# Data Reconciliation in Microcosm – Reducing DP Meter Uncertainty

**Allan Wilson, Accord ESL**
**Phil Stockton, Accord ESL**
**Richard Steven, DP Diagnostics**

## 1 INTRODUCTION

The hydrocarbon production industry runs large complex pipework systems with numerous and varied equipment such as multiple valves, pressure and temperature sensors, flow meters etc. However, due to the inherent uncertainty in each equipment setting and instrumentation output, the resulting massed raw data can be somewhat inconsistent. As such industry applies 'Data reconciliation' techniques on the macro overall pipe system. Such techniques involve mathematical procedures that combine a pipework's multiple instrumentation readings, equipment settings, associated uncertainties, and governing physical laws, to automatically validate data and reconcile measurements such that the whole makes physical sense. The technique can improve best estimates of not just measured system variables but even unmeasured variables. The technique transforms raw and sometimes inconsistent data sets into a single consistent data set representing the most likely truth.

In this paper, the technique often applied on this macro scale, is introduced to the micro scale of an individual meter system. In the macro scale data reconciliation, the flow meter system is treated like any other instrument output, i.e. as a single node, a single point measurement. There has not been any attempt to take a flow meter design's sub-systems and develop mathematical techniques specifically tailored to the internal operation of that specific metering system for the purpose of improving that the individual flow meter's performance, for all the advantages that would entail.

In this paper mathematical techniques, based on data reconciliation, have been developed and applied specifically to flow metering systems to improve the performance of the flow meter, including fine adjustments to the stated flowrate prediction while lowering its uncertainty. These techniques have been collectively described in this paper under the term: "Maximum likelihood uncertainty" (MLU).

MLU requires multiple instrument readings, but this is achievable with:

1. two meters in series making a metering system or,

2. a hybrid meter that incorporates two or more metering principles in one metering system, or

3. the use of standalone meters with either:
   a. suitable added instrumentation or,
   b. suitable existing diagnostic systems giving suitable secondary valuable information.

Though the main focus of this paper is Differential Pressure (DP) meters, which are covered under 3. above, MLU can be applied to any system of measurements satsifying any of 1. to 3. In fact, the description of MLU is most easily understood using a flow measurement system comprising two independent meters in series and this example is used in Section 2 to describe the technique. The more advanced, general method applied, (though not restricted to), DP meters is then developed in Section 3.

As the description and application of the MLU method is developed, multiple real world examples will be presented using a mixture of field and calibration facility orifice, Venturi, and cone DP meter data sets along with USM data. These examples show that the technique has significant implications regarding flow meter uncertainty and consequently reducing the associated financial risk exposure.

# 2 MAXIMUM LIKELIHOOD UNCERTAINTY (MLU) TECHNIQUE – INDEPENDENT METERS

## 2.1 MLU Description

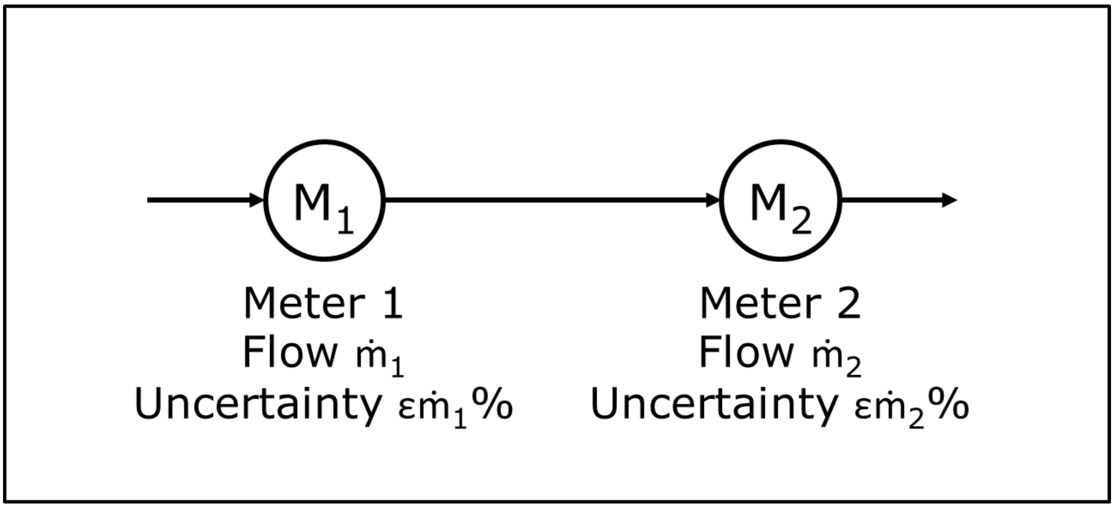


**Fig. 1 Two Flow Meters in Series**

Consider two flow measuring devices installed in series which are measuring the same mass flowrate as depicted schematically in Fig. 1.

Since both devices have inherent uncertainty, they will report different flowrates (only slightly different if the devices are of good quality). The true mass flow is not known with absolute precision. Each device, if functioning correctly, will report the flow within the bounds of its stated uncertainty in accordance with the probability upon which the uncertainty bounds are specified. Typically, uncertainties are quoted at the 95% confidence level, which means that the reported mass flow will be within 1.96 standard deviations of the true value with 95% probability.

Each device exhibits an uncertainty in its reported flow. The two measuring devices may have different uncertainties. The device with the lower uncertainty is more likely to be closer to the true value, but this is not guaranteed. However, for steady flow the physical law of conservation of mass states that the true mass flow passing through both meters in series is known to be the same with absolute certainty.

If one flow meter's flow prediction measured flow is assumed to be the most representative of the true flow (i.e. usually the flow meter with the lower uncertainty) and the other flow meter's flow prediction is ignored, then information about the true flow is being discarded. This additional information being ignored is in the form of the second measurement of the true flow, and the physical fact that the two meters are metering the same flowrate.

The method of check metering employed in industry does just this. Once the check meter is seen to agree with the primary meter within the combined meter uncertainties, often set as the root sum square value of the two meter uncertainties, the primary meter's correct operation is seen as confirmed.

Presently, industry makes no more use of the check meter information, and simply uses the primary meter output with its stated uncertainty. This is even true for paired reference flow meters in series at flow meter calibration facilities.

This proposed method utilizes the information from both flow meters, and the knowledge that they are measuring the same true flow, in such a way as to generate a model where probability theory can be applied to produce a statistical maximum likelihood estimate of that true flow given the available data. This statistical maximum likelihood estimate of that true flow is more likely to be closer to the true flowrate value than either of the two input flowrate predictions. Furthermore, this maximum likelihood estimate flowrate prediction also has a lower associated uncertainty than either of the two individual flowrate predictions.

The method can be extended to include more than two flow measurement devices in series. The method can even be extended to flow measurement devices in parallel as long as mass conservation relates their combined measured flows.

Indeed, the method is not restricted to the conservation of mass flow (or volume flow for constant density) but can be extended to utilise other relevant laws of physics. One example, which is exploited in this disclosure, is the rule of 'the equivalence of measured pressure differentials'.

The method termed 'MLU' is the combination of a system's multiple instrumentation readings, associated uncertainties, governing physical laws, and mathematical techniques, to automatically validate data and reconcile measurements such that the whole makes physical sense. The technique can improve best estimates of not just measured system variables but even unmeasured variables. The technique transforms raw and sometimes inconsistent data sets into a single consistent data set representing the most likely truth.

MLU techniques are typically used by the hydrocarbon production industry for product allocation in complex pipe networks owned by many parties or process plants where there are multiple independent measurements of distinct hydrocarbon streams. This macro system can be used to validate measurements as being compatible with respect to uncertainties and the relevant constraints, or to detect gross instrumentation errors. In this disclosure the technique often applied on this macro scale is introduced to the micro scale of flow meter systems.

A simple theoretical example is now presented to illustrate the method of combining two independent flow measurements to derive a adjusted measurement with a reduced associated uncertainty.

## 2.2 MLU – Mathematical Development for Two Independent Meters in Series

Returning to the system with two independent flow meters in series both measuring the same mass flowrate as depicted schematically in Fig. 1. These meters can be similar or dissimilar. Each meter is independent of the other in terms of the inputs used to calculate the mass flow. Denoting the first meter as meter 1, its mass flow prediction is denoted as $\dot{m}_1$. Denoting the second meter as meter 2, its mass flow prediction is denoted as $\dot{m}_2$. Each meter's reported mass flow will have an uncertainty associated with it denoted by by $U\dot{m}_1$ and $U\dot{m}_2$ respectively. These uncertainties are absolute and are related to the relative uncertainties $\varepsilon\dot{m}_1$ and $\varepsilon\dot{m}_2$ by equations (1) and (2):

$$U\dot{m}_1 = \varepsilon\dot{m}_1 * \dot{m}_1 \tag{1}$$

$$U\dot{m}_2 = \varepsilon\dot{m}_2 * \dot{m}_2 \qquad (2)$$

Both sets of uncertainties are expressed at some stated confidence level (usually 95%) and follow normal distributions. The probability density function below describes the probability of how the measured flow differs from the true flow $\dot{m}_t$:

$$Pd_1(\,\dot{m}_t \mid \dot{m}_1, \sigma_1\,) = \frac{1}{\sqrt{2\pi{\sigma_1}^2}} e^{-\left(\frac{(\dot{m}_t-\dot{m}_1)^2}{2{\sigma_1}^2}\right)} \qquad (3)$$

Where,

$Pd_1$ Probability density for the 1st meter
$\dot{m}_t$ True mass flow
$\sigma_1$ Standard deviation of the 1st meter mass flow

The standard deviation is related to the 95% measurement uncertainty by:

$$U\dot{m}_1 = 1.96 * \sigma_1 \qquad (4)$$

A similar expression describes the probability density of the 2nd meter:

$$Pd_2(\,\dot{m}_t \mid \dot{m}_2, \sigma_2\,) = \frac{1}{\sqrt{2\pi{\sigma_2}^2}} e^{-\left(\frac{(\dot{m}_t-\dot{m}_2)^2}{2{\sigma_2}^2}\right)} \qquad (5)$$

The product of these two probability density functions describes the probability of how the true value differs from the two measured values.

$$Pd_2(\,\dot{m}_t \mid \dot{m}_2, \sigma_2\,)Pd_1(\,\dot{m}_t \mid \dot{m}_1, \sigma_1\,) = \frac{1}{\sqrt{2\pi{\sigma_2}^2}} e^{-\left(\frac{(\dot{m}_t-\dot{m}_2)^2}{2{\sigma_2}^2}\right)} \frac{1}{\sqrt{2\pi{\sigma_1}^2}} e^{-\left(\frac{(\dot{m}_t-\dot{m}_1)^2}{2{\sigma_1}^2}\right)} \qquad (6)$$

This product of probabilities can be differentiated with respect to the true value and set to zero to find the most probable estimate of the true value given the two measured flows and their associated uncertainties. This most probable estimate of the true flow is termed the reconciled mass flowrate $\dot{m}_r$. It is more mathematically convenient to work with the negative of the natural logarithm of the probability densities. Since the logarithm is a monotonic transformation, its maximum will correspond with that of the product of the probability densities (L):

$$L = -\ln\left(\frac{1}{\sqrt{2\pi{\sigma_2}^2}} e^{-\left(\frac{(\dot{m}_t-\dot{m}_2)^2}{2{\sigma_2}^2}\right)} \frac{1}{\sqrt{2\pi{\sigma_1}^2}} e^{-\left(\frac{(\dot{m}_t-\dot{m}_1)^2}{2{\sigma_1}^2}\right)}\right) \qquad (7)$$

Therefore,

$$L = -\ln\left(\frac{1}{\sqrt{2\pi{\sigma_2}^2}}\frac{1}{\sqrt{2\pi{\sigma_1}^2}}\right) + \left(\frac{(\dot{m}_t - \dot{m}_2)^2}{2{\sigma_2}^2}\right) + \left(\frac{(\dot{m}_t - \dot{m}_1)^2}{2{\sigma_1}^2}\right) \qquad (8)$$

Differentiating:

$$\frac{dL}{d\dot{m}_t} = \left(\frac{(\dot{m}_t - \dot{m}_2)}{{\sigma_2}^2}\right) + \left(\frac{(\dot{m}_t - \dot{m}_1)}{{\sigma_1}^2}\right) \qquad (9)$$

When this differential equals zero $\dot{m}_t = \dot{m}_r$, and rearranging in terms of $\dot{m}_r$:

$$\dot{m}_r = \frac{{\sigma_1}^2\dot{m}_2 + {\sigma_2}^2\dot{m}_1}{{\sigma_2}^2 + {\sigma_1}^2} \tag{10}$$

Or in terms of the uncertainties:

$$\dot{m}_r = \frac{U\dot{m}_1^{\,2}\dot{m}_2 + U\dot{m}_2^{\,2}\dot{m}_1}{U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}} \tag{11}$$

The uncertainty in this reconciled flow ($U\dot{m}_r$) can also be determined using the Taylor Series Method (TSM) for the propagation of errors described in the "Guide to the Expression of Uncertainty in Measurement", aka "the GUM" [6]. The sensitivity coefficients are given by:

$$\frac{\partial\dot{m}_r}{\partial\dot{m}_2} = \frac{U\dot{m}_1^{\,2}}{U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}} \tag{12}$$

And,

$$\frac{\partial\dot{m}_r}{\partial\dot{m}_1} = \frac{U\dot{m}_2^{\,2}}{U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}} \tag{13}$$

$$U\dot{m}_r = \left(\left(\frac{\partial\dot{m}_r}{\partial\dot{m}_1}\right)^2 U\dot{m}_1^{\,2} + \left(\frac{\partial\dot{m}_r}{\partial\dot{m}_2}\right)^2 U\dot{m}_2^{\,2}\right)^{0.5} \tag{14}$$

Substituting for the partial differentials:

$$U\dot{m}_r = \left(\left(\frac{U\dot{m}_2^{\,2}}{U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}}\right)^2 U\dot{m}_1^{\,2} + \left(\frac{U\dot{m}_1^{\,2}}{U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}}\right)^2 U\dot{m}_2^{\,2}\right)^{0.5} \tag{15}$$

After simplification:

$$U\dot{m}_r = \frac{U\dot{m}_2 U\dot{m}_1}{\left(U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}\right)^{0.5}} \tag{16}$$

Inspection of this result reveals that the uncertainty in the reconciled flow is always less than either of the uncertainties in the individual meter measurements. If the above is re-expressed as:

$$U\dot{m}_r = x * U\dot{m}_2 \tag{17}$$

Where the factor $x$ is given by:

$$x = \frac{U\dot{m}_1}{\left(U\dot{m}_2^{\,2} + U\dot{m}_1^{\,2}\right)^{0.5}} \tag{18}$$

x is always less than 1, unless $U\dot{m}_2$=0 or $U\dot{m}_1$=∞, when x equals 1. Hence, $U\dot{m}_r$ is always less than $U\dot{m}_2$, for any real meter. A similar argument proves that $U\dot{m}_r$ is always less than $U\dot{m}_1$. Hence, for any real meter, $U\dot{m}_r$ is always less than either meter's uncertainty.

The principle is that the higher uncertainty measurement does contain independent additional information, and even if it is less accurate than the lower uncertainty information, when the information is combined, the higher uncertainty additional information supplements the lower uncertainty information, which inherently reduces the overall measurement uncertainty.

This is illustrated schematically in Fig. 2:

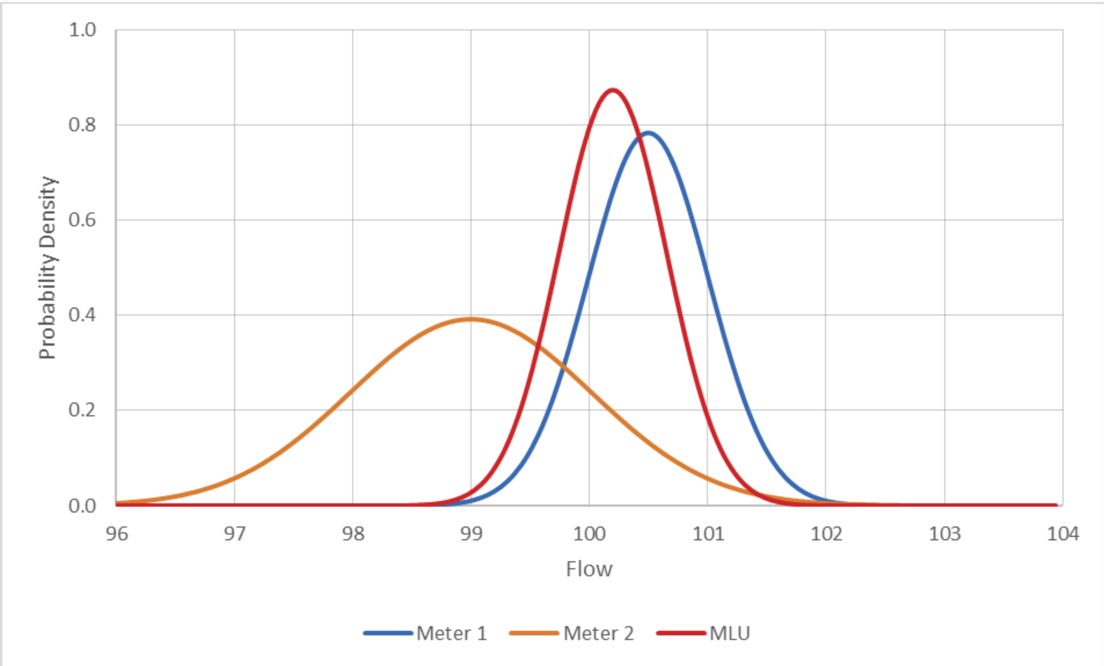


**Fig. 2 Probability Densities for MLU of Two Flow Meters in Series**

This is based on a nominal true flow of 100 units, being measured by two independent meters:

- Meter 1 reports a flow of 100.5 units and has an uncertainty of ±1%,
- Meter 2 reports a flow of 99 units and has an uncertainty of ±2%.

The MLU flow is 100.2 units and its associated uncertainty is ±0.89%. The probability density described by the red normal curve can be seen by inspection that it is derived from the product of the probability densities of the two meters (denoted by blue and orange lines).

## 2.3 Example Vortex plus Cone Meter in Series

This example applies ther MLU technique to determine the reconciled mass flow rate and its uncertainty for a single-phase flow measured by two meters in series. In this example, independent flow rate measurements are made by a vortex meter and a cone meter in series (see Fig. 3).

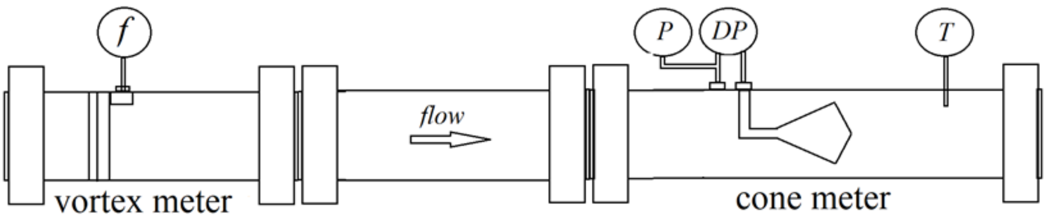


**Fig. 3 Vortex Meter and Cone DP Meter Installed in Series**

The vortex meter will report mass flow by $\dot{m}_v$ and the cone meter a mass flow by denoted by $\dot{m}_c$. Each meter's reported mass flow will have an uncertainty associated with it denoted by $U\dot{m}_v$ and $U\dot{m}_c$ respectively.

Substituting for Meter 1 and Meter 2 in equation (11), the reconciled mass flow rate is given by:

$$\dot{m}_r = \frac{U\dot{m}_v{}^2 \dot{m}_c + U\dot{m}_c{}^2 \dot{m}_v}{U\dot{m}_c{}^2 + U\dot{m}_v{}^2} \quad (19)$$

And based on equation (16), the reconciled uncertainty is given by:

$$U\dot{m}_r = \frac{U\dot{m}_c U\dot{m}_v}{\left(U\dot{m}_c{}^2 + U\dot{m}_v{}^2\right)^{0.5}} \quad (20)$$

The results of applying this approach to a real vortex meter and cone meter operating in series are shown below. The data is from a 6", schedule 80, gas flow laboratory with the vortex meter upstream of the DP cone meter. The meters were separated by a short spool piece. The data consist of points at nominally 60 bar and 15 bar. The standalone cone meter had a mass flow prediction uncertainty of ±0.6%. The standalone vortex meter had a mass flow prediction uncertainty of ±0.75%. After this method is applied the reconciled mass flowrate prediction uncertainty is reduced to ±0.47%, i.e. the combined metering system has a lower mass flowrate uncertainty than either individual meter.

An example of the approach applied to a real cone meter and vortex meter operating in series is illustrated below.

**Table 1 - Vortex and Cone Meter in Series with Applied MLU**

| Data Point | Cone Meter | | Vortex Meter | | Reconciled | | |
|---|---|---|---|---|---|---|---|
| | Rel. Uncertainty=> | 0.60% | Rel. Uncertainty=> | 0.75% | | | |
| | Measured Flow | Abs. Uncertainty | Measured Flow | Abs. Uncertainty | Flow | Abs. Uncertainty | Rel. Uncertainty |
| | kg/s | ±kg/s | kg/s | ±kg/s | ±kg/s | ±kg/s | ±% |
| 1 | 3.118 | 0.019 | 3.115 | 0.023 | 3.117 | 0.015 | 0.47% |
| 2 | 7.807 | 0.047 | 7.777 | 0.058 | 7.796 | 0.037 | 0.47% |
| 3 | 4.208 | 0.025 | 4.193 | 0.031 | 4.202 | 0.020 | 0.47% |
| 4 | 4.223 | 0.025 | 4.203 | 0.032 | 4.215 | 0.020 | 0.47% |
| 5 | 5.323 | 0.032 | 5.312 | 0.040 | 5.319 | 0.025 | 0.47% |
| 6 | 5.326 | 0.032 | 5.317 | 0.040 | 5.323 | 0.025 | 0.47% |
| 7 | 11.794 | 0.071 | 11.762 | 0.088 | 11.781 | 0.055 | 0.47% |
| 8 | 20.142 | 0.121 | 20.226 | 0.152 | 20.175 | 0.095 | 0.47% |

The probability densities of the meter readings and MLU reconciled flow for data point 8 in Table 1 are presented in Fig. 4:

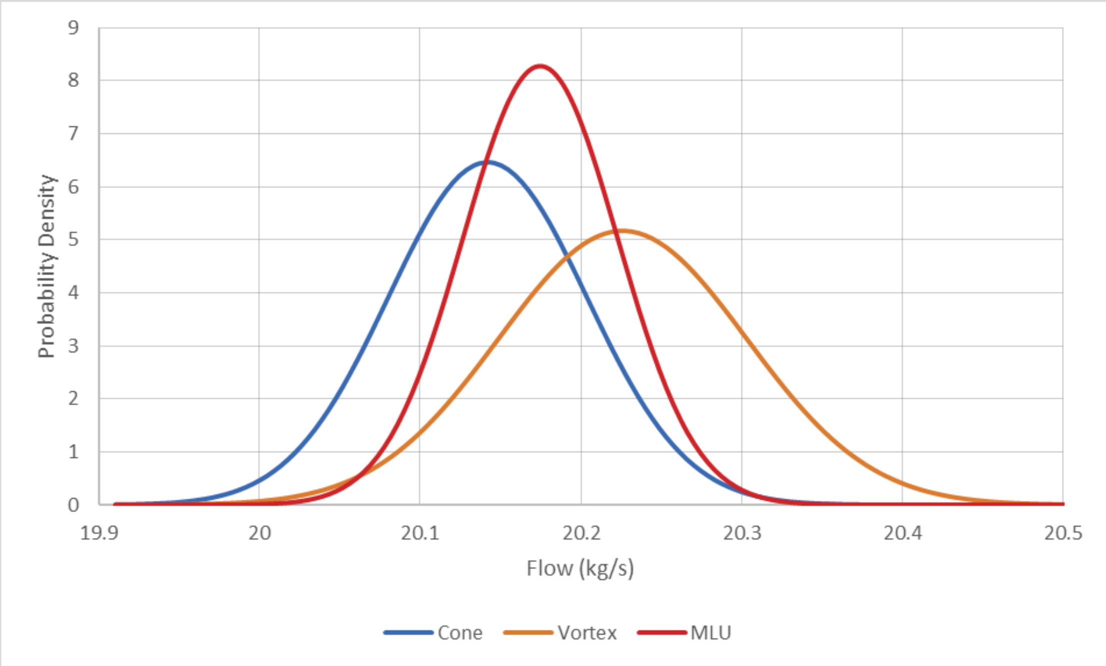


**Fig. 4 Probability Densities for MLU applied to Cone and Vortex Flow Meters in Series**

Say this metering system was for natural gas custody transfer. At 60 Bar and 20$^0$C, the standalone cone DP meter would predict a gas mass flow of 20.142 kg/s ± 0.6% (see point 8 in Table 1). In terms of Million Standard Cubic Feet per Day (MMSCFD), this is a custody transfer result of 83.48 MSCFD ± 0.50 MMSCFD. Considering the typical gas calorific value of 1000 BTU/SCF, and a gas price of $2.50 per million BTU, this is an annual flow value of $76.18 million ± $457K. The traditional method of metering the flow uses a single meter's uncertainty value. If the cone meter stood alone in the pipe this result would be used without any check meter verification. If there were two meters in series for check metering the standard practice is to use the meter with the lower uncertainty for billing, i.e. in this example the cone meter at 0.6% uncertainty. However, if this MLU technique is applied it is proven that the flow prediction and its associate uncertainty needs finely adjusted to 20.175 kg/s ±0.47% (see point 8 in Table 1). The custody transfer metering result has shifted to 83.62 MMSCFD ± 0.39 MMSCFD, i.e. an annual flow value of $76.30 million ± $359K. That is, the primary flowrate has been finely altered by +0.14 MMSCFD, i.e. a billing alteration of +$127.775K per annum, while the uncertainty in the flow metering system output has dropped by 0.13% from ± $457K to ± $359K per year, i.e. exposure to flow metering uncertainty is reduced by $98,000 per year.

## 2.4 MLU Applied to Hybrid Meters

The example in the previous section utilized the method for the case where there were two independent metering systems in series (see Fig. 3). The method will work regardless of whether the two (or more) flow meters are similar or dissimilar design. This example of two independent dissimilar flow meters in series is applicable in industry in several scenarios. It would work with check metering (including when a buyer and seller have separate meters in series), and it will work with hybrid meter designs, such as the vortex and cone meter combination in Fig. 5 (presented by Steven [4]).

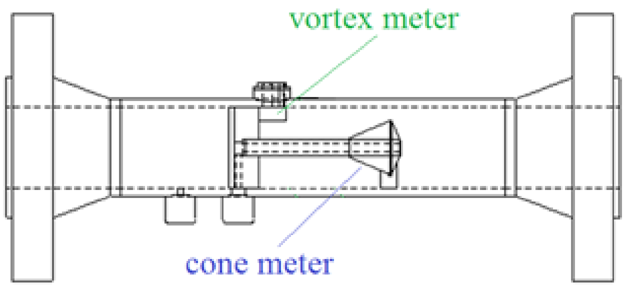


**Fig. 5 A Vortex Meter + Cone DP Hybrid Meter**

This is a specific example of a Boden [5] mass flow meter design. Boden taught that cross-referencing of a density sensitive volume meter (e.g. a DP meter), and a density insensitive meter (e.g. a turbine, ultrasonic, or vortex meter) produces a density prediction. Combing the density insensitive meter's volume flowrate prediction with this density prediction produces a mass flowrate prediction. The Boden concept is a mass meter design, i.e. a flow meter that can predict the mass flow without requiring an externally obtained fluid density input. However, although Boden metering system designs (such as Fig. 5) consist of two flow meter designs in series, or two flow metering principles in one meter body, they are not in any way used to reduce the flow metering systems fluid flow prediction uncertainty.

A hybrid meter is one where two different metering measurements are made by sharing at least some physical components. Fig. 5 shows an example of a hybrid meter produced by two dissimilar meters.

However, some ultrasonic meter (USM) systems have more than one 'ultrasonic meter system' in the same meter body. This could be described as a hybrid of two distinct independent ultrasonic meter designs sharing the same upstream flow conditioner and the same meter body. As ultrasonic meters are described by their number of paths, such designs are called by the two ultrasonic meter's number of paths. For example, a 4+1 ultrasonic metering system indicates one embedded four path ultrasonic metering system and one embedded one path ultrasonic metering system installed together in one meter body. Some ultrasonic meter designs have 'bounce paths' where the transducer pair are not directly opposite from each other but send and receive signals by bouncing the ultrasonic wave off the wall. This makes no difference to the application of this technique.

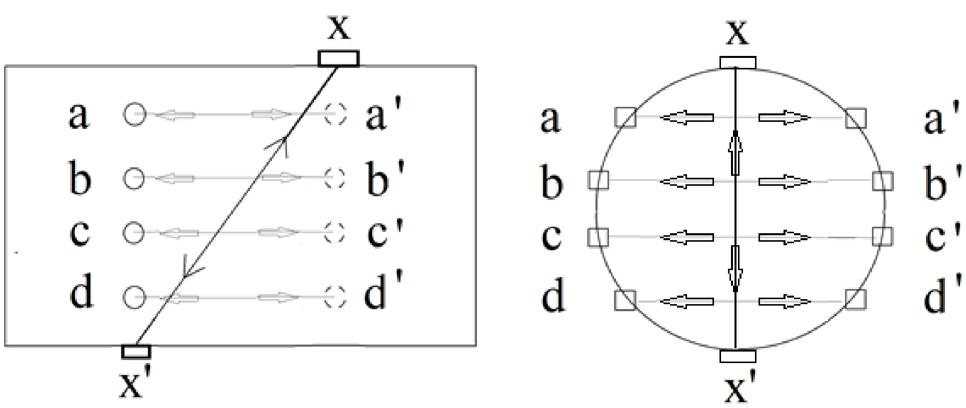


**Fig. 6 Schematic Sketch of a Four Path Plus One Bounce Path Ultrasonic Meter/s Design.**

The 4 path chordal design has four paths created by transducer pairs a & a', b & b', c & c', d & d'. This four path chordal design typically has its own dedicated flow computer (or 'head'). The 1 path design has a path created by transducer pairs x & x'. This one path design typically has a separate dedicated flow computer (or 'head'). That is the two ultrasonic metering systems are independent. Ultrasonic meter manufacturers use such meter designs to give an extra level of system redundancy, e.g. check metering, and to offer an extra diagnostic capability (e.g. the 1 path meter may be positioned in the vertical axis and can therefore be adversely affected by problems such as contamination or liquids at the base of the meter before the four path meter diagnostic system can see the problem). There is no attempt by ultrasonic meter manufacturers to cross reference the two (or more) independent meters in order to reduce the overall system flowrate prediction uncertainty. However, any such ultrasonic metering system with two such systems (e.g. 4+1, 4+2, 4+4 etc.) could be developed to utilize this uncertainty reduction method. The next example shows the method being applied to 4+1 path ultrasonic meter data.

## 2.5 Example 4 + 1 Path Ultrasonic Meter

Consider the case of a 4+1 path ultrasonic meter. There are various common path configurations, such as 4+1, 4+2, 4+4 etc. The method discussed here works for all such ultrasonic meter configurations.

The 4-path and 1-path measured flow rates are treated as being independent. Ultrasonic meters are used to determine the volume flow rate. Ancillary calculations are applied to derive the fluid density from which a mass flow rate is then derived. However, as the two volume meters are embedded in the same meter body at the same flow conditions the fluid density is the same for both meters and hence there is the same mass flow and the same volume flow past the two meters. The reconciliation calculations equations (11) and (16) apply in any situation where the same quantity is independently measured by different means. Thus, a reconciled volume or mass flow rate can be derived and its uncertainty calculated. Equations (11) and (16) can then be applied to obtain a reconciled volume or mass flow rate.

**Table 2 - 4+1 Ultrasonic Meter with Applied MLU**

| | USM 4-path | | USM 1-path | | MLU | | |
|---|---|---|---|---|---|---|---|
| | Rel Uncert 0.5% | | Rel Uncert 0.6% | | Reconciled | Reconciled | Reconciled |
| Data Point | Measured Flow | Abs. Uncert | Measured Flow | Abs. Uncert | Flow | Abs. Uncert | Rel. Uncert |
| | kg/s | kg/s | kg/s | kg/s | kg/s | kg/s | % |
| 1 | 5.321 | 0.027 | 5.327 | 0.032 | 5.323 | 0.020 | 0.38% |
| 2 | 4.708 | 0.024 | 4.713 | 0.028 | 4.710 | 0.018 | 0.38% |
| 3 | 4.163 | 0.021 | 4.143 | 0.025 | 4.155 | 0.016 | 0.38% |
| 4 | 3.582 | 0.018 | 3.583 | 0.021 | 3.582 | 0.014 | 0.38% |
| 5 | 2.972 | 0.015 | 2.975 | 0.018 | 2.973 | 0.011 | 0.38% |
| 6 | 2.366 | 0.012 | 2.368 | 0.014 | 2.367 | 0.009 | 0.38% |
| 7 | 1.767 | 0.009 | 1.769 | 0.011 | 1.768 | 0.007 | 0.38% |
| 8 | 1.154 | 0.006 | 1.155 | 0.007 | 1.154 | 0.004 | 0.38% |

The probability densities of the meter readings and MLU reconciled flow for data point 1 in Table 2 are presented in Fig. 7:

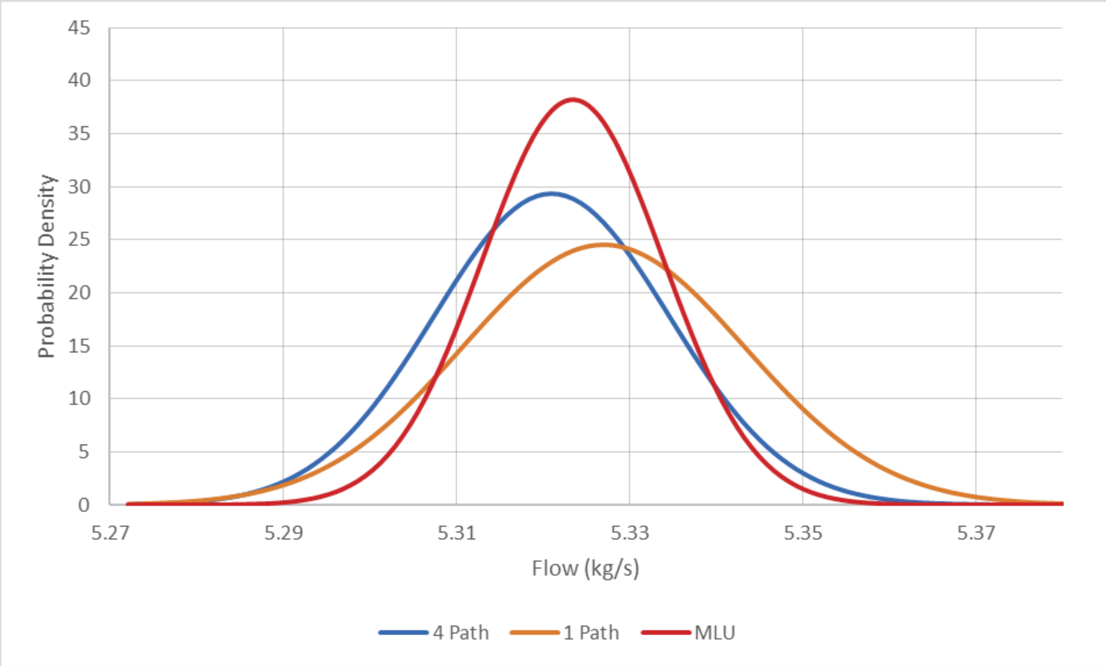


**Fig. 7 Probability Densities for MLU applied to 4 + 1 Ultrasonic Flow Meter**

The results of applying this approach to a real 4+1 path USM are shown in Table 2. This data comes from an 8", schedule 80 flow meter tested at a natural gas flow laboratory. The data consist of points nominally at 15 Bar. The calibrated 4 path and 1 path USM relative volume uncertainties were subsequently set (by consideration of the ISO 17089-1 ultrasonic meter standard uncertainty examples) as 0.5% and 0.6% respectively. In this particular example the shift in mass flowrate prediction is marginal, but the method reduces the flowrate uncertainty from the 4-path ultrasonic meter's 0.5% to 0.38%.

The data in Table 2 is for a rather low pressure and as a consequence the mass flowrate is low and the financial value of the flow relatively modest. However, the result is indicative of any 4+1 (or 4+2, 4+4 etc.) ultrasonic meter in that the flowrate prediction can be finely adjusted and the flowrate uncertainty can be significantly reduced. For many higher value custody transfer flows the advantages of this can be significant.

This example utilized the method for the case where the metering system inherently had two pre-existing independent but similar metering systems to compare. However, most standalone meters do not have two such independent sub-system flowrate predictions as a standard commercial offering. However, it is sometimes possible to add a supplemental system (or systems) to produce a second flowrate prediction. The next example shows a case where a standard flow meter with no inherent suitable extra instrumentation is modified to produce a second flow meter system such that the method can be applied.

## 2.6 Example Coriolis with DP Measurement

Coriolis meters produce a single mass flowrate prediction. Standard Coriolis meters have no second mass flow prediction in which to apply this method. But a second mass flowrate prediction can be created. The permanent pressure loss can be read across the Coriolis meter, see Fig. 8:

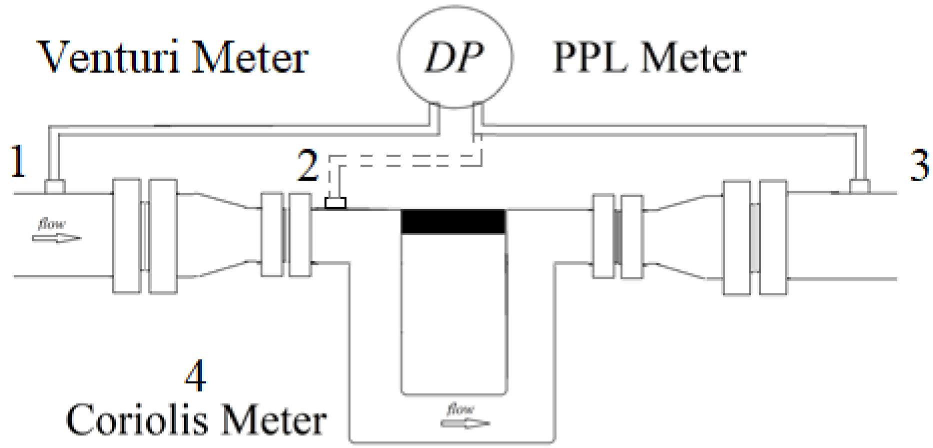


**Fig. 8 Coriolis Meter with Added Permanent Pressure Loss Reading**

This permanent pressure loss is directly related to the fluid velocity, and therefore the mass flow. The mass flowrate can therefore be calculated from this read permanent pressure loss DP. This flow metering concept is fundamentally the same as that discussed in the DP meter diagnostic / validation section 3.2 below. Equation (24) is applicable for the permanent pressure loss across any pipe obstruction, DP meter and Coriolis meter inclusive. Hence, reading the permanent pressure loss across the Coriolis meter, i.e. the differential pressure between pressure taps 1 and 3 (see Fig. 8), creates a second independent mass flowrate prediction. Furthermore, some Coriolis meters are installed with a reduced bore assembly, see Fig. 8. In this scenario there is the option to read the differential pressure across the reduction in bore, i.e. the differential pressure between pressure taps 1 and 2, thereby effectively creating an upstream Venturi meter.

Hence, say a Coriolis meter is calibrated to have a gas mass flowrate of 0.5%. This is a typical Coriolis meter uncertainty. The permanent pressure loss across a typical Coriolis meter is such that a calibrated permanent pressure loss Coriolis flow meter could have an uncertainty of 0.75%. Carrying out the MLU technique produces a reconciled (finely adjusted) flowrate prediction, and reduces the uncertainty to 0.42%.

For the specific case of a reduced bore flow meter design there is always the option to read the three DPs across the three pressure taps. This is in effect the same as the Venturi meter set up shown in Fig. 9 in section 3.2 below with the addition of there being a flow meter such as a Coriolis, ultrasonic or turbine meter etc. in the Venturi throat.

Fig. 8 shows the example of a Coriolis / Venturi meter hybrid meter. This would give three DP related flowrate predictions, to combine with the Coriolis meter flow prediction, but these DP meter predictions are not wholly independent of each other. Indeed, this is the case for the standard Venturi meter (and all DP meters such as the orifice, nozzle, cone, wedge DP meter etc.) with the three DPs as shown in Fig. 8. However, the above method of reducing uncertainty describes a linear system with one unknown and uncorrelated measurement uncertainties. If uncertainties are correlated (as in Section 3) the method can be extended to account for these correlations according to the GUM. This is now addressed and the MLU method is extended to include non-linear systems with multiple unknown variables with potentially correlated uncertainties.

# 3 MAXIMUM LIKELIHOOD UNCERTAINTY (MLU) TECHNIQUE – MULTIPLE DEPENDENT MEASUREMENTS

## 3.1 DP Flow Meters with Multiple Dependent Measurements

For differential pressure meters such as orifice, nozzle, Venturi, cone, and wedge DP meters (i.e. ISO 5167 Parts 2, 3, 4, 5, & 6 [1]) this technique can be applied if the system has multiple (two or more) DPs read.

This section specifically discussed DP meter diagnostic / verification systems. However, the 'Maximum Likelihood Uncertainty' system can be applied to any DP meter with such read DPs. This addition can be an add on to this diagnostic system, or it can be independent of the diagnostic system. That is, if the diagnostic system is present, once the diagnostic system verifies the meter is operating correctly, the Maximum Likelihood Uncertainty' system can then reduce the metering system's uncertainty. However, most flow meters are operated without diagnostic systems. Hence, the extra DP transmitter/s could be added to any DP meter without the diagnostics system, the meter can be assumed to be working correctly, and the Maximum Likelihood Uncertainty system will then reduce the metering systems flowrate prediction uncertainty.

MLU involves the use of multiple instruments in conjunction with physical laws to reduce the metering system's overall uncertainty. For the specific case of a DP meter with three DPs (as shown in Fig. 9) the multiple instruments are reading different but related signals, i.e. the three DPs are all 'dependent' on the meter body's pressure field. In this document the general term 'dependent' is here defined as systems of flow rate predictions that share some, or all, of a common set of input variables. Whereas a metering system with instrument readings which are dependent on each other is more complex than when they are independent, the fundamental principle remains the same.

## 3.2 DP Meter Diagnostics / Validation System Recap

Let us now consider this approach to DP meters with the 'Prognosis' validation system.

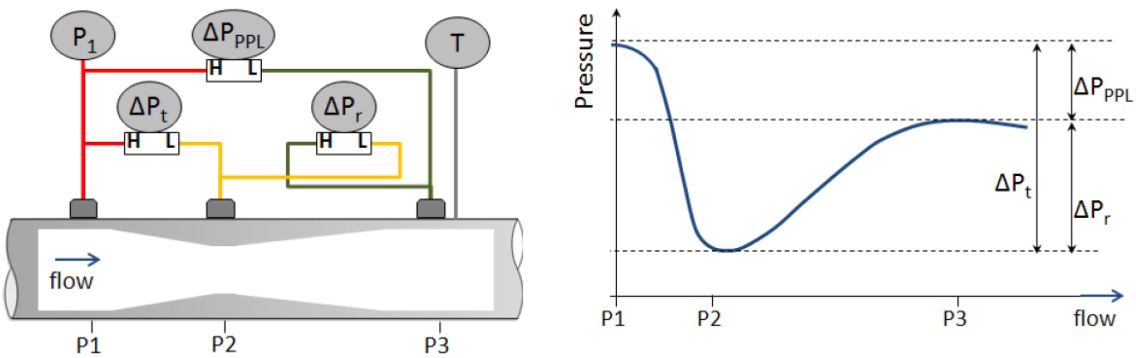


**Fig. 9 Venturi meter with instrumentation sketch and pressure field graph**

Steven 2008 [2] described a Differential Pressure (DP) meter diagnostic system that utilizes a third pressure tap downstream of the traditional DP meter. This allows the measurement of two or three DPs instead of the traditional single 'primary' DP.

Fig. 9 shows a sketch of a generic DP meter with three DP readings and the meter's pressure field. There is a third pressure tap ($P_3$) downstream of the two primary (or 'traditional') pressure ports ($P_1$ & $P_2$). This allows three DPs to be read, i.e. the primary or 'traditional' DP ($\Delta P_t$), recovered DP ($\Delta P_r$) and permanent pressure loss DP ($\Delta P_{PPL}$).

Each DP can be used to individually meter the flow rate. Hence, every DP meter with an added downstream pressure tap ($P_3$) is potentially three flow meters in one body. Inter-comparison of these flow rate predictions produces three diagnostic checks. There are three read DP ratios, i.e., the 'PLR' ($\Delta P_{PPL}/\Delta P_t$), the PRR ($\Delta P_r/\Delta P_t$), the RPR ($\Delta P_r/\Delta P_{PPL}$). DP meters have predictable reproducible DP ratios. Therefore, comparison of each read to expected DP ratio produces three diagnostic checks.

These checks, i.e three flowrate comparisons, three DP ratio comparisons to known baselines, the DP sum, and a signal / parameter stability check, create a 'diagnostic suite' whose output creates a diagnostic pattern. Pattern recognition allows the source of varous meter malfunctions to be predicted. Rather than primary malfunction identification, once the other seven diagnostic checks have identified a problem, the eighth diagnostic is primarily used to aid pattern recognition.

This DP meter validation system is specifically the use of a third downstream pressure tap, to facilitate the reading of two or three DPs, instead of the standard meter's single DP reading, such that the extra information can specifically be used to create a DP meter validation tool. The three flowrate predictions are in effect a check meter system. The expansion and PPL meter flowrate predictions are only used to verify the primary meter, they are not used in anyway to finely adjust and lower the uncertainty of the overall metering system's flowrate prediction. The system states if the primary meter is fully operational, or if it has malfunctioned. If it has malfunctioned it will give guidance via diagnostic pattern recognition on what the problem may be. It does not in any way attempt to reduce the overall system flow rate prediction uncertainty.

### 3.3 MLU – Mathematical Development for DP Meters with Multiple Dependent Measurements

The method requires redundancy in the set of measurements. For the case of a single DP meter with one differential pressure measurement there is just sufficient information to calculate the mass flow, (given other inputs such as discharge coefficient, dimensions, fluid density, etc.). The mass flow is termed as simply observable in that there is only one value it can be given the available measurements. If there are additional DP measurements associated with the meter, this allows more than one calculated mass flow to be obtained and the system exhibits redundancy.

As in the independent meter case, the redundant information arising from three DP measurements allows a reconciled mass flow rate to be calculated. Furthermore, these calculations enable the uncertainty in reconciled variables to be derived and in particular the mass flow rate uncertainty.

It is assumed the DP meter has been demonstrated to be functioning correctly; by Prognosis or any other equivalent means such as gross error detection techniques ([8], [9]). The meter must be 'healthy' otherwise data reconciliation will 'smear' gross errors in the measured data over the reconciled results and distort the calculated uncertainties.

Only single-phase flow is considered. It may be possible to extend this method to include two-phase flow in the future.

The principle behind the MLU approach is independent of DP meter type, because of the manner in which meters are operated. Venturis and cone meters are calibrated, whereas orifice meters are generally uncalibrated and ISO 5167 defines empirical relationships used to calculate mass or volume flow rates from the recovered and PPL DPs. The method is not limited to Venturi, Cone and Orifice meters. It can be applied generally to DP meters with redundant DP measurements.

The following is one example of the more complex mathematics required for the case where constraints are nonlinear in the measured, calibrated and unmeasured variables. It is also suitable where the instrument readings are not wholly independent. However, it is an example only, and the fundamental principle holds true if a different mathematical method was chosen.

For any DP meter the first law of thermodynamics requires that within uncertainties the measured traditional ($\Delta P_t$), recovered ($\Delta P_r$), and permanent pressure loss ($\Delta P_{PPL}$) differential pressures should satisfy:

$$\Delta P_t = \Delta P_r + \Delta P_{PPL} \tag{21}$$

To establish the principles of the data reconciliation technique let us assume we are dealing with a Venturi meter.

From ISO-5167 [1] and Steven [2] the 'traditional', 'recovered' and 'permanent pressure loss' mass flow rates $\dot{m}_t$, $\dot{m}_r$ and $\dot{m}_{PPL}$ are given by equations (22), (23) and (24):

$$\dot{m}_t = \dot{m}_t(d, D, Y, C_d, \rho, \Delta P_t) = EA_tYC_d(2\rho\Delta P_t)^{1/2} \tag{22}$$

$$\dot{m}_r = \dot{m}_r(d, D, K_r, \rho, \Delta P_r) = EA_tK_r(2\rho\Delta P_r)^{1/2} \tag{23}$$

$$\dot{m}_{PPL} = \dot{m}_{PPL}(D, K_{PPL}, \rho, \Delta P_{PPL}) = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2} \tag{24}$$

Where

- $D$ is the meter inlet diameter and the meter inlet area, $\mathrm{A} = \frac{\pi \mathrm{D}^2}{4}$
- $d$ is the meter throat diameter and the meter throat area, $A_t = \frac{\pi d^2}{4}$
- $C_d$ is the discharge coefficient
- $K_r$ is the expansion coefficient
- $K_{PPL}$ is the permanent pressure-loss coefficient
- $Y$ is the fluid's expansibility
- ρ is the fluid's density

The discharge, expansion and PPL coeffcients are normally calibrated at a test facility for a Venturi meter and in order to meet a client's uncertainty requirements may be Reynolds number dependent.

As usual, the meter beta ratio, β, and 'velocity of approach', $E$, are as defined in equations (25) and (26) respectively.

$$\beta = \sqrt{\frac{A_t}{A}} \tag{25}$$

$$E = \frac{1}{\sqrt{1-\beta^4}} \tag{26}$$

Equations (27), (28), (29) and (30) therefore provide four constraints which a reconciled mass flow rate, $\widehat{m}$, and variables in these equations should satisfy.

$$\Phi = \hat{\dot{m}} - \dot{m}_t\left(\hat{d}, \hat{D}, \hat{Y}, \hat{C}_d, \hat{\rho}, \Delta\hat{P}_t\right) = 0 \qquad (27)$$

$$\Xi = \hat{\dot{m}} - \dot{m}_r\left(\hat{d}, \hat{D}, \hat{K}_r, \hat{\rho}, \Delta\hat{P}_r\right) = 0 \qquad (28)$$

$$\Psi = \hat{\dot{m}} - \dot{m}_{PPL}\left(\hat{D}, \hat{K}_{PPL}, \hat{\rho}, \Delta\hat{P}_{PPL}\right) = 0 \qquad (29)$$

$$\Omega = \Delta\hat{P}_t - \Delta\hat{P}_r - \Delta\hat{P}_{PPL} = 0 \qquad (30)$$

Until now the three DP readings were treated separately giving three mass flow rates according to equations (22), (23) and (24). By applying MLU, a ***single*** reconciled mass flow rate consistent with constraints (27), (28), (29) and (30) is now calculated. Information from all DP readings is included by MLU in this single mass flow rate, which is more likely to be more representative of the true flow rate while at the same time having a lower uncertainty.

Constraints (22), (23) and (24) are nonlinear and contain the unmeasured mass flow rate, $\hat{\dot{m}}$. In a previous paper a weighted least-squares optimisation approach to data reconciliation with nonlinear constraints and all variables measured was applied to allocation on Maersk's Global Producer III [10]. That paper utilized a data reconciliation algorithm for a system with unmeasured variables developed by Britt and Luecke in [3], and it is this algorithm applied here.

The algorithm takes a vector x of measured or calibrated variables,

$$x = [\Delta P_t \quad \Delta P_r \quad \Delta P_{PPL} \quad Y \quad C_d \quad K_r \quad K_{PPL} \quad \rho] \qquad (31)$$

and unmeasured variables, mass flow rate in this instance,

$$u = [\dot{m}] \qquad (32)$$

and derives the corresponding reconciled variables:

$$x = [\Delta\hat{P}_t \quad \Delta\hat{P}_r \quad \Delta\hat{P}_{PPL} \quad \hat{Y} \quad \hat{C}_d \quad \hat{K}_r \quad \hat{K}_{PPL} \quad \hat{\rho}] \qquad (33)$$

and reconciled unmeasured mass flow rate,

$$\hat{u} = \left[\hat{\dot{m}}\right] \qquad (34)$$

Analogous to the maximum likelihood approach described in section 2, the reconciled values are obtained by minimising the weighted sum:

$$S = \sum_i \left(\frac{\hat{x}_i - x_i}{\sigma_{x_i}}\right)^2 \qquad (35)$$

subject to constraints (27), (28), (29) and (30). In equation (35) $\sigma_{x_i}$ is the uncertainty in each of the measured variables.

In the following analysis the uncertainties are derived from vendor product data sheets, or according to standard uncertainties quoted in ISO 5167, or from calibration results, depending on the variable.

The calculations require the iterative solving of equations (36) and (37) below until the differences between iterations in both the measured and unmeasured variables is less than some chosen threshold.

$$u_{i+1} = u_i - (J_u^T(J_xVJ_x^T)^{-1}J_u)^{-1}J_u^T(J_xVJ_x^T)^{-1}\big(J_xx_0 + J_x(x_i - x_0)\big) \tag{36}$$

$$x_{i+1} = x_0 - VJ_x^T(J_xVJ_x^T)^{-1}\big(J_xx_0 + J_u(u_i - u_0) + J_x(x_i - x_0)\big) \tag{37}$$

In equations (36) and (37) $V$ is the covariance matrix of measured variables:

$$V = \begin{bmatrix} \sigma_{\Delta P_t}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_\rho^2 \end{bmatrix} \tag{38}$$

and $J_x$ and $J_u$ are Jacobian matrices:

$$J_x = \begin{bmatrix} \frac{\partial\Phi}{\partial\Delta P_t} & \frac{\partial\Phi}{\partial\Delta P_r} & \frac{\partial\Phi}{\partial\Delta P_{PPL}} & \frac{\partial\Phi}{\partial Y} & \frac{\partial\Phi}{\partial C_d} & \frac{\partial\Phi}{\partial K_r} & \frac{\partial\Phi}{\partial K_{PPL}} & \frac{\partial\Phi}{\partial\rho} \\ \frac{\partial\Xi}{\partial\Delta P_t} & \frac{\partial\Xi}{\partial\Delta P_r} & \frac{\partial\Xi}{\partial\Delta P_{PPL}} & \frac{\partial\Xi}{\partial Y} & \frac{\partial\Xi}{\partial C_d} & \frac{\partial\Xi}{\partial K_r} & \frac{\partial\Xi}{\partial K_{PPL}} & \frac{\partial\Xi}{\partial\rho} \\ \frac{\partial\Psi}{\partial\Delta P_t} & \frac{\partial\Psi}{\partial\Delta P_r} & \frac{\partial\Psi}{\partial\Delta P_{PPL}} & \frac{\partial\Psi}{\partial Y} & \frac{\partial\Psi}{\partial C_d} & \frac{\partial\Psi}{\partial K_r} & \frac{\partial\Psi}{\partial K_{PPL}} & \frac{\partial\Psi}{\partial\rho} \\ \frac{\partial\Omega}{\partial\Delta P_t} & \frac{\partial\Omega}{\partial\Delta P_r} & \frac{\partial\Omega}{\partial\Delta P_{PPL}} & \frac{\partial\Omega}{\partial Y} & \frac{\partial\Omega}{\partial C_d} & \frac{\partial\Omega}{\partial K_r} & \frac{\partial\Omega}{\partial K_{PPL}} & \frac{\partial\Omega}{\partial\rho} \end{bmatrix} \tag{39}$$

$$J_u = \begin{bmatrix} \frac{\partial\Phi}{\partial\dot{m}} \\ \frac{\partial\Xi}{\partial\dot{m}} \\ \frac{\partial\Psi}{\partial\dot{m}} \\ \frac{\partial\Omega}{\partial\dot{m}} \end{bmatrix} \tag{40}$$

After evaluation of the partial differentials in equation (39), the Jacobian matrix $J_x$ becomes:

$$J_x = \begin{bmatrix} -m_t/2\Delta P_t & 0 & 0 & -m_t/Y & -m_t/C_d & 0 & 0 & -m_t/2\rho \\ 0 & -m_r/2\Delta P_r & 0 & 0 & 0 & -m_t/K_r & 0 & -m_r/2\rho \\ 0 & 0 & m_{PPL}/2\Delta P_{PPL} & 0 & 0 & 0 & -m_t/K_{PPL} & -m_{PPL}/2\rho \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

And the Jacobian matrix $J_u$ in the unmeasured variables becomes:

$$J_u = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{42}$$

Measured values are used as inputs to $x_0$ in the first iteration. An initial estimate of the mass flow rate used in the first iteration, $u_0$, can be set from any one, or any combination, of the traditional, recovered and PPL calculated mass flow rates, e.g. an arithmetic average or uncertainty weighted average.

The iterations stop when:

$$\sum |u_{i+1} - u_i| \leq \delta \tag{43}$$

And,

$$\sum |x_{i+1} - x_i| \leq \varepsilon \tag{44}$$

The set of equations are solved using the following iterative scheme:

1. Obtain the measured input data and enter into the vector x in equation (31);
2. Estimate an initial value of $\dot{m}$ and enter into (32);
3. Obtain the uncertainties in the measured input data and populate the covariance matrix V (38);
4. Calculate the entries in the Jacobian matrix $J_x$ according to (41), using the entries in vector x (the entries in the Jacobian matrix $J_u$ are constants in accordance with (42));
5. Obtain updated values of both the unmeasured and measured variables from (36) and (37) respectively;
6. Compare the values of the differences between the updated values and those from the previous iteration for the unmeasured and measured variables and check for convergence using (43) and (44);
7. If convergence not achieved then update vectors x and u with the updated values of the measured and unmeasured variables and return to 4, other wise the iteration stops;

If convergence is achieved then the latest values of the measured and unmeasured variables are the reconciled values according to (33) and (34).

Importantly, analogous to the examples in section 2, the ***uncertainty in the reconciled mass flow rate*** can be calculated from:

$$\sigma_u = (J_u^T (J_x V J_x^T)^{-1} J_u)^{-1} \tag{45}$$

### 3.4 Calibrated Cone DP Meter with Pressure Field DP Readings

In this section we present the results of applying this method to a cone DP meter with three DP readings as shown in Fig. 10:

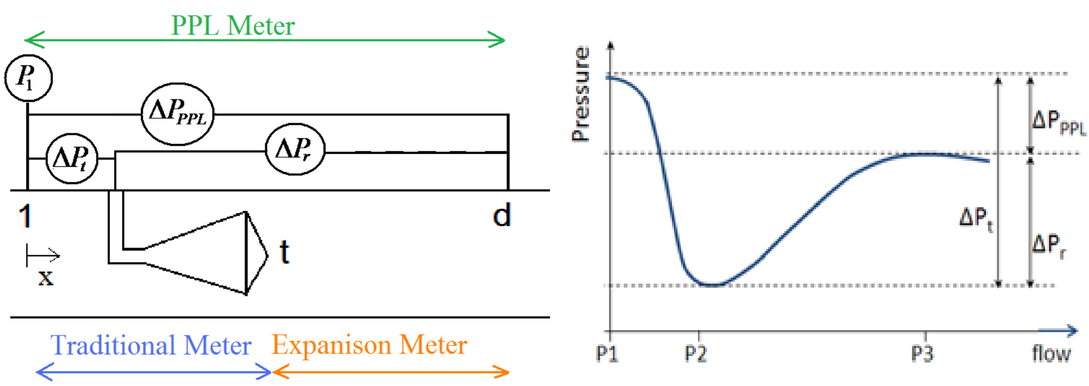


**Fig. 10 Cone DP Meter with instrumentation sketch and pressure field graph**

As with the Venturi meter in Fig. 9 the three DPs can each be used to predict the flow (via equations (22), (23) and (24)) but as the DPs are related these three flowrate predictions are not independent.

This example's data set is from a 14" 0.56 beta cone DP meter calibrated at a natural gas calibration facility (see Fig. 11).


**Fig. 11 - 14" Cone DP Meter Under Test**

The meter had an inlet diameter of 0.337 m and cone diameter of 0.28 m. Cone meter geometry uncertainty is accounted for in the calibrated flow coefficient data. This standalone cone meter's mass flowrate prediction uncertainty is 0.78% at the flow rate in this example.

Measured input variables, relative uncertainties and absolute uncertainties are listed in Table 3.

**Table 3 - 14", 0.56 Beta Cone DP Meter Variable and Parameter Uncertainties**

| Measurement | Unit | Value | Relative Uncertainty | Absolute Uncertainty |
|---|---|---|---|---|
| $DP_t$ | Pa | 2759 | 1.17% | 32.3 |
| $DP_r$ | Pa | 948 | 0.86% | 8.1 |
| $DP_{PPL}$ | Pa | 1774 | 1.82% | 32.3 |
| Y | Dimensionless | 0.9996 | 0.004% | 0.00004 |
| $C_d$ | Dimensionless | 0.8514 | 0.50% | 0.004257 |
| $K_r$ | Dimensionless | 1.440 | 2.50% | 0.0360 |
| $K_{PPL}$ | Dimensionless | 0.344 | 1.00% | 0.00344 |
| P | $kg/m^3$ | 33.6 | 0.27% | 0.09 |

These measurements comprise the inputs into the vector x in equation (31) and the covariance matrix, V, in equation (38). Uncertainties are stated at the 95%

confidence level. The covariance matrix V is diagonal with entries given by the squares of the absolute uncertainties listed in Table 3.

V= diag (1045, 66.2, 1045, 1.60E-09, 1.81E-05, 1.30E-03, 1.18E-05, 8.22E-03)

The derived traditional, recovered and PPL mass flow rates calculated according to Equations (22), (23) and (24) are shown in Table 4 along with their arithmetic average. The arithmetic average may be used as the initial estimate of the reconciled mass flow rate and input into vector u of equation (38). (Alternative methods to calculate the iniitial eatimate of the mass flow rate for input into vector u could be used, such as an uncertainty weighted average mass flow rate, or simply choosing the traditional mass flow rate as it typically has the lowest uncertainty).

**Table 4 - 14", 0.56 Beta Cone DP Meter Three Related Flowrate Predictions**

| Mass Flow Rate | Value (kg/s) |
|---|---|
| $m_{trad}$ | 10.6135 |
| $m_{exp}$ | 10.5263 |
| $m_{PPL}$ | 10.5855 |
| Arithmetic Average | 10.5751 |

The iterative scheme described in section 3.3 was applied and the algorithm converged after 3 iterations using threchild values of $\delta = \varepsilon = 10^{-6}$. The results are presented in Table 5:

**Table 5 – MLU Results for 14", 0.56 Beta Cone DP Meter**

| Measured Variable | Initial Value | Adjustment | Reconciled Value |
|---|---|---|---|
| $DP_t$ | 2759.4650 | 20.1387 | 2739.3263 |
| $DP_r$ | 948.1350 | -1.9463 | 950.0813 |
| $DP_{PPL}$ | 1774.7270 | -14.5179 | 1789.2449 |
| Y | 0.9996 | -5.6E-08 | 0.9996 |
| $C_d$ | 0.8514 | -0.00074 | 0.8521 |
| $K_r$ | 1.4400 | -0.00640 | 1.4464 |
| $K_{PPL}$ | 0.3441 | 0.00145 | 0.3426 |
| P | 33.5792 | 0 | 33.5792 |
| | | | |
| Unmeasured Variable | Initial Value | Adjustment | Reconciled Value at end of iteration |
| M | 10.6135 | 7.63E-08 | 10.5839 |

Since the method has converged the mass flow rate uncertainty may now be calculated. It is calculated according to equation (45) which gives the variance of the mass flow rate var(m) = 6.12E-03. The absolute uncertainty in mass flow rate is therefore σ_m = sqrt(var(m)) = 0.078 kg/s. Thus, the relative uncertainty in reconciled mass flow rate is ε_m = 0.51%, compared to 0.59% relative uncertainty in the traditional mass flow rate.

Fig. 12 shows the meter's mass flow rate relative uncertainty calculated according to ISO 5167-5 for the calibration data set. This is referred to as the 'Traditional' uncertainty. Also shown is the 'Reconciled' mass flow rate uncertainty, calculated according to equation (45).

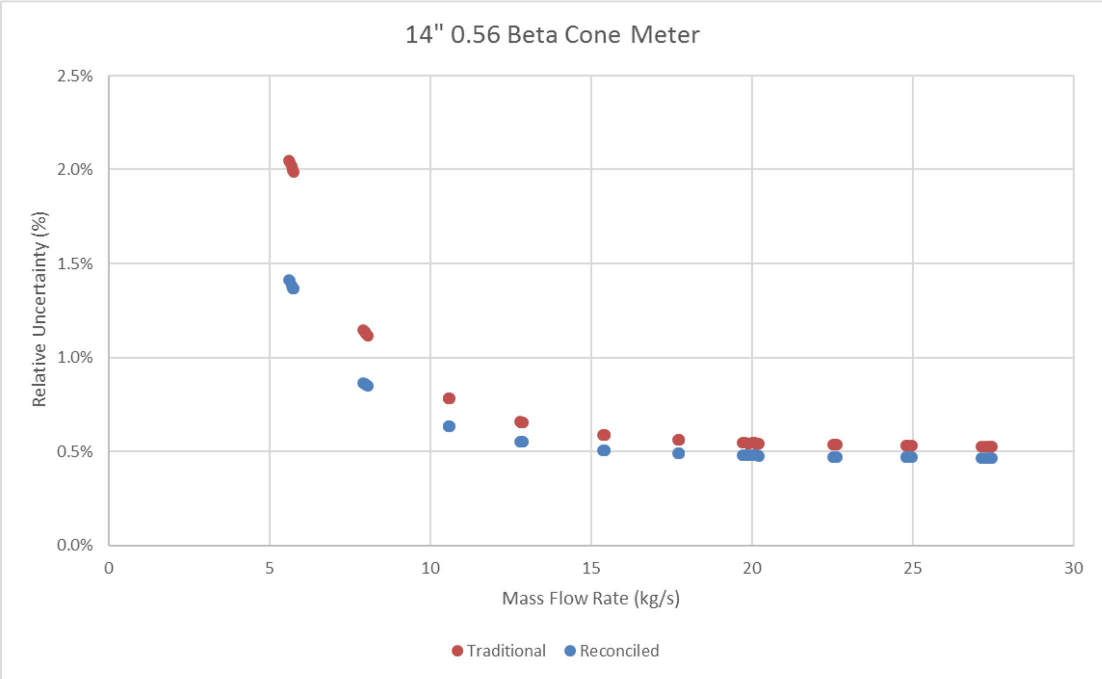


**Fig. 12 - 14" Cone DP Meter Traditional and Reconciled mass flow rate uncertainties versus mass flow rate**

At high flow rates / Reynold's number, the traditional and reconciled mass flow rates are close, although as expected the reconciled flow rate uncertainty is slightly lower than the traditional flwo rate uncertainty. Use of the additional DP measurements affects the mass flow rate uncertainty much more significantly at low flow rates/Re.

The relative and abolute reduction in uncertainty obtained by MLU is plotted against flow rate in Fig. 13:

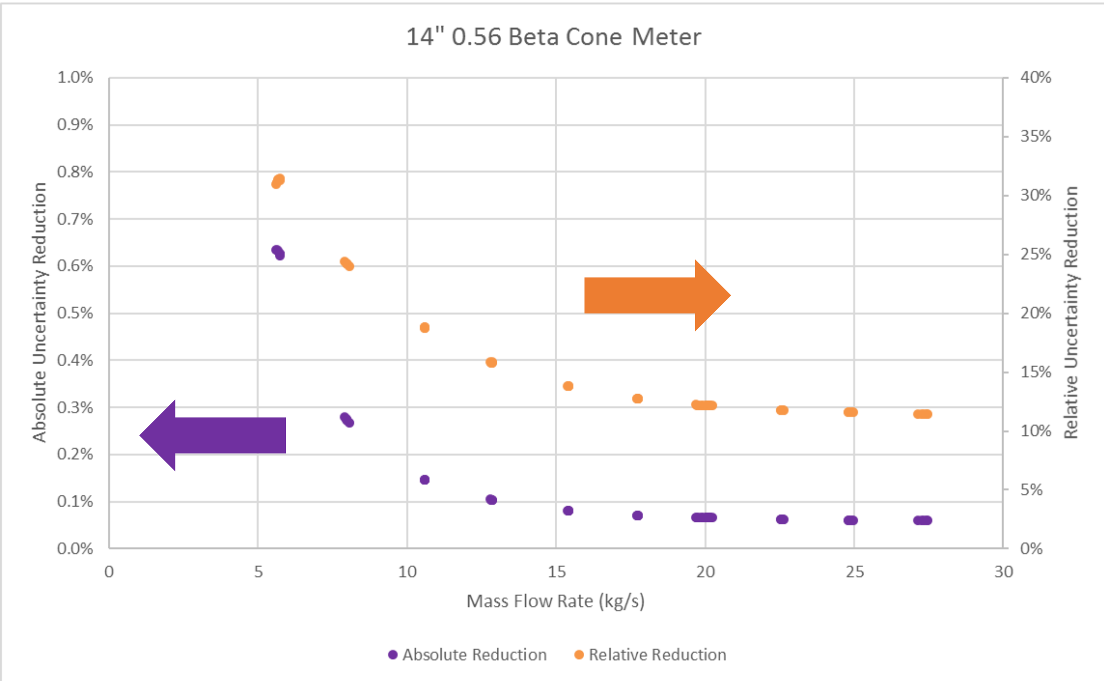


**Fig. 13 - 14" Cone DP Meter Relative and Absolute Reduction in the mass flow rate uncertainty**

Furthermore, the gas amount billing has been finely adjusted. The stand-alone 14" cone meter reads 10.613 kg/s at 0.78% relative uncertainty. The combined system MLU technique calculates 10.584 kg/s at 0.64%. That is the system has shown that the standalone cone meter is most probably over-reading the gas flow by i.e. 0.029 kg/s, i.e. 2.51 tonnes of gas per day, or 914.5 tonnes of gas /annum. This is a positive shift of 43.8 MMSCF per year (about $110,000 / per year at $2.5 / million BTU prices). Without the MLU technique this issue goes unchecked for the life of the meter, often many years. Note that this example shows a negative shift in flowrate prediction. For all MLU techniques always reduce the flowrate prediction uncertainty, the flowrate prediction results for any given application can be positive or negative shifts.

## 3.5 Uncalibrated Orifice DP Meter with Pressure Field DP Readings

In this example, we consider an uncalibrated 4", 0.5 beta orifice meter with an inlet diameter of 0.102 m and throat diameter of 0.0508 m. The standalone uncertainty of the uncalibrated meter (based on AGA Report 3 [7]) is 0.79%. For this example, as the orifice meter is not normally calibrated the inlet and throat diameters, *D* and *d*, are also considered measured variables in the reconciliation. They are therefore required to be included in the vector of measurements, *x*, the Jacobian, $J_x$, and the covariance matrix, *V.*

Measured input variables, relative uncertainties and absolute uncertainties are listed in Table 6.

**Table 6 - 4", 0.5 Beta Orifice DP Meter Variable and Parameter Uncertainties**

| Measurement | Unit | Value | Percent Uncertainty | Absolute Uncertainty |
|---|---|---|---|---|
| $DP_t$ | Pa | 90059.66 | 1.00% | 900.597 |
| $DP_r$ | Pa | 23751.81 | 1.00% | 237.518 |
| $DP_{PPL}$ | Pa | 66282.69 | 1.00% | 662.827 |
| D | M | 0.051 | 0.05% | 0.000 |
| D | M | 0.102 | 0.25% | 0.000 |
| Y | Dimensionless | 0.991 | 0.30% | 0.003 |
| $C_d$ | Dimensionless | 0.605 | 0.50% | 0.003 |
| $K_r$ | Dimensionless | 1.162 | 1.50% | 0.017 |
| $K_{PPL}$ | Dimensionless | 0.178 | 1.00% | 0.002 |
| P | kg/m$^3$ | 36.304 | 0.27% | 0.098 |

These measurements comprise the inputs into the vector x in equation (31) and the covariance matrix, V, in equation (38).

Uncertainties are stated at the 95% confidence level. The covariance matrix *V* is diagonal with entries given by the squares of the absolute uncertainties listed in Table 6:

V= diag(811074, 56414, 439339, 6.5E-10, 6.5E-08, 9E-06, 9.2E-06, 3.04E-4, 3.2E-06, 9.6E-03).

The derived traditional, recovered and PPL mass flow rates calculated according to Equations (22), (23) and (24) are shown in Table 7 along with their arithmetic average.

**Table 7 - 4", 0.5 Beta Orifice DP Meter Three Related Flowrate Predictions**

| Mass Flow Rate | Value (kg/s) |
|---|---|
| $m_{trad}$ | 10.6135 |

| | |
|---|---|
| $m_{exp}$ | 10.5263 |
| $m_{PPL}$ | 10.5855 |
| Arithmetic Average | 10.5751 |

The iterative scheme described in section 3.3 was applied and the algorithm converged; the results are presented in Table 5:

**Table 8 – MLU Results for 4", 0.5 Beta Uncalibrated Orifice DP Meter**

| Measured Variable | Initial Value | Adjustment | Reconciled Value |
|---|---|---|---|
| $DP_t$ | 90059.66 | 38.4760 | 90021.1884 |
| $DP_r$ | 23751.81 | -23.1928 | 23775.0006 |
| $DP_{PPL}$ | 66282.69 | 36.5002 | 66246.1878 |
| d | 0.0508 | -3.64E-07 | 0.0508 |
| D | 0.1023 | 1.83E-05 | 0.1022 |
| Y | 0.9914 | 4.63E-05 | 0.9913 |
| $C_d$ | 0.605 | 7.86E-05 | 0.6049 |
| $K_r$ | 1.162 | -0.004860 | 1.1669 |
| $K_{PPL}$ | 0.178 | 0.000241 | 0.1781 |
| ρ | 36.304 | 0 | 36.3039 |
| | | 1.88E-07 | Converged |
| | | | |
| Unmeasured Variable | Initial Value | Adjustment | Reconciled Value |
| m | 3.204 | 1.8E-12 | 3.2064 |

Applying the Since the MLU method has converged the mass flow rate uncertainty may now be calculated. It is calculated according to equation (45) which gives the variance of the mass flow rate var($m$) = 3.6E-04. The absolute uncertainty in mass flow rate is therefore = sqrt(var($m$)) = 0.019 kg/s. Thus, the relative uncertainty in reconciled mass flow rate is = 0.59%, compared to 0.79% relative uncertainty in the traditional mass flow rate.

Fig. 14 shows the meter's mass flow rate relative uncertainty calculated according to ISO 5167-5 for the calibration data set. This is referred to as the 'Traditional' uncertainty. Also shown is the 'Reconciled' mass flow rate uncertainty, calculated according to equation (45).

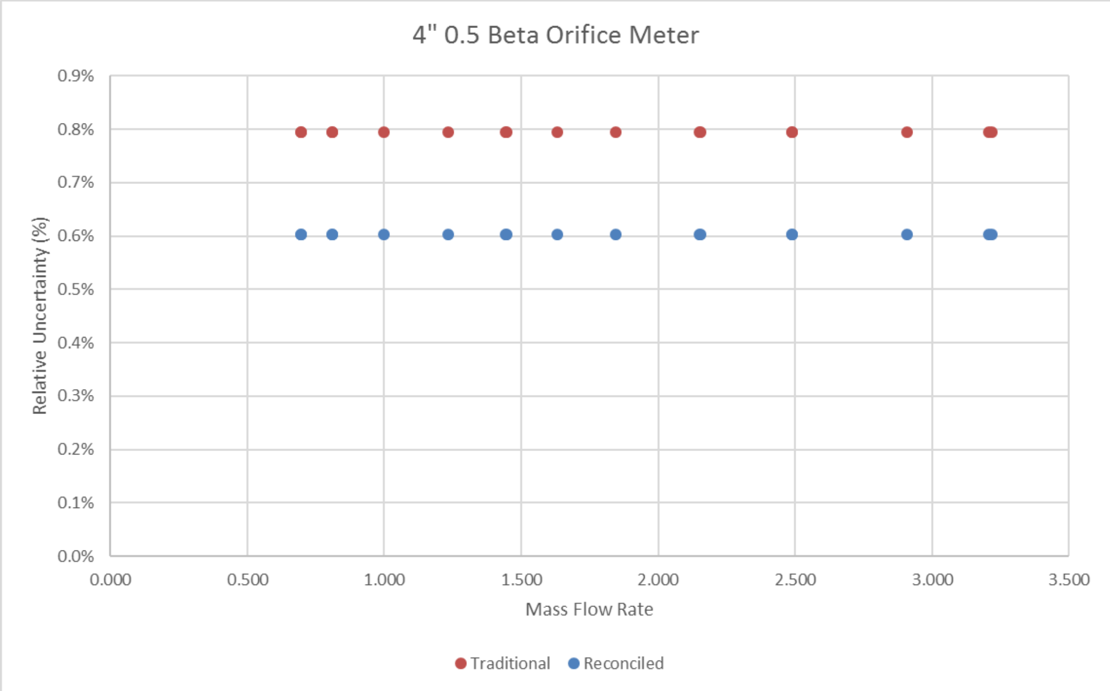


**Fig. 14 - 4" Uncalibrated Orifice DP Meter Traditional and Reconciled mass flow rate uncertainties versus mass flow rate**

There is a consistent and significant reduction in uncertainty at all flow rates / Reynold's number, for the reconciled flow when compared to the traditional value. The relative and abolute reduction in uncertainty obtained by MLU is plotted against flow rate in Fig. 15:

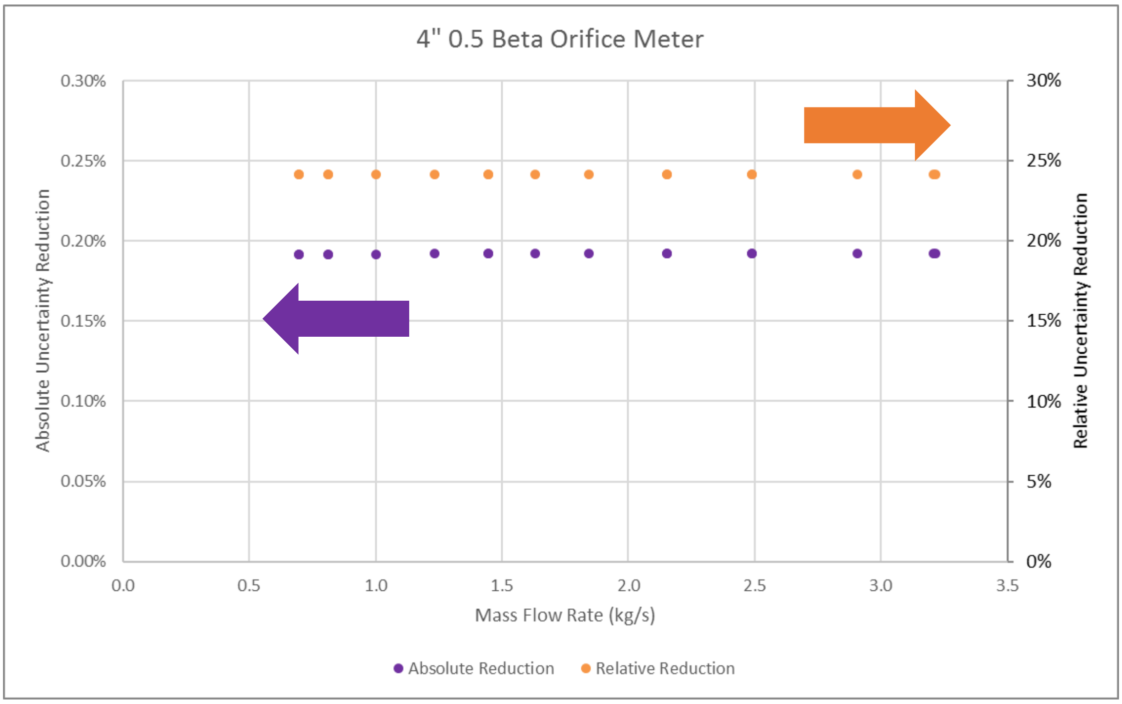


**Fig. 15 - 14" Uncalibrated Orifice DP Meter Relative and Absolute Reduction in the mass flow rate uncertainty**

Furthermore, the gas amount billing has been finely adjusted. The stand-alone 4" orifice meter reads 3.208 kg/s at 0.74%. The combined system's MLU technique calculates 3.206 kg/s at 0.497%. That is the system has shown that the standalone cone meter is most probably over-reading the gas flow by i.e. 0.002 kg/s, i.e. 172.8

tonnes of gas per day, or 63.1 tonnes of gas /annum. This is a positive shift of 27.2 MMSCF per year (about $68,000 / per year at $2.5 / million BTU prices). Without the MLU technique this issue goes unchecked for the life of the meter, often many years. Note that this example shows a positive shift, but MLU results of any given application can be positive or negative shifts in flowrate prediction. This example is for a small (i.e. 4”) meter. The financial significance of this technology applied to larger meters is significantly greater

The method is not limited to DP meters with three independent DP measurements, but applies to any meter with two or more measurements.

## 4 CONCLUSIONS

The patent pending MLU method in essence is the idea of utilizing macro scale pipework process ‘MLU’ techniques to the micro scale of individual flow metering systems. Existing macro pipe network systems use MLU techniques to gauge the overall state of the entire process. Here the flow meters in the network are not subject to any internal MLU techniques, and simply supply flowrate predictions as a data point. That data point input is assumed as correct and as accurate and reliable as is possible. However, in this approach data analysis techniques are applied within individual metering systems with multiple data readings such that this metering systems flow rate prediction is improved.

## 5 NOTATION

| | |
|---|---|
| $A$ | Meter inlet area , $A = \frac{\pi D^2}{4}$ |
| $A_t$ | Meter throat area, $A_t = \frac{\pi d^2}{4}$ |
| $C_d$ | Discharge coefficient |
| $d$ | Meter throat diameter |
| $D$ | Meter inlet diameter |
| $E$ | ‘Velocity of Approach’ |
| $K_r$ | Expansion coefficient |
| $K_{PPL}$ | Permanent Pressure-Loss coefficient |
| $J_x$ | Jacobian of measured variables |
| $J_u$ | Jacobian of unmeasured variables |
| L | Log likelihood |
| ṁ | Flow rate |
| Pd | Probability density |
| S | Sum of squares |
| u | Vector of unmeasured variables |
| Uṁ | Absolute uncertainty in flow rate |
| V | Covariance matrix |
| x | Factor, vector of measured variables |
| $Y$ | Expansibility |

**Greek**

| | |
|---|---|
| β | Meter beta-ratio |
| δ | Threshold |
| ΔP | Differential pressure |
| ε | Threshold |
| εṁ | Relative uncertainty in flow rate |
| ρ | Density |
| σ | Standard deviation, absolute uncertainty |
| Φ | Constraint variable |
| Ξ | Constraint variable |
| Ψ | Constraint variable |
| Ω | Constraint variable |

**Superscripts**

| | |
|---|---|
| ˆ | Reconciled |

**Subscripts**

| | |
|---|---|
| 1 | Meter 1, Pressure tap 1 |
| 2 | Meter 2, Pressure tap 2 |
| 3 | Pressure tap 3 |
| c | Cone |
| i | Iteration number |
| r | Reconciled |
| t | True |
| v | Vortex |

## 6 REFERENCES

[1] International Standard Organisation Measurement of Fluid Flow by Means of Pressure Differential Devices, Inserted in circular cross-sections running full, no. 5167 2003.

[2] Steven R., Diagnostic Methodologies for Generic Differential Pressure Flow Meters, North Sea Flow Measurement Workshop 2008, St. Andrews, Scotland, UK.

[3] Britt, H. I., Luecke, R. H., The Estimation of Parameters in Nonlinear, Implicit Models Technometrics Vol 15 No 2 (May 1973) 233 – 247.

[4] Steven, R., US Patent 2010/0224009 A1, Hybrid Flowmeter that Includes an Integral Vortex Flowmeter and a Differential Flow Meter.

[5] Boden, US Patent 2772567A, Granted 1956.

[6] Guide to the Expression of Uncertainty in Measurement, International Organisation for Standardisation, ISO/IEC Guide 98:1995

[7] AGA Report 3, Orifice Metering of Natural Gas and Other Related Hydrocarbon Fluids, 4th Edition, 2000

[8] Drysdale, E., Little, H., Stockton, P., Wilson, A., Gross Meter Error Detection and Elimination by Data Reconciliation, North Sea Flow Measurement Workshop 2011, Tønsberg, Norway.

[9] Corbett, N., Sibbald, R., Stockton, P., Wilson, Gross Error Detection: Maximising the Use of Data with UBA on Global Producer III (Part 2) North Sea Flow Measurement Workshop 2015, Tønsberg, Norway.

[10] Corbett, N., Johnston, J., Sibbald, R., Stockton, P., Wilson, Allocation in an Uncertain World: North Sea Flow Measurement Workshop 2013, Tønsberg, Norway.