The Introspective Orifice Meter – Uncertainty Improvements

by

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This is what I'm going to talk about

North Sea FLOW Measurement Workshop

1. Recap Maximum Likelihood Uncertainty (MLU)
2. The next dimension - TIME
3. Kalman Filter
4. Thought experiment – theoretical example
5. Real data
6. Conclusions – the introspective orifice meter

Heads and tails sequences

What Consider the two patterns HTH and HTT

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Which of the following is true:

A The average number of tosses until HTH is **larger** than the average number of tosses until HTT

B The average number of tosses until HTH is the **same** as the average number of tosses until HTT

C The average number of tosses until HTH is **smaller** than the average number of tosses until HTT

Recap Maximum Likelihood Uncertainty (MLU)

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1. **Recap Maximum Likelihood Uncertainty (MLU)**
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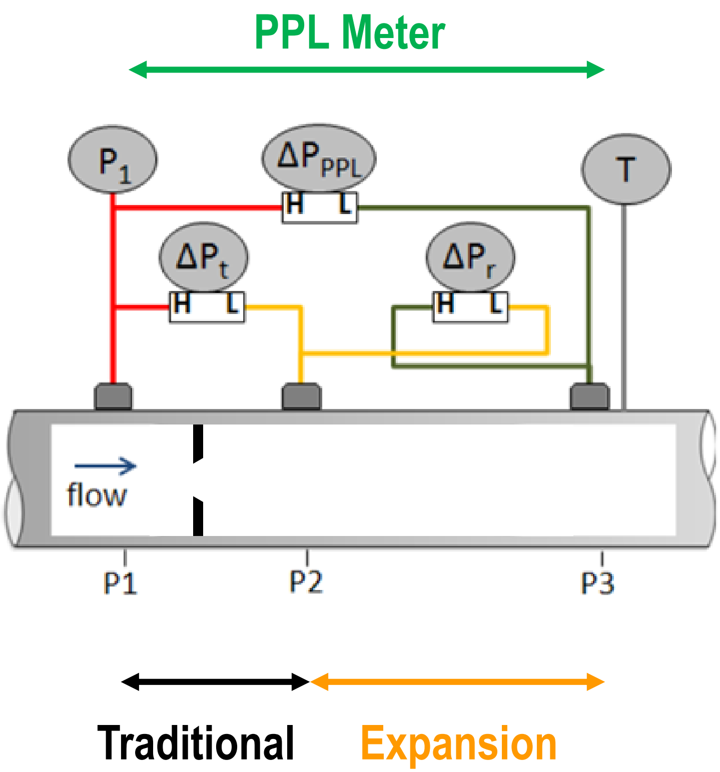
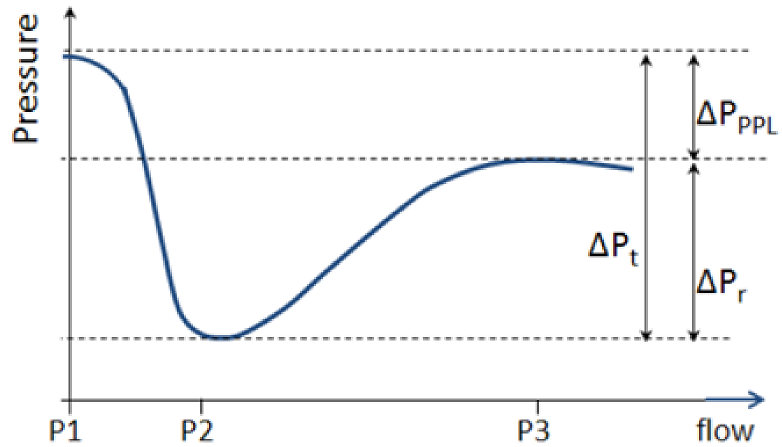
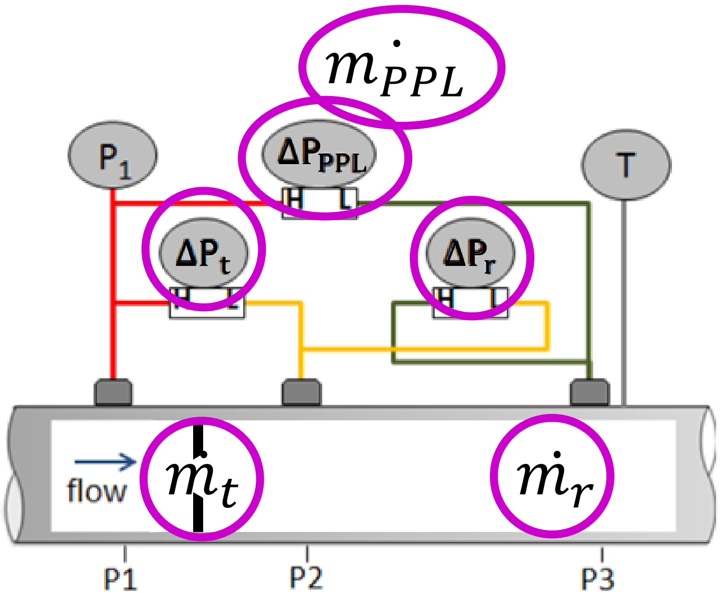
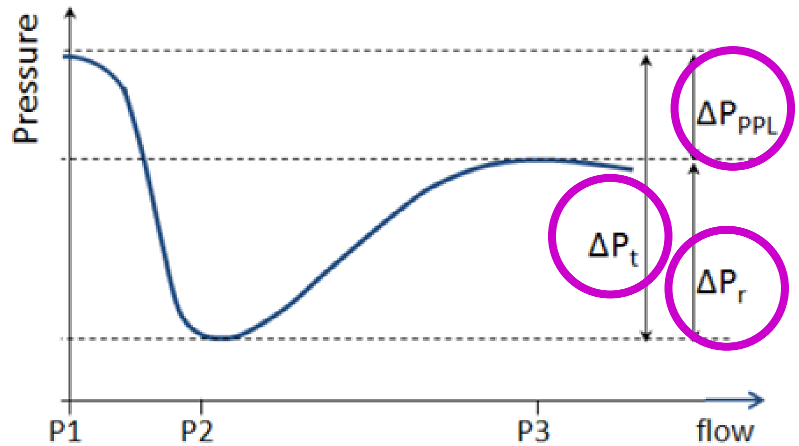
DP Diagnostics



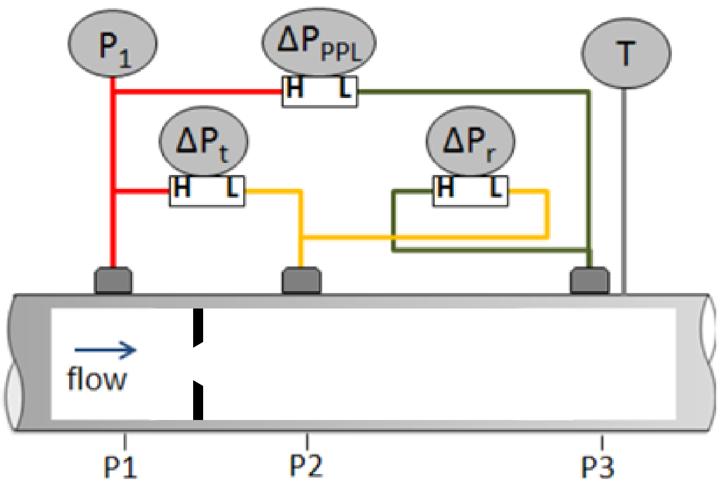


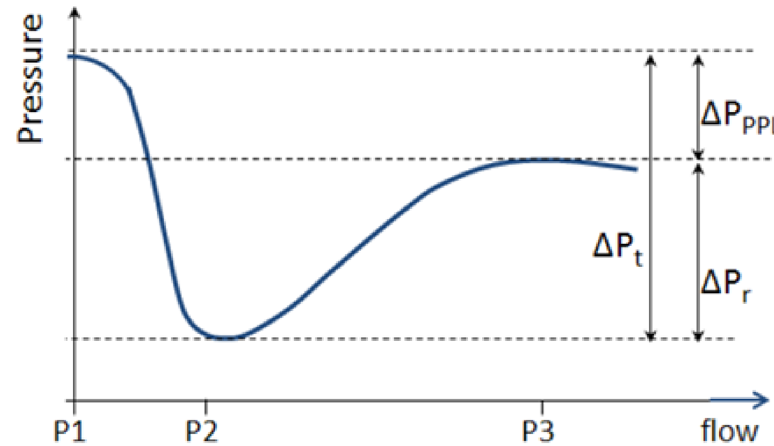
Three DP Meters





Three DP Meters

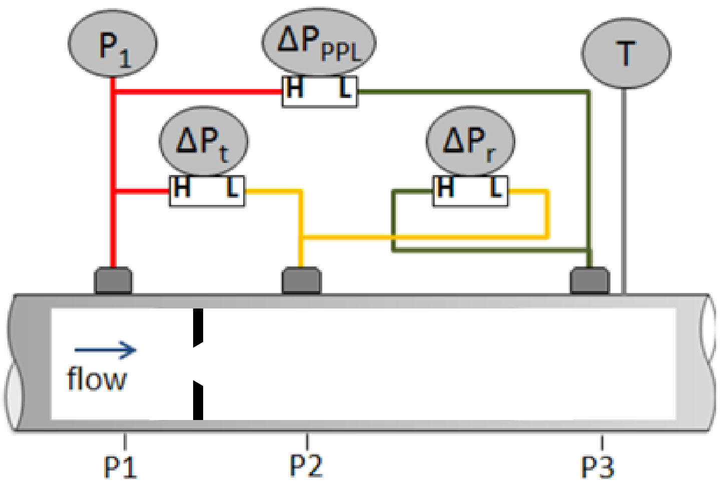


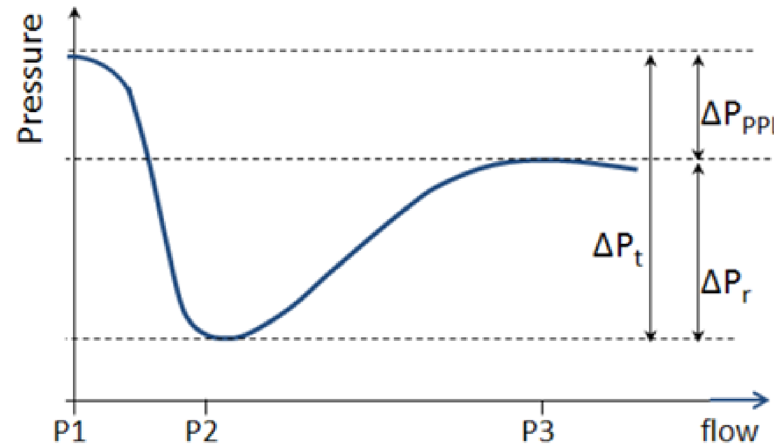


$\dot{m}_t$ $\Delta \mathrm{P_t}$ $\dot{m}_r$ $\Delta \mathrm{P_r}$

$\dot{m}_{PPL}$ $\Delta \mathrm{P_{PPL}}$

Three DP Meters

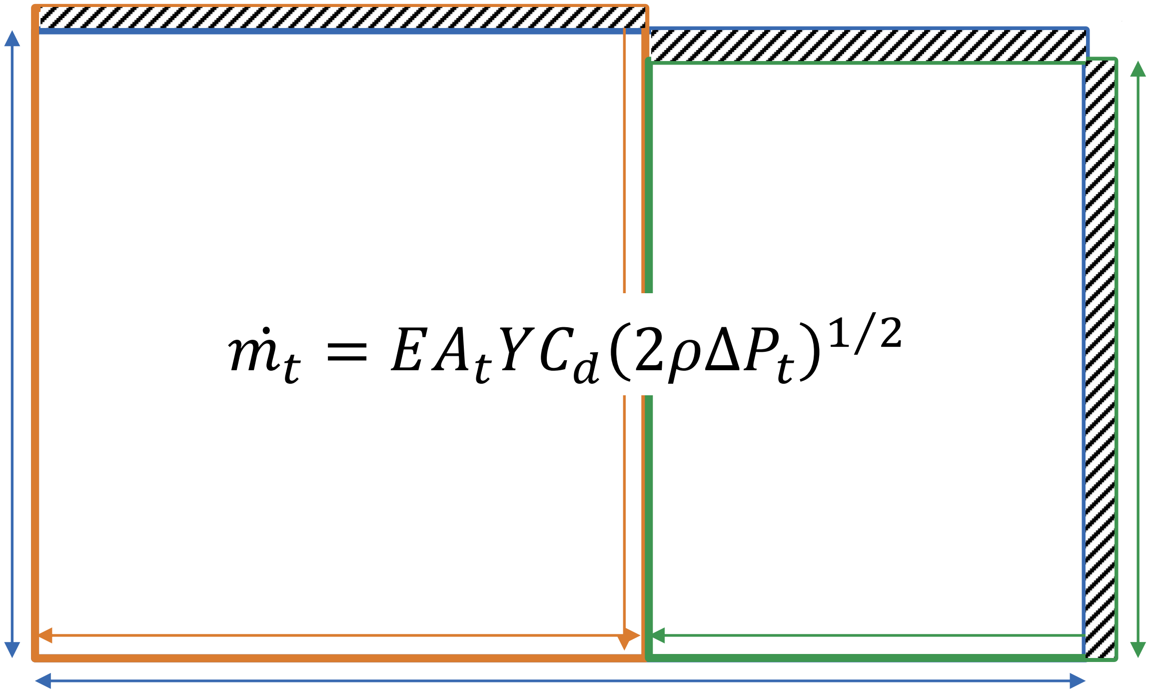




$$\dot{m}_t = EA_tYC_d(2\rho\Delta P_t)^{1/2} \qquad \dot{m}_r = EA_tK_r(2\rho\Delta P_r)^{1/2}$$

$$\dot{m}_{PPL} = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2}$$

Mass Flow and Pressure Drop Constraints



$$\dot{m}_t = EA_tYC_d(2\rho\Delta P_t)^{1/2}$$

100% certainty – all 3 DP meters are measuring the same flow rate

100% certainty – ΔPs must be consistent with each other

Pressure Drop Constraint

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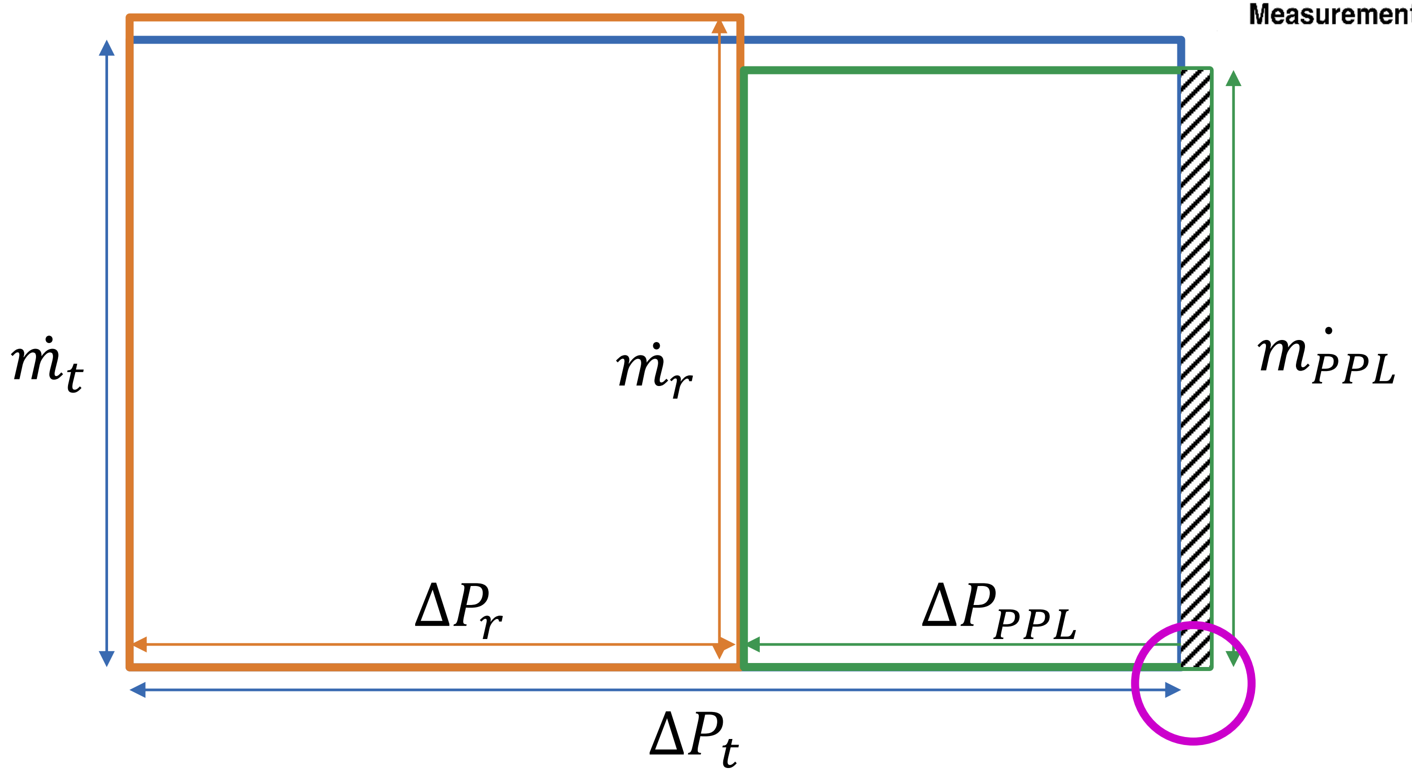
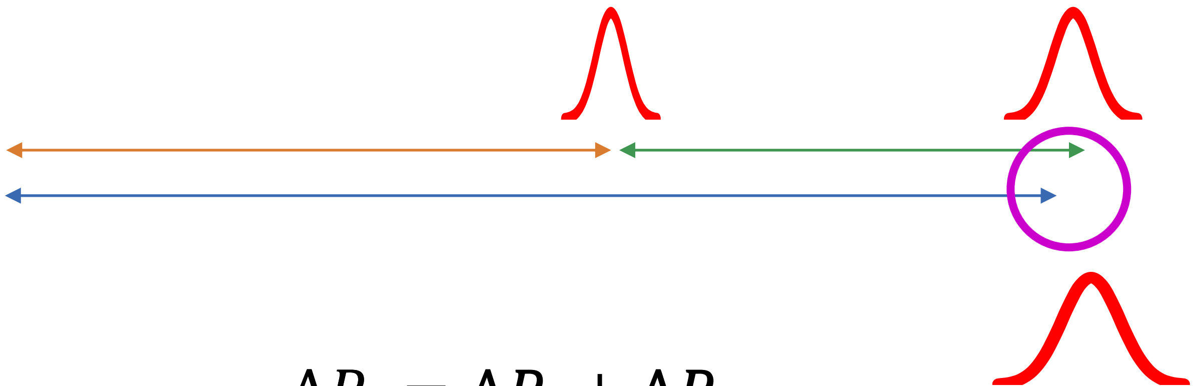
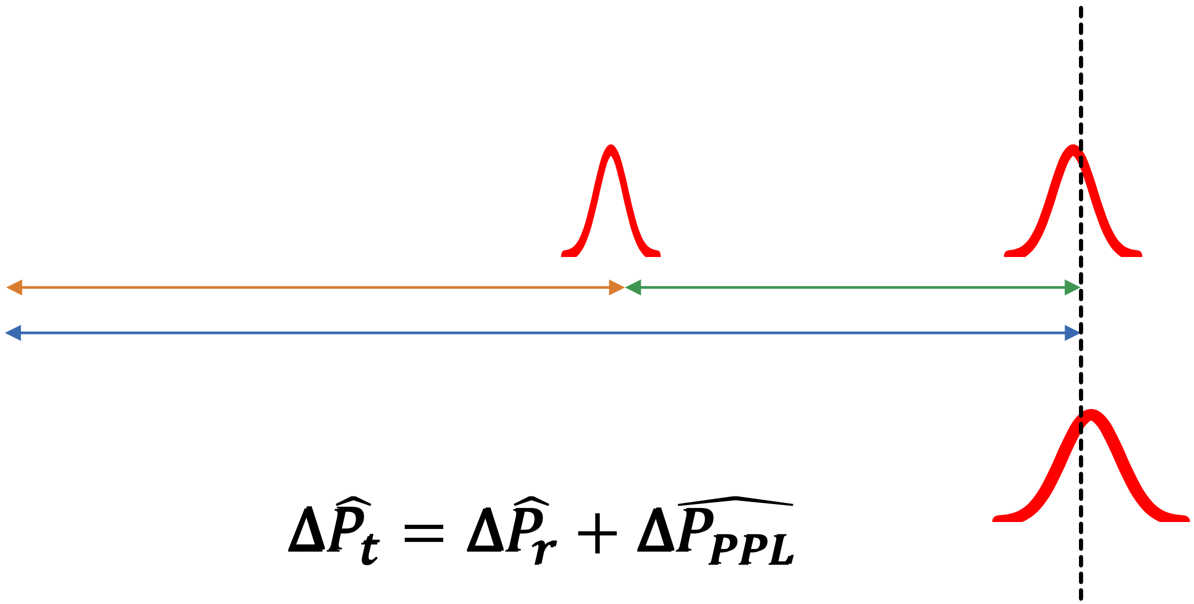


Pressure Drop Constraint



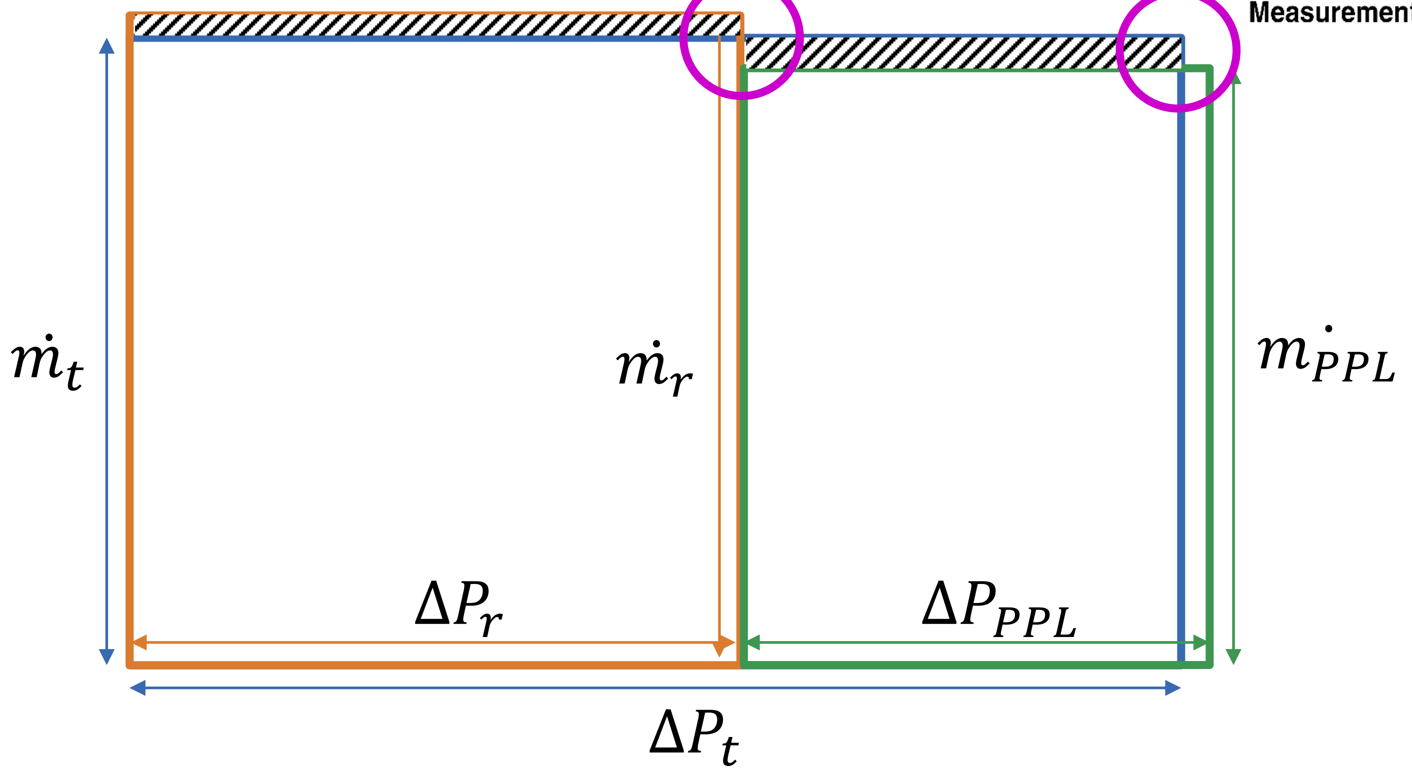
$$\Delta P_t = \Delta P_r + \Delta P_{PPL}$$

Pressure Drop Constraint



$$\Delta\widehat{P}_t = \Delta\widehat{P}_r + \Delta\widehat{P_{PPL}}$$

Mass Balance



Non-Linear Constrained Maximum Likelihood

Mass Balances

$$\Phi = \hat{\dot{m}} - \dot{m}_t\left(\hat{d}, \hat{D}, \hat{Y}, \hat{C}_d, \hat{\rho}, \Delta\hat{P}_t\right) = 0$$

$$\Xi = \hat{\dot{m}} - \dot{m}_r\left(\hat{d}, \hat{D}, \hat{K}_r, \hat{\rho}, \Delta\hat{P}_r\right) = 0$$

$$\Psi = \hat{\dot{m}} - \dot{m}_{PPL}\left(\hat{D}, \hat{K}_{PPL}, \hat{\rho}, \Delta\hat{P}_{PPL}\right) = 0$$

Pressure Balance

$$\Omega = \Delta\hat{P}_t - \Delta\hat{P}_r - \Delta\hat{P}_{PPL} = 0$$

Measured Variables

$$x = [\Delta P_t \quad \Delta P_r \quad \Delta P_{PPL} \quad Y \quad C_d \quad K_r \quad K_{PPL} \quad \rho]^t$$

Unmeasured Variables

$$\mathrm{u} = \hat{\dot{m}}$$

$$S = \sum_i \left(\frac{\hat{x}_i - x_i}{\sigma_{x_i}}\right)^2$$

Minimise weighted sum

Non-Linear Constrained Maximum Likelihood

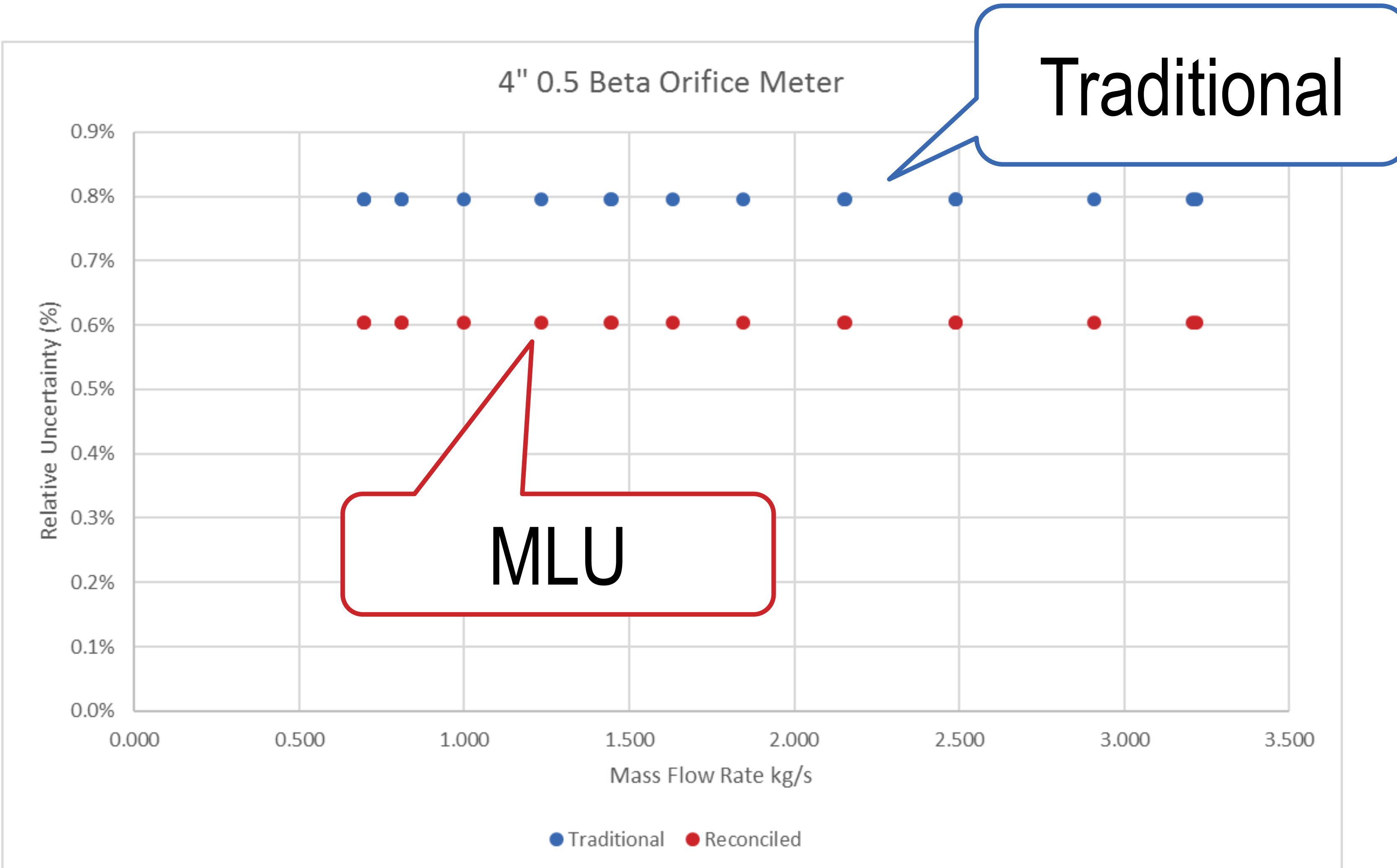
$$u_{i+1} = u_i - (J_u^T(J_x V J_x^T)^{-1} J_u)^{-1} J_u^T (J_x V J_x^T)^{-1} \left(J_x x_0 + J_x(x_i - x_0)\right)$$

$$x_{i+1} = x_0 - V J_x^T (J_x V J_x^T)^{-1} \left(J_x x_0 + J_u(u_i - u_0) + J_x(x_i - x_0)\right)$$

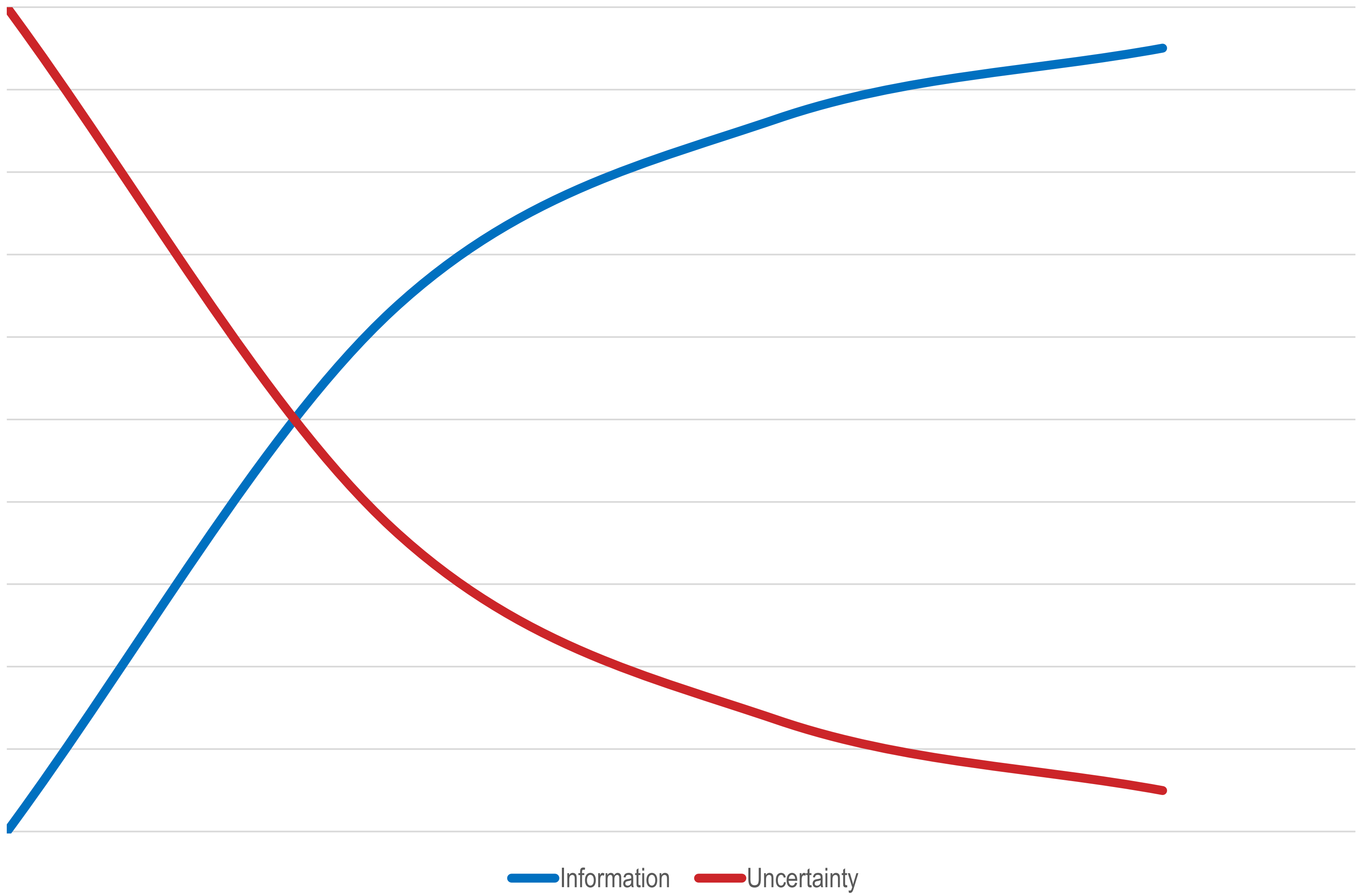
Determines MLU mass flow $\quad \mathrm{u} = \hat{\dot{m}}$

Determines MLU mass flow uncertainty $\quad \sigma_u = (J_u^T(J_x V J_x^T)^{-1} J_u)^{-1}$

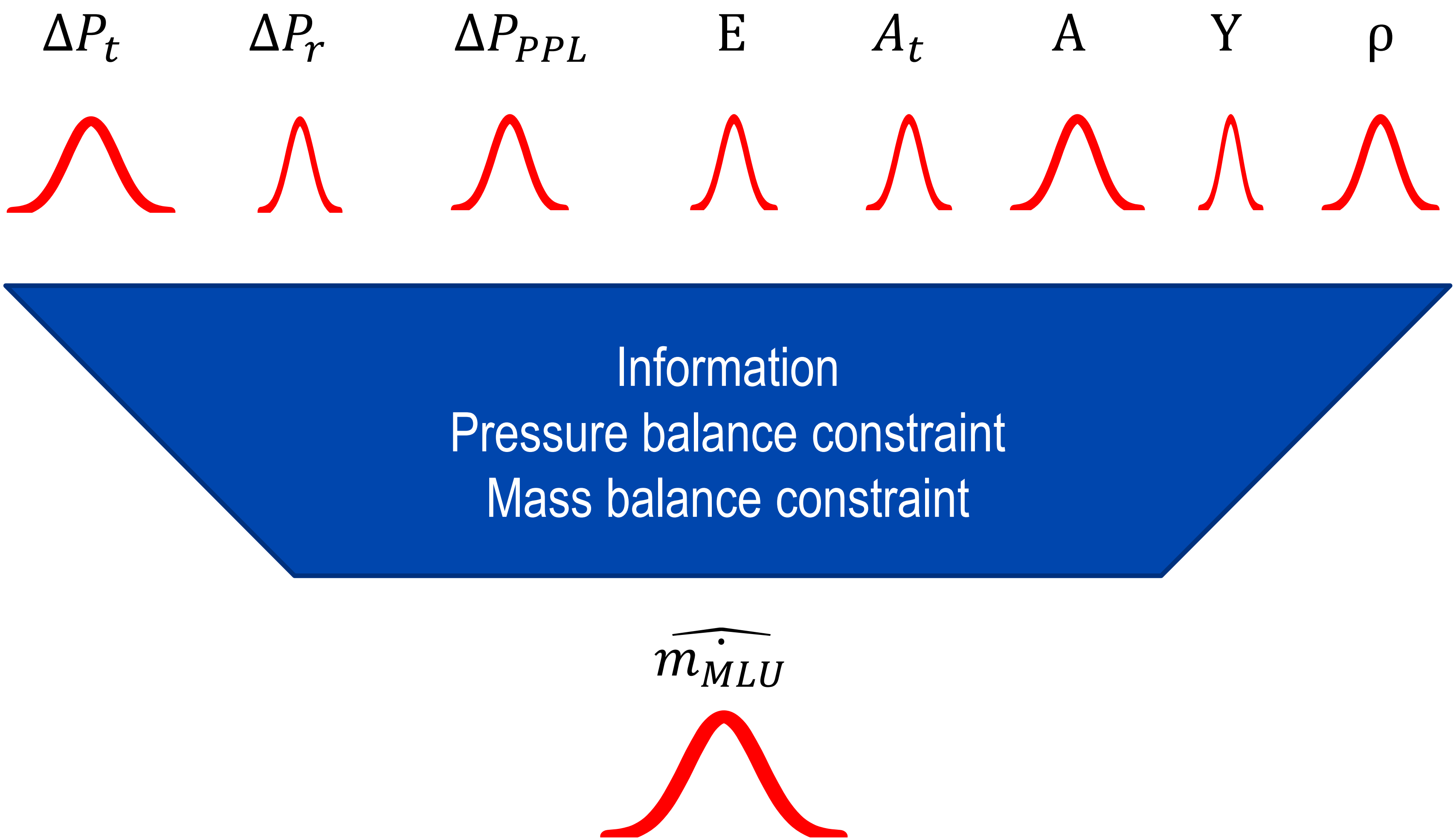
Uncertainty over a Range of Flows



Information and Uncertainty



Information and Uncertainty



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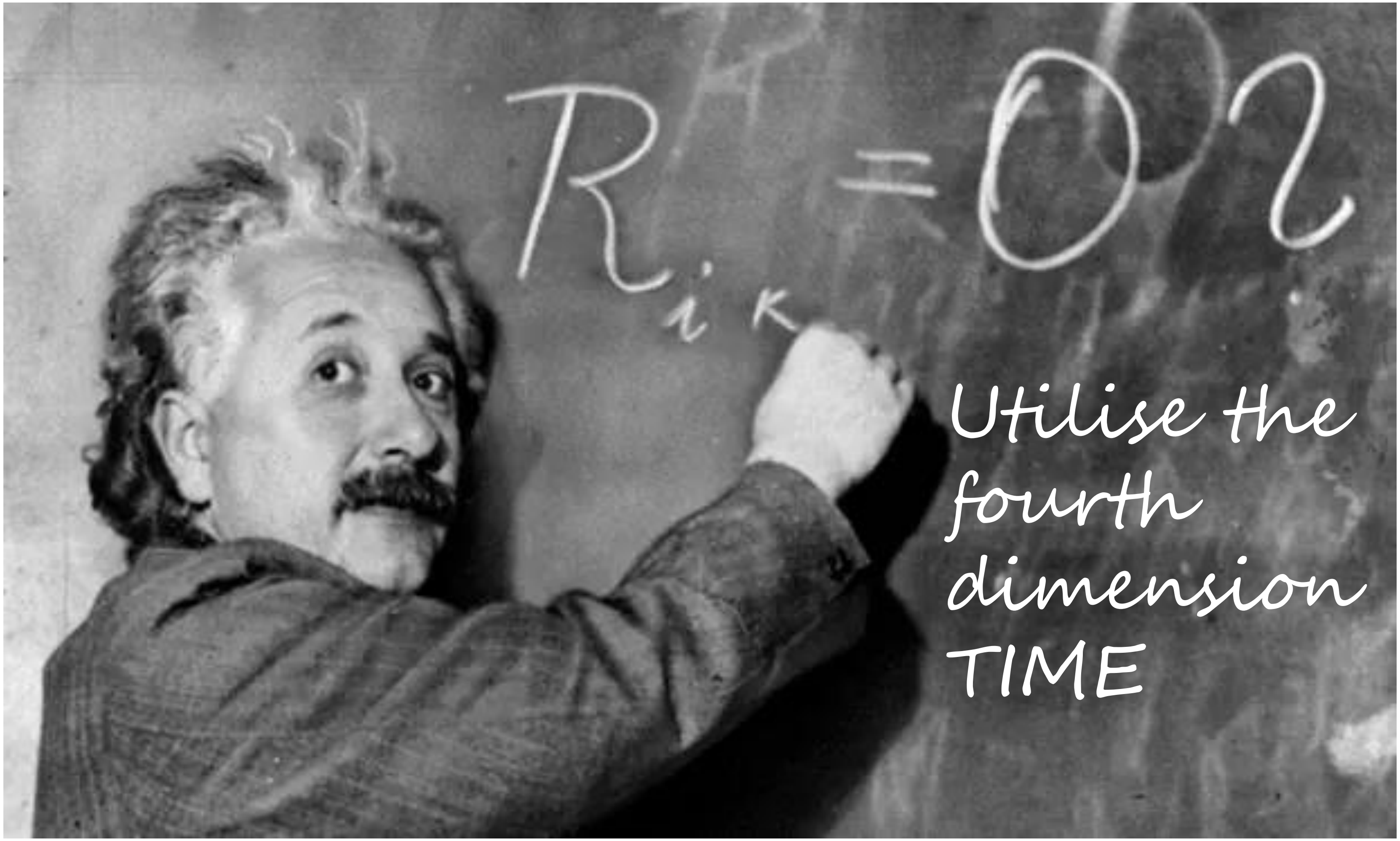
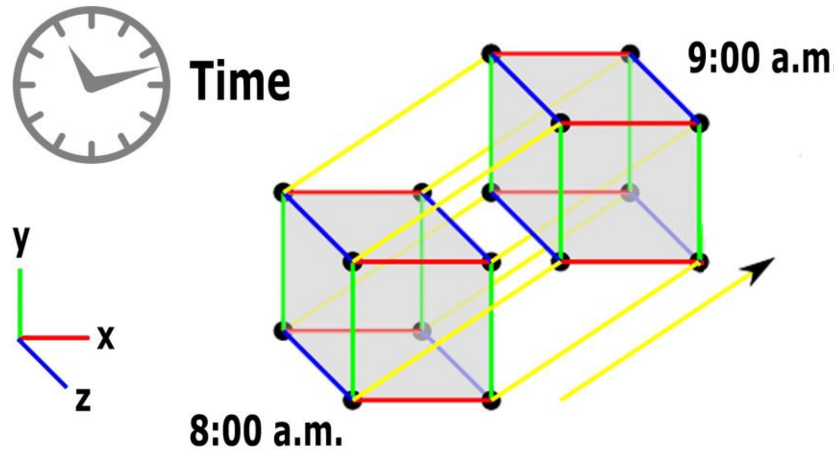
The next dimension



Special Theory of Relativity 1905



Return to the Mass Balance

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$$\dot{m}_t = EA_tYC_d(2\rho\Delta P_t)^{1/2} \qquad \dot{m}_r = EA_tK_r(2\rho\Delta P_r)^{1/2}$$

$$\dot{m}_{PPL} = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2}$$

$$\dot{m}_t = \dot{m}_r = \dot{m}_{PPL}$$

Mass Balance

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$$EA_tYC_d(2\rho\Delta P_t)^{1/2} = EA_tK_r(2\rho\Delta P_r)^{1/2} = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2}$$

$$\dot{m}_t = \dot{m}_r = \dot{m}_{PPL}$$

Re-writing the Mass Balance

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$$E A_t Y C_d (\cancel{2\rho} \Delta P_t)^{1/2} \quad = E A_t K_r (\cancel{2\rho} \Delta P_r)^{1/2} \quad = A K_{PPL} (\cancel{2\rho} \Delta P_{PPL})^{1/2}$$

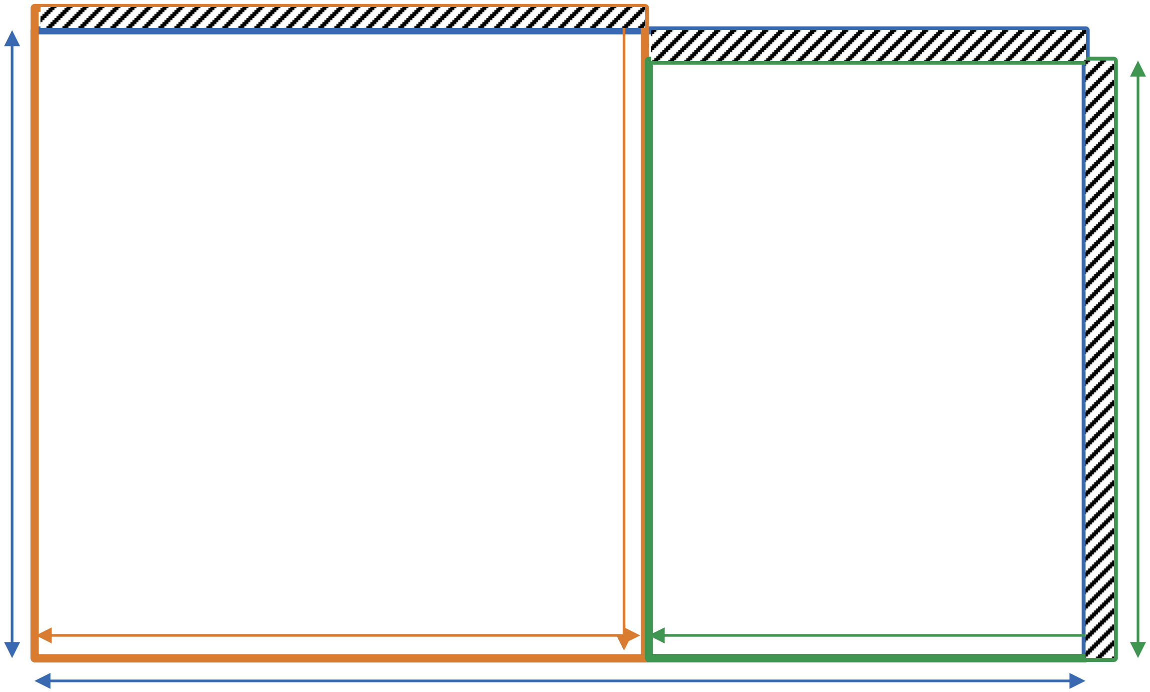
$$\dot{m}_t = \dot{m}_r = \dot{m}_{PPL}$$

Modified Flow Coefficients

$$\underbrace{E A_t Y C_d}_{C'_d}(\quad \Delta P_t)^{1/2} = \underbrace{E A_t K_r}_{K'_r}(\quad \Delta P_r)^{1/2} = \underbrace{A K_{PPL}}_{K'_{PPL}}(\quad \Delta P_{PPL})^{1/2}$$

$$\dot{m}_t = \dot{m}_r = \dot{m}_{PPL}$$

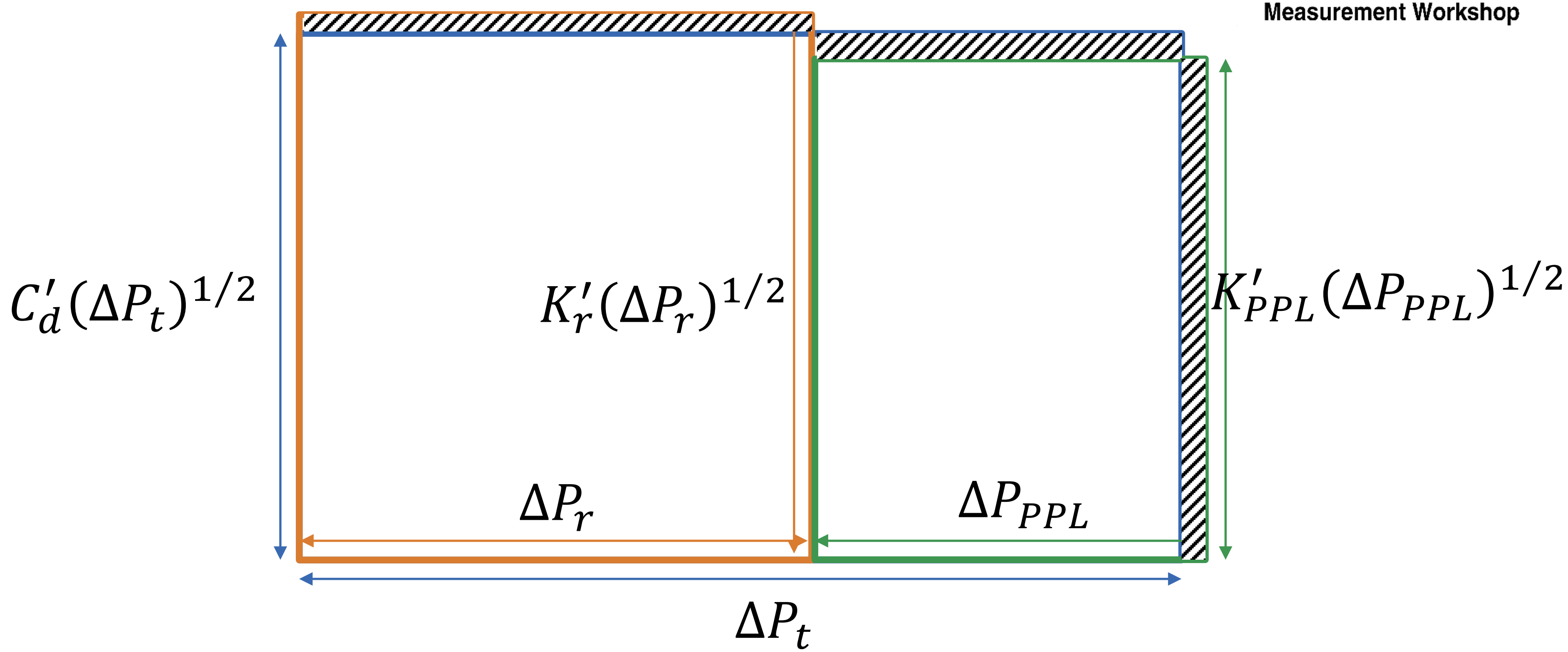
Mass Flow and Pressure Drop Constraints



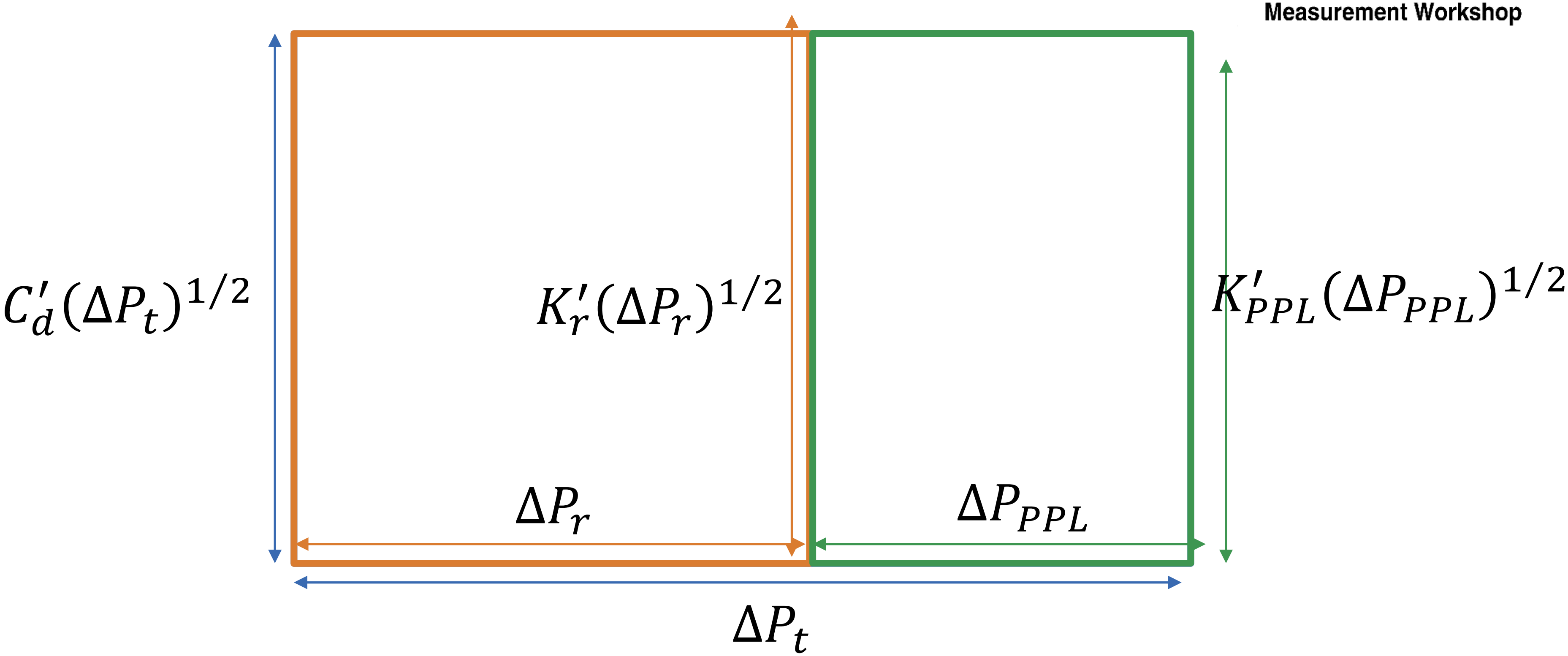
$$C'_d(\Delta P_t)^{1/2} = K'_r(\Delta P_r)^{1/2} = K'_{PPL}(\Delta P_{PPL})^{1/2}$$

$$\dot{m}_t = \dot{m}_r = \dot{m}_{PPL}$$

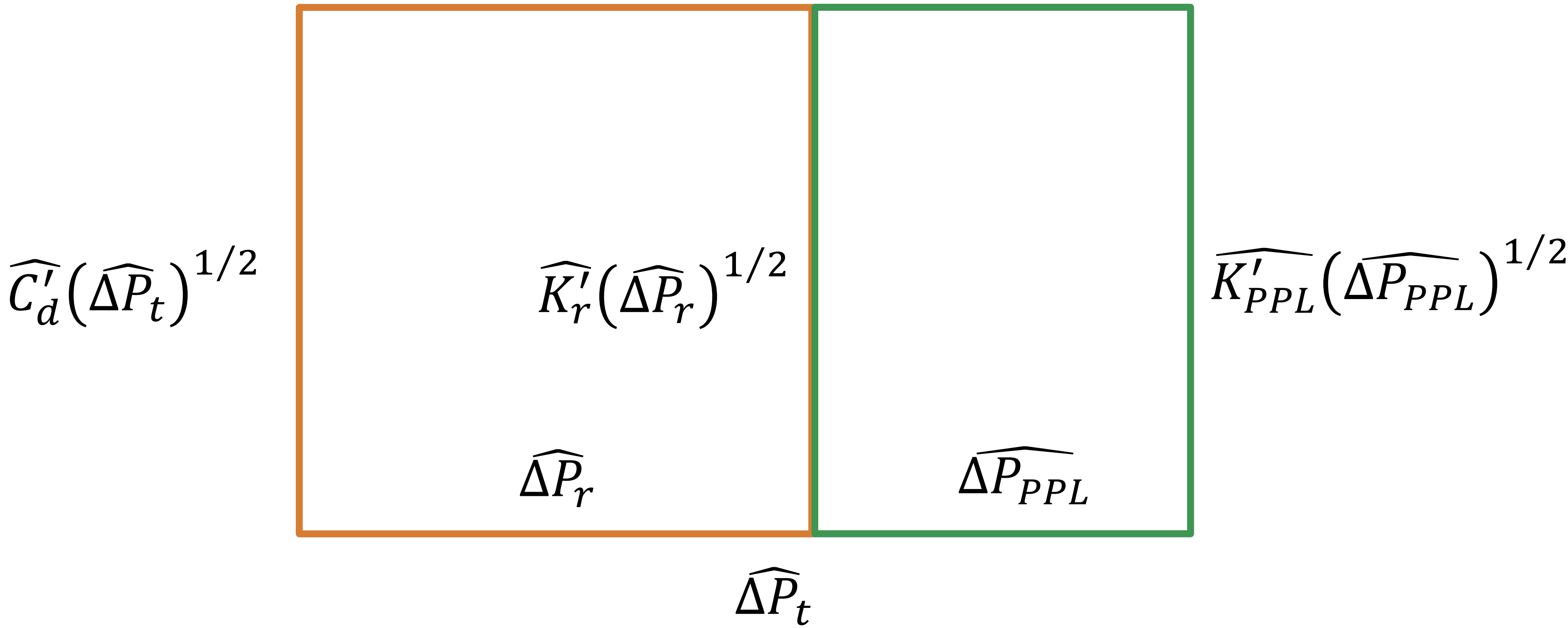
Mass Flow and Pressure Drop Constraints



Maximum Likelihood Uncertainty



Maximum Likelihood Uncertainty



Mass Flow and Uncertainty

$$\widehat{K'_r}\left(\widehat{\Delta P_r}\right)^{1/2}$$

$$\widehat{K'_{PPL}}\left(\widehat{\Delta P_{PPL}}\right)^{1/2}$$

$$\widehat{\Delta P_r}$$

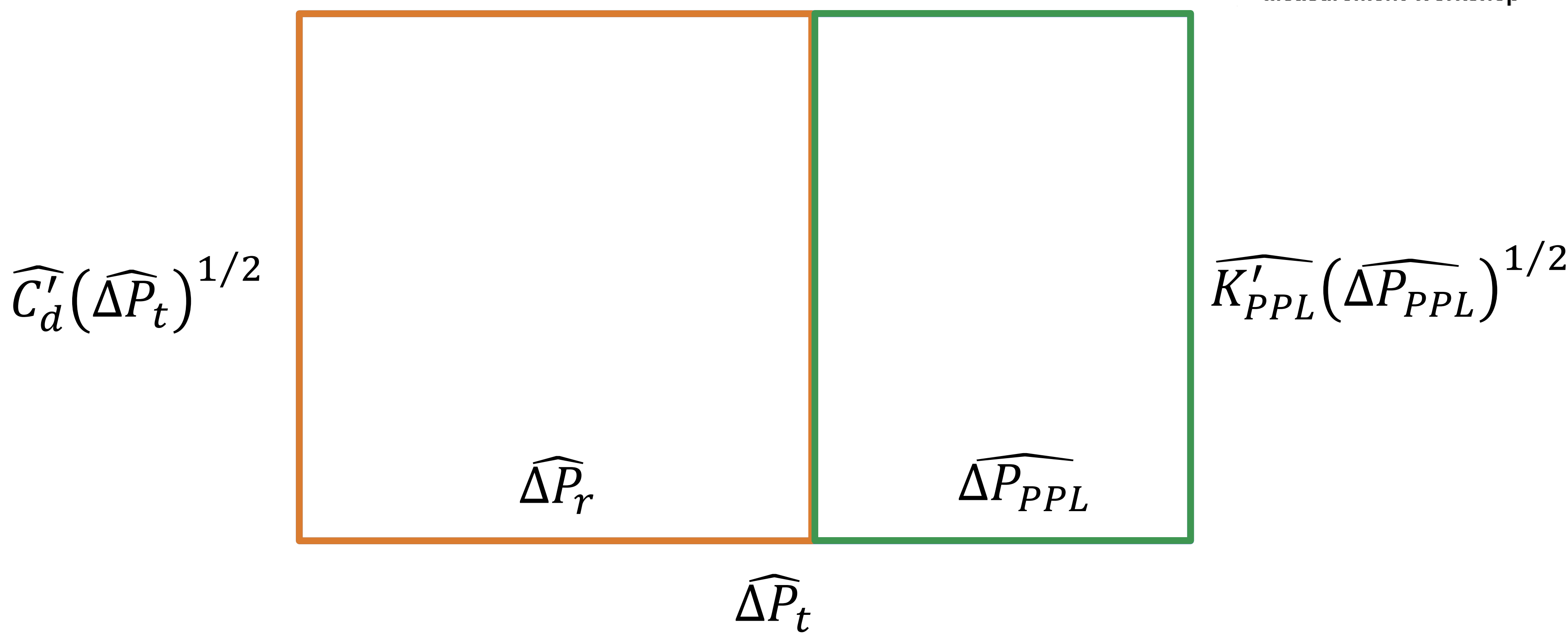
$$\widehat{\Delta P_{PPL}}$$

$$\widehat{\Delta P_t}$$

$$\widehat{\dot{m}_{MLU}} = \widehat{C'_d}\left(\widehat{\Delta P_t}\right)^{1/2}(2\rho)^{1/2}$$

MLU mass flow, reduced uncertainty

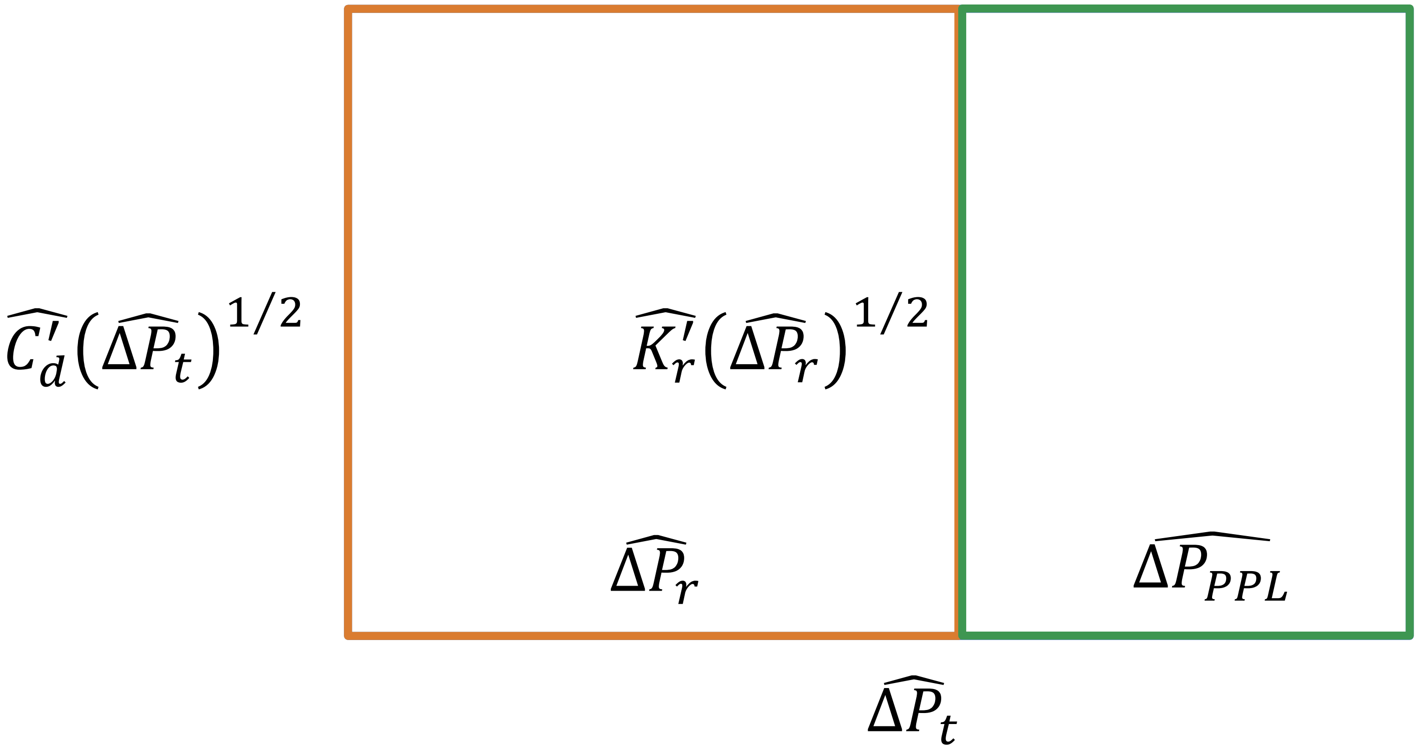
Mass Flow and Uncertainty



$$\widehat{\dot{m}_{MLU}} = \widehat{K'_r}\left(\widehat{\Delta P_r}\right)^{1/2}(2\rho)^{1/2}$$

MLU mass flow, reduced uncertainty

Mass Flow and Uncertainty



$$\widehat{\dot{m}_{MLU}} = \widehat{K'_{PPL}}\left(\widehat{\Delta P_{PPL}}\right)^{1/2}(2\rho)^{1/2}$$

MLU mass flow, reduced uncertainty

MLU time, t=1

$$\widehat{C'_d}\left(\widehat{\Delta P_t}\right)^{1/2}$$

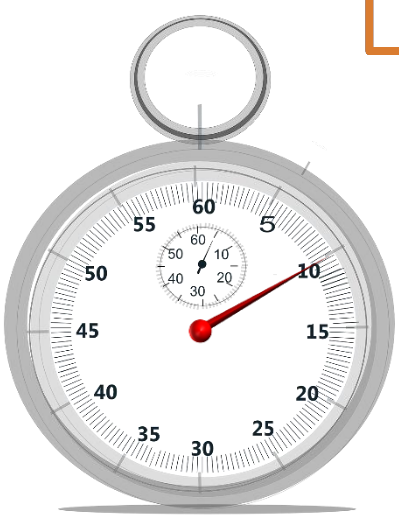
$$\widehat{K'_r}\left(\widehat{\Delta P_r}\right)^{1/2}$$

$$\widehat{K'_{PPL}}\left(\widehat{\Delta P_{PPL}}\right)^{1/2}$$

$$\widehat{\Delta P_r}$$

$$\widehat{\Delta P_{PPL}}$$

$$\widehat{\Delta P_t}$$



MLU time, t=1

$$\widehat{C'_{d,1}}\left(\widehat{\Delta P_{t,1}}\right)^{1/2}$$

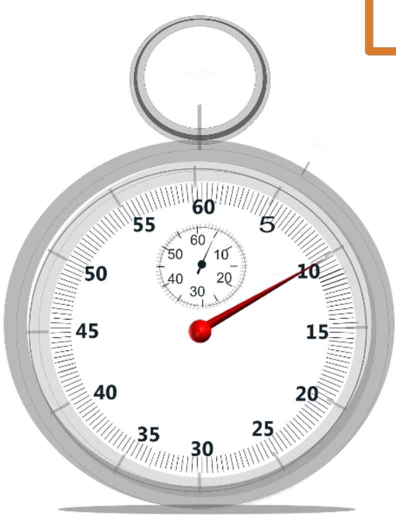
$$\widehat{K'_{r,1}}\left(\widehat{\Delta P_{r,1}}\right)^{1/2}$$

$$\widehat{K'_{PPL,1}}\left(\Delta\widehat{P_{PPL,1}}\right)^{1/2}$$

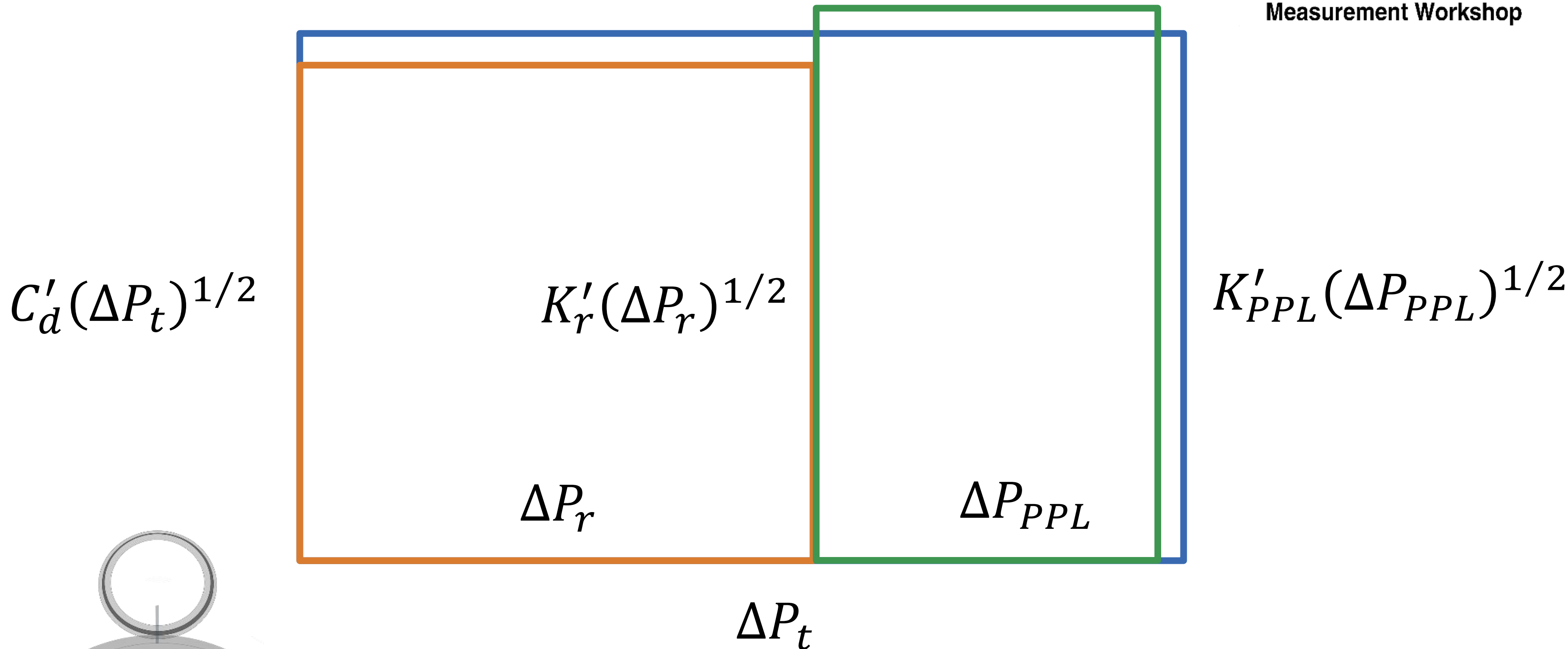
$$\widehat{\Delta P_{r,1}}$$

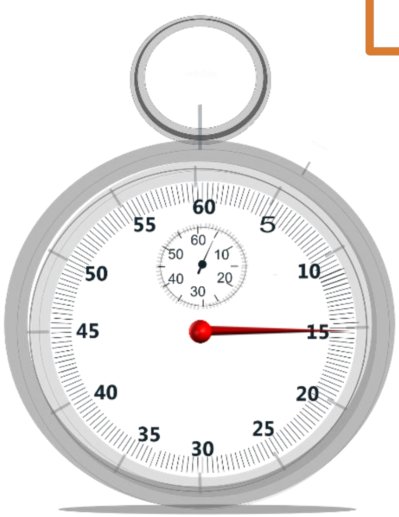
$$\Delta\widehat{P_{PPL,1}}$$

$$\widehat{\Delta P_{t,1}}$$



MLU time, t=1


$C'_d(\Delta P_t)^{1/2}$
$K'_r(\Delta P_r)^{1/2}$
$K'_{PPL}(\Delta P_{PPL})^{1/2}$
ΔP_r
ΔP_{PPL}
ΔP_t


60
5
10
15
20
25
30
35
40
45
50
55

MLU time, t=2

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$$\widehat{C'_d}\left(\widehat{\Delta P_t}\right)^{1/2}$$

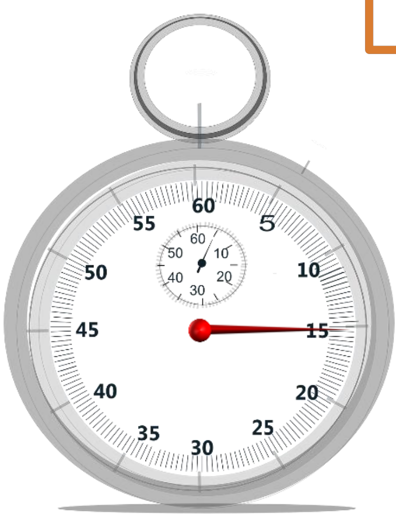
$$\widehat{K'_r}\left(\widehat{\Delta P_r}\right)^{1/2}$$

$$\widehat{K'_{PPL}}\left(\widehat{\Delta P_{PPL}}\right)^{1/2}$$

$$\widehat{\Delta P_r}$$

$$\widehat{\Delta P_{PPL}}$$

$$\widehat{\Delta P_t}$$



MLU time, t=2

$\widehat{C'_{d,2}}\left(\widehat{\Delta P_{t,2}}\right)^{1/2}$

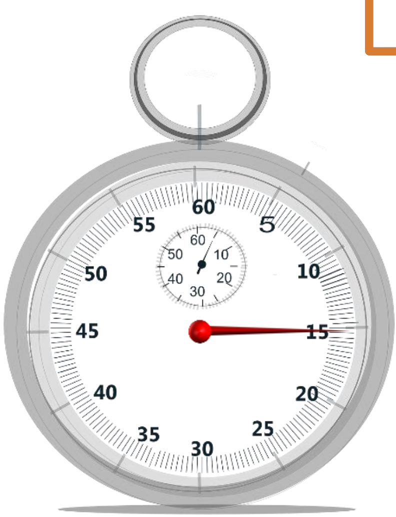
$\widehat{K'_{r,2}}\left(\widehat{\Delta P_{r,2}}\right)^{1/2}$

$\widehat{K'_{PPL,2}}\left(\widehat{\Delta P_{PPL,2}}\right)^{1/2}$

$\widehat{\Delta P_{r,2}}$

$\widehat{\Delta P_{PPL,2}}$

$\widehat{\Delta P_{t,2}}$

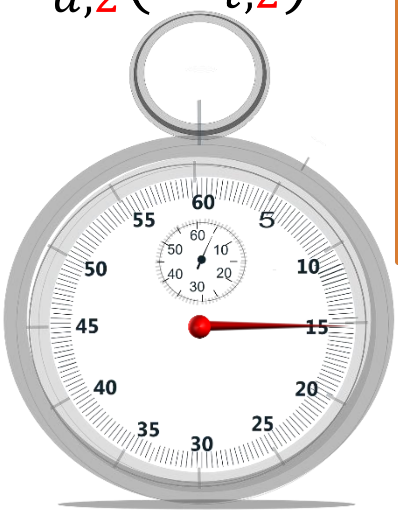


MLU time, t=1

$$\widehat{C'_{d,2}}\left(\widehat{\Delta P_{t,2}}\right)^{1/2}$$

$$\widehat{K'_{r,2}}\left(\widehat{\Delta P_{r,2}}\right)^{1/2}$$

$$\widehat{K'_{PPL,2}}\left(\widehat{\Delta P_{PPL,2}}\right)^{1/2}$$



$$\widehat{\Delta P_{r,2}}$$

$$\widehat{\Delta P_{PPL,2}}$$

$$\widehat{\Delta P_{t,2}}$$

MLU time, t=1 and t=2

$$\widehat{C'_{d,1}}\left(\widehat{\Delta P_{t,1}}\right)^{1/2} \qquad \widehat{K'_{r,1}}\left(\widehat{\Delta P_{r,1}}\right)^{1/2} \qquad \widehat{K'_{PPL,1}}\left(\widehat{\Delta P_{PPL,1}}\right)^{1/2}$$

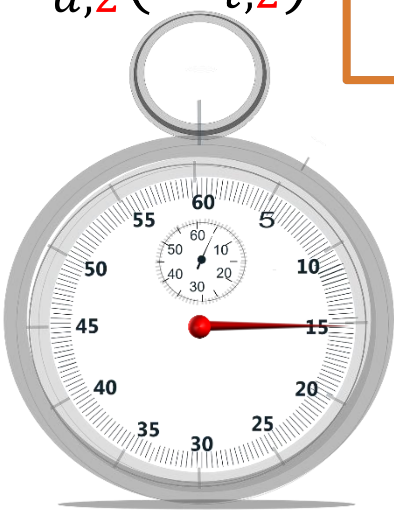
$$\widehat{C'_{d,2}}\left(\widehat{\Delta P_{t,2}}\right)^{1/2} \qquad \widehat{K'_{r,2}}\left(\widehat{\Delta P_{r,2}}\right)^{1/2} \qquad \widehat{K'_{PPL,2}}\left(\widehat{\Delta P_{PPL,2}}\right)^{1/2}$$

$\widehat{\Delta P_{r,1}}$ $\widehat{\Delta P_{PPL,1}}$

$\widehat{\Delta P_{t,1}}$

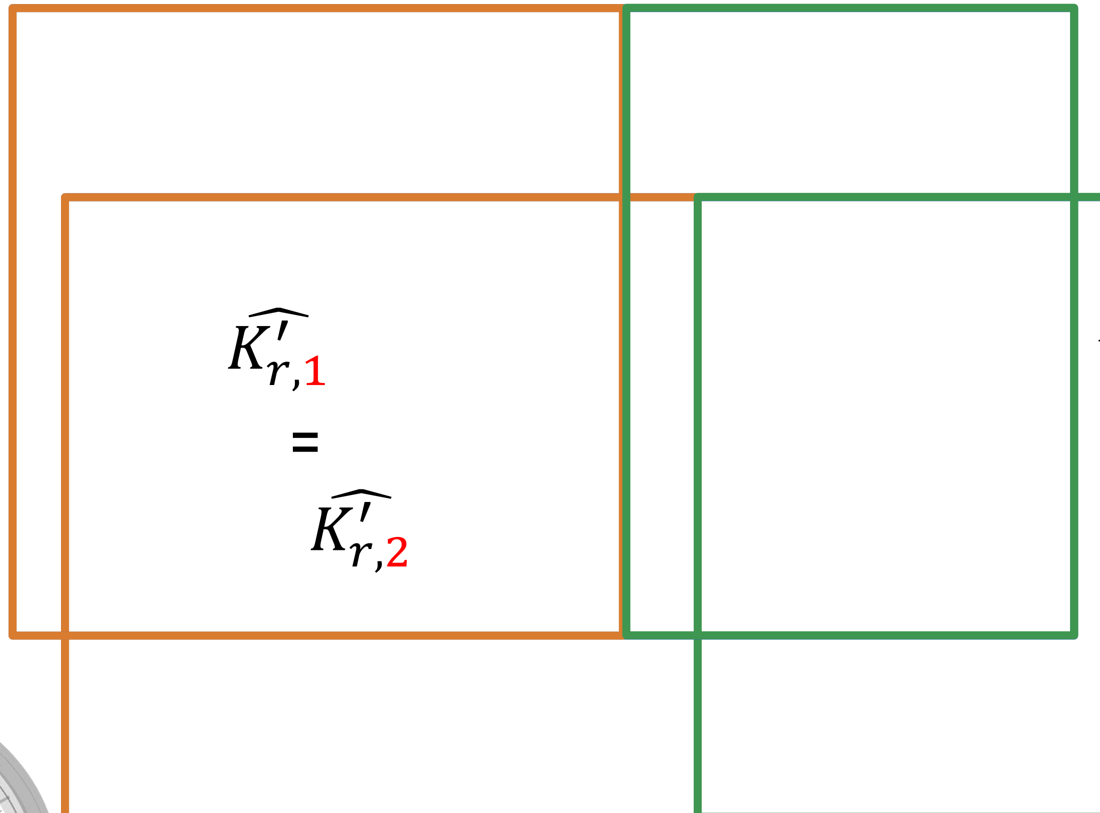
$\widehat{\Delta P_{r,2}}$ $\widehat{\Delta P_{PPL,2}}$

$\widehat{\Delta P_{t,2}}$



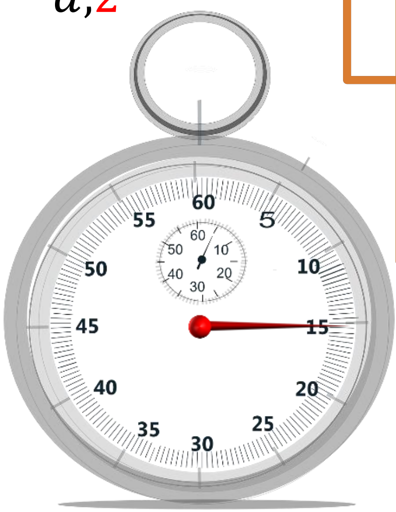
MLU time, t=1 and t=2

$$\widehat{C'_{d,1}} = \widehat{C'_{d,2}}$$

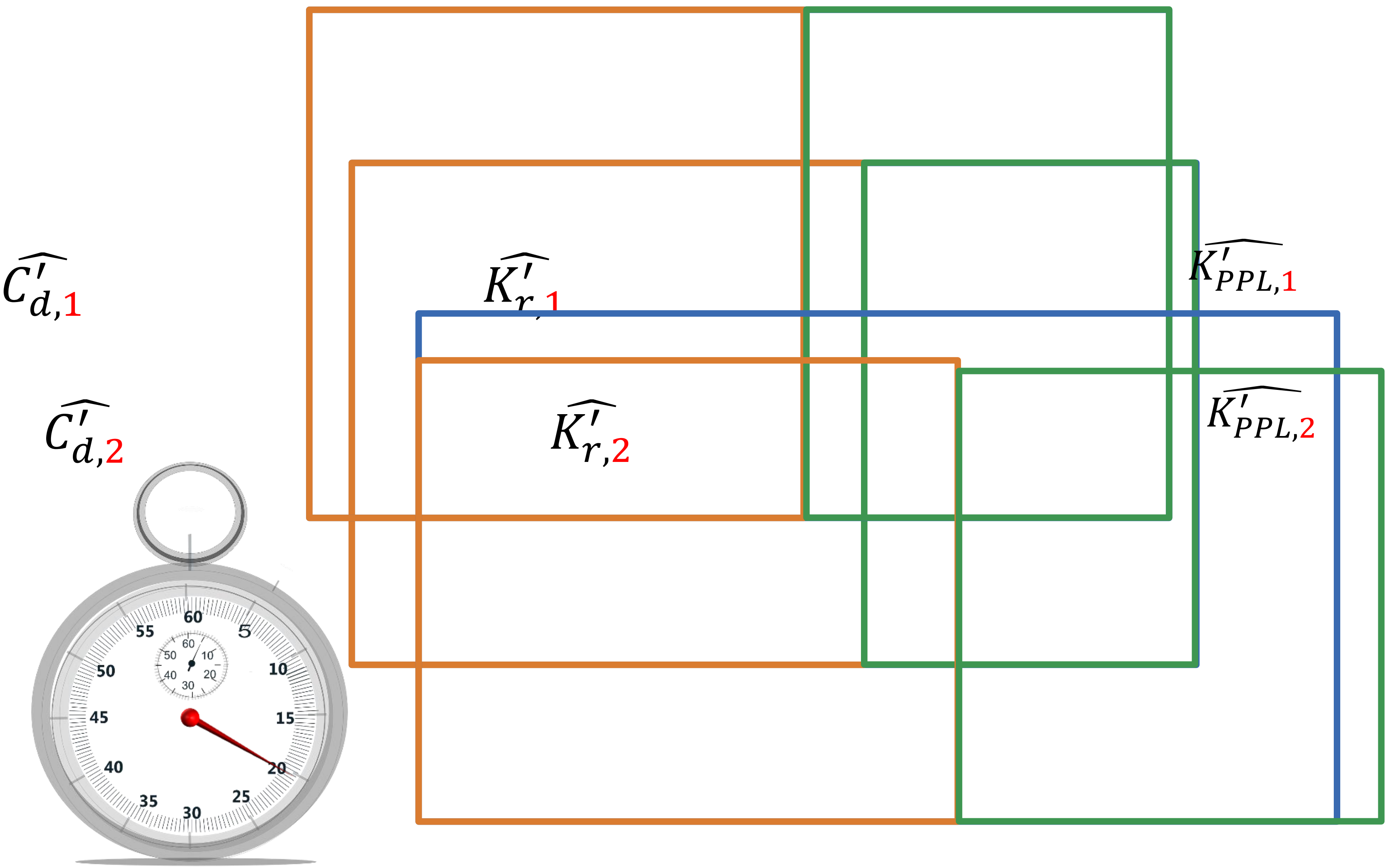


$$\widehat{K'_{r,1}} = \widehat{K'_{r,2}}$$

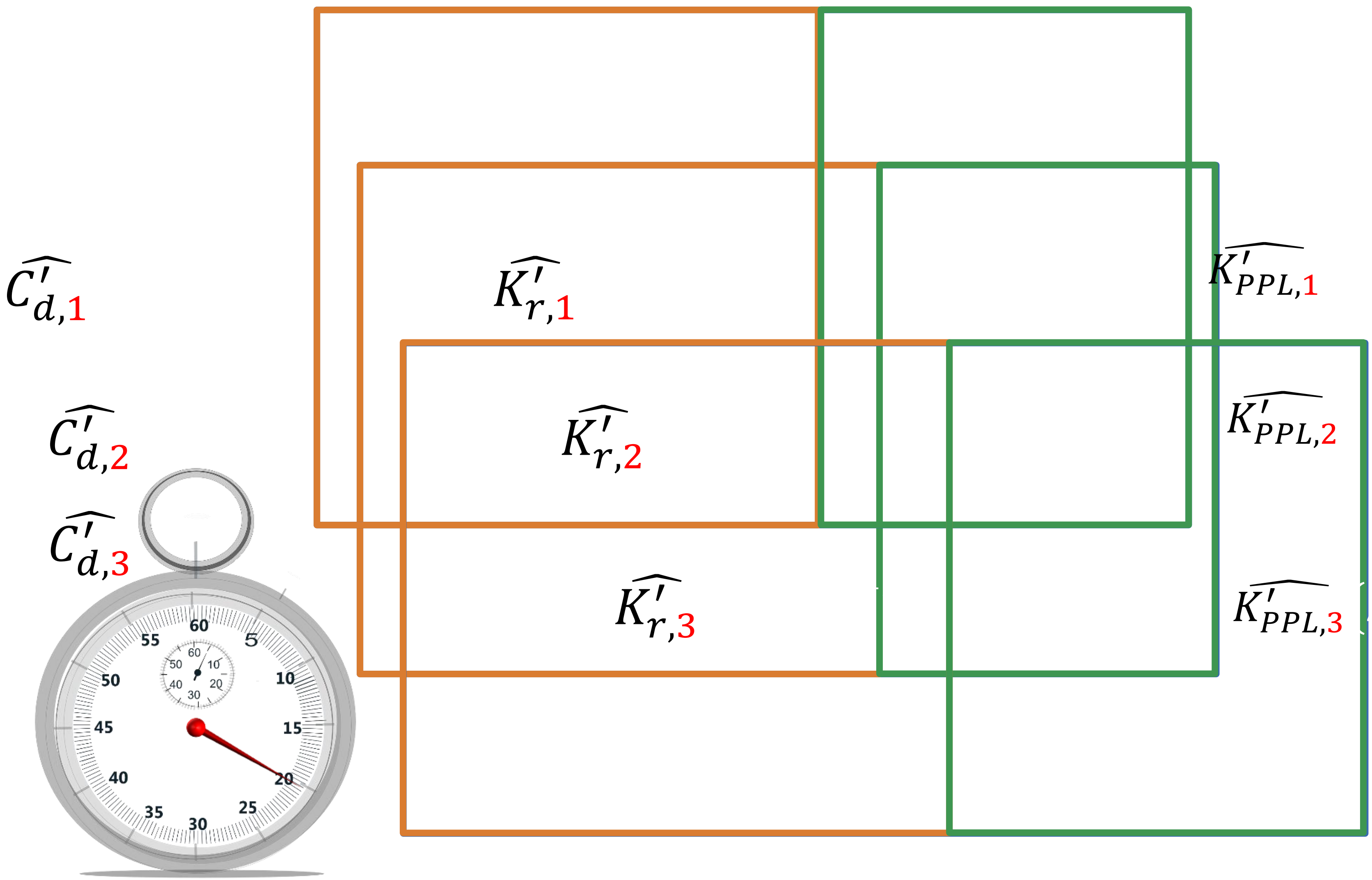
$$\widehat{K'_{PPL,1}} = \widehat{K'_{PPL,2}}$$



MLU time, t=1, t=2 and t=3



MLU time, t=1, t=2 and t=3



Flow Coefficient Constraints

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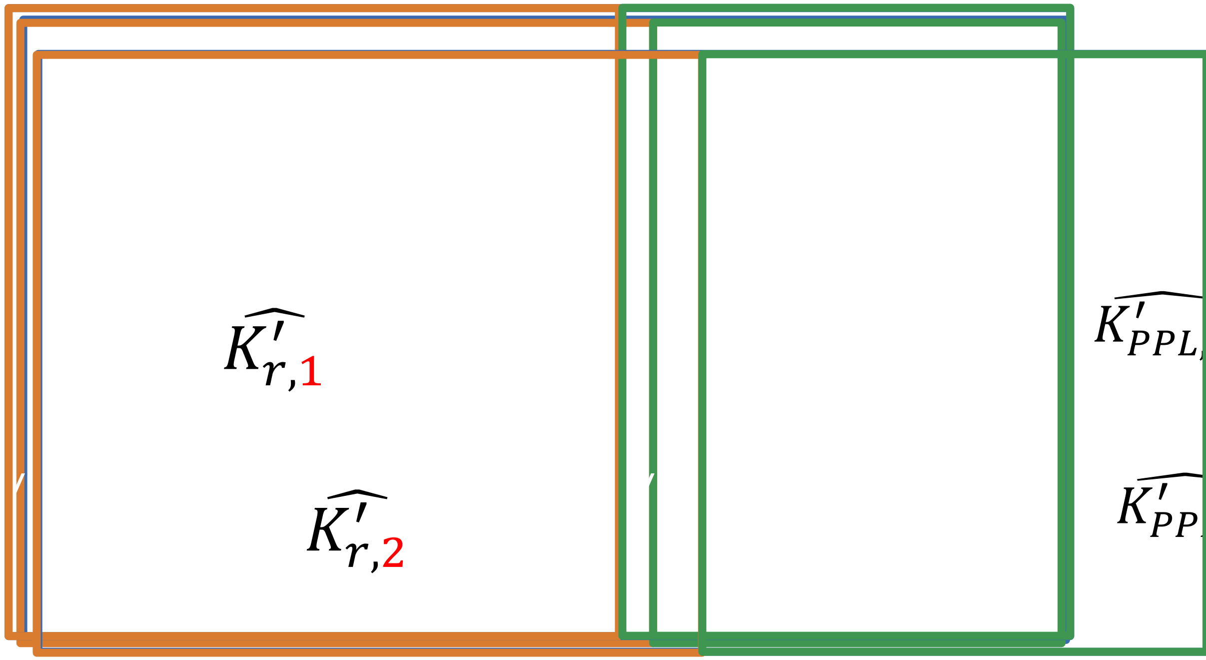


$\widehat{C'_{d,1}}$

$\widehat{K'_{r,1}}$

$\widehat{K'_{PPL,1}}$

$\widehat{C'_{d,2}}$

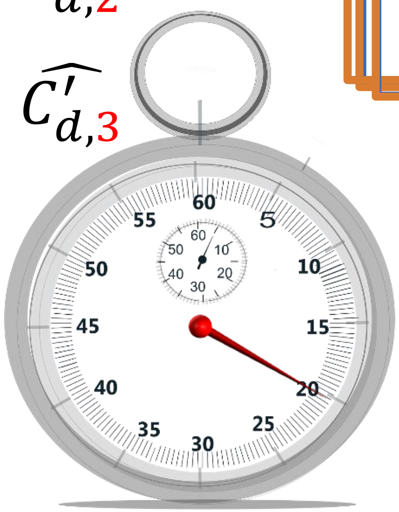
$\widehat{K'_{r,2}}$

$\widehat{K'_{PPL,2}}$

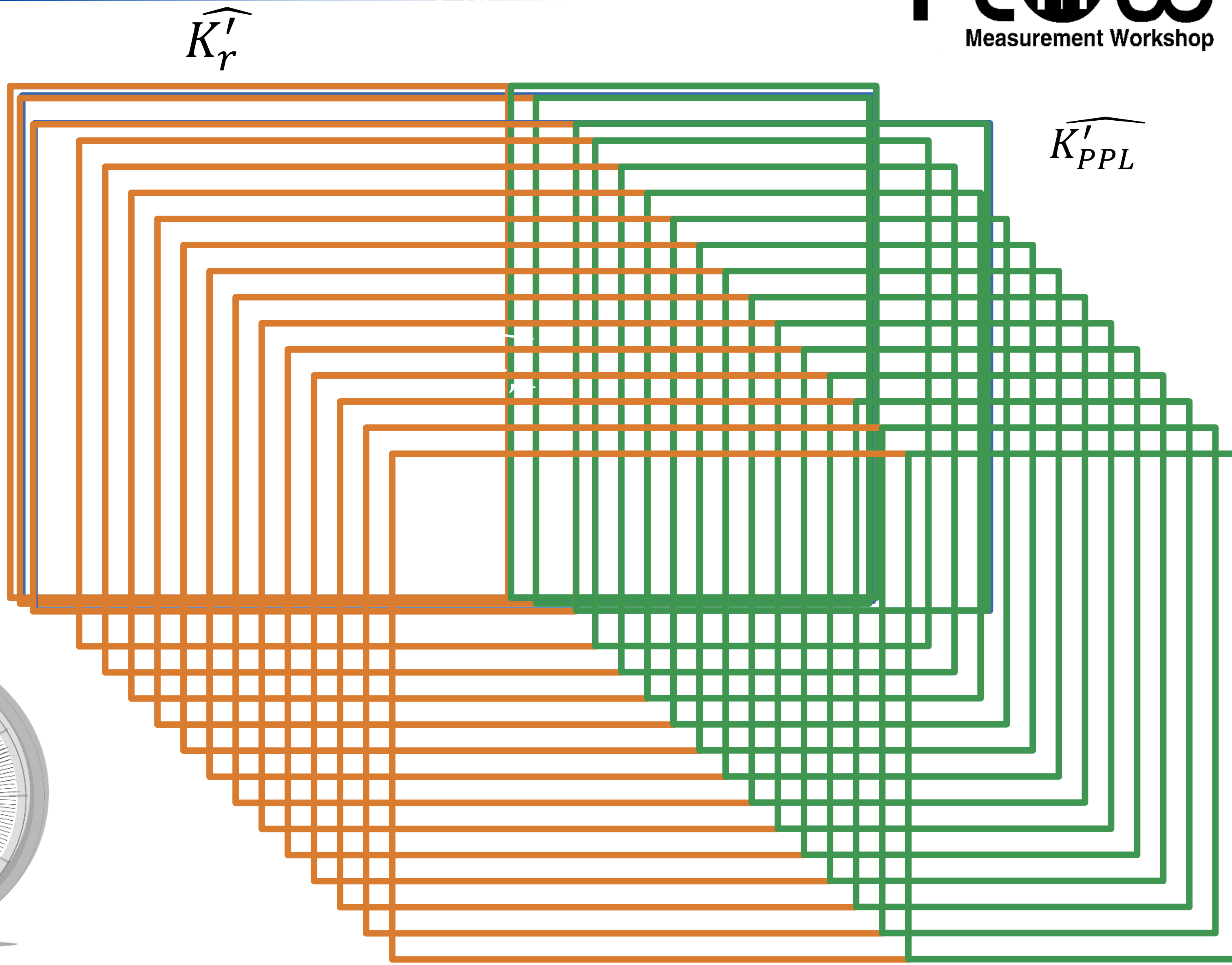
$\widehat{C'_{d,3}}$

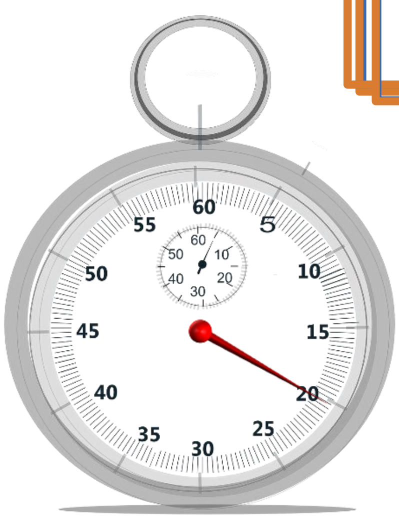
$\widehat{K'_{r,3}}$

$\widehat{K'_{PPL,3}}$



Flow Coefficient Constraints





Kalman Filter

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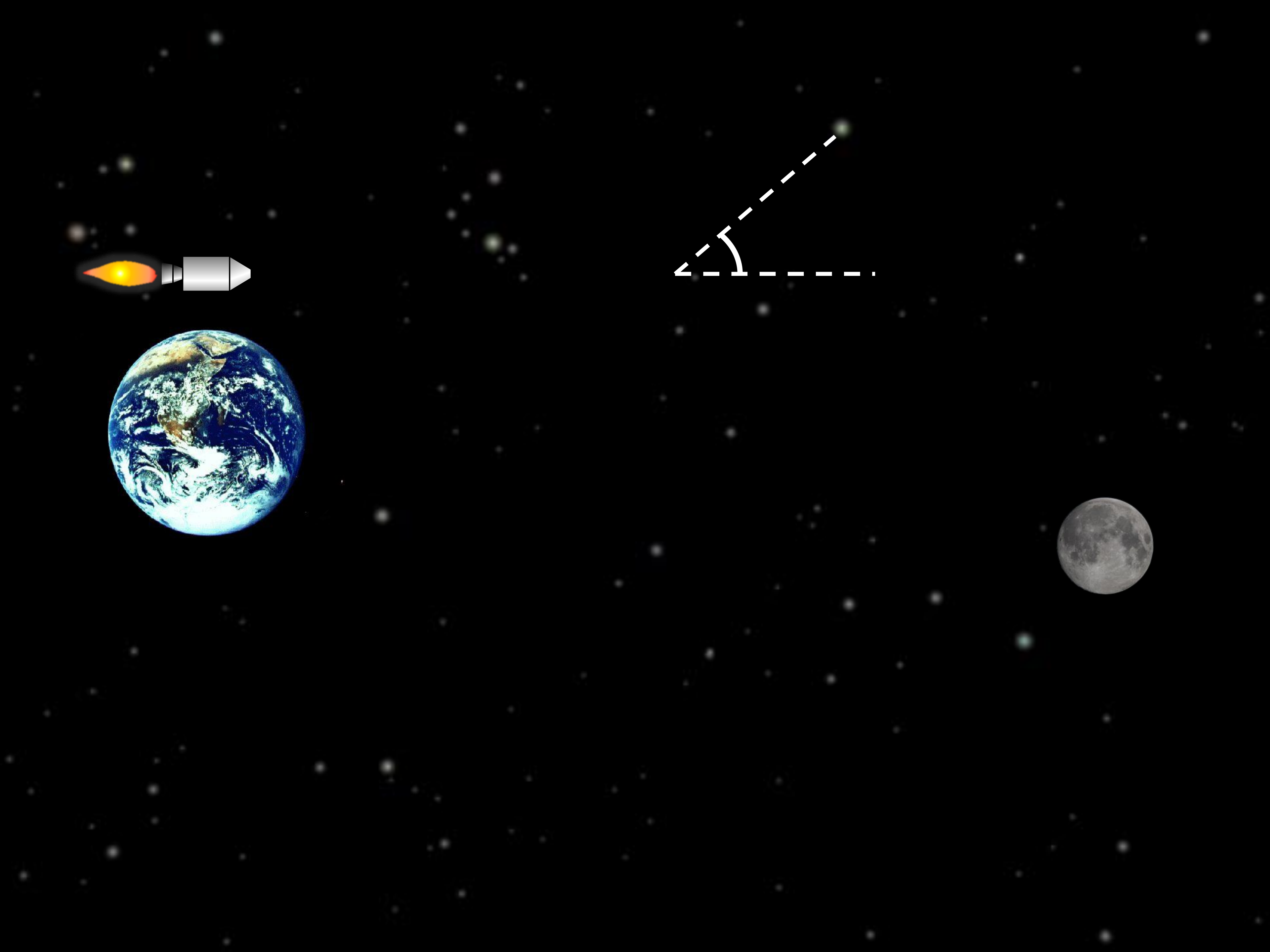
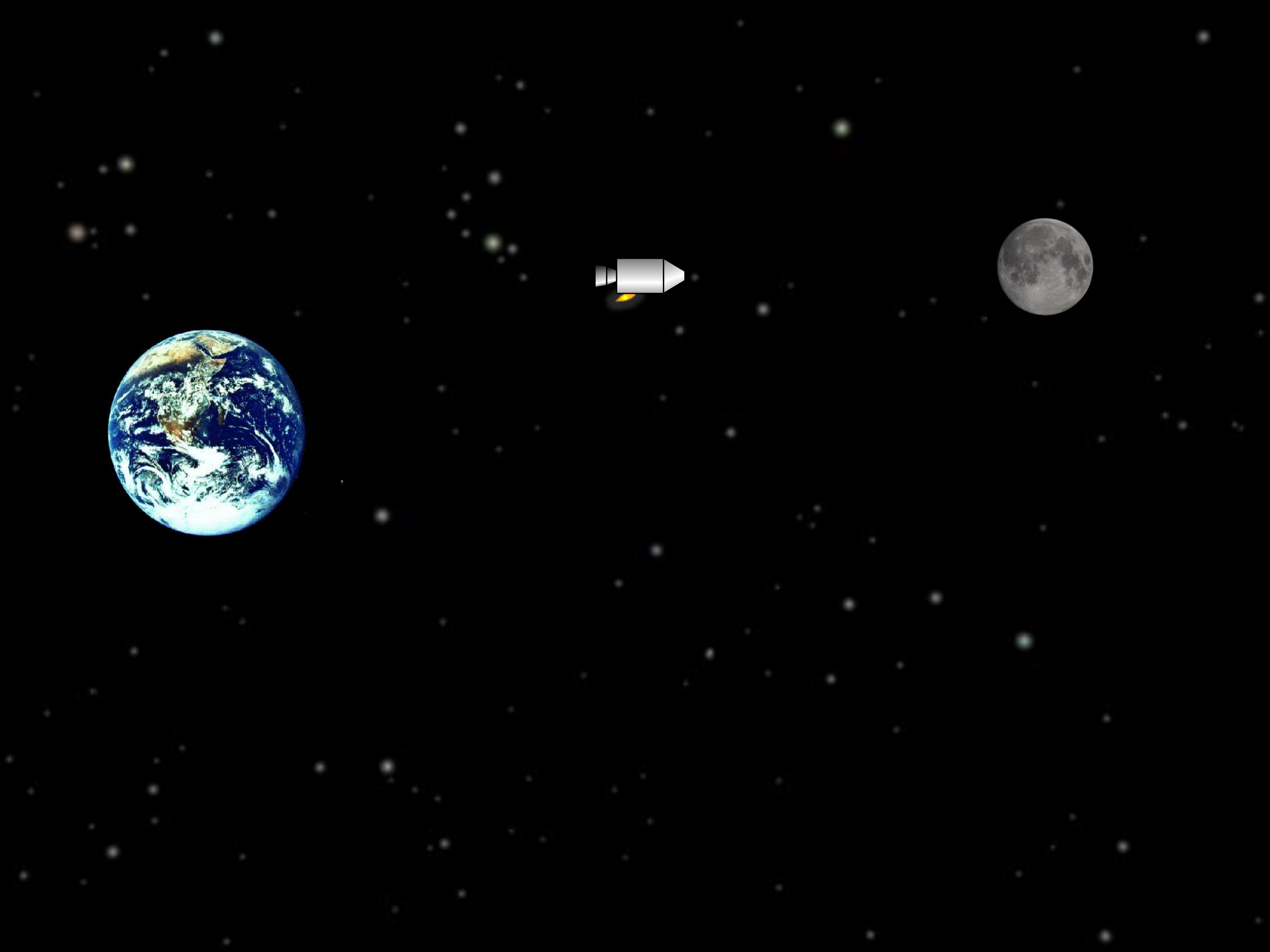
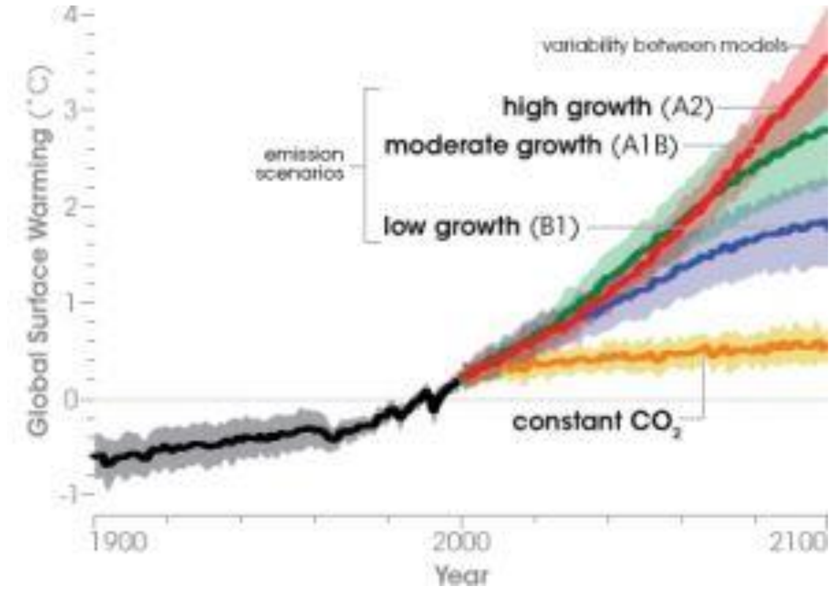
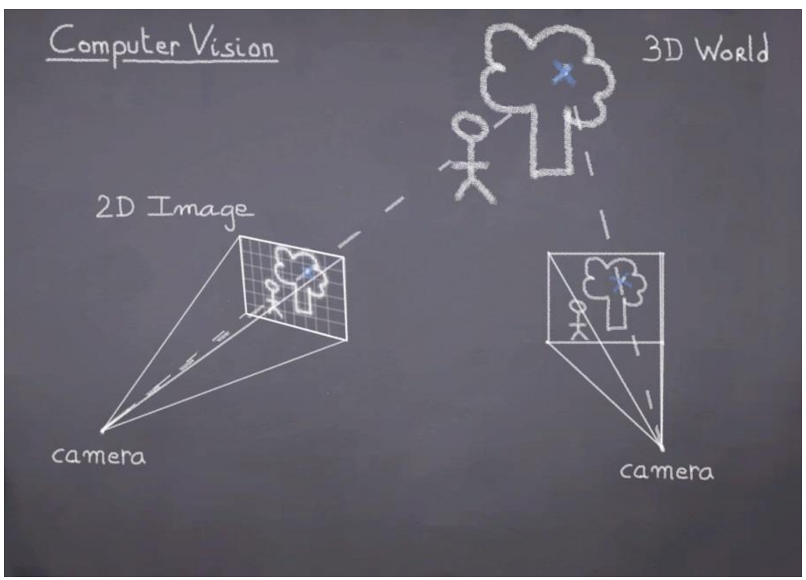
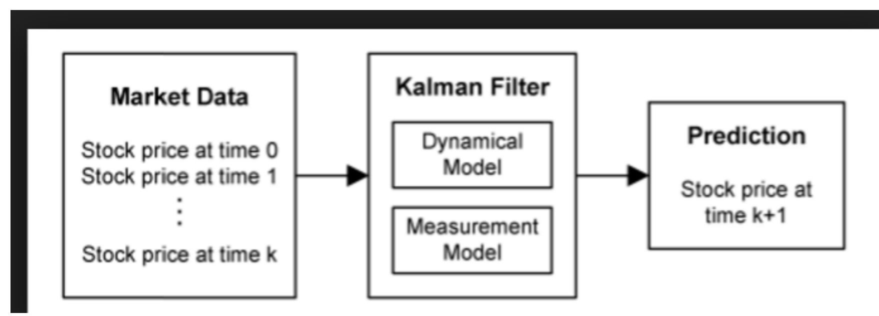




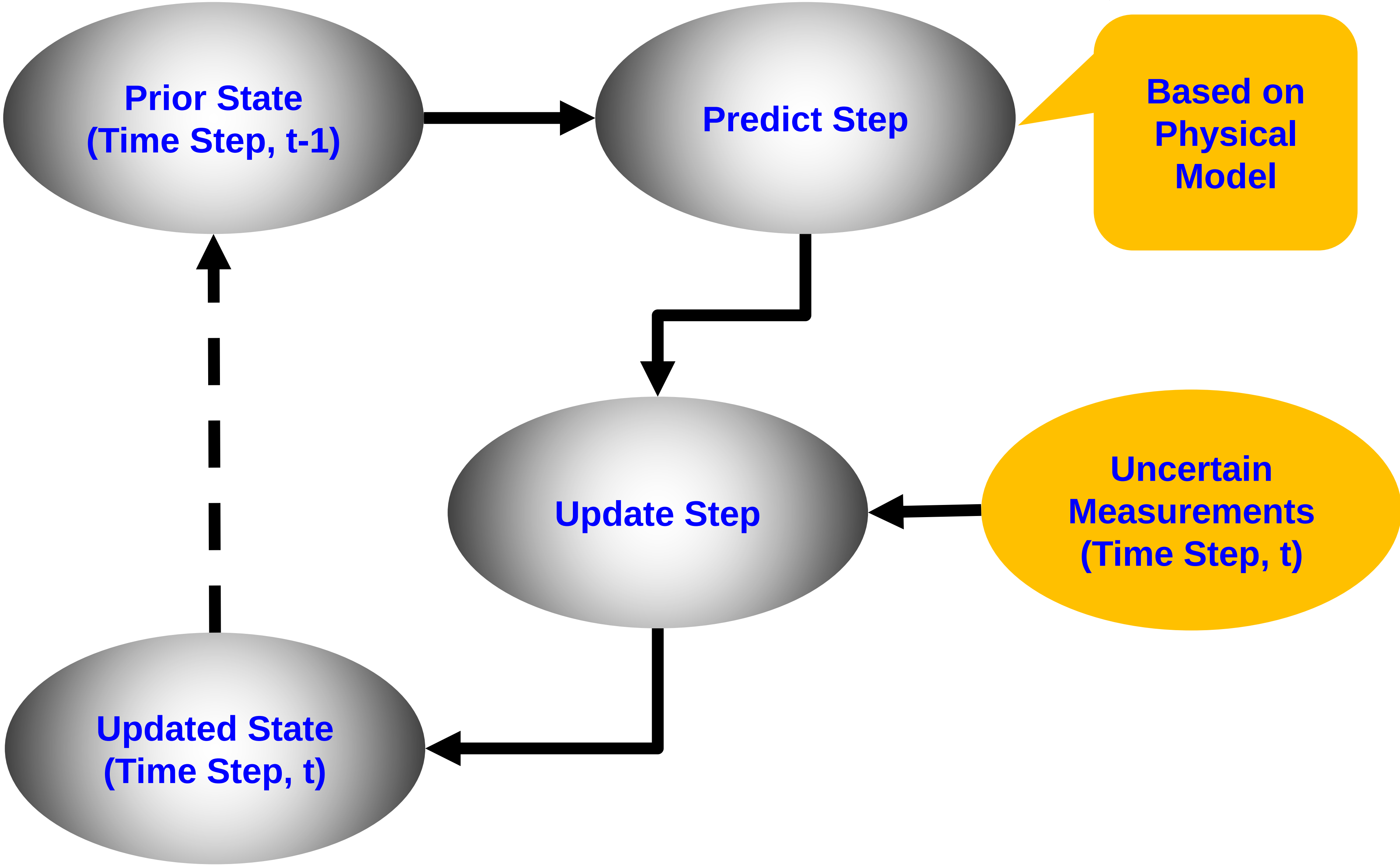
Kalman Filter Applications



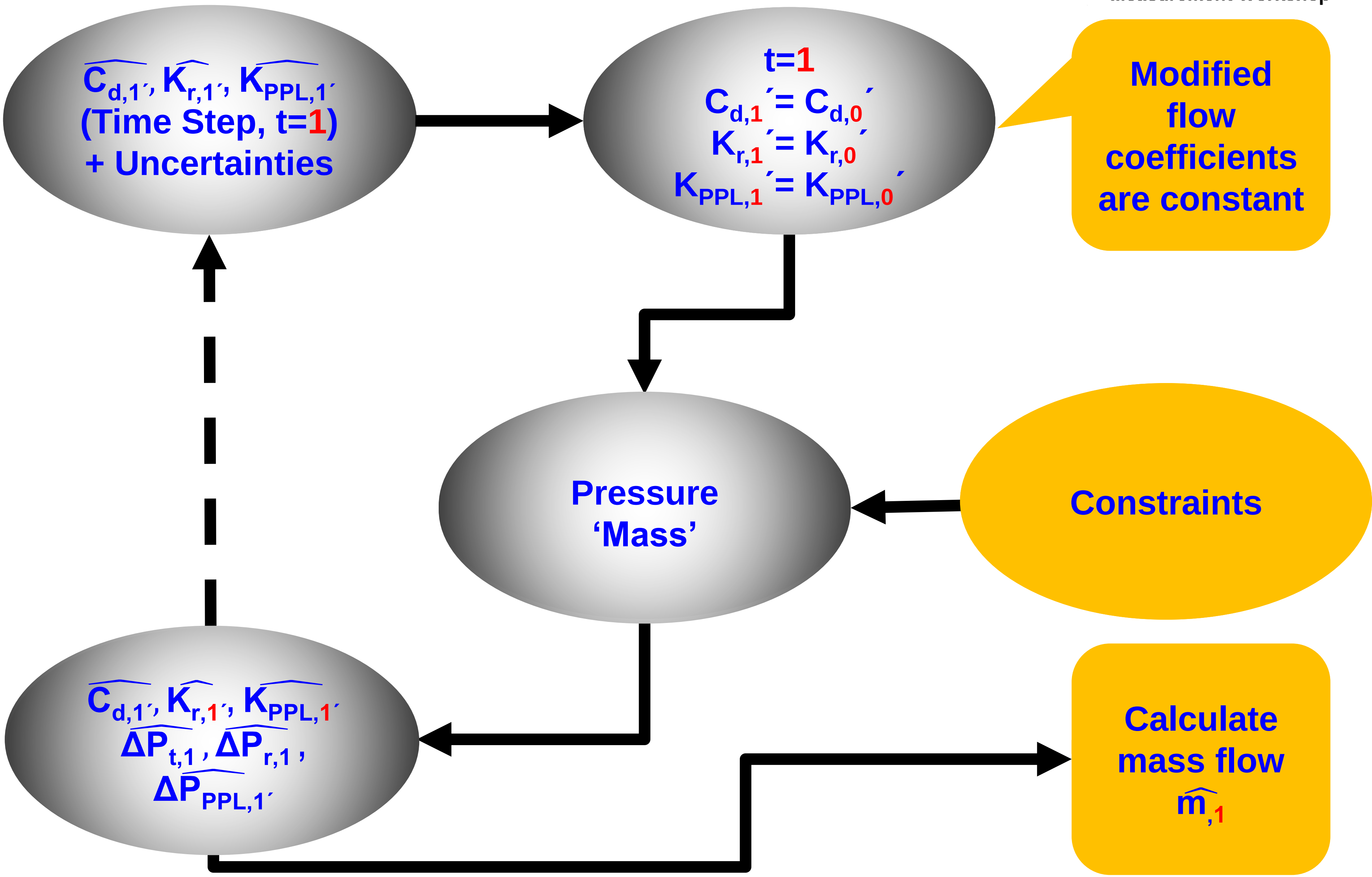




Kalman Filter Algorithm



Kalman Filter Algorithm - MLU



Thought experiment – theoretical example

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1. Recap Maximum Likelihood Uncertainty (MLU)
2. The next dimension - TIME
3. Kalman Filter
4. **Thought experiment – theoretical example**
5. Real data
6. Conclusions – the introspective orifice meter

Thought Experiment

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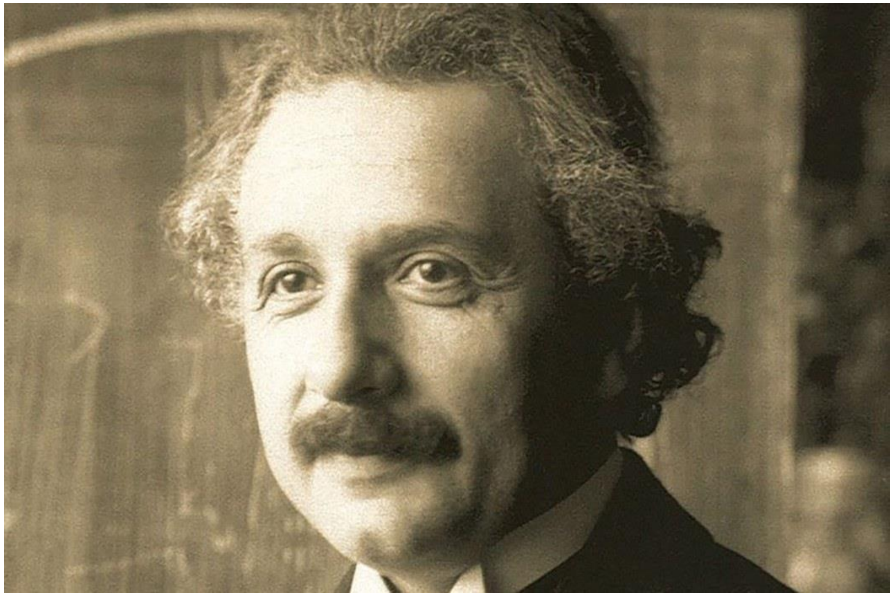

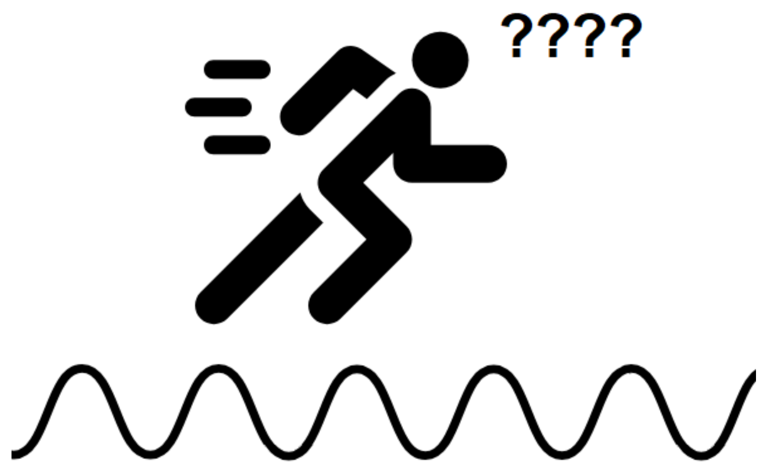

Three DP Meters – Perfect Information

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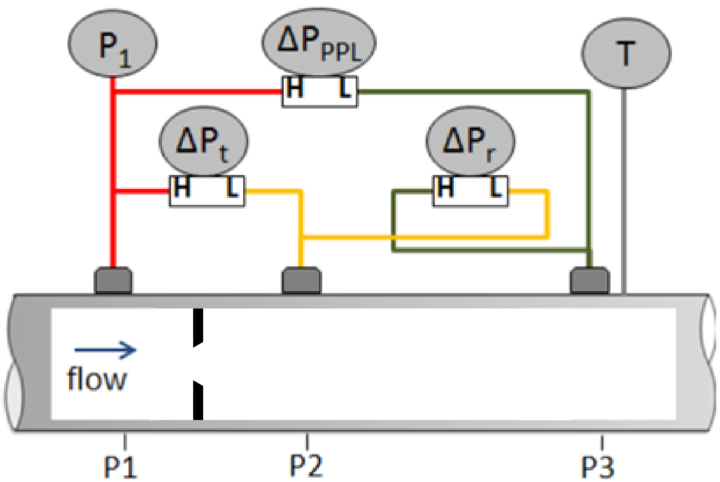
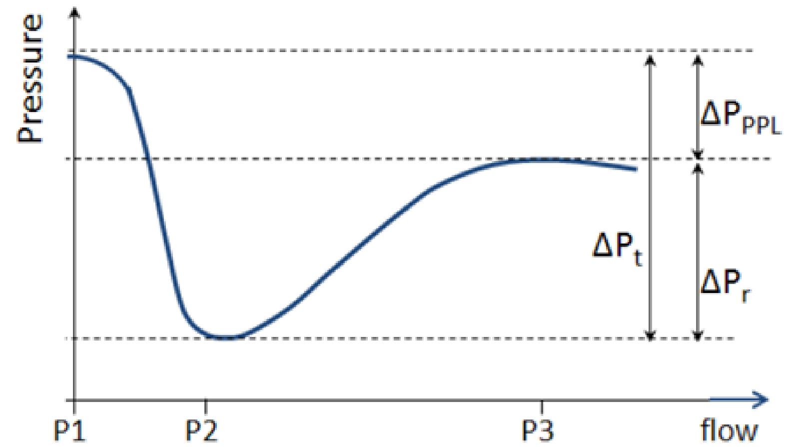
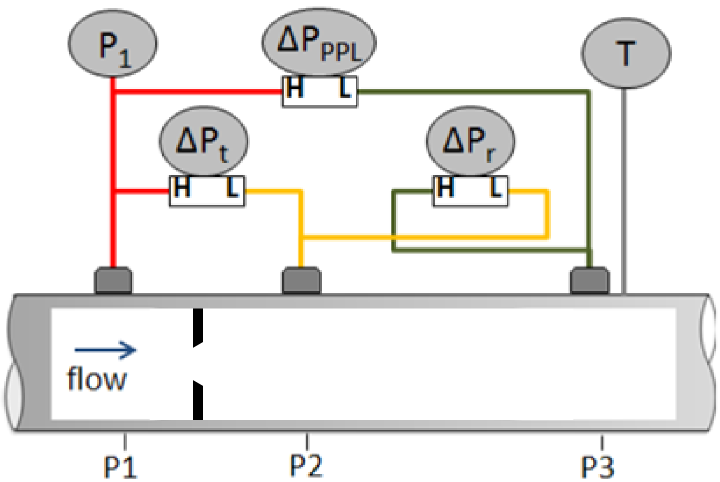


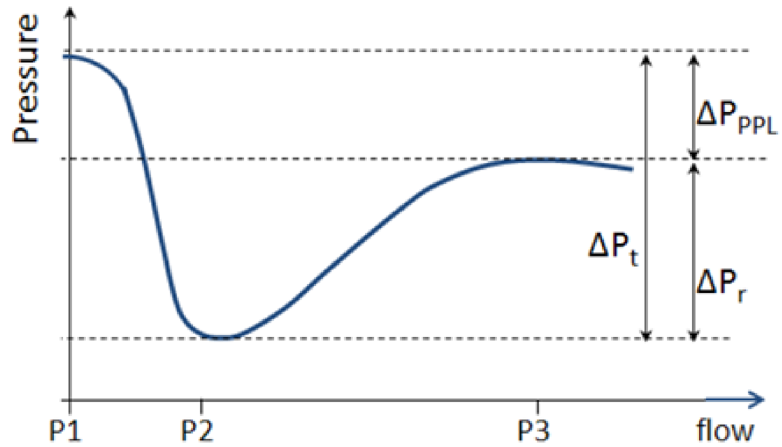


$$\dot{m}_t = EA_tYC_d(2\rho\Delta P_t)^{1/2} \qquad \dot{m}_r = EA_tK_r(2\rho\Delta P_r)^{1/2}$$

$$\dot{m}_{PPL} = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2}$$

Three DP Meters – Perfect Information





$$\dot{m} = EA_tYC_d(2\rho\Delta P_t)^{1/2} \qquad \dot{m} = EA_tK_r(2\rho\Delta P_r)^{1/2}$$

$$\dot{m} = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2}$$

4”, 0.5 Beta Orifice DP Meter

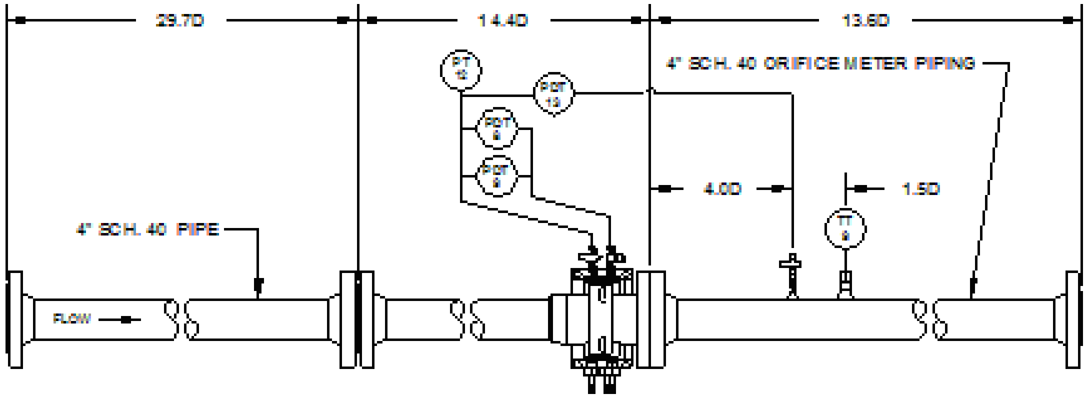


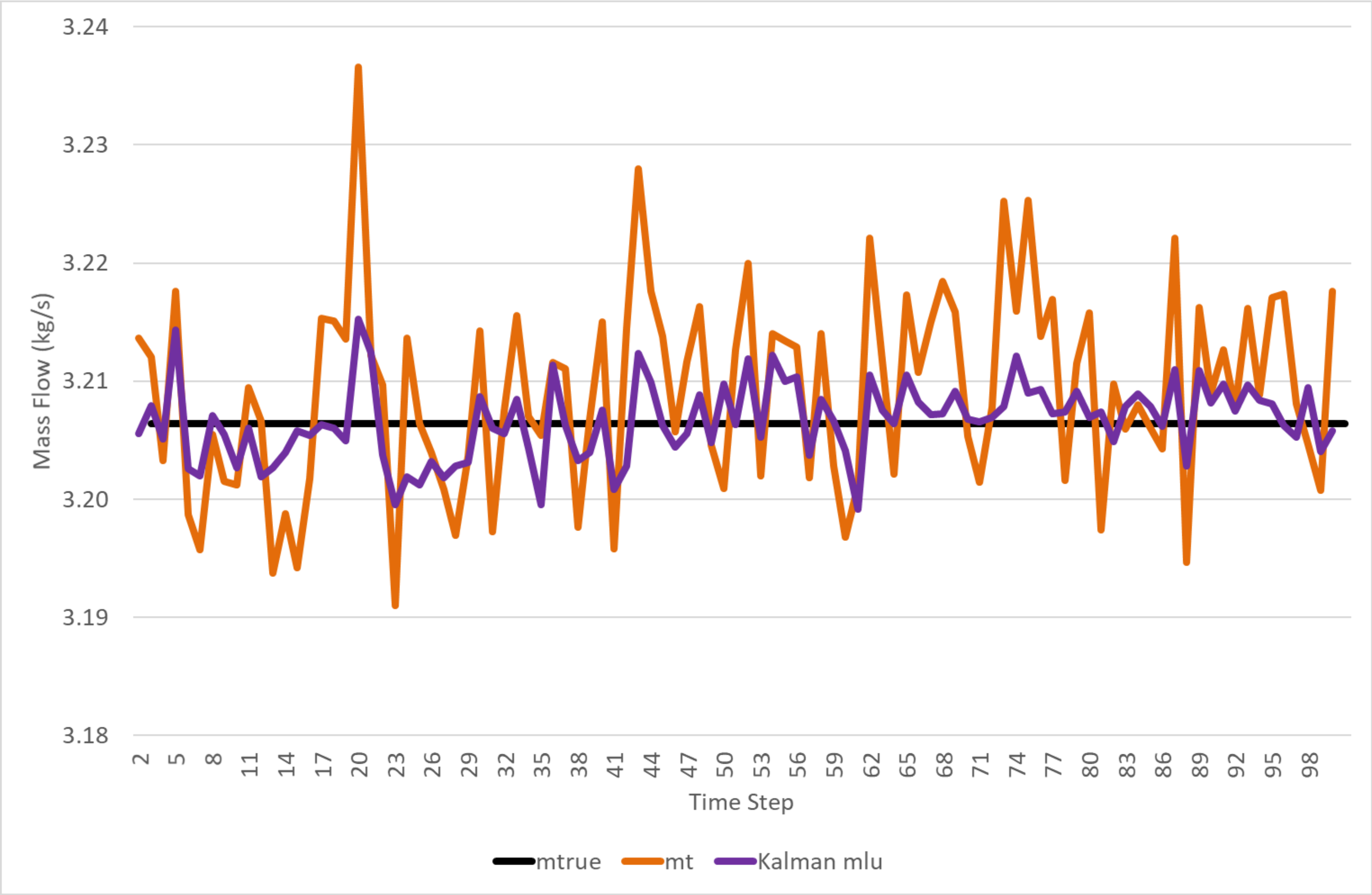
Table 1 – Hypothetical 4”, 0.5 Beta Orifice DP Meter Variable and Parameter Uncertainties

Variable / Parameter	Unit	‘True’ Value	Percent Uncertainty	Absolute Uncertainty
Mass Flow	kg/s	3.2064		
DP_t	Pa	90,796	1.0%	908
DP_r	Pa	23,931	1.0%	239
DP_{PPL}	Pa	66,866	1.0%	669
d	M	0.0508	0.05%	0.000025
D	m	0.102	0.25%	0.00026
Y	Dimensionless	0.991	0.30%	0.0030
C_d	Dimensionless	0.602	0.5%	0.003
K_r	Dimensionless	1.163	2.9%	0.017
K_{PPL}	Dimensionless	0.177	1.2%	0.002
ρ	kg/m^3	36.304	0.27%	0.098

Simulated DP Measurements

	ΔP_t	ΔPr	ΔP_{PPL}
	Pa	Pa	Pa
True DP	90,796	23,931	66,866
Uncertainty (±%)	1.00%	1.00%	1.00%

Hypothetical Example – Mass Flow Rate



Hypothetical Example – Mass Flow Rate Uncertainty

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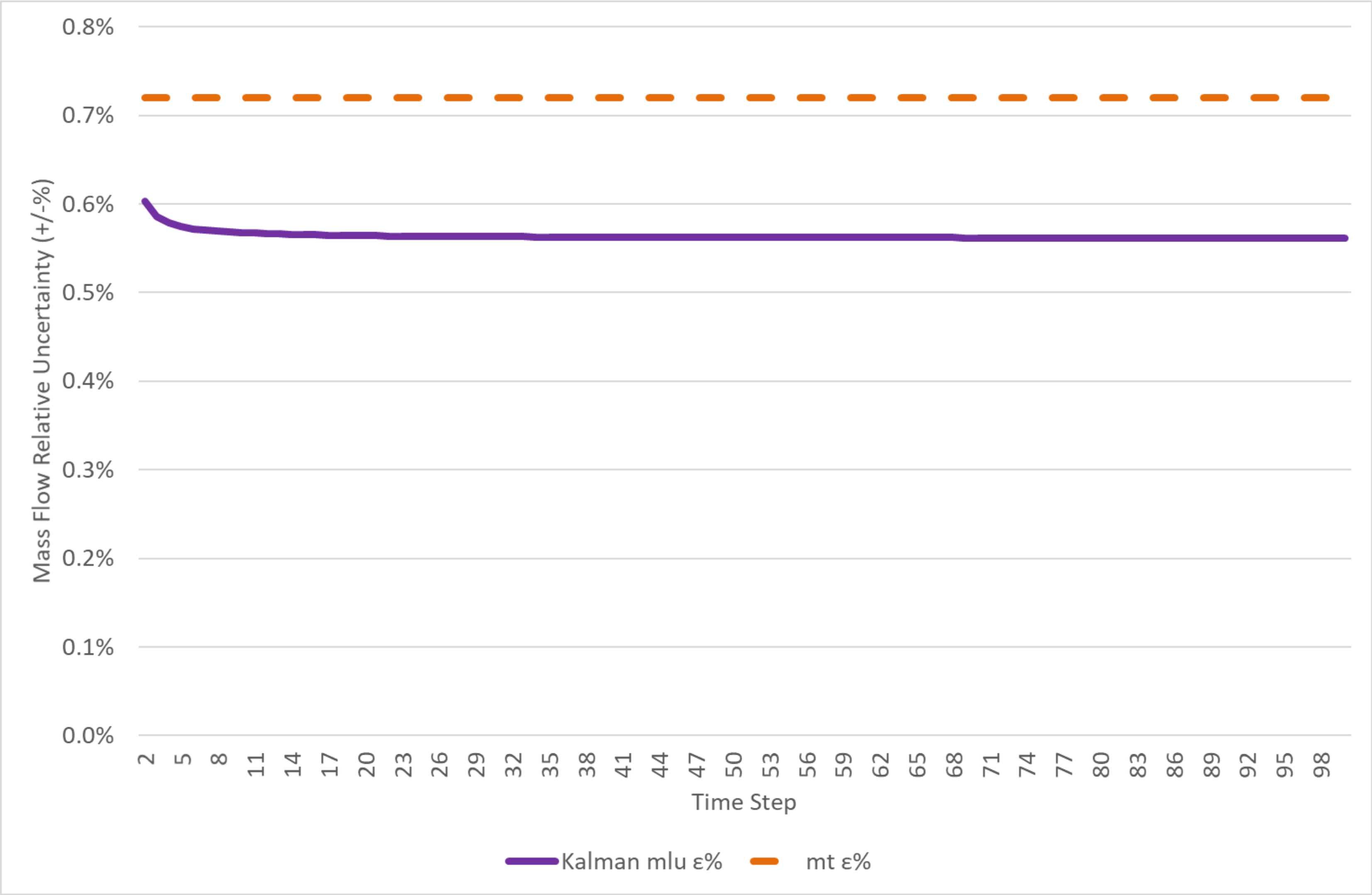

Hypothetical Example – Mass Flow Rate Uncertainty

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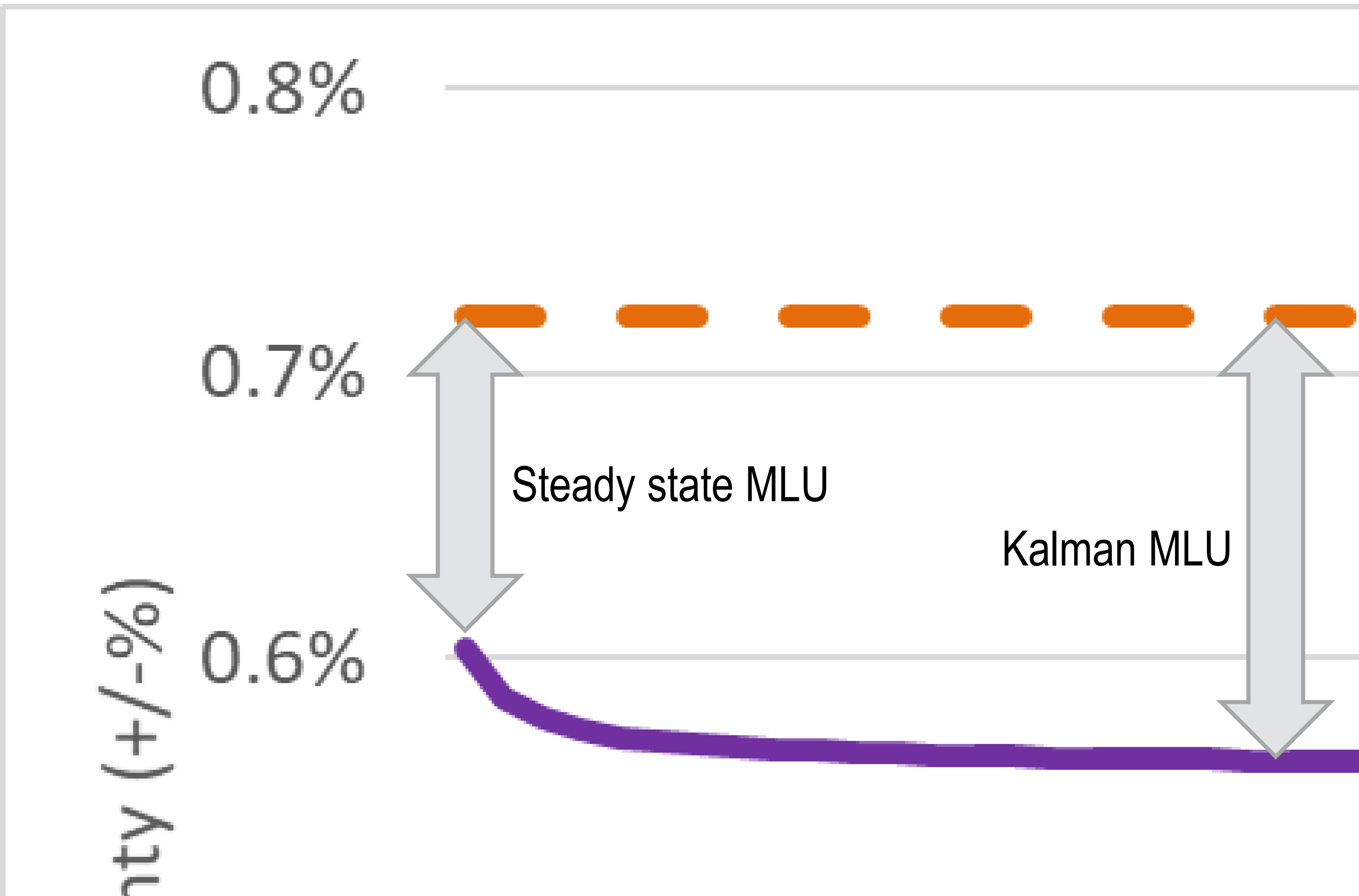
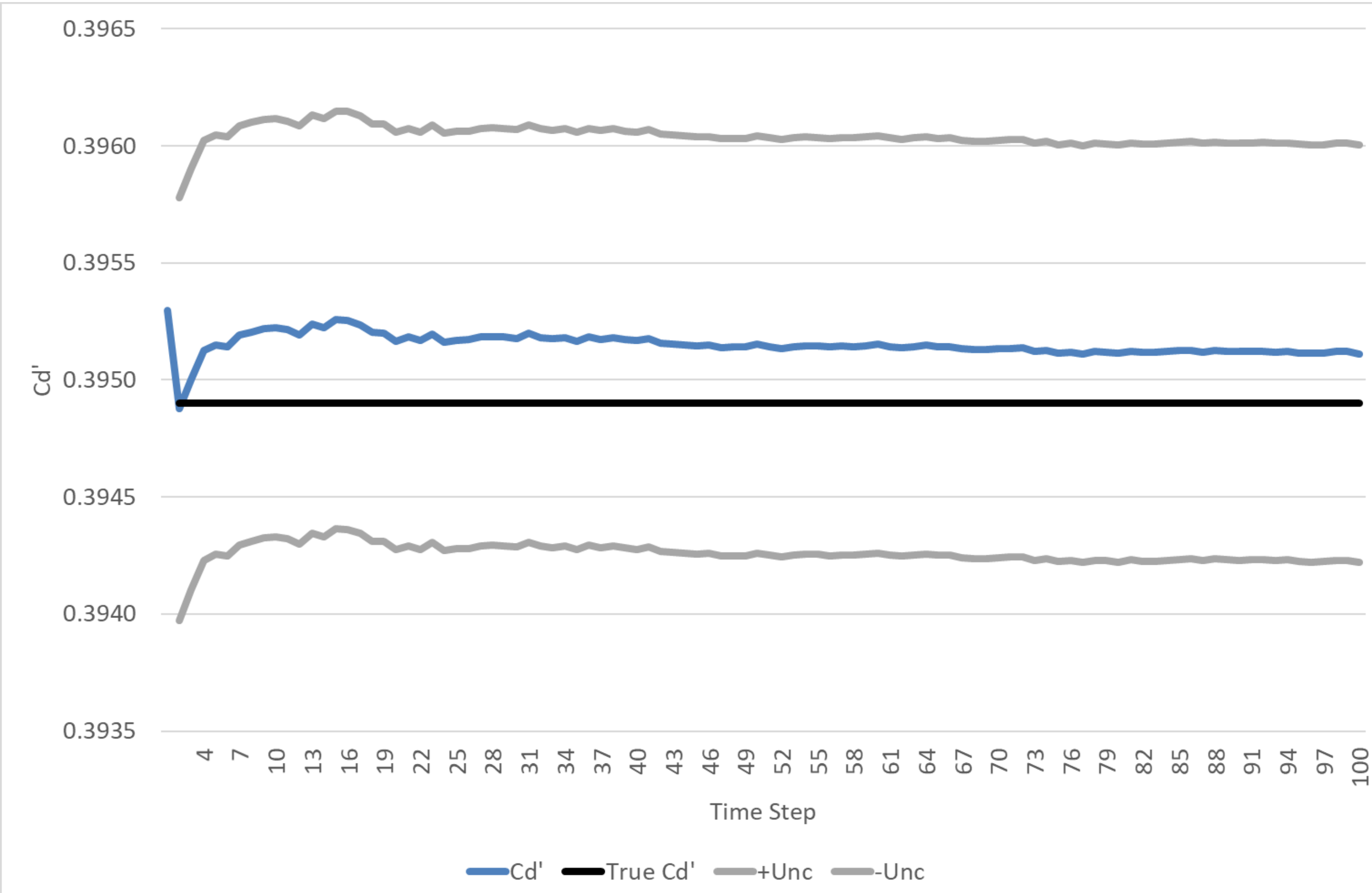
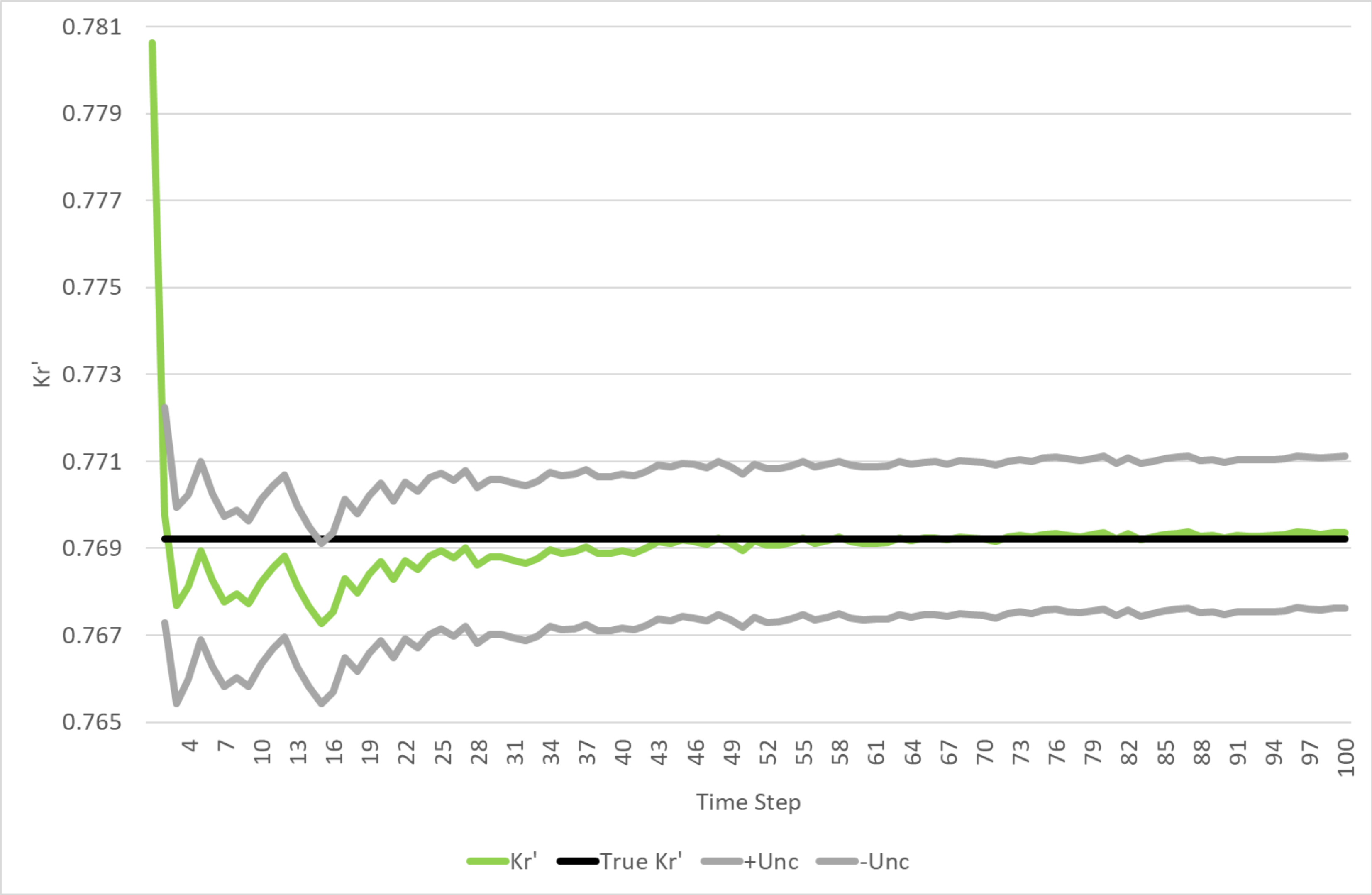


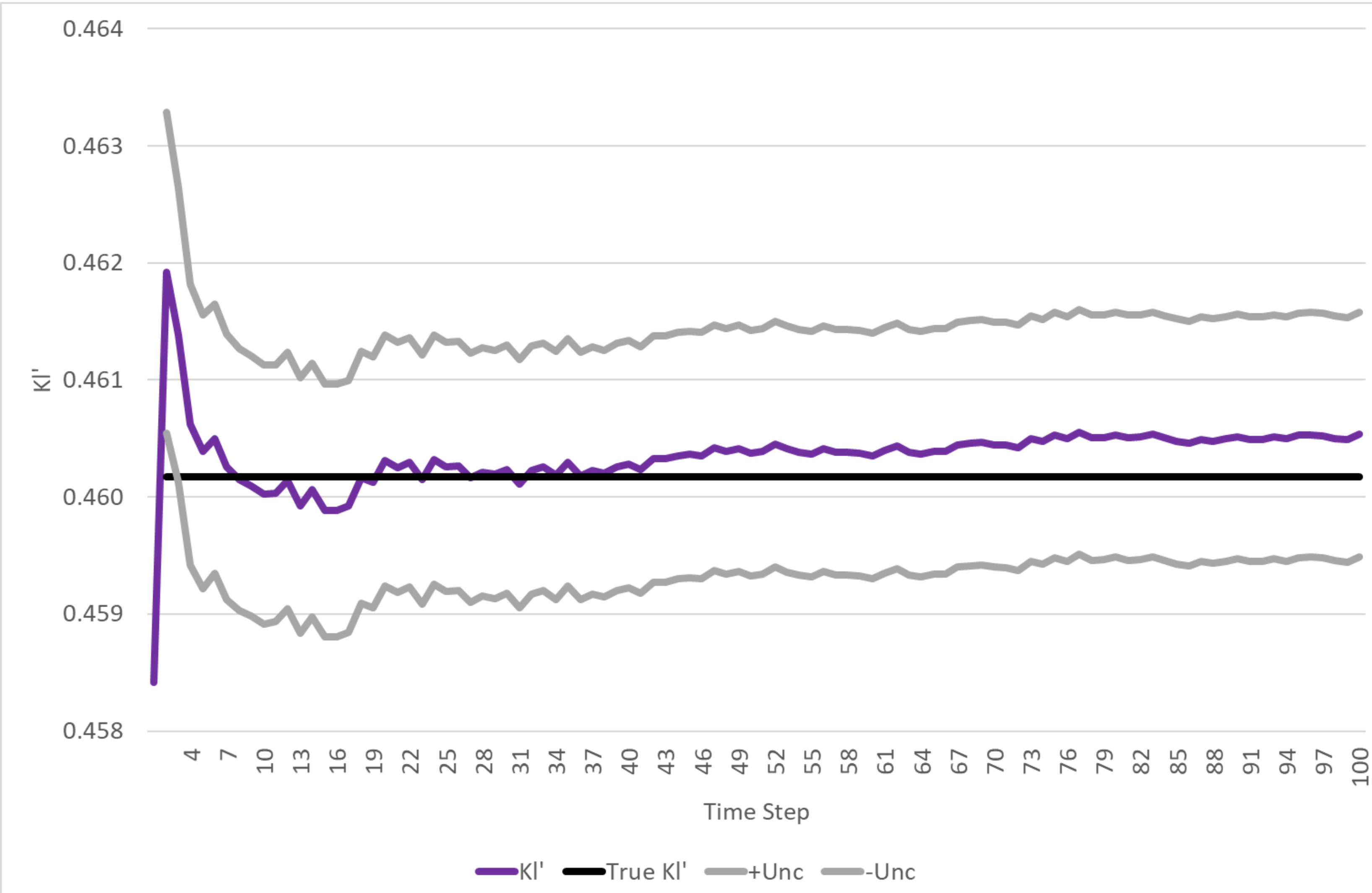
Hypothetical Example – Cd' Evolution



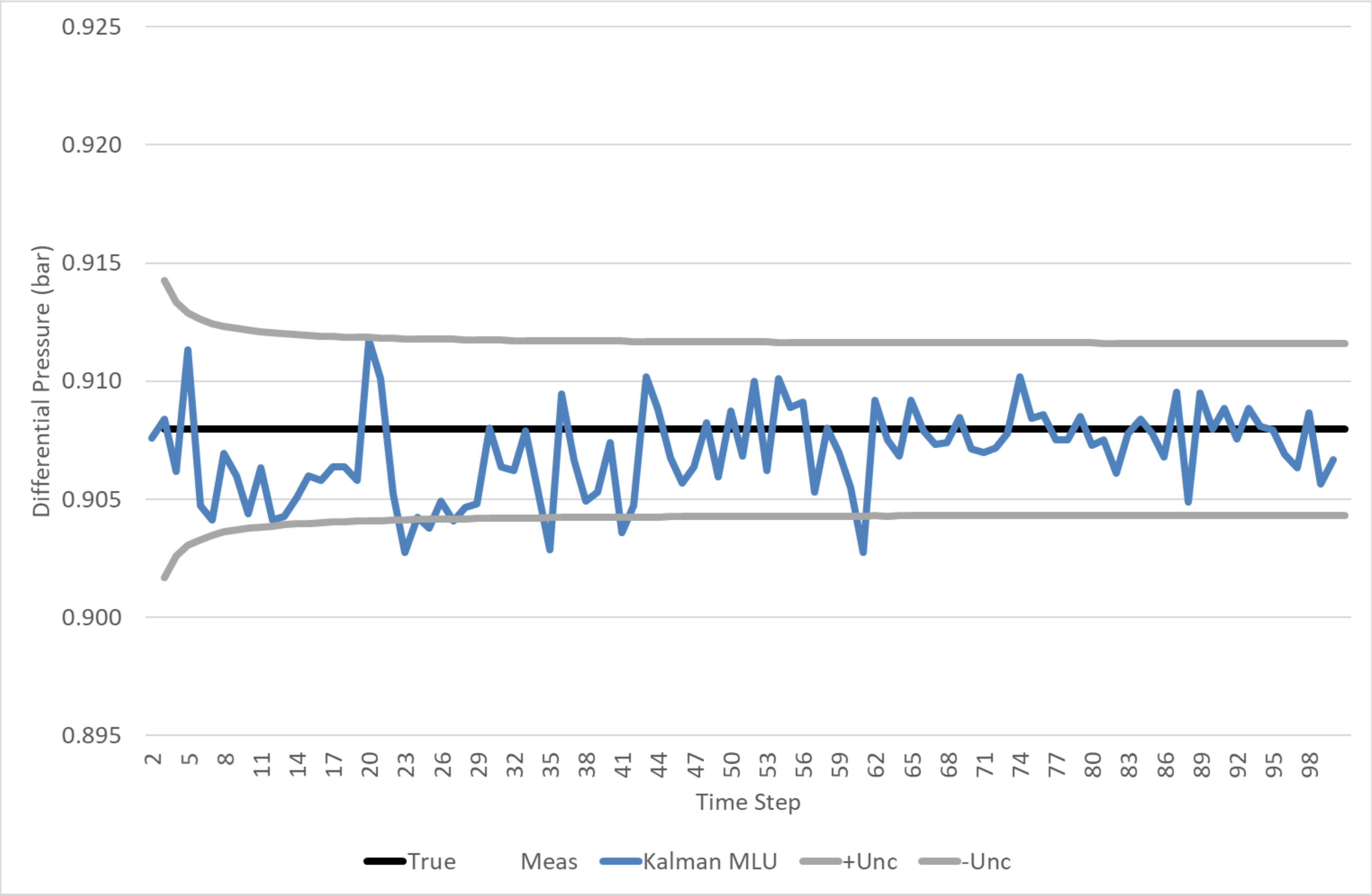
Hypothetical Example – Kr’ Evolution



Hypothetical Example – K_{PPL}' Evolution



Hypothetical Example – ΔP_t Evolution



Hypothetical Example – ΔP_r Evolution

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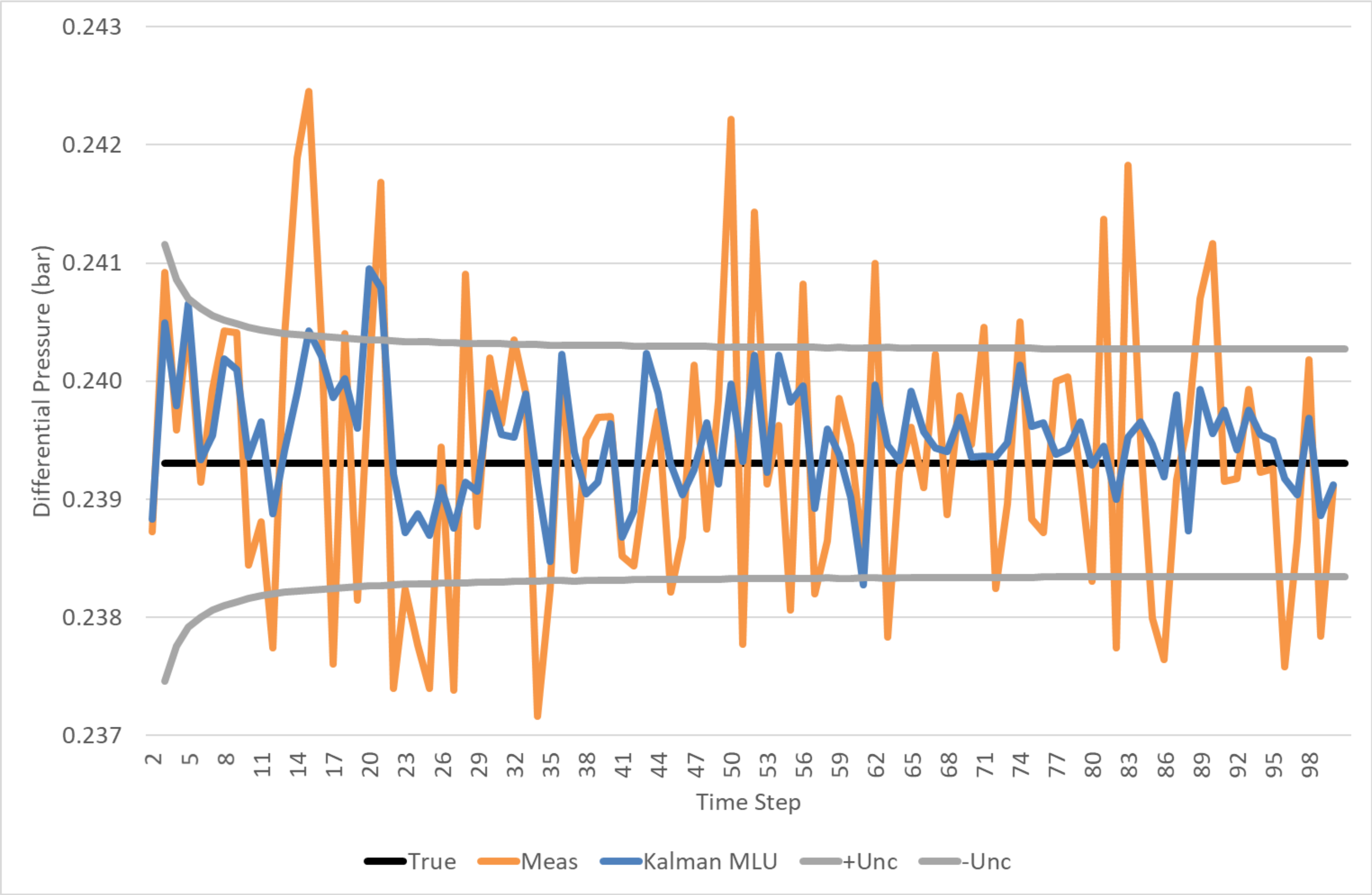

Hypothetical Example – ΔP_{PPL} Evolution

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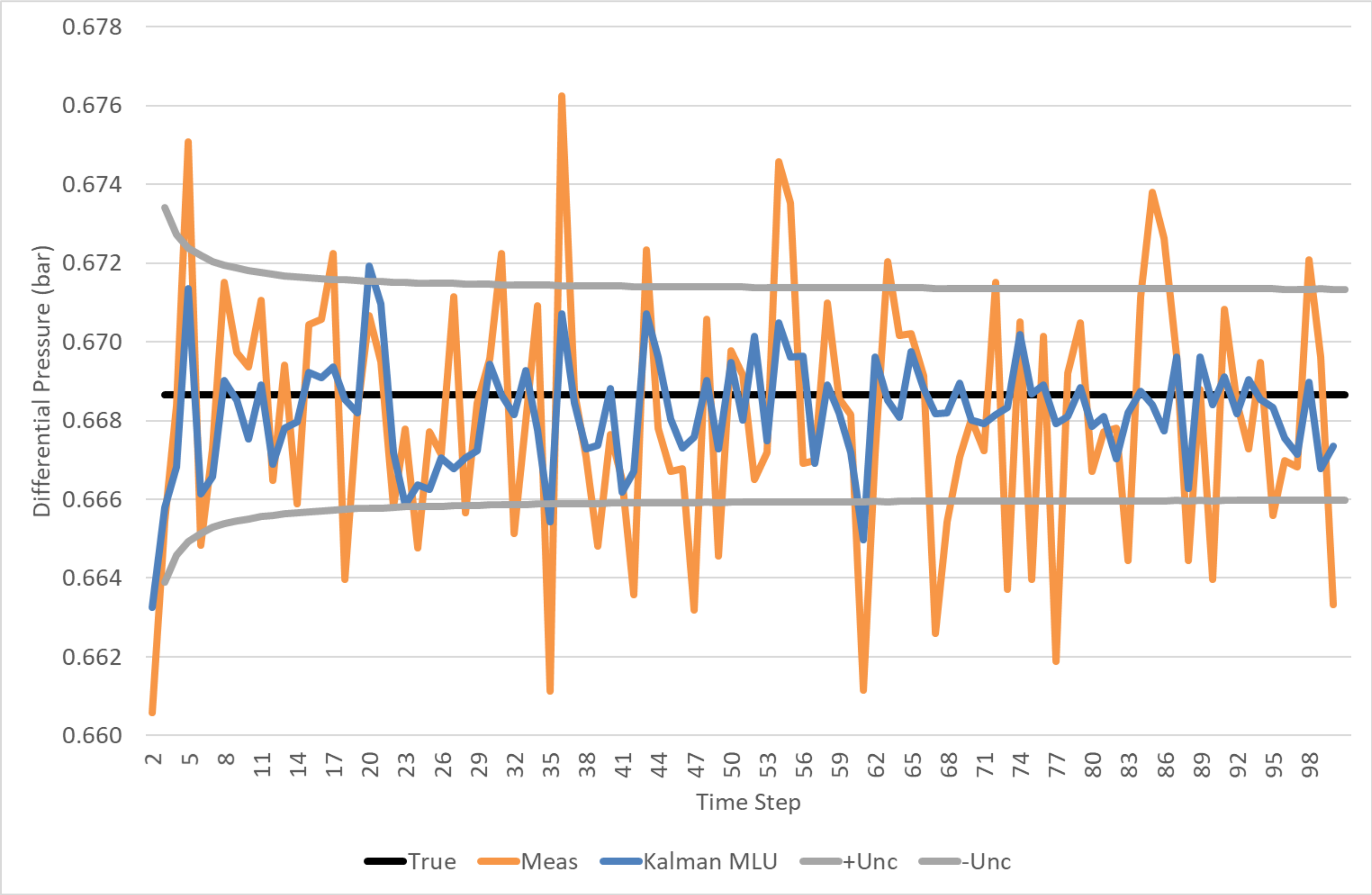

Real Data

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1. Recap Maximum Likelihood Uncertainty (MLU)
2. The next dimension - TIME
3. Kalman Filter
4. Thought experiment – theoretical example
5. **Real data**
6. Conclusions – the introspective orifice meter

Real Data – 6”, 0.6 Beta Orifice DP Meter

Real Data – Mass Flow Rate

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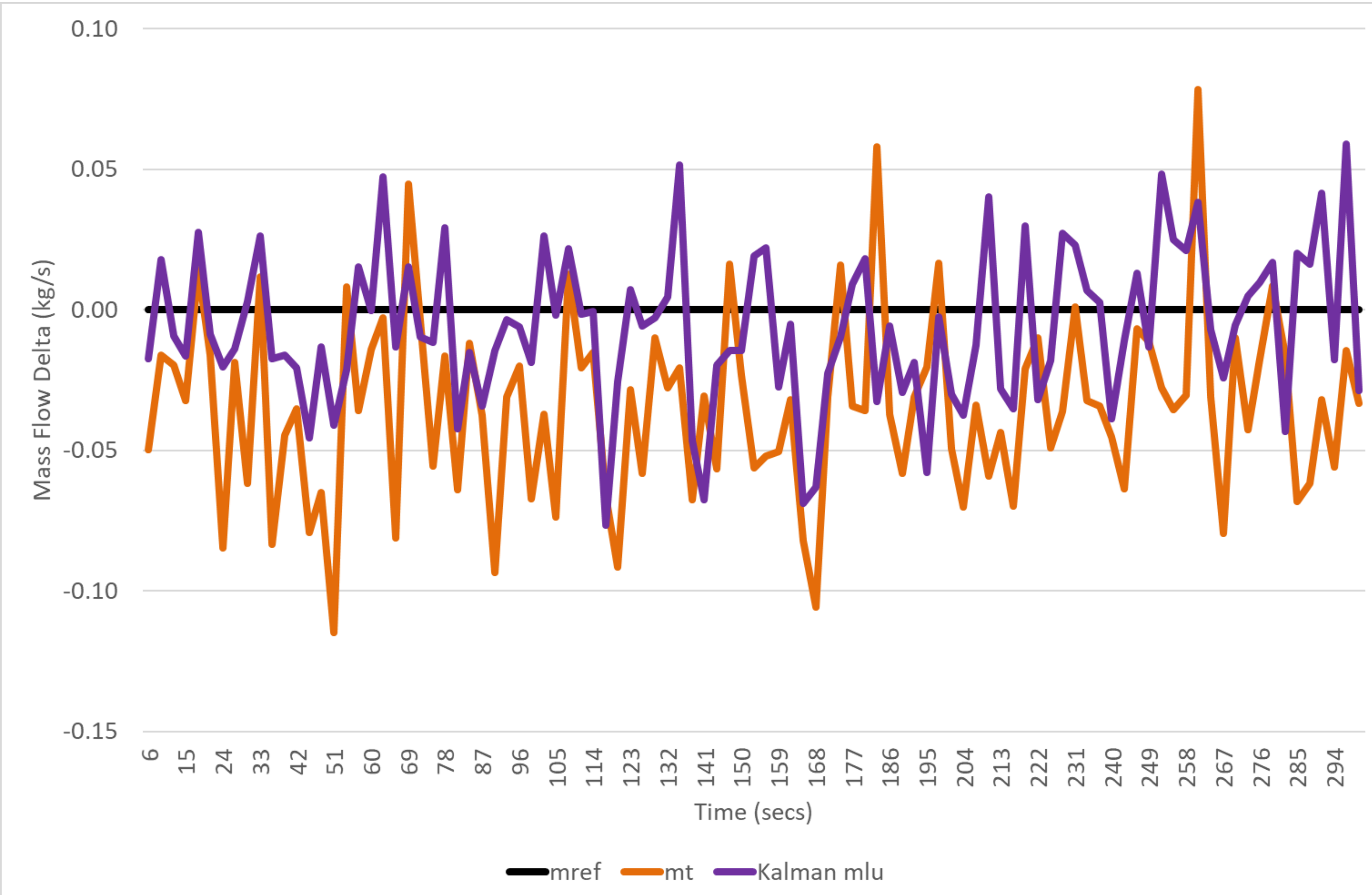
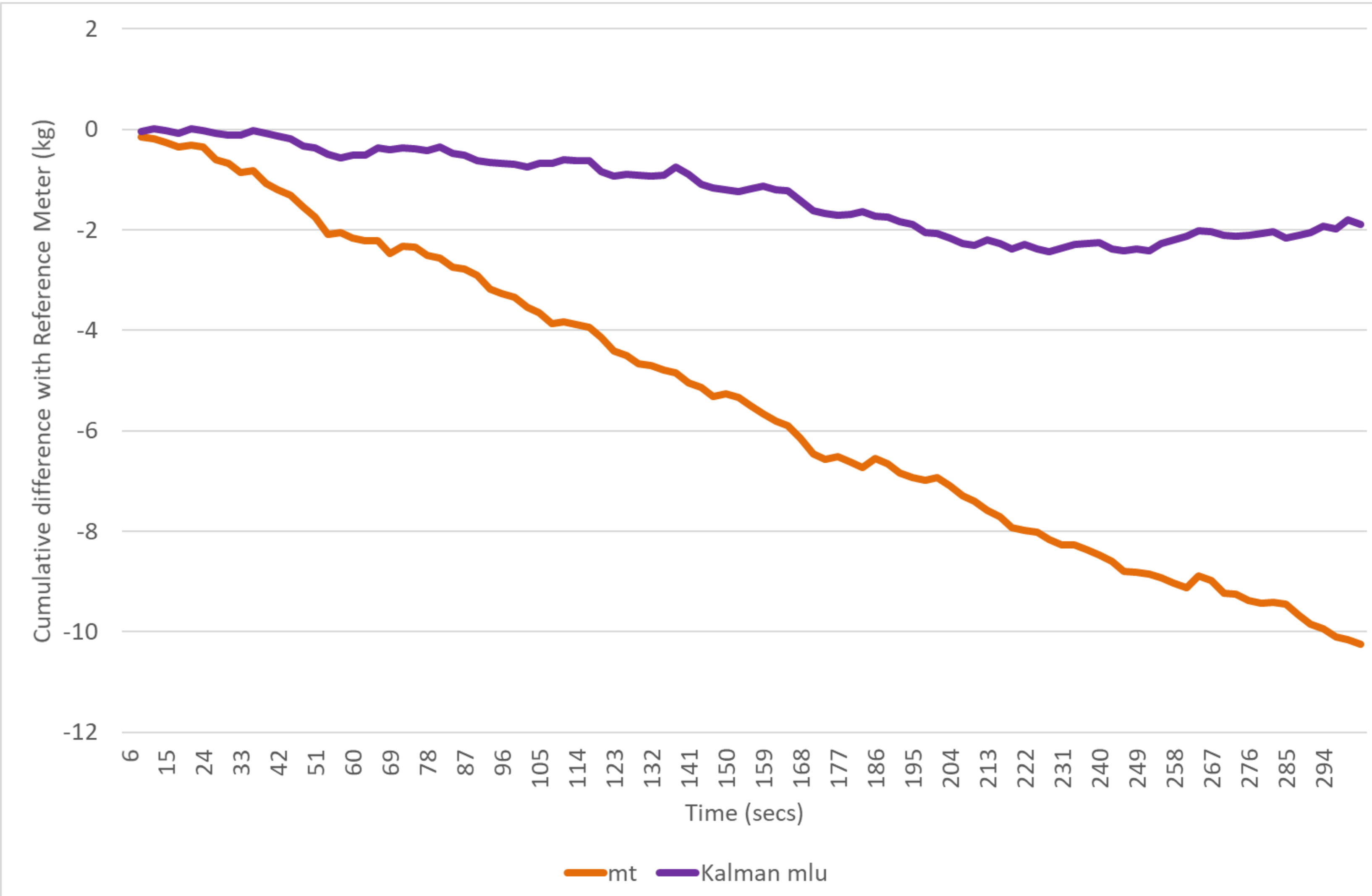
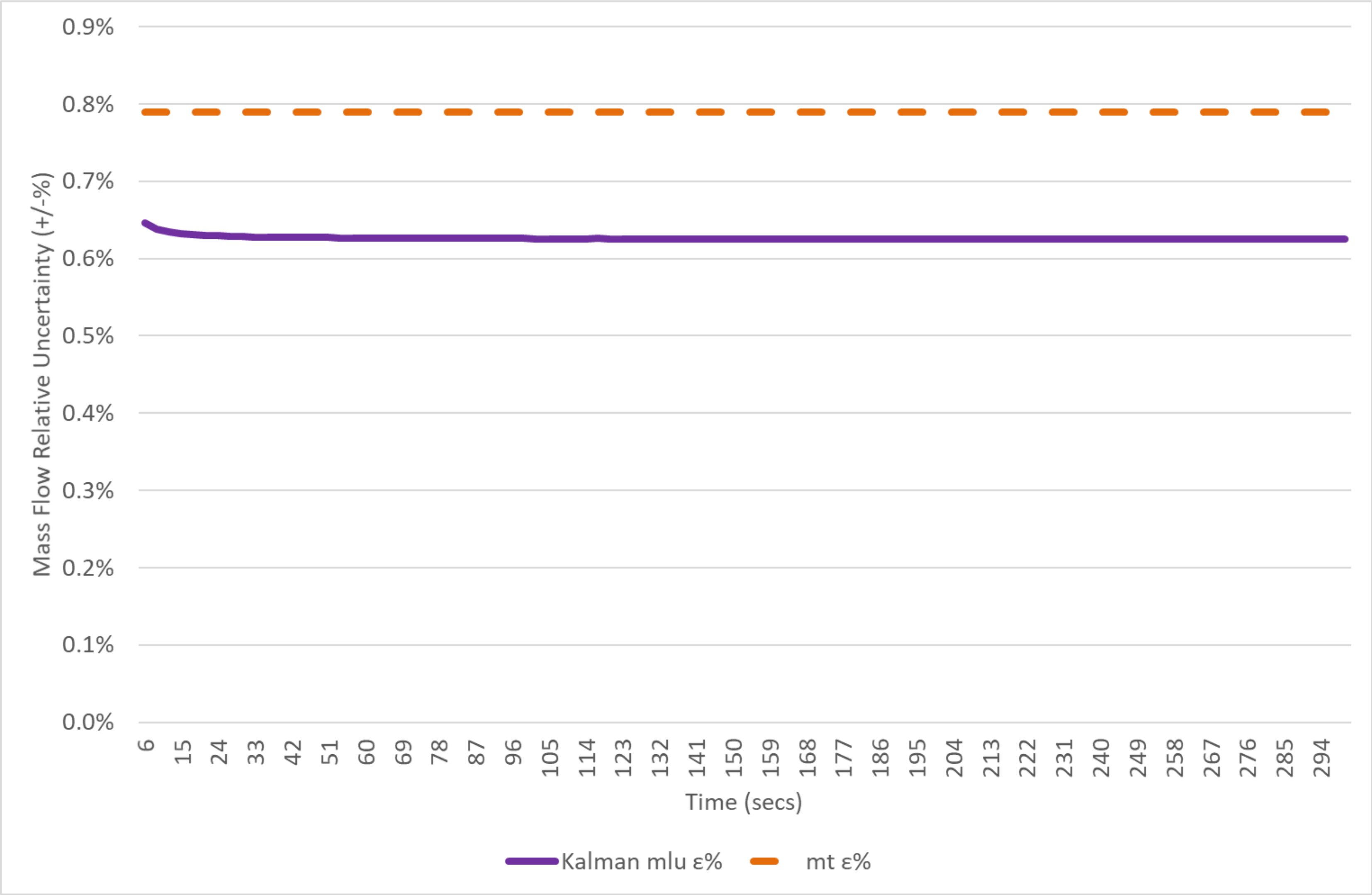


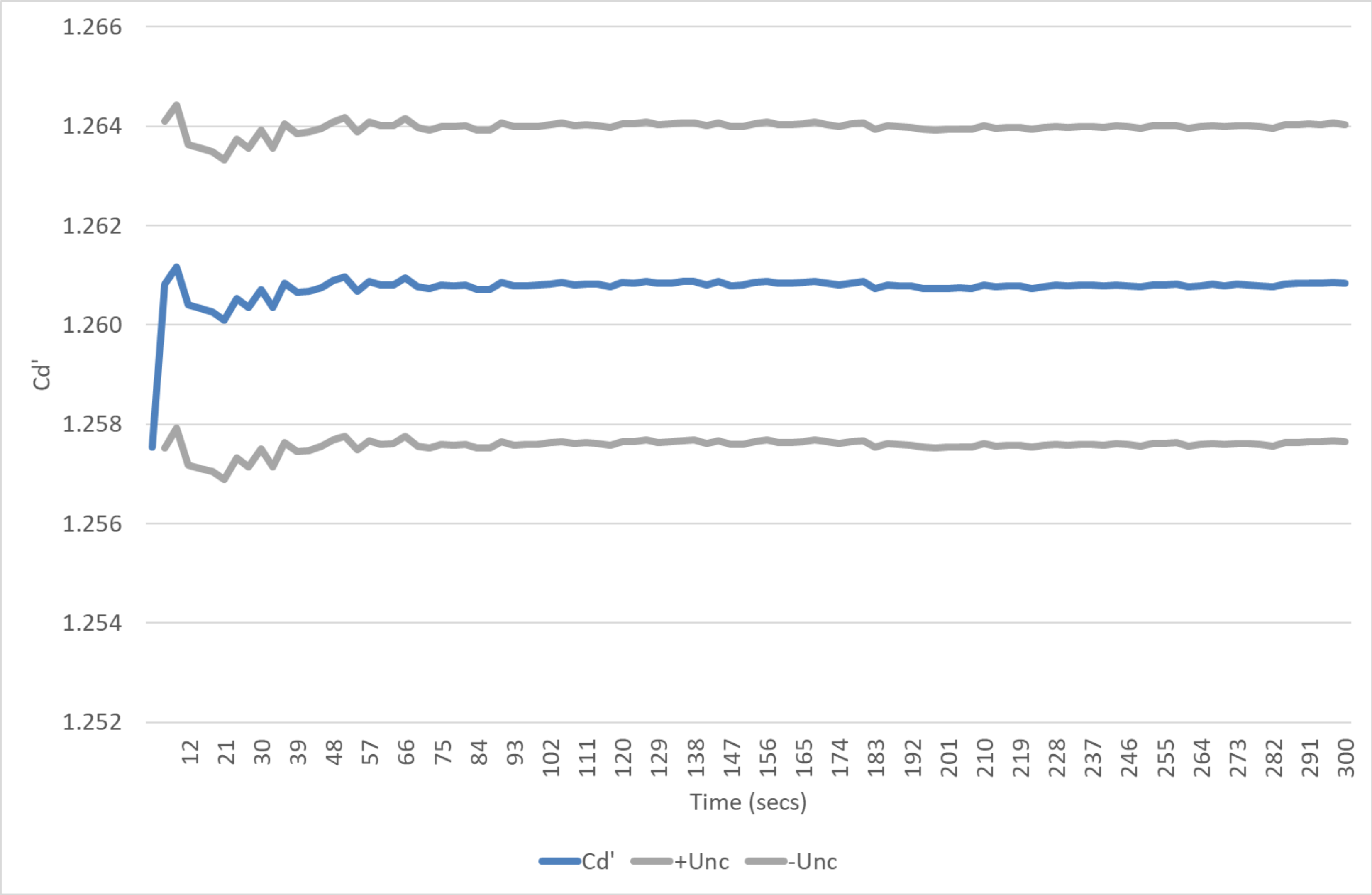
Real Data – Cumulative Mass Difference



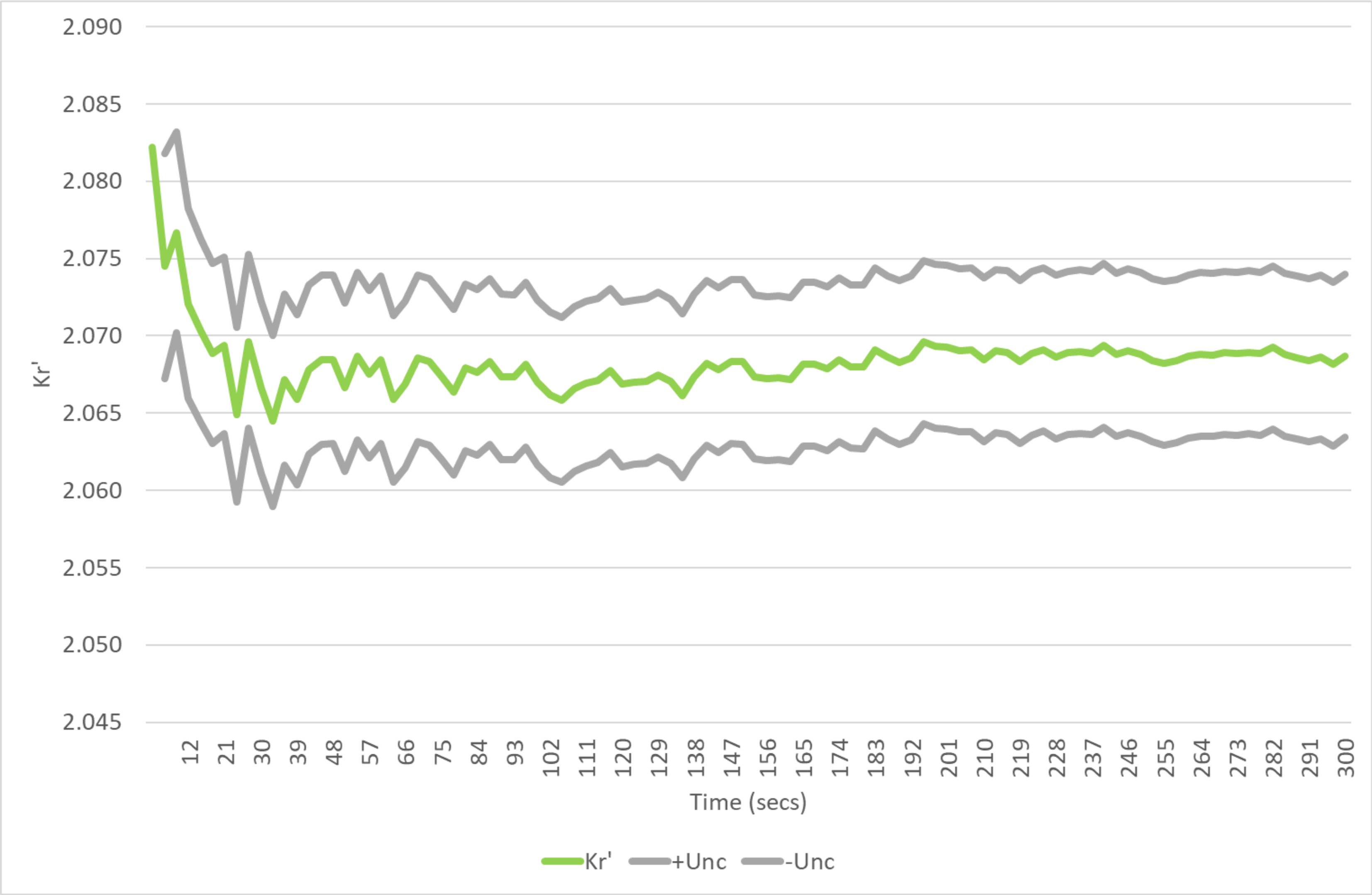
Real Data – Mass Flow Rate Uncertainty



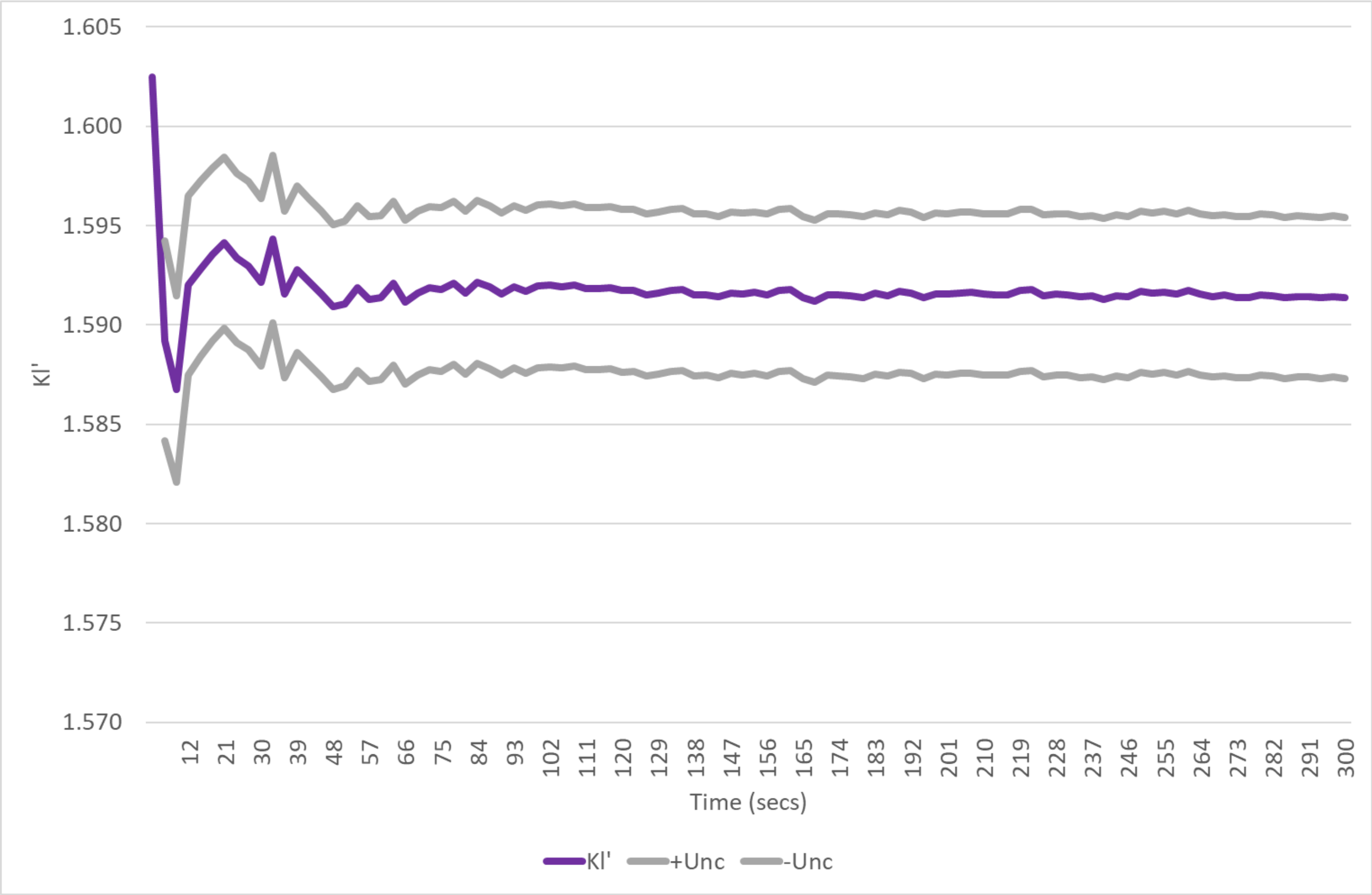
Hypothetical Example – Cd' Evolution



Hypothetical Example – Kr’ Evolution



Hypothetical Example – K_{PPL}' Evolution



Conclusions – the introspective orifice meter

North Sea FLOW Measurement Workshop

1. Recap Maximum Likelihood Uncertainty (MLU)
2. The next dimension - TIME
3. Kalman Filter
4. Thought experiment – theoretical example
5. Real data
6. **Conclusions – the introspective orifice meter**

The Introspective Orifice Meter

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- DP meter, well established
- Data: DP signal variation in time
- Kalman MLU
- Utilises the time dimension
- Demonstrated using theoretical and real data
- The introspective orifice meter

What Consider the two patterns HTH and HTT

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Which of the following is true:

A The average number of tosses until HTH is **larger** than the average number of tosses until HTT

B The average number of tosses until HTH is the **same** as the average number of tosses until HTT

C The average number of tosses until HTH is **smaller** than the average number of tosses until HTT

Heads and tails sequences

H T H

H T T H T H

H T T

H T H T T

To raise new questions, new possibilities, to regard old problems from a new angle, requires creative imagination and marks real advance in science.