When By-Difference Allocation is the Best Choice

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1 INTRODUCTION

It is often assumed that pro rata allocation is always superior to by-difference allocation in terms of uncertainty. In this paper, it is shown that for some combinations of production and gas lift rates, by-difference allocation may result in uncertainties significantly lower than pro rata allocation.

The objective of this work is to show how the total economic risk obtained by different allocation methods heavily depend on the production profiles. It is focused upon the importance of gas lift and gas production rates. The aim is to establish some useful "rules of thumb" to identify cases where the uncertainty of an input parameter/measurement is amplified (through the sensitivity coefficients) to such an extent that the calculations would have been more accurate if the measurement had been omitted. Such measurements will increase the allocation uncertainty and thus the financial risk of loss related to misallocation, even when the cost of collecting the information is not included.

Analytic equations, which may be used to estimate the allocation uncertainty and thus the economic risk for different allocation principles and production profiles, are presented. Based upon the analytical equations, it is demonstrated how different allocation principles (pro rata vs by-difference) may be optimal for different sets of realistic production profiles.

It is shown that for certain gas lift/gas production ratios and allocation setups, a by-difference allocation principle may lead to lower allocation uncertainties than a more costly pro rata alternative. Thus, expected production and gas lift profiles should always be taken into account when evaluating allocation systems in a risk-cost-benefit perspective.

2 UNCERTAINTY MODELS - ALLOCATION SYSTEM WITH TWO FIELDS

We will study the case where the production from two fields A and B, both with gas lift, are commingled before processed and exported from a shared production facility (e.g. an offshore platform) as illustrated in Figure 1. In section 2.1 we show how the produced gas from each field can be calculated based on measured gas into the shared production facility, subtracted measured gas lift for each field. In section 2.2 and section 2.3 we give the allocation equations and the associated uncertainty models for a by-difference and pro rata allocation system, respectively.

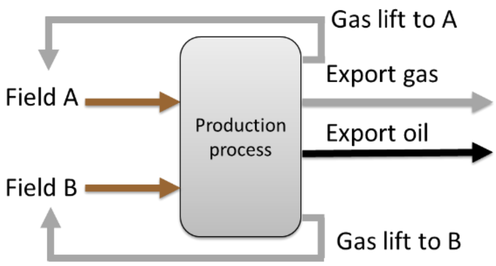

Fig 1: **Schematic illustration of simple allocation system**. Measurement points are not indicated on the illustration. In a pro-rata case, both field A and field B gas production is measured either from MPFMs or inlet separators. In a by difference case, only field A gas production is measured.

2.1 Calculation of produced gas

The *produced* gas from each field ($A_{gas,prod}$, $B_{gas,prod}$) can be calculated as the total measured gas from each field into the production process ($A_{gas,tot}$, $B_{gas,tot}$), subtracted the measured gas lift to each field ($A_{gaslift}$, $B_{gaslift}$):

$$A_{gas,prod} = A_{gas,tot} - A_{gaslift}$$
$$B_{gas,prod} = B_{gas,tot} - B_{gaslift} \quad (1)$$

The exported gas from the shared production facility is the sum of the produced gas from each field (fuel and flare gas not taken into account here):

$$E_{gas} = A_{gas,prod} + B_{gas,prod} \quad (2)$$

Note that the equations are generic, and the quantities can be measured either in mass, volume at standard conditions or calorific value.

Following ISO GUM [1], assuming that the different measurements can be expressed as Gaussian distributions around a true value, with systematic errors corrected for, the uncertainty model for the produced gas from each field can be written as:

$$\left(\frac{U(A_{gas,prod})}{A_{gas,prod}}\right)^2 = \left(\frac{A_{gas,tot}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2$$
$$\left(\frac{U(B_{gas,prod})}{B_{gas,prod}}\right)^2 = \left(\frac{B_{gas,tot}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gas,tot})}{B_{gas,tot}}\right)^2 + \left(\frac{B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gaslift})}{B_{gaslift}}\right)^2 \quad (3)$$

Here $\frac{U(x)}{x}$ is the relative uncertainty of the variable x. Note that that $A_{gas,prod}$ and $B_{gas,prod}$ are not measured quantities, but are calculated as shown in Equation (1). As the total gas from each field is often measured after a 1st stage separator, or using a multiphase flow meter combined with oil recovery factors, shrinkage factors or similar from PVT-simulations, we assume that there are no correlations between the gas measurements for each field and gas lift measured towards the end of the production process where the separate phases have stabilized.

The factors coloured blue in equation (3) represent the relative sensitivity coefficients for the different uncertainty contributors. It can be seen from equation (3) and (1) that in the cases where the gas lift to one field is high compared with the total measured gas from the same field, this results in small denominators for the relevant sensitivity coefficients (in the case of field A: $A_{gas,tot} - A_{gaslift}$). This will in turn lead to high sensitivity coefficients, both for the uncertainty contribution from measured total production (first term in uncertainty model) and measured gas lift (second term in the uncertainty model).

In other words, for fields with much gas lift compared to produced gas, the uncertainty contributions from measured gas into production process and measured gas lift will be amplified by high sensitivity coefficients, causing a high relative uncertainty of the produced gas.

In the following subchapters we will study how this amplification will affect the allocation uncertainty in a by-difference and a pro rata allocation system.

2.2 By-difference allocation (only field A is measured)

In the case where only field A's production into the shared process plant is measured, whereas field B is allocated by-difference, the allocation equations can be written as:

$$A_{gas,allok,bd} = A_{gas,prod}$$
$$B_{gas,allok,bd} = E_{gas} - A_{gas,prod} \quad (4)$$

The gas allocated to field A, $A_{gas,allok,bd}$ is thus equal to $A_{gas,prod}$ which is calculated as described in equation (1) from measured A gas into the shared process plant, subtracted the measured gas lift to field A.

The gas allocated to field B, $B_{gas,allok,bd}$, is the difference between the measured total gas out of the shared production process E_{gas} and $A_{gas,prod}$.

The uncertainty model for the gas allocated to field A is equal to the uncertainty model for the calculated produced gas from field A, as given in equation (3):

$$\left(\frac{U(A_{gas,allok,bd})}{A_{gas,allok,bd}}\right)^2 = \left(\frac{U(A_{gas,prod})}{A_{gas,prod}}\right)^2 = \left(\frac{A_{gas,prod} + A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gasprod}}\right)^2 \left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2 \quad (5)$$

The relative sensitivity coefficients are summarized in Table 1, along with a comment on the aspect ratio.

Table 1 – Relative sensitivity coefficients allocated gas to field A, by-difference.

Uncertainty source	Relative sensitivity coefficient	Comment
Total gas from field A into shared production facility	$\frac{A_{gas,prod} + A_{gaslift}}{A_{gas,prod}}$	$A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →1
Gas lift to field A	$\frac{A_{gaslift}}{A_{gas,prod}}$	$A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →0

For field B, on the other hand, the uncertainty model for the by-difference allocated gas can be written as:

$$\left(\frac{U(B_{gas,allok,bd})}{B_{gas,allok,bd}}\right)^2 = \left(\frac{E_{gas}}{B_{gas,prod}}\right)^2 \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{A_{gas,prod}}{B_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,prod})}{A_{gas,prod}}\right)^2 \quad (6)$$

The sensitivity coefficient of the uncertainty contribution from calculated produced gas from field A, $\frac{A_{gas,prod}}{B_{gas,prod}}$, will be high in cases where the production of field A is high compared to the production of field B, and low in cases where field B production is relatively high.

The uncertainty model for field B can be developed further by inserting the uncertainty model for $A_{gas,prod}$ from equation (5) into equation (6):

$$\left(\frac{U(B_{gas,allok,bd})}{B_{gas,allok,bd}}\right)^2 = \left(\frac{E_{gas}}{B_{gas,prod}}\right)^2 \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{A_{gas,prod}}{B_{gas,prod}}\right)^2 \left[\left(\frac{A_{gas,prod} + A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2\right] \quad (7)$$

As discussed in section 2.1, the sensitivity coefficients $\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}$ and $\frac{A_{gaslift}}{A_{gas,prod}}$ will be high when there is relatively much gas lift compared with produced gas from field A.

To sum up for by-difference allocated gas to field B, the combined sensitivity coefficient $\frac{A_{gas,prod}}{B_{gas,prod}} \cdot \frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}} = \frac{A_{gas,prod}+A_{gaslift}}{B_{gas,prod}}$ will be high in cases where the production of field A is high compared to the production of field B, and $\frac{A_{gas,prod}}{B_{gas,prod}} \cdot \frac{A_{gaslift}}{A_{gas,prod}} = \frac{A_{gaslift}}{B_{gas,prod}}$ will be high in cases when field A has relatively much gas lift compared with produced gas from field B. This is a very clear-defined, specific situation, and not the case for many of the fields which are allocated by-difference. However, the situation is not unrealistic and may very well be encountered in cases where one field has a relatively high production compared to another, but still relies on an important amount of gas lift.

Table 2 – Relative sensitivity coefficients allocated gas to field B, by-difference.

Uncertainty source	Relative sensitivity coefficient	Comment
Gas exported from production process	$\frac{E_{gas}}{E_{gas} - A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →high $A_{gas,prod} << B_{gas,prod}$ →1
Total gas from field A into shared production facility	$\frac{A_{gas,prod}}{B_{gas,prod}} \cdot \frac{A_{gas,prod} + A_{gaslift}}{A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →high $A_{gas,prod} << B_{gas,prod}$ →0 $A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →1
Gas lift to field A	$\frac{A_{gas,prod}}{B_{gas,prod}} \cdot \frac{A_{gaslift}}{A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →high $A_{gas,prod} << B_{gas,prod}$ →0 $A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →0

2.3 Pro rata allocation

In the case where both the production of field A and B into the shared process plant are measured, the allocation equations can be written as:

$$A_{gas,allok,pr} = \frac{A_{gas,prod}}{A_{gas,prod} + B_{gas,prod}} E_{gas}$$
$$B_{gas,allok,pr} = \frac{B_{gas,prod}}{A_{gas,prod} + B_{gas,prod}} E_{gas} \tag{8}$$

Note that although $\frac{E_{gas}}{A_{gas,prod}+B_{gas,prod}} = 1$ according to equation (2), the term is included in equation (8) because of the uncertainties related to each quantity.

The corresponding uncertainty models can be written as:

$$\left(\frac{U(A_{gas,allok,pr})}{A_{gas,allok,pr}}\right)^2 = \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{B_{gas,prod}}{A_{gas,prod} + B_{gas,prod}}\right)^2 \left[\left(\frac{U(A_{gas,prod})}{A_{gas,prod}}\right)^2 + \left(\frac{U(B_{gas,prod})}{B_{gas,prod}}\right)^2\right] \tag{9}$$

$$\left(\frac{U(B_{gas,allok,pr})}{B_{gas,allok,pr}}\right)^2 = \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{A_{gas,prod}}{A_{gas,prod} + B_{gas,prod}}\right)^2 \left[\left(\frac{U(A_{gas,prod})}{A_{gas,prod}}\right)^2 + \left(\frac{U(B_{gas,prod})}{B_{gas,prod}}\right)^2\right]$$

The uncertainty models can be developed further by inserting the uncertainty model for $A_{gas,prod}$ and $B_{gas,prod}$ from equation (5) into equation (9):

$$\left(\frac{U(A_{gas,allok,pr})}{A_{gas,allok,pr}}\right)^2 = \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{B_{gas,prod}}{A_{gas,prod} + B_{gas,prod}}\right)^2 \left[\left(\frac{A_{gas,prod} + A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2 + \left(\frac{B_{gas,prod} + B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gas,tot})}{B_{gas,tot}}\right)^2 + \left(\frac{B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gaslift})}{B_{gaslift}}\right)^2\right] \tag{10}$$

$$\left(\frac{U(B_{gas,allok,pr})}{B_{gas,allok,pr}}\right)^2 = \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}}\right)^2 \left[\left(\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2 + \left(\frac{B_{gas,prod}+B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gas,tot})}{B_{gas,tot}}\right)^2 + \left(\frac{B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gaslift})}{B_{gaslift}}\right)^2\right]$$

The sensitivity coefficients for the different uncertainty contributions for field A are detailed in table 3 for field A and in table 4 for field B.

Table 3 – Relative sensitivity coefficients allocated gas to field A, pro-rata

Uncertainty source	Relative sensitivity coefficient	Comment
Gas exported from production process	1	
Total gas from field A into shared production facility	$\frac{B_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →0 $A_{gas,prod} << B_{gas,prod}$ →1 $A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →1
Gas lift to field A	$\frac{B_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{A_{gaslift}}{A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →0 $A_{gas,prod} << B_{gas,prod}$ →1 $A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →0
Total gas from field B into shared production facility	$\frac{B_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{B_{gas,prod}+B_{gaslift}}{B_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →0 $A_{gas,prod} << B_{gas,prod}$ →1 $B_{gaslift} >> B_{gas,prod}$ →high $B_{gas,lift} << B_{gas,prod}$ →1
Gas lift to field B	$\frac{B_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{B_{gaslift}}{B_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →0 $A_{gas,prod} << B_{gas,prod}$ →1 $B_{gaslift} >> B_{gas,prod}$ →high $B_{gas,lift} << B_{gas,prod}$ →0

Table 4 – Relative sensitivity coefficients allocated gas to field B, pro-rata

Uncertainty source	Relative sensitivity coefficient	Comment
Gas exported from production process	1	
Total gas from field A into shared production facility	$\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →1 $A_{gas,prod} << B_{gas,prod}$ →0 $A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →1
Gas lift to field A	$\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{A_{gaslift}}{A_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →1 $A_{gas,prod} << B_{gas,prod}$ →0 $A_{gaslift} >> A_{gas,prod}$ →high $A_{gas,lift} << A_{gas,prod}$ →0
Total gas from field B into shared production facility	$\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{B_{gas,prod}+B_{gaslift}}{B_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →1 $A_{gas,prod} << B_{gas,prod}$ →0 $B_{gaslift} >> B_{gas,prod}$ →high $B_{gas,lift} << B_{gas,prod}$ →1
Gas lift to field B	$\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}} \cdot \frac{B_{gaslift}}{B_{gas,prod}}$	$A_{gas,prod} >> B_{gas,prod}$ →1 $A_{gas,prod} << B_{gas,prod}$ →0 $B_{gaslift} >> B_{gas,prod}$ →high $B_{gas,lift} << B_{gas,prod}$ →0

3 EXAMPLES - ALLOCATION SYSTEM WITH TWO FIELDS

In this chapter we illustrate how the difference in flow rates between field A and field B, as well as the ratio between gas lift and produced gas influence the allocation uncertainties in pro rata and by-difference allocation systems.

There are a number of parameters input to the uncertainty models for by-difference and pro rata allocation in equations (5), (7) and (10). In order to illustrate how the relation between the different flow rates are important for the relative sensitivity coefficients and thus allocation uncertainties, we have here chosen to keep the measurement uncertainties constant.

In the following examples, the measurement uncertainty of total gas from field A or field B is set to 2 % of gas mass. This could correspond to a gas measurement system at the outlet of a 1st stage/inlet or test separator, or to the gas measurement uncertainty of a multiphase flow meter which is regularly calibrated against a test separator. Note that the example uncertainty is set higher than the Norwegian Petroleum Directorate (NPD) requirements, as the gas phase out of a 1st stage separator is typically less stable than the stabilized gas flow at the fiscal measurement station at the end of the production process.

The export and gas lift gas mass measurement uncertainty is set to 1 %, according to NPD requirements. All uncertainties are given as relative expanded, with 95 % confidence level. Figure 2 gives a schematic illustration of the allocation system.

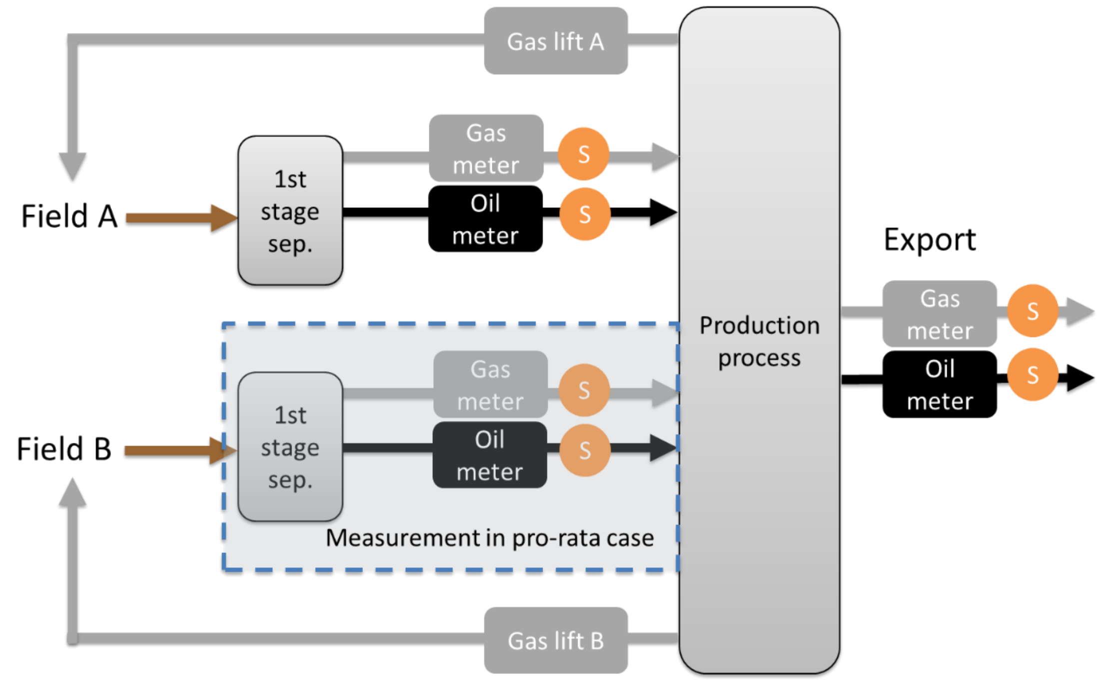

Fig. 2 – **Schematic illustration of the example allocation system.**

We study the example allocation system by varying the ratio between field A and field B produced gas (excluding gas lift), from $\frac{B\,produced}{A\,produced} = 0{,}25$ (field B production 4 times less than field A) to $\frac{B\,produced}{A\,produced} = 4$ (field B production 4 times more than field A production). This is shown along the x-axis in Figure 3. . We do this for different gas lift fractions.

3.1 Field A gas lift fraction 0,25-0,75, Field B gas lift fraction = 0

Figure 3 and Figure 4 show field A and field B allocation uncertainties in the case where field A gas lift fraction is 0,25, 0,50 and 0,75, whereas field B gas lift fraction remains 0.

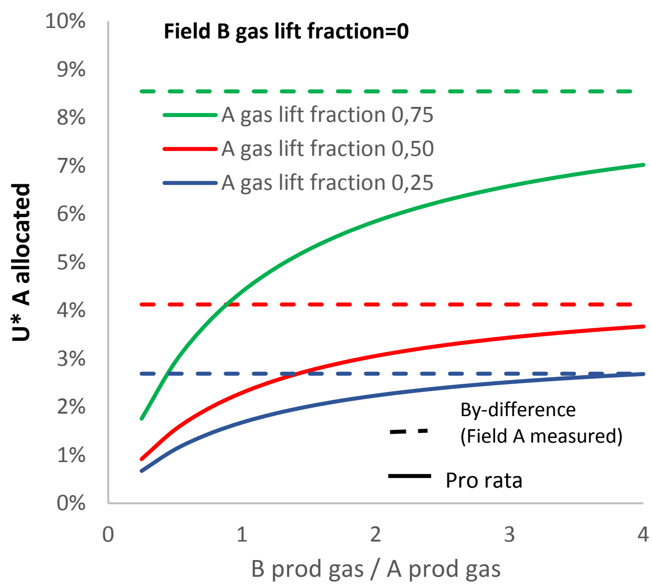

Fig. 3 – **Gas allocation uncertainty for field A**, as a function of fraction of produced gas from field B versus produced gas from field A, **for different field A gas lift fractions**.
Solid lines show pro rata allocation uncertainty, whereas the dotted lines show by-difference allocation uncertainty.

As shown in Figure 3, field A by-difference allocation uncertainty (dotted lines) does not depend on the ratio between field B and field A produced gas. This is because in the by-difference allocation case, field A is allocated directly based on measurements of field A total production (including gas lift) and field A gas lift.

Both the by-difference and pro rata field A allocation uncertainties increase with field A gas lift fraction. This is explained by the sensitivity coefficients $\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}$ and $\frac{A_{gaslift}}{A_{gas,prod}}$ in the uncertainty models for field A allocated by-difference (equation 5) and pro rata (equation 10). Due to the increasing gas lift in the nominator, both sensitivity coefficients increase rapidly with increasing field A gas lift fraction, and thus amplify the uncertainty contribution from field A measurements.

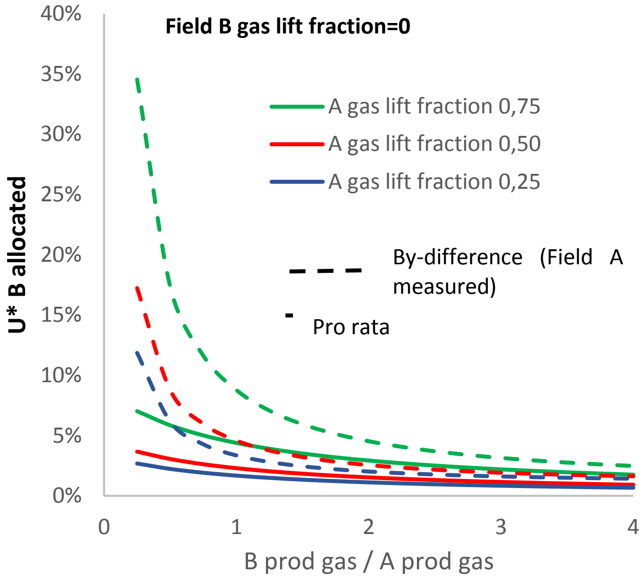

Fig. 4 – **Gas allocation uncertainty for field B**, as a function of fraction of produced gas from field B versus produced gas from field A, **for different field A gas lift fractions**. Solid lines show pro rata allocation uncertainty, whereas the dotted lines show by-difference allocation uncertainty.

For field B, shown in Figure 4, both by-difference (dotted lines) and pro rata (solid lines) allocation uncertainties depend on the ratio between field B and field A produced gas. This is explained by the produced flow rates (exclusive gas lift) appearing in the sensitivity coefficient in both uncertainty models:

- In the **field B by-difference** uncertainty model (equation 7), the sensitivity coefficient $\frac{A_{gas,prod}}{B_{gas,prod}}$ regulates the contribution of field A production uncertainty (total measured production minus measured gas lift). When field B production (excluding gas lift) is small compared with field A production this sensitivity coefficient increases rapidly and strongly amplify this uncertainty contribution.

- In the **field B pro-rata** allocation model (equation 10), the sensitivity coefficient $\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}}$ regulates both the contributions of field A and field B production uncertainty (total measured production for each field minus measured gas lift). When field B production (excluding gas lift) is small compared with field A production, this sensitivity coefficient approaches 1. When field B production is high compared with field A production, this sensitivity coefficient is smaller and thus attenuates the mentioned uncertainty contributions.

Both the by-difference and pro rata field B allocation uncertainties increase with increasing field A gas lift fraction. This is explained by the sensitivity coefficients $\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}$ and $\frac{A_{gaslift}}{A_{gas,prod}}$ in the uncertainty models for field B allocated by-difference (equation 7) and pro rata (equation 10). Due to the increasing gas lift in the nominator, both sensitivity coefficients increase rapidly with increasing field A gas lift fraction.

Note that for both field A and field B, the by-difference allocation uncertainties are higher than the pro-rata allocation uncertainties. This conforms to the common belief that by-difference allocation uncertainty is always higher than pro rata allocation uncertainty, which will be challenged in the next section.

3.2 Field A gas lift fraction 0, Field B gas lift fraction 0,25 - 0,75

Figure 5 and Figure 6 show field A and field B allocation uncertainty in the case where field A gas lift fraction is 0, for different field B gas lift fractions.

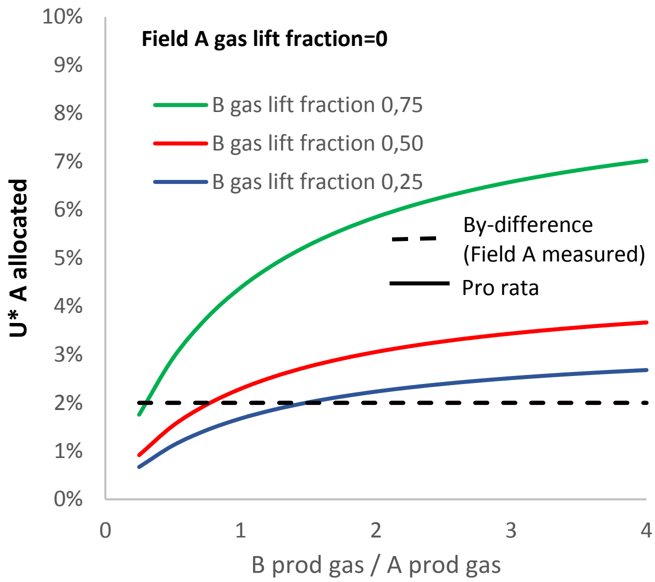

Fig. 5 – **Gas allocation uncertainty for field A**, as a function of fraction of produced gas from field B versus produced gas from field A, **for different field B gas lift fractions**. Solid lines show pro rata allocation uncertainty, whereas the dotted line shows by-difference allocation uncertainty. *Note that the by-difference uncertainty is independent of field B gas lift fraction, and lower than the pro rata uncertainty for high field B gas lift fractions or high field B production compared to field A production.*

As shown in Figure 5, field A by-difference allocation uncertainty (dotted lines) does neither depend on the ratio between field B and field A produced gas, nor field B gas lift. This is because in the by-difference allocation case, field A is allocated only based on field A measurements.

The pro rata field A allocation uncertainties increase with field B gas lift fraction. This is explained by the sensitivity coefficients $\frac{B_{gas,tot}}{B_{gas,tot}-B_{gaslift}}$ and $\frac{B_{gaslift}}{B_{gas,tot}-B_{gaslift}}$ in the uncertainty models for field A allocated pro rata (equation 10). Due to the subtraction of gas lift in the denominator, both sensitivity coefficients increase rapidly with increasing field B gas lift fraction, and amplify thus the uncertainty contribution from field B measurements.

As a result from the fact that field A by-difference allocation uncertainty does not increase with field B gas lift fraction, contrary to field A pro rata allocation uncertainty, the field A by-difference is lower than the pro rata allocation uncertainty for a large set of field B/ field A production ratios and field B gas lift fractions.

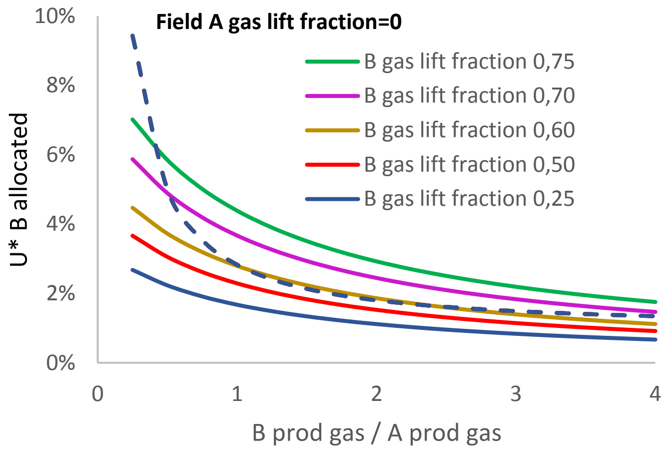

Fig. 6 – **Gas allocation uncertainty for field B**, as a function of fraction of produced gas from field B versus produced gas from field A, **for different field B gas lift fractions**. Solid lines show pro rata allocation uncertainty, whereas the dotted line shows by-difference allocation uncertainty. *Note that the by-difference uncertainty is independent of field B gas lift fraction, and* ***lower than the pro rata uncertainty for high field B gas lift fractions*** *unless field B production is low compared with field A production.*

For field B, shown in Figure 6, the pro rata field B allocation uncertainties increase with increasing field B gas lift fraction. This is explained by the sensitivity coefficients $\frac{B_{gas,prod}+B_{gaslift}}{B_{gas,prod}}$ and $\frac{B_{gaslift}}{B_{gas,prod}}$ in the uncertainty models for field B allocated pro rata (equation 10). Due to the increasing field B gas lift in the nominator, both sensitivity coefficients increase rapidly with increasing field B gas lift fraction. As field B gas lift is not included in any sensitivity coefficients in the field B by-difference allocation uncertainty model (equation 7), the same effect is not observed for the by-difference case.

As a result from the fact that field B by-difference allocation uncertainty does not increase with field B gas lift fraction, contrary to field B pro rata allocation uncertainty, the field B by-difference is lower than the pro rata allocation uncertainty for high field B gas lift fractions and high field B production compared with field A production. For field B gas lift fractions above 0,60, Figure 6 shows that for the measurement uncertainties used in the example, the by-difference allocation uncertainty will be lower than the pro rata when the production of field B is slightly higher than the production of field A.

4 INDUSTRY CASE

In this chapter we describe an anonymized analysis carried out for a real-life allocation system under development on the Norwegian Continental Shelf. We first describe the allocation and measurement system, then we show the production profiles and the resulting gas allocation uncertainties, both for a by-difference and pro rata allocation case.

Contrary to the simplified example described in chapter 2 and 3, this real-life system consists of two fields both producing oil and gas, and the uncertainty analysis has been carried out for the whole expected lifetime of the two fields. In addition, flare and fuel gas is included in the analysis.

The oil mass per component measured for each field is calculated from the total single phase volume measured at the inlet separator together with densities and component mass fractions from sampling and Component Oil Recovery Factors (CORFs). These factors are results from simulations in a process simulation software based on actual composition (as sampled at outlet of the separators) and representative process conditions in the production process.

The oil and gas masses per component for each field, which are input to the allocation calculations, are found from single phase volumetric measurements after 1st stage separators, and the composition (mass fraction of each component) is found from monthly samples. The gas lift to each field is calculated based on measured gas lift volume and fuel gas composition and density, and then subtracted from the total measured hydrocarbon production from each field.

The input uncertainties used in this industry case is 3.0 % for gas mass and 1.0 % oil mass measured at the outlet of the inlet separators, and 0.3 % for gas density and 0.5 % for oil density. The export metering uncertainties are set to NPD regulations [2]. In addition, sampling uncertainties after inlet separators are set to 1,5 % and analysis uncertainties are set to 1,5 % weight absolute.

Figure 7 shows the production profiles for this industrial case, expressed as the ratio between production from field B and field A, excluding gas lift, as well as the gas lift fractions for each field. The figure shows that in the first few years of production, field A and field B production excluding gas lift are comparable. Beyond this, field B production increases compared with field A production, and for a few years field B production is 3 times higher than field A, decreasing to 2 times higher towards the end. The figure also shows that field A gas lift fraction increases from zero to approximately 90 %, whereas field B gas lift fraction varies between approximately 60 % and 90 %. Both gas lift fractions stabilises at approximately 90 % towards the end of the lifetime period.

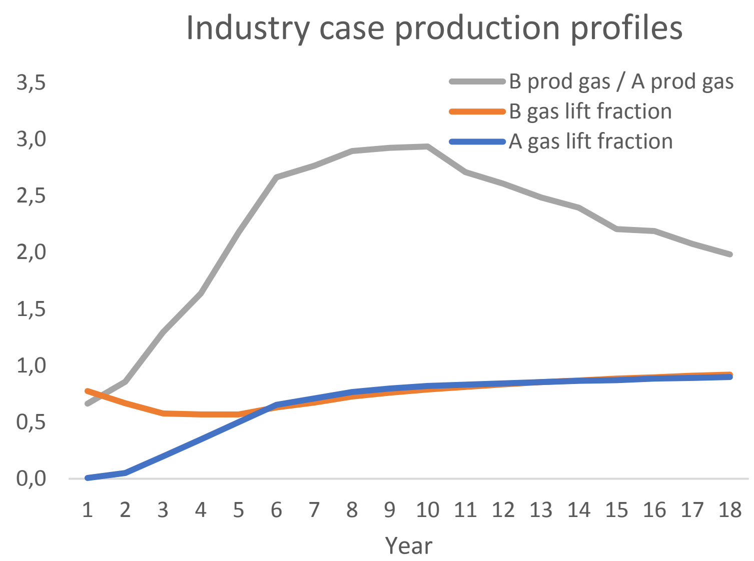

Fig. 7 – **Production profiles** for the industry case, shown as ratio between field B and field A production (exclusive gas lift) in grey, B gas lift fraction in red and A gas lift fraction in blue.

Figure 8 shows the calculated gas volume allocation uncertainties, both for a pro rata case where both fields are measured, and for a by-difference case where only field A is measured.

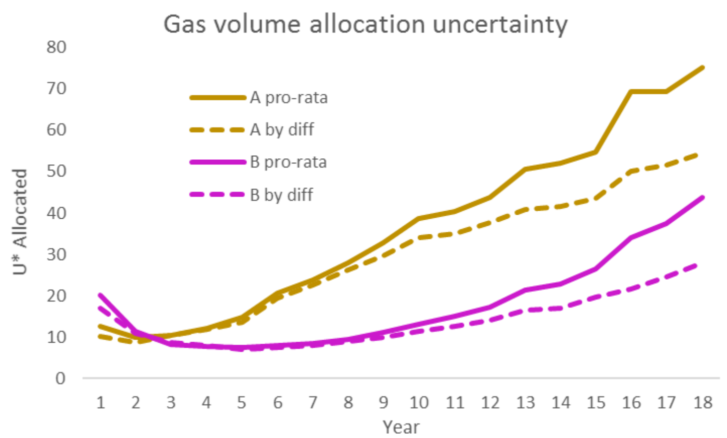

Fig. 8 – **Gas volume allocation uncertainties** for field A (brown) and field B (purple). The pro rata allocation uncertainties where both fields are measured are plotted as solid lines. The by-difference allocation uncertainties, where only field A is measured, are plotted as dotted lines.

The allocation in the industry case is based on calculated oil and gas per component at export conditions using Component Oil Recovery Factors combined with mass fractions from samples. The uncertainty models for the industrial case are thus more complex than the uncertainty models for the simplified systems described in chapter 2.2 and chapter 2.3. It is, however, possible to *qualitatively* explain the allocation uncertainties observed in Figure 8 using the uncertainty models in equations (5), (7) and (10).

Figure 8 shows that in the first few years, field B allocation uncertainties are higher than field A allocation uncertainties. However, as the production of field B relative to field A increases, field A allocation uncertainties become higher than field B allocation uncertainties, both for the by-difference and pro rata case. For the by-difference case, this may be explained by the simplified field B by-difference allocation uncertainty model in equation (7), repeated below for convenience. Here the sensitivity coefficient in front of the field A measurement uncertainties decreases as field A production decreases compared with field B production.

$$\left(\frac{U(B_{gas,allok,bd})}{B_{gas,allok,bd}}\right)^2 = \left(\frac{E_{gas}}{B_{gas,prod}}\right)^2\left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{A_{gas,prod}}{B_{gas,prod}}\right)^2\left[\left(\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}\right)^2\left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2\left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2\right] \qquad (7, rep.)$$

It is interesting to see that for most years and for both fields, the pro rata allocation uncertainties are higher than the by-difference allocation uncertainties. The pro-rata uncertainty models for the simplified case, as shown in equation (10), repeated below for convenience, are useful for explaining this:

- For field A, the high pro-rata allocation uncertainties are due to the high field A and field B gas lift fractions after year 5, and the high field B gas production compared with field A gas production
- For field B, the high pro-rata allocation uncertainties are due to the high field A and field B gas lift fractions after year 5, but slightly reduced due the high field B gas production compared with field A gas production

Note that compared to the by-difference allocation uncertainty model which only includes the uncertainty contributions from field A measurements, the pro-rata uncertainty model includes uncertainty contributions from measurements on both fields.

$$\left(\frac{U(A_{gas,allok,pr})}{A_{gas,allok,pr}}\right)^2 = \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{B_{gas,prod}}{A_{gas,prod}+B_{gas,prod}}\right)^2\left[\left(\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}\right)^2\left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2\left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2 + \left(\frac{B_{gas,prod}+B_{gaslift}}{B_{gas,prod}}\right)^2\left(\frac{U(B_{gas,tot})}{B_{gas,tot}}\right)^2 + \left(\frac{B_{gaslift}}{B_{gas,prod}}\right)^2\left(\frac{U(B_{gaslift})}{B_{gaslift}}\right)^2\right] \qquad (10, rep.)$$

$$
\left(\frac{U(B_{gas,allok,pr})}{B_{gas,allok,pr}}\right)^2 = \left(\frac{U(E_{gas})}{E_{gas}}\right)^2 + \left(\frac{A_{gas,prod}}{A_{gas,prod}+B_{gas,prod}}\right)^2 \left[\left(\frac{A_{gas,prod}+A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gas,tot})}{A_{gas,tot}}\right)^2 + \left(\frac{A_{gaslift}}{A_{gas,prod}}\right)^2 \left(\frac{U(A_{gaslift})}{A_{gaslift}}\right)^2 + \left(\frac{B_{gas,prod}+B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gas,tot})}{B_{gas,tot}}\right)^2 + \left(\frac{B_{gaslift}}{B_{gas,prod}}\right)^2 \left(\frac{U(B_{gaslift})}{B_{gaslift}}\right)^2\right]
$$

5 CONCLUSION

In this paper we have shown that it is important to take expected production and gas lift profiles into account when evaluating allocation systems in a risk-cost-benefit perspective over field lifetime. We have demonstrated how gas lift rates may have a great impact on allocation uncertainty, and how this may lead to counter-intuitive results.

We have compared a simplified pro-rata and a by-difference allocation system. In the pro-rata case both fields A and B are measured, whereas in the by difference case only field A's production into the shared process plant is measured, and field B is allocated by-difference.

Both the by-difference and pro rata field B allocation uncertainties increase with increasing field A gas lift fraction. However, for both fields, *the by-difference allocation uncertainties are independent of field B gas lift fraction*.

We have shown that since field B gas lift fraction does not affect the by-difference allocation uncertainties, then:

- for field B, the by-difference allocation uncertainty is lower than the pro rata uncertainty for
    - high field B gas lift fractions or
    - high field B production compared to field A production.

- the field A by-difference is lower than the pro rata allocation uncertainty
    - for a large set of field B/ field A production ratios and field B gas lift fractions.

Furthermore, we have presented an anonymized analysis carried out for a real-life allocation system under development on the Norwegian Continental Shelf. Here a by-difference allocation principle lead to lower allocation uncertainties than a more costly pro rata alternative. This counter-intuitive conclusion shows the importance of conducting allocation uncertainty studies to get a clear and knowledge-based picture in a risk-cost-benefit perspective, taking all expected rates into consideration.

7 REFERENCES

[1] ISO/IEC, “Guide to the expression of uncertainty in measurement,” ISO/IEC, Geneva, 2008.

[2] The Norwegian Petroleum Directorate, “Regulations relating to measurement of petroleum for fiscal purposes and for calculation of CO2-tax,” NPD, 2001.