35th International North Sea Flow Measurement Workshop
24. - 26. October 2017

Systematic bias in pro rata allocation schemes

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ABSTRACT

Misallocation due to allocation uncertainty may result in increased exposure to economic risk for owners or stakeholders in hydrocarbon fields. ***It is often assumed that allocation errors are random and that they will "even out" over time, irrespective of the system setup and allocation uncertainty. In this paper, we show that this is normally not the case, even for simple allocation systems using standard pro rata allocation.*** *For instance, a two-field pro rata allocation setup with a high measurement uncertainty for one of the meters compared to the other, causes the field with the highest allocation uncertainty to be* ***systematically under-allocated****. We show that this misallocation is inherent to the allocation system, and will occur even without any systematic measurement error present.*

Since pro rata allocation systems are widely used, either as general allocation principle or as part in a multi-tier allocation, this inherent misallocation should be of particular interest to the industry. The financial loss associated with systematic misallocation can only be evaluated based on a correct quantification of the misallocation. Therefore, it is important to be aware of how ***systematic misallocation may be a direct consequence of the setup of a pro rata allocation system and the maintenance scheme of the different metering stations****.*

The objective of our work is to quantify the systematic misallocation in pro rata allocation setups, and identify in which cases this effect is ***economically significant****. Furthermore, the aim is to establish some useful "rules of thumb" that may be used to evaluate if an allocation setup is subject to systematic misallocation.*

We explain the mechanisms behind systematic misallocation, illustrating the effect with a few simple examples. Then we analytically show how the statistical expected value in pro rata allocation differs from the actual production rate. As it may be practically unmanageable to express the systematic misallocation analytically for more complex systems, we show how this can be done using ***numerical methods*** *instead.*

Finally, we demonstrate the calculation of systematic misallocation for a ***realistic measurement setup and allocation scenario*** *in a multi-field setting based on experience from industrial projects.*

Our work shows that the pro rata allocation principle inherently leads to systematic misallocation, particularly in cases where there is a significant difference between the uncertainties of the allocated fields. This misallocation is systematic and does not cancel out over time. ***Therefore, pro rata allocation systems should always be evaluated for any inherent systematic misallocation.***

1 INTRODUCTION

The overall goal of this paper is to show that systematic misallocation may occur in pro rata allocation systems, and that this bias does not cancel out over time. This allocation bias occurs even without any systematic measurement error present.

In order to contribute to a more thorough understanding of allocation bias, we start out in Section 2 discussing some mechanisms behind systematic misallocation. Then, in Section 3, we go on showing how the statistically expectated allocated quantity may be estimated analytically, and we compare this value with the "true" production. As shown in the same section, we find that in many cases a systematic allocation bias will occur. As the analytic calculation may be laborious for more complex systems, we proceed in Section 4 to show how the systematic bias can be calculated numerically as well. Applying the theory and methodology of the previous chapters, we illustrate in Section 5 how this allocation bias may occur in a realistic measurement setup and allocation scenario in a multi-field setting based on industrial projects, and we give a rough estimation of the associated financial risk.

In Section 6, we establish a few "rules of thumb" which may be useful in predicting when misallocation is likely to occur and further study is needed. Finally, we summarize and conclude our work.

The here presented work shows that:

- The pro rata allocation principle **inherently** leads to **systematic misallocation.**
- **Systematic** implies that these misallocations **do not cancel out over time.**
- The systematic misallocation is **most significant** in cases with considerable **differences in the measurement uncertainties and/or production rates** of the different fields.
- For simple allocation setups, the allocation bias may be estimated **analytically**. For more complex setups, **efficient numerical methods** are available.

2 THE MECHANISM BEHIND SYSTEMATIC MISALLOCATION

A common misconception related to allocation uncertainty is that "you lose some, you win some", i.e. that misallocation related to allocation uncertainty will "even out" over time. For instance, [1, p. 45] states that *"The exposure due to random uncertainty approaches zero in the long term"*. This is in contrast to biases, which accumulate over time, and thus should be corrected for and minimized wherever possible [1, p. 45] [2].

In [3, pp. 3-5], Stockton reports results from a Monte Carlo simulation of a pro rata allocation system that suggests that (allocation) biases exist even in absence of systematic measurement errors. The examples below are based on the same paper:

Masking of systematic errors/meter bias: One reason why the uncertainty may not "even out" after a while is that <u>a high uncertainty may</u> <u>mask a systematic error</u>. If the uncertainty of a metering station for instance is 10 %, it may be difficult to detect a systematic error of for instance 2 %. If the value of the product flowing through this metering station is for example 100 000 USD/day, this masked, uncorrected systematic error could result in a daily economic loss of 2 000 USD/day.

The masking of systematic errors/meter bias is mentioned here for completeness, but is not further studied in the remaining of this paper.

Allocation bias: In [3, pp. 3-5], Stockton presents an example of allocation bias, which inspired our investigation in this paper. Here a version of this example with simplified numbers is presented:

Let A and B be two fields producing 100 units each per day. Allocation follows the pro rata principle. Field A's production and the export measurement have a negligible uncertainty and Field B has an uncertainty of 10 %.
The first day 110 units are measured from Field B, and $\frac{110}{100+110} \cdot 200 = 104.8$ units are allocated to Field B. The next day 90 units are measured, and only $\frac{90}{100+90} \cdot 200 = 94.7$ units are allocated to Field B. Even though the sum measured at Field B the two days equals 200, only $104.8 + 94.7 = 199.5$ units are allocated to Field B. This equals to an under-allocation of 0.25 %. Note that this under-allocation occurs despite that the underlying measurement errors cancel out. Hence, the deviation between true and allocated production may constitute a systematic bias.

The above example uses a simplified distribution where the measured production for Field B is either 10 % above or 10 % below the "true" produced value. In [3, pp. 3-5], Stockton also calculates the systematic allocation bias for a more realistic case where the measurement uncertainty of Field B's production has a normal (Gaussian) distribution. In this case the bias is smaller than for the simplified example (0.0625 % instead of 0.25 %), but still present. If these deviations indeed represent a systematic bias, they will accumulate over time and hence become non-negligible, despite their limited size.

In this paper, we will show that there exists a systematic bias in pro rata allocation. We will establish analytical and numerical methods for estimating this bias and develop some "rules of thumb" in order to quickly identify allocation systems where systematic allocation bias may be an issue.

3 ANALYTIC CALCULATION OF STATISTICAL EXPECTED BIAS

Due to the high values of the product in oil and gas systems, we find the systematic allocation bias in pro rata allocation systems discussed in the previous chapter remarkable and rather startling effect which needs further investigation. In order to explain this effect from a mathematical perspective, we have derived analytical expressions of the allocation bias, starting from the basic statistical definitions of expected value. In [4] we show these calculations in detail. In this chapter we present the basic definitions used in the calculations and the resulting analytic expressions of the bias.

3.1 Definitions

According to the law of large numbers, the arithmetic mean of a number of draws from a distribution with mean μ will approach the expected value as the number of draws approaches infitity:

$$E(X) = \mu = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i \tag{1}$$

In practice this means that if a variable X_i is measured an infinite number of times, the statistical expected value $E(X)$ of that variable equals its mean value μ, which is found by adding all the measured values and dividing by the number of measurements.

An allocation system may be described by an allocation function f which defines how the share of the total production is allocated to a specific source. In a pro rata allocation system consisting of two fields A and B, the allocated quantity to field B may be expressed by the allocation function $f_B(A,B) = \frac{B}{A+B} \cdot export$. Here A denotes the measured production from field A, B the measured production from field B and $export$ denotes the measured commingled production.

If the allocation is carried out n times, then the law of large numbers states that the expected value of the allocated value to field B can be written as:

$$E(f_B) = \lim_{n\to\infty} \frac{1}{n}\sum_{l=1}^{n} {}^{l}f_B \tag{2}$$

Here ${}^{l}f_B$ is allocated value to field B the l^{th} time the allocation is carried out.

Since each ${}^{l}f_B$ is calculatede based on measurements subject to random measurement errors, the value allocate to field B will fluctuate to some extend and hence deviate from the true production of field B (which we denote by f_B^*), which is assumed to be constant in this context. If these misallocations should even out, the average allocated value has to approach f_B^* as the number of allocations carried out increases. In other words, the expected value of the quantity allocated to field B has to be equal to the true production. This is the same as to say that the allocation function is an unbiased estimator for the true produced value of field B.

In order to assess the biasedness of a pro rata allocation system, we need to find an expression for the expected allocated value $E(f_B)$. For non-linear function such as the pro rata allocation function, no closed form expression can be derived, but good approaximations can be found using Taylor expansions, for further details please confer [4].

3.2 Estimation bias in allocation systems

Using the definitions introduced in the previous section, together with the mentioned Taylor expansions, the following expression is found for the allocation bias[1] for a system with n sources/fields and a generic allocation function f_B:

$$E(f_B) - f_B^* \approx \sum_{i=1}^{n} \left(\frac{1}{2} \frac{\partial^2 f_B}{\partial x_i^2} \big(E(X)\big) Var(X_i) + \sum_{j \neq i} \frac{\partial^2 f_B}{\partial x_i \partial x_j} \big(E(X)\big) Cov\big(X_i, X_j\big) \right) \quad (3)$$

where $E(f_B)$ is the expected value of the allocated value to a given field B and f_B^* is the true value. $Var(X_i)$ and $Cov(X_i, X_j)$ represent the variation in measurement X_i and the covariance between the measurements X_i and X_j, respectively.

In the case with only two fields, A and B, the expression of the bias simplifies to:

$$\begin{aligned} E(f_B) - f_B^* \approx {} & \frac{1}{2} \frac{\partial^2 f_B}{\partial A^2} \big(E(A)\big) Var(A) + \frac{1}{2} \frac{\partial^2 f_B}{\partial B^2} \big(E(B)\big) Var(B) \\ & + \frac{\partial^2 f_B}{\partial A \partial B} \big(E(B)\big) Cov(A, B) \end{aligned} \quad (4)$$

3.2.1 By-difference and uncertainty-based allocation systems

We show in [4] that by-difference and uncertainty-based allocation systems are *linear* and thus the allocation bias is shown to be zero for both these systems. Pro rata allocation systems, on the other hand, are *non-linear* and will only result in zero allocation bias in certain cases, further discussed in the next section.

3.2.2 Pro rata allocation systems

If the fields are allocated pro rata, and measured with uncorrelated measurement systems, the estimate for the allocation bias bias given in equation (1) simplifies to:

$$E(f_B) - f_B^* \approx \frac{1}{(\sum_j \mu_j)^2} \left(\mu_i \sum_{j \neq i} \sigma_j^2 - \sum_{j \neq i} \mu_j \, \sigma_i^2 \right) \quad (5)$$

Here μ_i is the expected value of the measured production from source i, and σ_i the standard deviation of the measurement distribution[2].

For an example case with three fields, A, B and C, with uncorrelated measurements and pro rata allocation, the expression of the bias simplifies further to:

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[1] *Estimation up to second order.*

[2] *In the case of a normal distribution, this equals relative expanded uncertainty with 95 % confidence level divided by 1.96 and multiplied by the measured value.*

$$E(f_B) - f_B^* \approx \frac{1}{(\mu_A + \mu_B + \mu_C)^2}[\mu_B(\sigma_A^2 + \sigma_C^2) - (\mu_A + \mu_C)\sigma_B^2] \tag{6}$$

Figure 1 and Figure 2 illustrate the analytically calculated allocation bias as a function of field B production and measurement uncertainty, respectively.

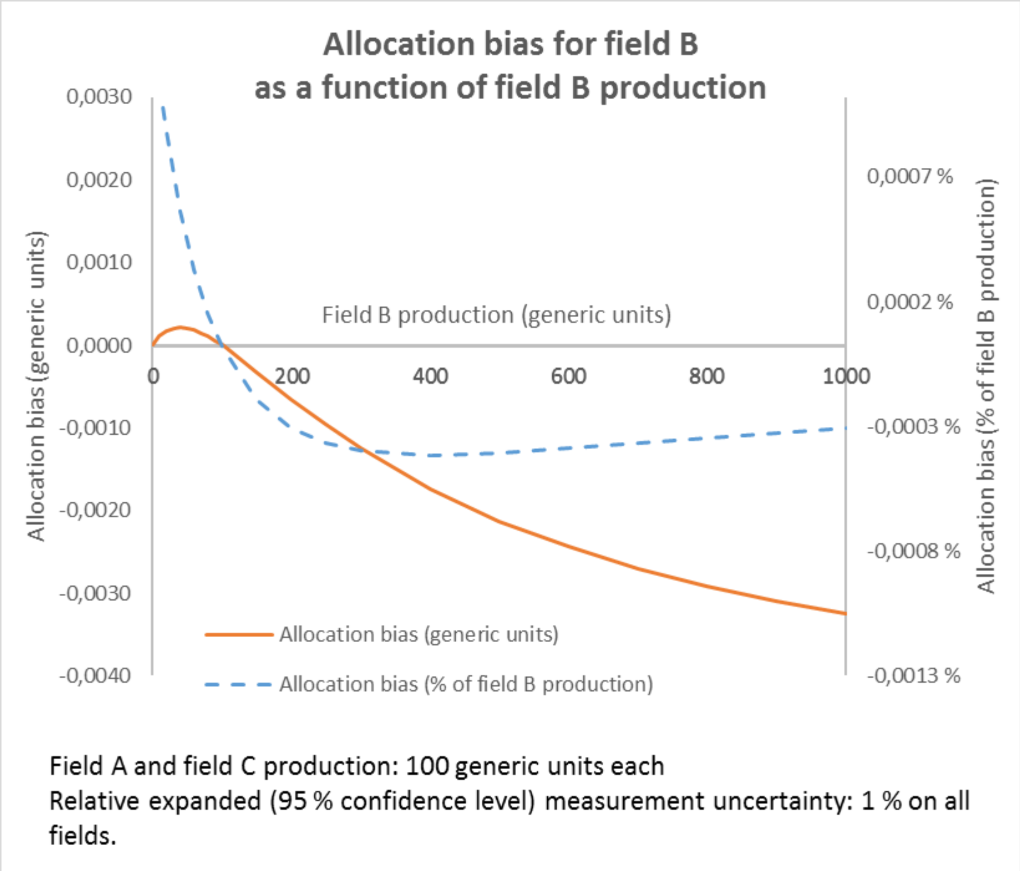

Figure 1: Illustration of analytically calculated allocation bias for field B as a function of field B production. Example for a pro rata allocation system for fields A, B and C, all with uncorrelated measurement uncertainties.

Figure 1 and **Error! Reference source not found.** shows that in the case where all fields have similar measurement uncertainties, Field B will be subject to a systematic **over-allocation** if it has less production than each of fields A and C. On the other hand, if Field B has a higher production than each of fields A and C, then field B production will be systematically **under-allocated**.

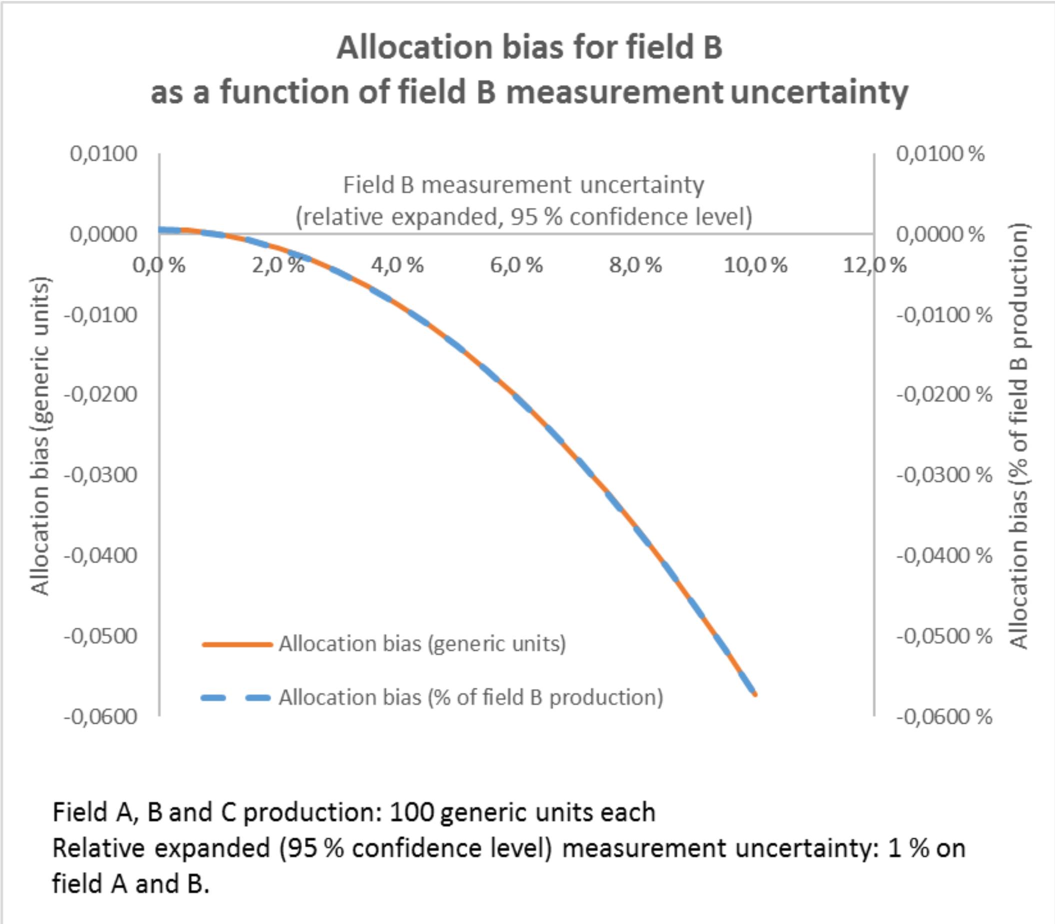

Figure 2: Illustration of analytically calculated allocation bias for field B as a function of field B measurement uncertainty. Example for a pro rata allocation system for fields A, B and C, all with uncorrelated measurement uncertainties.

Figure 2 and **Error! Reference source not found.** show that in the case where all fields have similar production rates, Field B will be subject to a systematic **over-allocation** if it has a lower relative measurement uncertainty than each of fields A and C. On the other hand, if Field B has a higher measurement uncertainty than each of fields A and C, then field B production will be systematically **underallocated**.

Allocation bias (generic units)		Field B measurement uncertainty (relative expanded, 95 % confidence level)										
		0,0 %	**1,0 %**	**2,0 %**	**3,0 %**	**4,0 %**	**5,0 %**	**6,0 %**	**7,0 %**	**8,0 %**	**9,0 %**	**10,0 %**
Field B production (generic units)	**0**	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
	100	0,0269	0,0267	0,0260	0,025	0,023	0,022	0,019	0,016	0,013	0,009	0,005
	200	0,0452	0,0445	0,0423	0,039	0,034	0,027	0,019	0,010	-0,001	-0,013	-0,027
	300	0,0578	0,0564	0,0522	0,045	0,036	0,023	0,008	-0,010	-0,031	-0,055	-0,081
	400	0,0664	0,0643	0,0579	0,047	0,032	0,013	-0,010	-0,038	-0,070	-0,106	-0,146
	500	0,0723	0,0694	0,0607	0,046	0,026	0,000	-0,032	-0,069	-0,113	-0,162	-0,217
	600	0,0763	0,0726	0,0616	0,043	0,018	-0,015	-0,056	-0,103	-0,158	-0,220	-0,290
	700	0,0788	0,0744	0,0612	0,039	0,008	-0,032	-0,080	-0,137	-0,204	-0,279	-0,363
	800	0,0803	0,0752	0,0598	0,034	-0,002	-0,048	-0,105	-0,172	-0,249	-0,336	-0,434
	900	0,0811	0,0753	0,0578	0,029	-0,012	-0,065	-0,129	-0,205	-0,293	-0,392	-0,503
	1000	0,0813	0,0748	0,0553	0,023	-0,023	-0,081	-0,153	-0,238	-0,335	-0,446	-0,569

Table 1: Illustration of analytically calculated allocation bias for field B as a function of field B measurement uncertainty and production rate. Example for a pro rata allocation system for fields A, B and C, all with uncorrelated measurement uncertainties. Field A and C have a production of 500 generic units, with relative expanded measurement uncertainties of 5 %. The green and read colour represent over- and under-allocation, respectivly. Darker colours indicate larger (absolute) values.

3.3 Conditions for unbiased pro rata allocation

If the measured production from the different fields/sources in an allocation system are uncorrelated, we see, by setting eq. (4) to zero, that the allocation bias will be zero if the following relation is true:

$$\frac{\mu_i}{\sum_{j \neq i} \mu_j} = \frac{\sigma_i^2}{\sum_{j \neq i} \sigma_j^2} \tag{7}$$

Here μ_i is the mean or the expected value of the measured quantity from field i, and σ_i is the standard deviation[3] of the measurement.

In the case with three fields, with uncorrelated measurements and allocated pro rata, the this condition simplifies to:

$$\frac{\mu_B}{\mu_A + \mu_C} = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_C^2} \tag{8}$$

In practice this means that if all three sources have similar production and are measured with the same measurement uncertainty, the allocation bias will approach zero. If fields A and C produce 100 units each, with uncertainty of 1 %, and field B produces 200 units, the measurement uncertainty of field B production would need to be $\approx 0{,}71\%$ in order to avoid systematic allocation bias. If field B produces only 50 units, a measurement uncertainty of field B production of $\approx 1{,}4\ \%$ would result in zero expected systematic allocation bias.

[3] *For a normally distributed measurement with a relative expanded uncertainty of 10 % (with 95 % confidence level and thus a coverage factor of 1.96) and a mean of for example 100 units, the corresponding standard deviation is* $\frac{100 \cdot 10\%}{1.96} \approx 5$ *units.*

4 NUMERICAL CALCULATION OF MISALLOCATION

4.1 Numerical model of allocation systems

In chapter 3, the statistically expected allocation bias was calculated analytically for a simple pro rata allocation set-up with three fields with uncorrelated measurement uncertainties. For more complex allocation systems, the expression of the allocation bias in equation 3 soon becomes difficult to write out analytically.

In order to estimate any systematic allocation bias in more complex, real-life scenarios, it is therefore useful to use a numerical method. The major advantages of using a numerical Monte Carlo (MC) approach, as compared to the analytical calculations, are the following:

- More complex allocation systems can be analysed using the MC approach, while the analytical approach is laborious and cumbersome for complex systems.
- Any correlations between input parameters are easily taken into account by generating correlated input distributions.

In our NSFMW paper from 2016 [5] we introduced an ISO GUM [6] compliant Monte Carlo-based numerical method which can be used to calculate allocation uncertainties for arbitrarily complex systems. The method was further detailed in [7]. Roughly speaking, we model each input parameter by a probability distribution instead of only one value. The standard deviations of these distributions are set based on the uncertainty of each of the parameters. The distributions are then combined and commingled according to the allocation calculations, and the standard deviations of the resulting distributions indicate the uncertainty of the allocated quantities. Figure 3 illustrates this method.

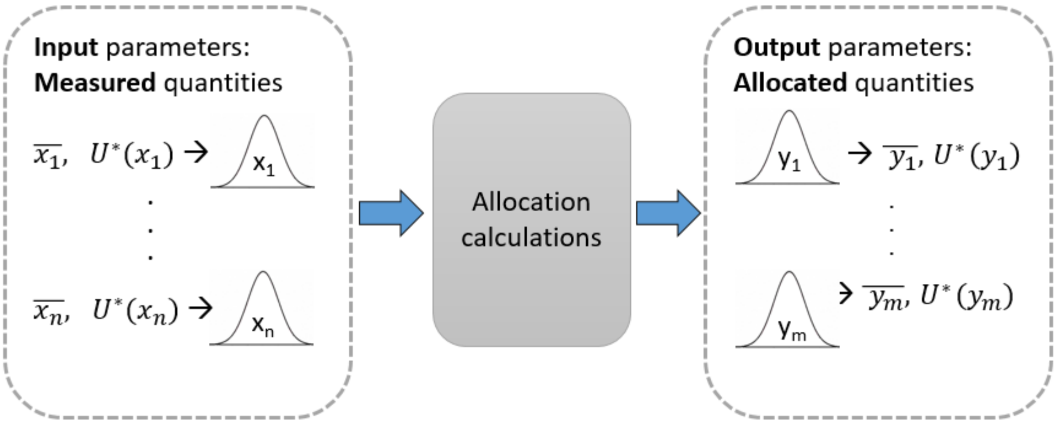

Figure 3: Illustration of a Monte Carlo based method for calculating allocation uncertainties **[5]**. *Whereas the input distributions are depicted as normal distributions in the figure, this is not a requirement for using the Monte Carlo method. In the figure* $\boldsymbol{U}^*(\boldsymbol{x_i})$ *symbolizes the relative expanded uncertainty of a quantity* $\boldsymbol{x_i}$, *and* $\bar{\boldsymbol{x}}_{\boldsymbol{i}}$ *its average value. Similar symbolism applies to* $\boldsymbol{y_i}$.

4.2 Numerical calculation of allocation bias

Once the allocation system has been modelled as described in section 4.1, the allocation bias for each field may be calculated directly from the model allocated values. Using the notations introduced in Figure 3, where $\overline{y_i}$ denote the average allocated value to field *i* (model output), and $\overline{x_i}$ the measured value for field *i* (model input), the bias may be expressed as:

$$Allocation\ bias\ field\ i = \overline{y_i} - \overline{x_i} \tag{9}$$

Note that the number of samples in the Monte Carlo distributions must be sufficiently large to provide stable results. In practice, this limit may be determined by repeating the simulation with an increasing number of samples, until the simulation outputs do not change from one simulation to another. This approach is, in essence, a direct application of the law of large numbers and often referrred to as naïve Monte Carlo estimator [8]. While theortically sound, this method typically requires a large number of simulation runs and one additional decimal place in presicion requires increacing thenumber of simulations runs by a factor of 100 [8].

In [4], we devise a general-purpose method for allocation bias estimation based on multiple linear regression. The advantages of this method is that it converges towards the analytical calculated bias values faster than the Monte Carlo simulation referred to above, so that fewer simulation runs are neccessary.

5 SYSTEMATIC MISALLOCATION FOR A REALISTIC MEASUREMENT SETUP

Figure 4 shows an example allocation system consisting of three fields with a common production process. The production from field A is processed through a 1st stage separator with metering equipment meeting fiscal standards. Field B's production is measured by a subsea multiphase flow meter, with no possibility for testing or verification towards topside metering system. Field C is measured by a topside multiphase flow meter, with the possibility of periodic calibration against a test separator.

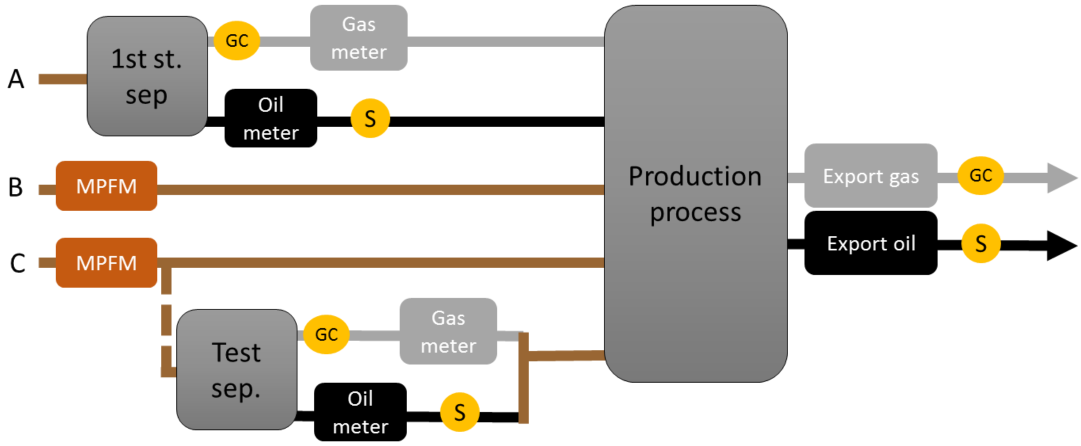

Figure 4: Example allocation system consisting of three fields with a common production process. The yellow circles represents gas chromatograph (GC) and sampling (S). MPFM denotes Multiphase Flow Meter.

The following relative expanded metering uncertainties (95 % confidence level) are used in the example:

- Field A : Oil volume 0.30 %, oil standard density 0.07 %, gas mass 1 %
- Field B : Hydrocarbon mass 10 %
- Field C : Hydrocarbon mass 1 %
- Export : Oil volume 0.30 %, oil standard density 0.07 %, gas mass 1 %

The following production rates are used in the example:

- Field A : 1000 tons/day
- Field B : 1000 tons/day
- Field C : 1000 tons/day

The multiphase meters at field B and C are from the same manufacturer, with the same built-in PVT-models. It is therefore assumed that there is some correlation between the uncertainties in the multiphase meter measurements.

If the correlation coefficient of the multiphase meter uncertainties between field B and C is set to 0.5, and the simulation is carried out with $2 \cdot 10^6$ simulation runs, the simulation shows that field B will be systematically underallocated with approximately 600 kg/day. This is a small number compared with the daily production from field B (≈0.06 % of 1000 tons/day), but **the effect is systematic and cumulative** and should not be ignored.

Over a year, the statistically expected under-allocation to field B would be approximately 200 tons of hydrocarbon mass. If field B produces mainly oil, at an oil price of 50 USD/bbl, this would result in a yearly loss of approximatively 85 000 USD.

Note that this yearly loss is not a potential loss due to systematic errors that are not revealed, but a direct result of the pro rata allocation setup in combination with difference in measurement uncertainty between the different fields.

It is possible that the uncertainty in the field B measurement system may also cover systematic errors as illustrated in Figure 5. The allocation uncertainty to field B is calculated to be 6.5 % (relative expanded, 95 % confidence level).

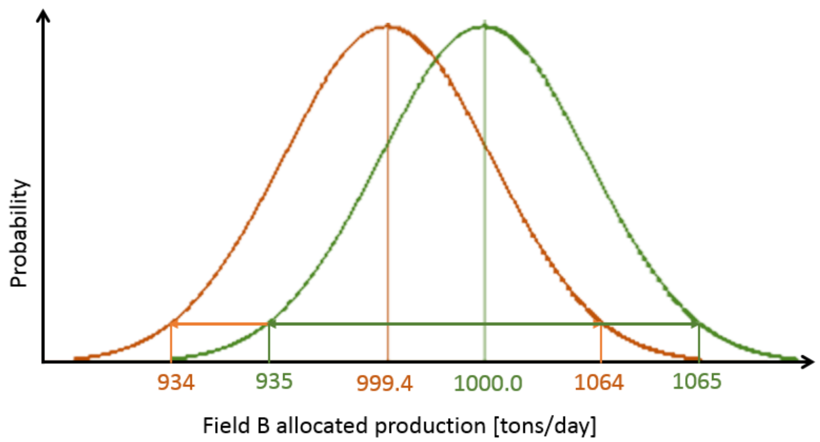

Figure 5: Illustration of how the systematic allocation bias affects the field B allocation distribution (not to scale). The green line shows how the allocation distribution would have been if there were no systematic bias. The orange line shows the allocation distribution taking into account the systematic bias. The 95 % confidence levels for each distribution are marked together with the lower and upper limits of the intervals.

6 RULES OF THUMB - PARAMETERS THAT INDICATE POSSIBLE MISALLOCATION

Below are some “rules of thumb” which may help determining whether or not a field is likely to be subject to an under- or over-allocation compared to the other fields in the allocation system.

- By-difference and uncertainty based allocation systems are linear and therefore not subject to systematic allocation bias.

For pro rata allocation systems, the following rules of thumb, summarized in Figure 6, hold:

- If one of the fields has a **lower production** than the other fields, this field would be subject to an expected systematic over-allocation, unless its production is measured with a correspondingly **higher measurement uncertainty** than the other fields.
- If one of the fields have a **lower measurement uncertainty** than the other fields, this would result in a systematic **over-allocation** to the field, unless the field has a correspondingly higher production than the other fields.
- If one of the fields have a **higher production** than the other fields, this field would be subject to an expected systematic **under-allocation**, unless its production is measured with a correspondingly lower measurement uncertainty than the other fields.

- If one of the fields have a **higher measurement uncertainty** than the other fields, this would result in a systematic **under-allocation** to the field, unless the field has a correspondingly lower production than the other fields.

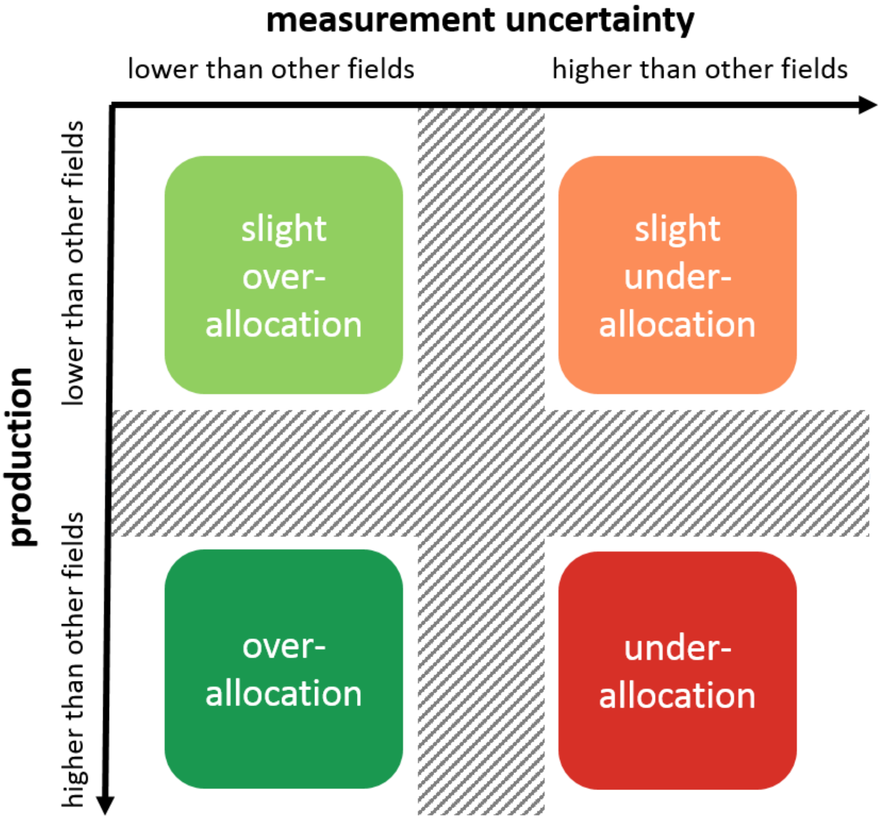

Figure 6: Rules of thumb for determining if field B is expected to be under- or over-allocated, as a function of measurement uncertainty and production rate compared to other fields in the allocation system.

7 SUMMARY AND CONCLUSIONS

In this paper we have discussed the mechanisms behind systematic misallocation. We have presented an analytical expression which may be used to calculate the statistical expected allocation bias. From this expression we have found that pro rata allocation systems are subject to systematic bias, unless certain conditions are met.

Furtheron, we have calculated the allocation bias for some example cases as a function of field production and measurement uncertainty.

We show how the allocation bias may be calculated numerically for more complex, real-life systems.

Finally, we establish some rules of thumb which may be useful when evaluating wheter an allocation system is subject to systematic allocation bias.

Our work shows that:

- The pro rata allocation principle **inherently** leads to **systematic misallocation.**
- **Systematic** implies that these misallocations **do not cancel out over time.**
- The systematic misallocation is **most significant** in cases with considerable **differences in the measurement uncertainties and/or production rates** of the different fields.
- For simple allocation setups, the allocation bias may be estimated **analytically**. For more complex setups, **efficient numerical methods** are available.

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