \boldsymbol{EL} &= \Delta Q_p \cdot \frac{2}{\sqrt{\pi}} \int_{\frac{\Delta_p - m}{\sigma \cdot \sqrt{2}}}^{+\infty} e^{-z^2} dz = \Delta Q_p \cdot \left[erf(+\infty) - erf\left(\frac{\Delta_p - m}{\sigma \cdot \sqrt{2}}\right) \right] \\ &= \Delta \boldsymbol{Q_p} \cdot \left[\boldsymbol{1 - erf}\left(\frac{\boldsymbol{\Delta_p - m}}{\boldsymbol{\sigma \cdot \sqrt{2}}}\right) \right] \end{aligned} \tag{39}$$

where erf is the error function and $erf(+\infty) = 1$.

A.5 Conclusion

In terms of relative expanded uncertainty U at a defined confidence level the expected loss can be expressed as shown in Table 1. In practice, the most common is the confidence level of 95 %.

Table 1 – Expected Loss in terms of Uncertainty

Confidence Level	Coverage Factor k (for Normal Distribution)	Expected Loss		
		Squared (25)	Absolute (29)	Absolute hybrid (33)
68 %	1.0	$1 \cdot c \cdot U^2$	$0.8 \cdot c \cdot U$	$0.4 \cdot c \cdot U$
90 %	1.6	$0.39 \cdot c \cdot U^2$	$0.5 \cdot c \cdot U$	$0.25 \cdot c \cdot U$
95 %	**2.0**	$\boldsymbol{0.25 \cdot c \cdot U^2}$	$\boldsymbol{0.4 \cdot c \cdot U}$	$\boldsymbol{0.2 \cdot c \cdot U}$
99 %	2.58	$0.15 \cdot c \cdot U^2$	$0.31 \cdot c \cdot U$	$0.15 \cdot c \cdot U$
99.7 %	3.0	$0.11 \cdot c \cdot U^2$	$0.27 \cdot c \cdot U$	$0.13 \cdot c \cdot U$

It is not shown in the paper but it should be noted that if the normal distribution is considered for the measurement bias without a good reason, then the calculated expected loss may differ from real value by 2 to 3 times [10]. Thus, prior to choosing a type of distribution the behaviour of the random variable (measurement bias) needs to be investigated.

It has to be noted that the confidence level of 90 % has a unique property in that the coverage factor does not depend on the type of distribution and is equal to $k = 1.6$ or $U = 1.6 \cdot \sigma$. If the distribution is not known, it is recommended to report uncertainty at the 90 % confidence level [11].

8 APPENDIX B

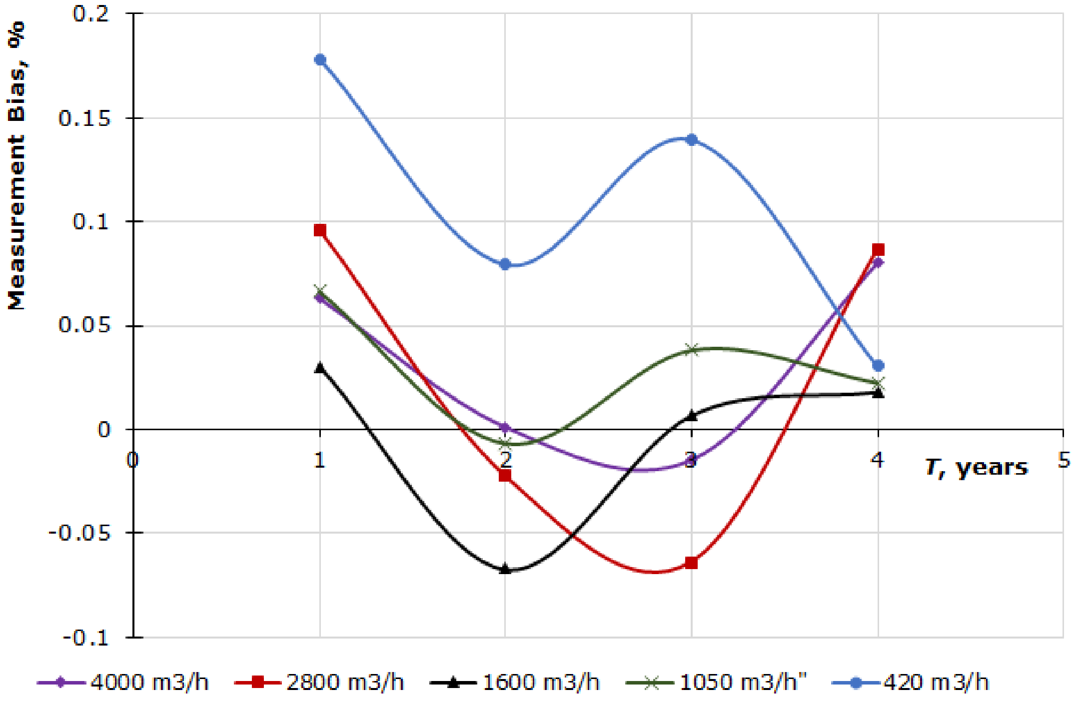

Figure 10 – Ultrasonic Flow Meter Calibration Results

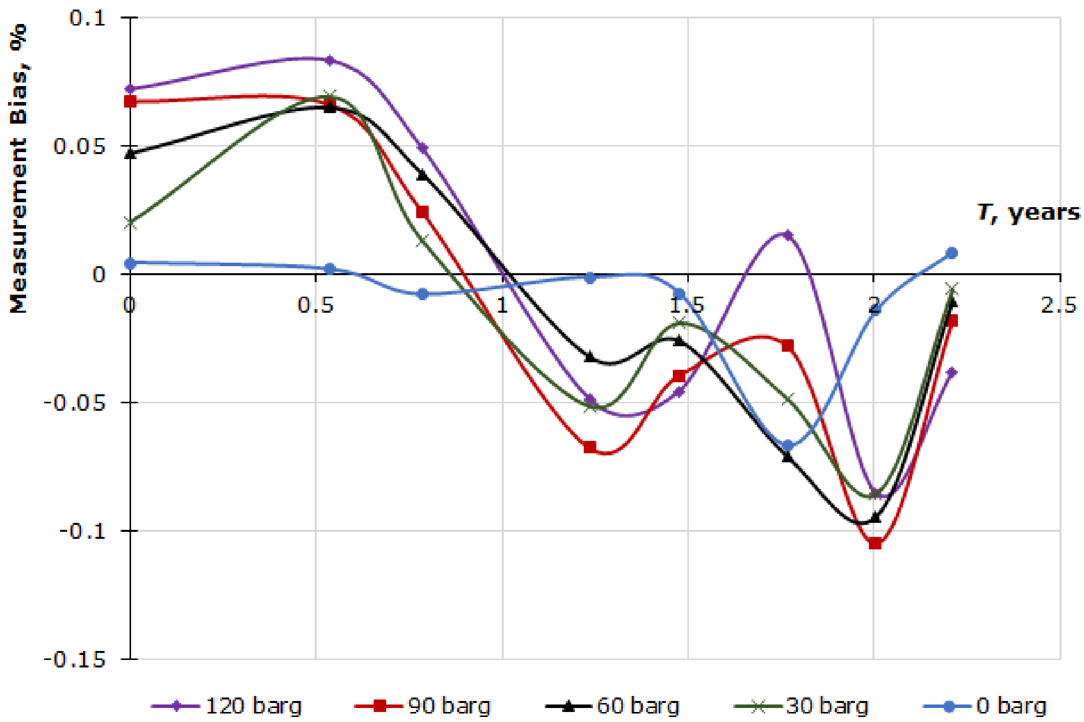

Figure 11 – Pressure Transmitter Calibration Results

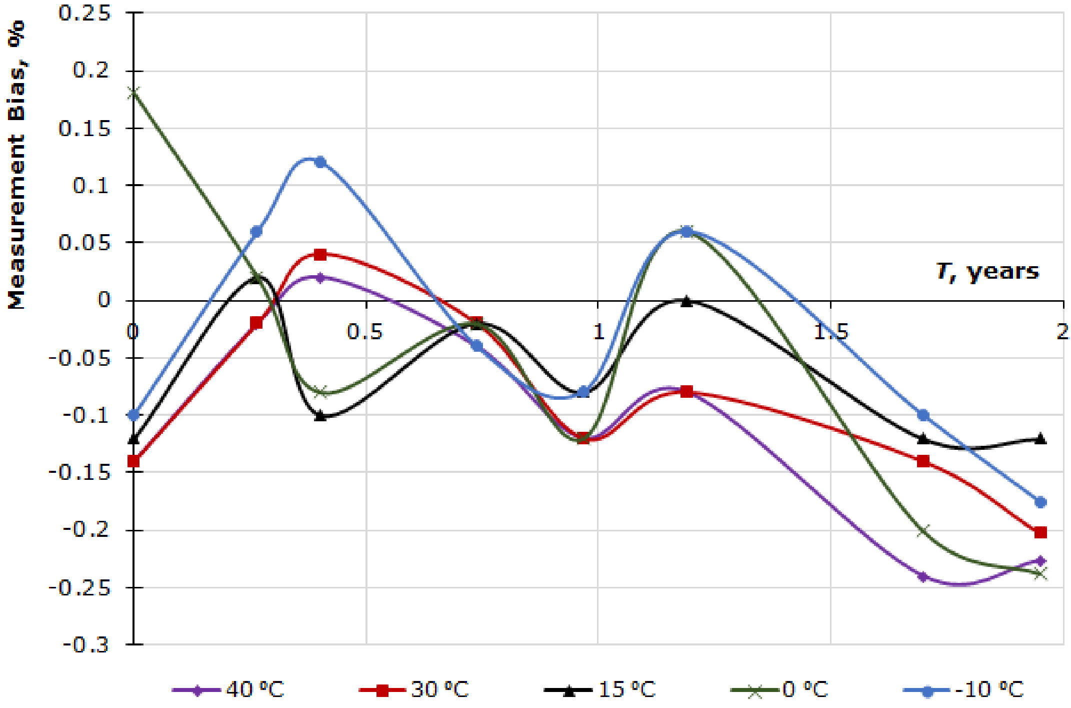

Figure 12 – Temperature Transmitter Calibration Results