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Offshore flares: measurement and calculation of combustion efficiency, methane and CO2e emissions

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1 INTRODUCTION

This paper is an analysis of the implications of a semi-empirical flare combustion efficiency (CE) equation, developed by the University of Alberta (UoA), on methane and carbon dioxide emissions from oil and gas installation flares.

Currently, for the reporting of CO_2 emissions, the flares on offshore oil and gas platforms are assumed to have a combustion efficiency of 98%. The 2% unburnt gas, however, is principally methane. According to IPCC (Intergovernmental Panel on Climate Change) data, methane has a Global Warming Potential (GWP) considerably greater than of CO_2 (84 times over 20 years but 28 times over 20 years [12]). The GWP is a measure of how much energy the emission of 1 ton of a gas will absorb over a given period of time, relative to the emission of 1 ton of carbon dioxide (CO_2). Hence, recently there has been an increased focus on the determination of combustion efficiencies and the consequent implications for total CO_2e emissions from flares when correctly accounting for the contribution of methane.

The UoA conducted extensive research into the combustion efficiency of hydrocarbon flares. One of the outcomes of this work was a semi-empirical equation, which expressed the combustion efficiency as a function of several variables including: gas exit velocity, wind speed and the Lower Heating Value (LHV) of the un-combusted gas. There is a physical basis for this equation and hence it provides a mechanism to study the effects of the key input parameters on the combustion efficiency and total emissions due to carbon dioxide and the un-combusted methane.

As an alternative or to complement sampling, the paper explores the use of process simulations to estimate the composition of flare gas and hence its LHV. In particular, the uncertainty in the LHV associated with this approach is assessed.

Efficiencies of over 98% are achievable at low to average wind speeds. However, due to the exponential nature of the equation, high wind speeds significantly reduce its value. The paper considers the wind speed probability distribution (typically modelled as Weibull) encountered in the North Sea and integrates it with the UoA CE equation to calculate an expected CE value. The simplistic assumption of a single average wind speed leads to an over-estimation of the CE value.

The paper explores the use of varying purge rates to the flare to improve CE and mitigate the deleterious impact of wind. Increasing purge rate in the form of fuel gas will increase CO_2 emissions due to combustion but by increasing the exit velocity it will increase CE and hence reduce methane emissions which may thereby reduce the total CO_2e from the flare.

2 BACKGROUND

2.1 Regulatory and Industry Initiative Information

Environmental and regulatory bodies around the world have recognised the impact of methane emissions upon global warming and have identified the combustion efficiency of gas being flared from oil and gas operations as an area where improvements need to be made.

Emissions from oil and gas operations add significant quantities of greenhouse gases to the atmosphere and a reduction in emissions will therefore contribute towards the Net Zero targets set by governments globally.

Emissions from flaring include carbon particles (soot), unburned hydrocarbons, CO, and other partially burned hydrocarbons. Also emitted are NOx and, if Sulphur-containing material such as hydrogen sulphide or mercaptans is flared, sulphur dioxide (SO2).

The quantities of hydrocarbon emissions generated relate to the degree of combustion; and the degree of combustion depends largely on the rate and extent of fuel-air mixing and on the flame temperatures which can be achieved and maintained.

Properly operated flares may achieve at least 98 percent combustion efficiency in the flare plume, meaning that hydrocarbon and CO emissions amount to less than 2 percent of hydrocarbons in the gas stream [13].

Methane emissions reduction is being targeted by a variety of regulatory and industry initiatives, supported by governments and oil and gas companies, with proposals for legislative controls being discussed [14].

One major industry initiative is being driven by the Oil and Gas Methane Partnership (OGMP) which was launched at the 2014 United Nations (UN) Secretary General's Climate Summit; having been created by the Climate and Clean Air Coalition (CCAC) and the United Nations Environmental Programme (UNEP) as a voluntary initiative to help companies reduce methane emissions in the oil and gas sector. The OGMP is supported by the Norwegian and United Kingdom governments and has approximately 83 national and international oil and gas companies as members at the time of writing.

Member companies are required to report on methane emissions along five levels of reporting, increasing in granularity by quantification methodology, level and sources of geography, and uncertainty in quantification. Companies progress along the reporting framework, increasing the accuracy of reported emissions with each level, with the goal of achieving "Gold Standard" reporting, the highest reporting level under OGMP. Gold Standard reporting is achieved once companies empirically reconcile measurements at source (Level four) and site (Level five) level for the vast majority of their assets.

Under the reporting framework, member companies have three years to achieve Gold Standard compliance for operated assets and five years for non-operated assets. However, in the interim, Gold Standard pathway is awarded to companies that demonstrate a credible and explicit path towards Level 4 and 5 reporting within 3 years for operated assets, and 5 years for non-operated assets

The OGMP has produced a Technical Guideline Document (TGD) for Flare Efficiency [15] which provides information on the current methods available to determine flare combustion efficiency at the different reporting levels.

2.2 Measurement

Accurate measurement, or calculation, of combustion efficiency is a challenging activity and, at present, is being performed on a retrospective basis using data gathered from numerous sources and which may have a high level of uncertainty.

As flare combustion efficiency is dependent upon a number of moving variables such as wind speed and gas composition (amongst others), the configuration of the plant for optimal flare combustion efficiency needs to be conducted on a dynamic basis rather than the current static, retrospective, basis.

Accord ESL have developed an application named 'Combustor' which will facilitate the direct measurement of flare combustion efficiency based on our own process simulation model CHARM [4], combined with academic research conducted by the University of Alberta in Canada [1], and which can be embedded within existing flare measurement and plant control systems as feedback for flare combustion optimisation. Our Combustor application will provide companies with a tool which will help to achieve level 4 Gold standard reporting under the OGMP framework.

2.3 Flare Combustion Efficiency

Combustion efficiency is often used interchangeably with destruction efficiency and they are, therefore, often confused. Destruction efficiency is a measure of how much of the original hydrocarbons are destroyed (to form CO_2 and CO), while combustion efficiency is a measure of how much of the original hydrocarbons burn completely and are transformed into CO_2 and water vapor.

Combustion efficiency is defined herein, in accordance with [1], as the ratio of the mass of fully oxidized carbon (i.e., mass of carbon within carbon dioxide) produced by combustion to the mass of carbon in the form of hydrocarbons in the fuel stream.

$$\mu = \frac{M_{C,CO2}}{M_{C,f}} \tag{1}$$

Where,

μ	combustion efficiency
$M_{C,CO2}$	mass rate of carbon in the form of CO_2 produced by the flame
$M_{C,f}$	mass rate of carbon in the form of hydrocarbon in the flare gas prior to combustion

For information, the related destruction removal efficiency (DRE) of methane (or any other hydrocarbon) is defined as:

$$DRE_{CH4} = 1 - \frac{M_{CH4,unc}}{M_{CH4,f}} \tag{2}$$

Where,

DRE_{CH4}	destruction removal efficiency of methane
$M_{CH4,unc}$	mass rate of methane uncombusted in the flame
$M_{CH4,f}$	mass rate of methane in the flare gas prior to combustion.

3 UOA EQUATION

3.1 Development

Currently, it appears that the most controlled method of measuring combustion efficiency from flares in a cross wind employs the use of a wind tunnel [11].

The UoA conducted research on CE using a wind tunnel over several years, the results of which are described in a report published in 2004 [1]. One of the products of the research was the development of a semi-empirical equation that calculates combustion efficiency as a function of fuel type, wind speed, flare jet exit velocity, flare stack outside diameter and the specific energy content of the fuel mixture, expressed in terms of the mass based lower (or net) heating value (LHV).

For natural gas flares the equation is:

$$\mu = 1 - 0.00166 \left(\frac{LHV_{CH4}}{LHV_f} \right)^3 e^{\left(\frac{0.317 U_w}{(g d U_f)^{1/3}} \right)} \quad (3)$$

Where,

μ	combustion efficiency
LHV_{CH4}	mass based lower heating value methane (MJ/kg)
LHV_f	mass based lower heating value flare stream (MJ/kg)
U_w	wind speed (m/s)
U_f	flare jet exit speed (m/s)
U_w	wind speed (m/s)
g	acceleration due to gravity (m/s^2)
d	flare outer diameter (m)

Though units are indicated, these are not mandatory providing the units employed are dimensionally consistent within the various terms.

The OGMP Technical Guideline Document (TGD) for Flare Efficiency [15] includes Equation (3) in its Level 4 Quantification Methodologies.

The equation is applicable to low momentum diffusion flames, such as those encountered during routine flaring on offshore platforms. It is not applicable during emergency flaring, or to steam or air assisted flares, in which cases the mechanism of flame stabilisation is significantly different.

Equation (3) is semi-empirical and defined in terms of two dimensionless groups. The argument of the exponential function is a modified Richardson number. The Richardson number expresses the ratio of the buoyancy flux, (i.e. the density difference of the flare jet and the ambient air), to the momentum flux of the crosswind. The form of the modified Richardson number presented above was developed by Johnson and Kostiuk [3]. The UoA research team found their experimental data collapsed the CE onto a single line when expressed as a function of the Richardson number for a gas of fixed composition.

The modified Richardson number is given by:

$$Ri_{mod} = \frac{U_w}{\left(gdU_f\right)^{1/3}} \qquad (4)$$

They found that the dimensionless LHV term was also required to account for the impact of the energy density of flare gases of differing compositions [1].

Plots of the CE versus the modified Richardson number, for various LHVs, are presented in Figure 1:

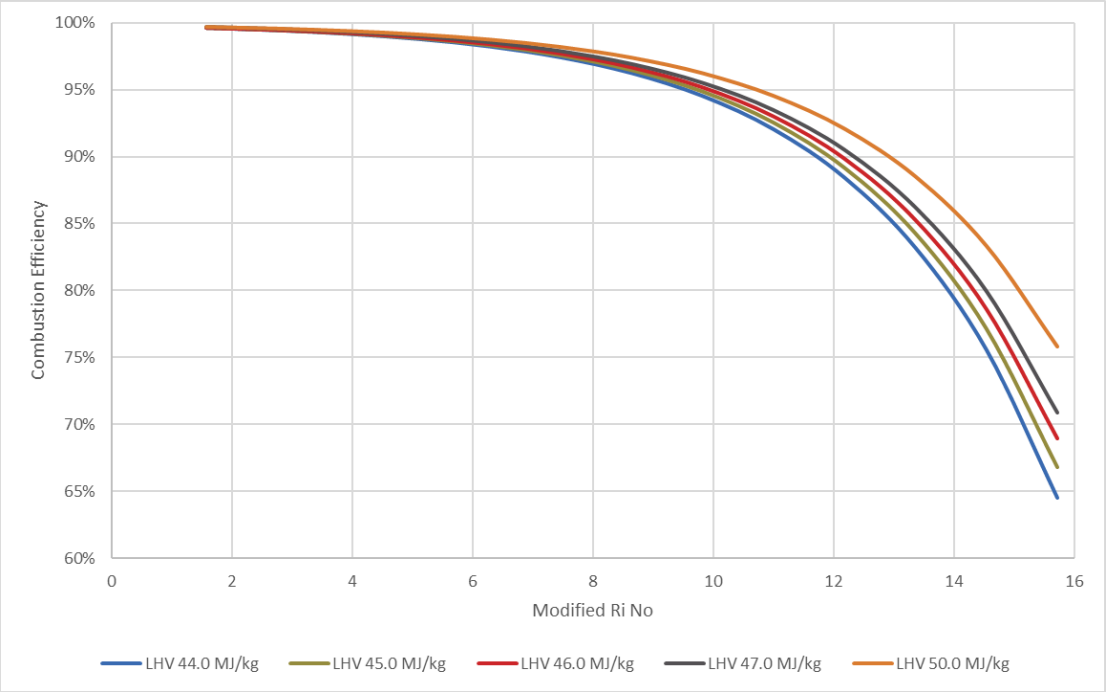

Figure 1 CE is a function of modified Richardson number

The lines for LHVs in the range 44.0 to 47.0 MJ/kg are representative of real data encountered by the authors and of the results generated by process simulation discussed in Section 4.2. The final value of 50.0 MJ/kg is representative of a gas of virtually pure methane and forms an upper limit of the mass based LHV for hydrocarbons.

3.2 Flare Physics

The UoA research team found that the same basic underlying physics applied, no matter the size of flare that they tested. They tested their conclusions and equation on data collected at UoA and the Canadian National Research Council (NRC) facility on flares of outside diameter up to 114.3 mm (4" NB, (nominal bore)). They found good agreement with the proposed mechanism and equation.

The UoA equation was developed using dimensional analysis and resulted in the characterisation of the CE in terms of two dimensionless numbers: the modified Richardson number and the energy density term expressed as the ratio of LHVs.

The Richardson number represents the ratio of the crosswind momentum (numerator) to the flare gas buoyancy terms (denominator). At relatively low flare flowrates (i.e. during routine and not emergency flaring) these were found to be the dominant effects. At higher flow rates the momentum of the flare jet itself becomes important.

Dimensionless numbers reduce the number of variables that describe a system, thereby reducing the amount of experimental data required to make correlations of physical phenomena to scalable systems.

3.3 Scale Up

Offshore platforms have flares installed with diameters significantly greater than 4" NB, which was the maximum diameter that the UoA research team could work with when developing their equation.

However, the use of dimensionless groups in semi-empirical equations for scaling up experimental or pilot plant to full scale industrial processes is well established in the chemical industries, ship design, etc. In racing car development wind tunnels are used extensively to test designs on scale models which are then scaled up.

Hence, it is a reasonable hypothesis to anticipate that the equation will be applicable to larger diameter flares for the following reasons:

- The UoA research team developed an initial CE equation based on flares whose diameters ranged from 12.1 to 49.8 mm. They subsequently tested the equation on larger diameter NRC flares (up to 114.3 mm) and found it was applicable. Hence, they had established that the equation could successfully be scaled up.
- The basic underlying physics determining combustion efficiency are the same for the wind tunnel and an offshore flare, i.e. the relative flame buoyancy force (driven by density differences) and the momentum of the crosswind along with the energy density of the fuel.
- A requirement for scale up is that the systems have to exhibit geometric similarity and dynamic similarity which is the case as both are systems with a vertically discharged combusted gas in a crosswind.

3.4 Incomplete Combustion Mechanism

According to [1] and [11] flare inefficiencies result from two mechanisms:

- Partial oxidation of the hydrocarbons to produce carbon monoxide rather than carbon dioxide. They found that any hydrocarbons present tend to be dominated by methane.
- Fuel stripping in which the fuel is stripped from the flare stream without any participation in the combustion. The unburnt hydrocarbons escaping have the same composition as the flare gas.

At low crosswind speeds the two mechanisms were of the same order of magnitude, but as windspeed increases the fuel stripping mechanism becomes the dominant cause of the inefficiency.

3.5 Sensitivity Analysis

The UoA research team estimated the uncertainty in the measured CE obtained in the wind tunnel was approximately ±0.6% absolute. The uncertainty introduced by the fitting of the data to the equation form and calculation of the coefficients, is not provided in the report but graphically appears to be a good fit.

In operation the measurement or calculation of the flare LHV, exit velocity, flare diameter and wind speed will introduce additional uncertainty into the calculated CE. The contribution of these parameters to the total CE uncertainty comprises two elements (as described in the GUM [6]):

- The uncertainty in the measured or estimated parameter (U_i)
- The sensitivity of the CE to the parameter (C_i)

These four input parameters are all individually measured/estimated and hence are independent of one another.

The sensitivity coefficients (partial differentials) can be determined analytically and have been calculated for an example case to illustrate the relative contribution of the input parameters to the uncertainty in the CE:

Table 1 – Sensitivity of CE Uncertainty to Input Parameters

Input Parameter	Value	Sensitivity Coefficient C_i	Absolute Uncertainty U_i	Relative Uncertainty (%)	$C_i * U_i$
LHV	45.0 MJ/kg	0.000834	±0.450	±1%	0.000375
Flare Jet Velocity	3.0 m/s	0.002367	±0.060	±2%	0.000142
Flare Diameter	0.2 m	0.035507	±0.0004	±0.2%	0.000014
Wind Speed	9.7 m/s	-0.002196	±0.194	±2%	-0.000426

The uncertainties in the input parameters in the table are typical estimates for illustrative purposes and are expressed at the 95% confidence level, (i.e. coverage factor of 1.96).

The combined effect of these input quantities results in a contribution to the uncertainty in the CE equation itself of ±0.06% (absolute) which is small compared

to the wind tunnel measurement uncertainty. The combined effect does depend on the magnitude of the input values, for example at a wind speed of 20 m/s its value increases to ±0.6% (absolute), comparable with the wind tunnel uncertainty.

The $C_i * U_i$ term (product of the sensitivity coefficients and absolute uncertainties) is indicative of the relative influence that each parameter has on the input component of CE uncertainty.

As can be seen the contribution of the flare diameter and measurement of the exit velocity are relatively small compared with that of the LHV and wind speed.

The value of the wind speed is an estimated average for the North Sea and is discussed further in Section 5.1.

The next two sections discuss the impact of the LHV and wind speed respectively in more depth.

4 IMPACT OF LOWER HEATING VALUE

4.1 Introduction

To determine the LHV, the flare gas may be sampled and its composition determined, via an online gas chromatograph if one is installed or, more commonly, sampled intermittently and analysed in a laboratory.

Since gas can be routed to the flare from many points in the process, the compositional variation could potentially be wide.

The problem with intermittent samples is that they may not adequately capture the variation in the flare gas composition. However, retrofitting online chromatographs on existing flares may be costly or difficult to achieve.

4.2 Use of Process Simulation

An alternative method to calculate the LHV is by process simulation. The OGMP Technical Guideline Document (TGD) for Flare Efficiency [15] includes process simulation in its Level 4 Quantification Methodologies.

The use of a process simulation model will provide a timely, cost effective and continuous method of calculating a representative LHV for the flare stream.

To test the feasibility of this approach a simulation of a real process was used to determine the variability and hence uncertainty in the calculated LHV of gas streams at various points within the process. This is important as, already discussed in Section 3.5, the uncertainty in LHV contributes to the uncertainty in the calculation of CE using Equation (3).

The process simulated, comprised two trains of separation, multistage compression facilities along with NGL liquid and lift gas recycles. A simplified schematic of the process is presented in
Figure 2:

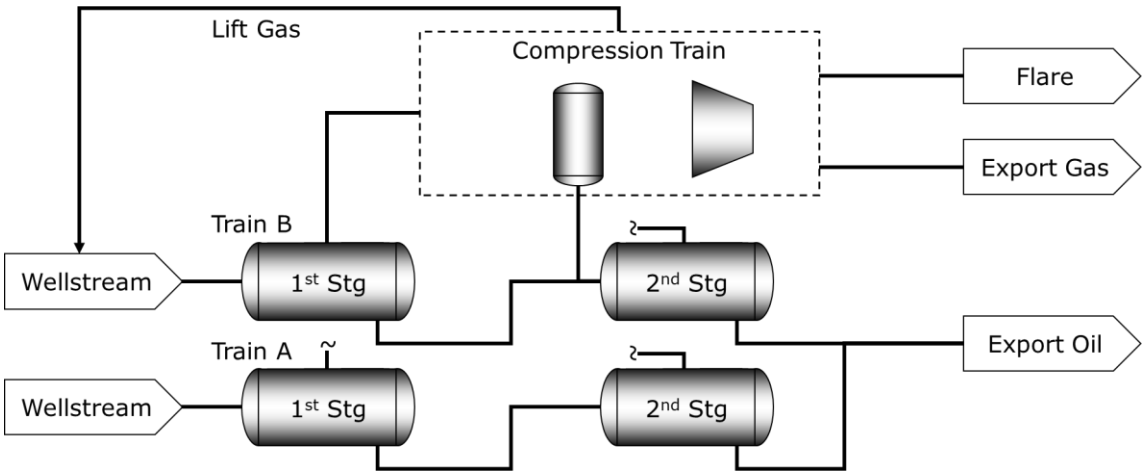

Figure 2 Simplified Offshore Process Schematic

The modelling was performed using the CHARM process simulator [4].

The complexity of the process model equations means that calculating the uncertainties in model outputs e.g. gas stream LHVs, using the analytical Taylor Series Method (TSM) as described in the GUM [6], is not practicable. Instead, the Monte Carlo Method (MCM), which is described in a Supplement to the GUM [7], has been employed.

MCM is a powerful tool for performing uncertainty analysis. The basic methodology is described below as applied to a process simulation:

- The average values of the process input parameters (temperatures, pressures, compositions, properties, etc.) are obtained.
- The random uncertainties of the input parameters are obtained.
- Appropriate probability distribution functions are assumed to describe the variation of the random uncertainties – usually these will be Gaussian (normal).
- A random number generator is used to produce a value of the random error independently for each input variable which is consistent with the random uncertainty and probability distribution functions.
- These random errors are applied to the average values to obtain "measured values" for the input parameters.
- The "measured" inputs are entered into the process model, which is then solved and the desired outputs obtained, e.g. gas LHV, etc.

This process corresponds to running the simulation once. The process is repeated M (where M may be 1,000 or 100,000 or …, etc. depending on the problem) times to obtain a distribution of the output result. The standard deviation and hence uncertainty can then be obtained for the output parameter of interest from the distribution of the simulation results generated.

The CHARM software has the input/output flexibility and speed of performance to afford the possibility of running a Monte Carlo simulation in a practical timeframe.

In summary the model inputs and their associated uncertainties were:

- Vessel pressures ±0.5 bar for 1st stage and compression train
- Vessel pressures ±0.1 bar for 2nd stage
- Vessel temperatures ±3°C
- Wellstream component flowrates (±10%)

Additionally, the physical properties of the components, pure and hypothetical, were varied in accordance with their estimated uncertainties. A complete breakdown of the uncertainties is presented in [5].

After the completion of the Monte Carlo simulation, the LHVs on both a volumetric and mass basis of gas streams within the process, along with the observed variation or uncertainty (at 95% confidence level) were calculated and are presented in Table 2:

Table 2 – CHARM Monte Carlo Simulation LHV Data

Stream	Volumetric LHV Average Value (MJ/Sm^3)	Relative Uncertainty (%)	Mass LHV Average Value (MJ/kg)	Relative Uncertainty (%)
Flare	41.8	±1.1%	45.1	±0.7%
Train A 1st Stage Gas	41.2	±2.0%	44.2	±1.0%
Train A 2nd Stage Gas	76.3	±3.4%	44.8	±0.5%
Train B 1st Stage Gas	41.6	±1.5%	45.5	±0.6%
Train B 2nd Stage Gas	73.5	±4.3%	45.1	±0.3%

The uncertainty or variation in the LHV for the simulated flare stream is relatively modest on both a volumetric and mass basis. To some extent this may not be that surprising as in the simulation the flare stream is assumed to be similar to the fuel or export gas. As such, it has been processed to remove heavier components and stabilise its composition.

In practice gas from any part of the process could be routed to the flare (at various times). Hence, the LHV data for the first and second stage separators in both trains are also presented. These are the most variable gas streams in terms of their composition and this is reflected in the increased variability and value of the volumetric LHV. However, the mass based LHV values are similar to the flare stream and their variability is remarkably low.

The mass based LHV is relatively stable due to two main reasons:

- Any variation in the concentration of one component has to be compensated by changes in the other component concentrations, since they must sum to 100%. So even if the LHV of the components differ, the impact of the variation on the mixture LHV is attenuated to some extent.

- As the carbon number of the hydrocarbons increases, the addition of carbon atoms increases the calorific energy of the molecule. Consequently, the LHV on a molar or volume basis rises significantly as illustrated in Figure 3 (data from ISO 6976 [8]). In contrast, the mass based LHV asymptotically falls towards to a constant value (with increasing carbon number) as illustrated in Figure 4. This is because, on a mass basis, the rise in calorific energy is also accompanied by a rise in the molecular mass.

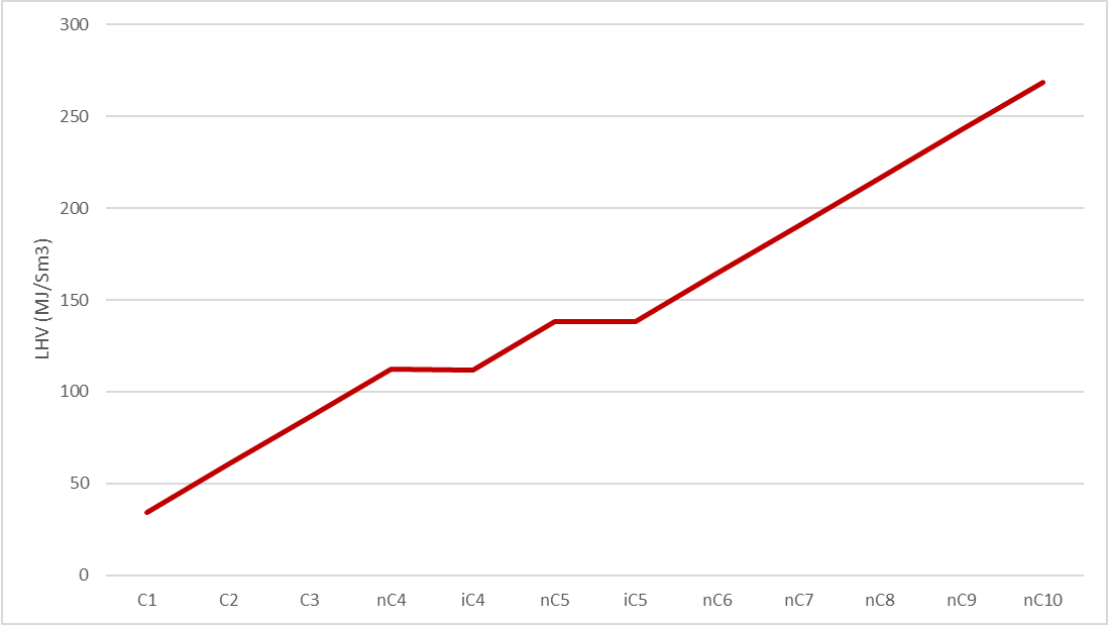

Figure 3 Volumetric Based LHV Hydrocarbon Gases

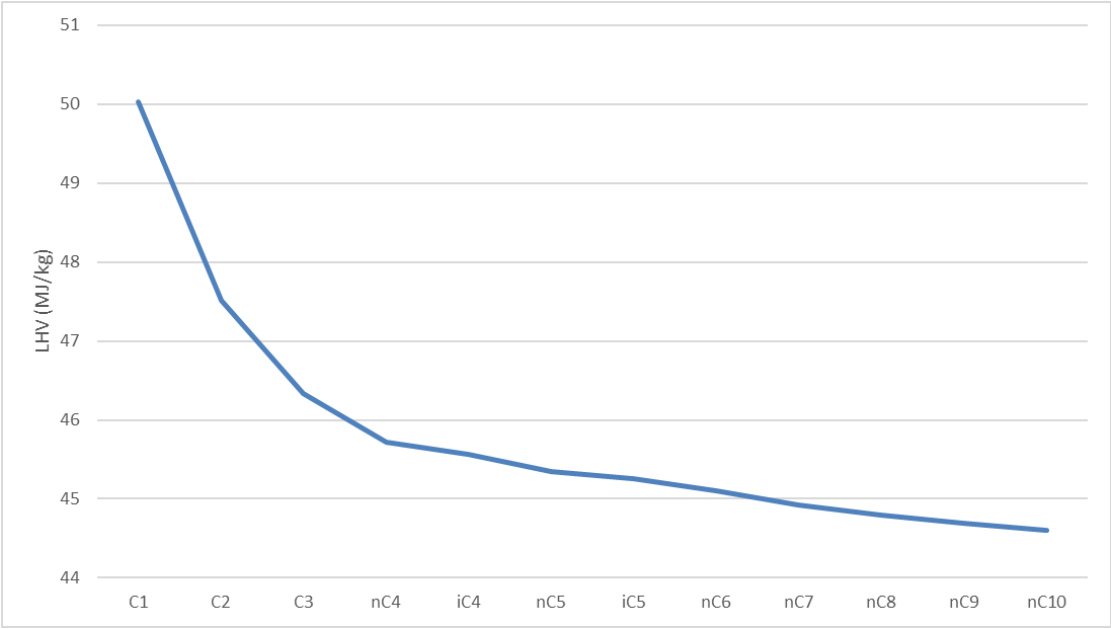

Figure 4 Mass Based LHV Hydrocarbon Gases

This is the reason that the mass based LHV is so stable for hydrocarbon rich streams and hence why the uncertainty is relatively low compared to the uncertainty in the composition and the molar or volumetric LHV. It is the mass based LHV that is used in the UoA equation and hence the above results illustrate the viability of using a process simulation to predict the LHV for combustion efficiency calculations.

Process simulation provides an advantage over intermittent sampling in that the relative contributions to the combined flare stream from variable sources across the plant can be continuously captured and which may be missed by the intermittent samples.

Depending on where an online chromatograph is located, this too may not capture all the variation in flare composition.

5 IMPACT OF WIND SPEED

5.1 North Sea Wind Speed Distribution

As highlighted in Section 3.5, wind speed has a major impact on CE. The wind speed in the North Sea varies throughout the year. The probability distribution of wind speed is typically described by the Weibull distribution:

$$P(U_w) = \left(\frac{\beta}{\alpha}\right)\left(\frac{U_w}{\alpha}\right)^{(\beta-1)} e^{\left(\left(\frac{U_w}{\alpha}\right)^{\beta}\right)} \quad (5)$$

Where,

$P(U_w)$ Probability density function which is proportional to the probability of occurrence of wind speed, U_w
α Scale parameter
β Shape parameter

This distribution is plotted in Figure 5:

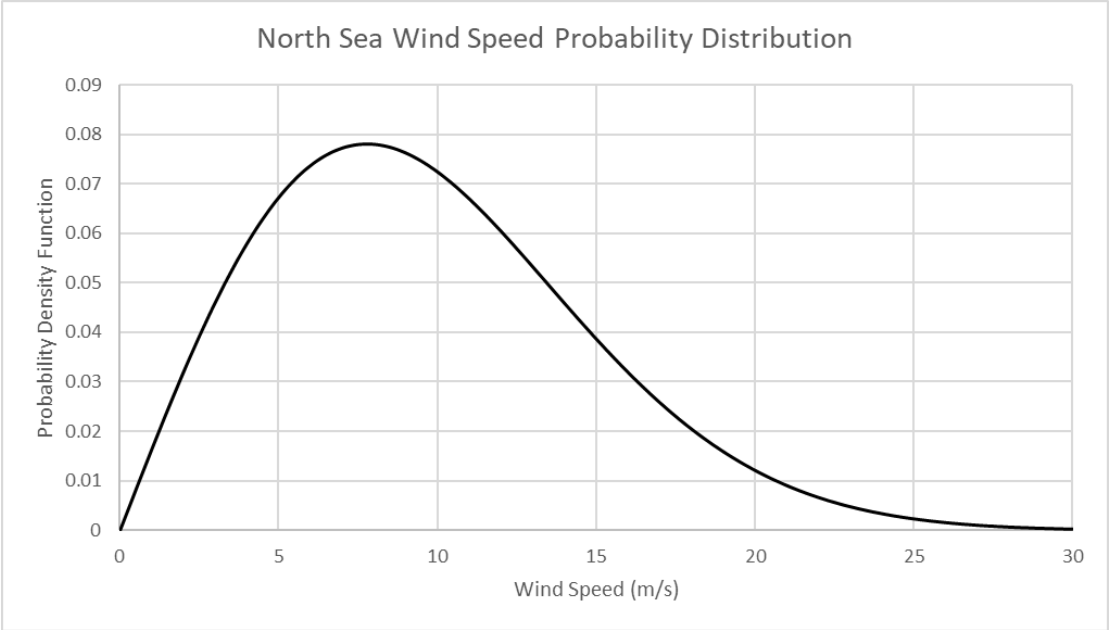

Figure 5 Weibull distribution of the typical wind speed in the North Sea

As can be observed, the Weibull distribution has a minimum value of zero, is asymmetric and asymptotically falls to zero with increasing wind speed. In the plot the scale parameter $\alpha = 11$ and shape parameter $\beta = 2$; these are typical values for the North Sea [9], [10].

The average yearly wind speed is obtained by integrating the product of the wind speed and the probability density function from zero to infinity.

$$U_{w,ave} = E[U_w] = \int_0^\infty U_w \, P(U_w) dU_w \tag{6}$$

Substituting for $P(U_w)$ from (5) into (6), yields:

$$U_{w,ave} = \int_0^\infty U_w \left(\frac{\beta}{\alpha}\right)\left(\frac{U_w}{\alpha}\right)^{(\beta-1)} e^{\left(\left(\frac{U_w}{\alpha}\right)^\beta\right)} dU_w \tag{7}$$

This equation can be integrated to obtain a solution in terms of an incomplete gamma function. However, a much simpler result is obtained if the shape parameter β has a value of 2 as is typical for the North Sea:

$$U_{w,ave} = \int_0^\infty U_w \left(\frac{2}{\alpha}\right)\left(\frac{U_w}{\alpha}\right) e^{\left(\left(\frac{U_w}{\alpha}\right)^2\right)} dU_w \tag{8}$$

$$U_{w,ave} = \left[\frac{\alpha\sqrt{\pi}}{2} erf\left(\frac{U_w}{\alpha}\right) - U_w e^{\left(-\left(\frac{U_w}{\alpha}\right)^2\right)}\right]_0^\infty \tag{9}$$

The error function term (erf) is a standard mathematical function, which is readily calculated, for example, in Microsoft Excel. The value of erf(0) = 0 and erf(∞) = 1. The right-hand term has a value of zero when U_w = 0 and = ∞. Hence, $U_{w,ave}$ is simply given by:

$$U_{w,ave} = \frac{\alpha\sqrt{\pi}}{2} \tag{10}$$

As indicated above, a typical value for α is 11, and hence the typical yearly average wind speed for the North Sea is calculated to be:

$$U_{w,ave} = 9.7\, m/s \tag{11}$$

It should be noted that the magnitude and probability distribution of wind speeds do vary with location across the North Sea and with height above sea level. The value above is provided above for information purposes and to illustrate the fact that the mean wind speed is greater than the most likely wind speed (peak of the plot in Figure 5), or mode, which is:

$$U_{w,mode} = 7.8\, m/s \tag{12}$$

However, neither of these values should be used to calculate an average yearly combustion efficiency which is the subject of the next section.

5.2 Average Combustion Efficiency

To calculate or forecast an average combustion efficiency say for a whole year, it may appear tempting to use a single average wind speed in equation (3) but this is incorrect and results in an over-estimation of the combustion efficiency. This is because the CE is not linearly dependent on wind speed.

In [2] Johnson M observed that it is useful to convert an instantaneous efficiency at one wind-speed to a meaningful "average" value. This could be used to provide the concept of "Yearly Averaged Efficiency" and "Yearly Averaged GHG Equivalent Emission." This requires the statistically weighted efficiency taking into account widely varying wind conditions.

The mean expected combustion efficiency over a typical year, μ_{ave}, is calculated from:

$$\mu_{ave} = E[\mu] = \int_0^\infty \mu\, P(U_w) dU_w \tag{13}$$

This equation was presented by Johnson [2]. A solution to this equation for the wind distribution described above in Section 5.1 is provided below.

To calculate the mean expected combustion efficiency over a typical year, μ_{ave}, it is not correct simply to set U_w in (3) equal to $U_{w,ave}$. The average combustion efficiency is obtained by integrating the product of the combustion efficiency and the probability density function from a wind speed of zero to infinity.

A solution is proposed below based on the wind behaving in accordance with a Weibull distribution. Substituting equations (3) and (5) into (13):

$$E[\mu] = 1 - \int_0^{\infty} \left(0.00166 \left(\frac{LHV_{CH4}}{LHV_f} \right)^3 e^{\left(\frac{0.317 U_w}{(g d U_f)^{1/3}} \right)} \right) \left(\frac{\beta}{\alpha} \right) \left(\frac{U_w}{\alpha} \right)^{(\beta-1)} e^{\left(\left(\frac{U_w}{\alpha} \right)^{\beta} \right)} dU_w \qquad (14)$$

If all other parameters remain constant, U_w is the only variable in the integration and if it is assumed the shape parameter $\beta = 2$, then the integral simplifies somewhat and becomes:

$$\mu_{ave} = 1 - .00166 \left(\frac{LHV_{CH4}}{LHV_f} \right)^3 \left(1 + \sqrt{\pi} \theta e^{(\theta)} \left(1 - \text{erf}\,(-\theta) \right) \right) \qquad (15)$$

Where,

$$\theta = \frac{0.317\alpha}{2\left(g d U_f\right)^{1/3}}$$

The error function term (erf) is standard mathematical function, which is readily calculated, for example, in Microsoft Excel.

To illustrate the importance of integrating the CE equation with respect to the wind distribution, the CE is calculated for the example case presented in Section 3.5:

$$\mu_{ave} = 98.0\% \qquad (16)$$

If the average wind speed of 9.7 m/s is simply inserted into equation (3), the calculated CE is:

$$\mu_{ave} = 98.8\% \qquad (17)$$

Which significantly underestimates the unburnt hydrocarbons (by 40%). If the mode wind speed of 7.8 m/s, CE becomes:

$$\mu_{ave} = 99.1\% \qquad (18)$$

Underestimating the unburnt hydrocarbons by more than 50%.

As stated, these assume that the other input parameters remain constant. However, this equation provides a potential mechanism with which to forecast unburnt hydrocarbon emissions.

The next section discusses adjusting the flare exit velocity to minimise emissions.

6 FLARE EMISSIONS OPTIMISATION

6.1 Minimisation of Unburnt Hydrocarbons at Constant Flare Rate

As wind speed increases the Richardson number rises, resulting in a reduced CE (see Equations (3) and (4) and Figure 1). To mitigate the effect of higher wind speeds, the flare gas LHV or the purge flow rate to the flare could be increased (thereby increasing the flare exit velocity). Increasing the purge flow rate appears a more readily practical possibility.

If the purge gas is fuel gas, then increasing its flow rate will improve combustion efficiency but will also increase the total flare flow rate. There will be an optimum flow rate where these two competing effects result in minimum unburned hydrocarbons. This may be determined mathematically. To simplify the mathematics presented here, it has been assumed that the purge and flared gas have the same LHV. (In practice, LHV differences can be included in a live calculation).

Equation (15) can be recast in terms of the inefficiency which is simply $1-\mu_{ave}$ and this represents the fraction of the flow that is unburnt flare. If multiplied by the flare actual flow, we have a formula for the yearly expected emissions of unburnt flare, in actual m^3/s:

$$Q_{UHC,ave} = \left(\frac{\pi d^2 U_f}{4}\right) * 0.00166 \left(\frac{LHV_{CH4}}{LHV_f}\right)^3 \left(1 + \sqrt{\pi}\frac{\varphi}{U_f^{1/3}} e^{\left(\frac{\varphi}{U_f^{1/3}}\right)} \left(1 - \text{erf}\left(-\frac{\varphi}{U_f^{1/3}}\right)\right)\right) \quad (19)$$

Where,

$Q_{UHC,ave}$ volumetric flow of unburnt hydrocarbons (m^3/s)

$$\varphi = \frac{0.317\alpha}{2(gd)^{1/3}}$$

Strictly the diameter d in Equation (19) above is the internal diameter of the flare, whereas the d in φ is the flare outside diameter. The volumetric flow is presented as actual m3/s, though this value will be close to the standard flow as the unburnt flare gas exiting the flare will be close to atmospheric pressure and temperatures.

When multiplied by seconds in a year, $Q_{UHC,ave}$ is the quantity that would be emitted on average for a fixed flare gas flow / velocity.

For the example case presented in Section 3.5, the yearly volumetric flow of unburnt hydrocarbons has been calculated and plotted as a function of flare exit velocity in Figure 6:

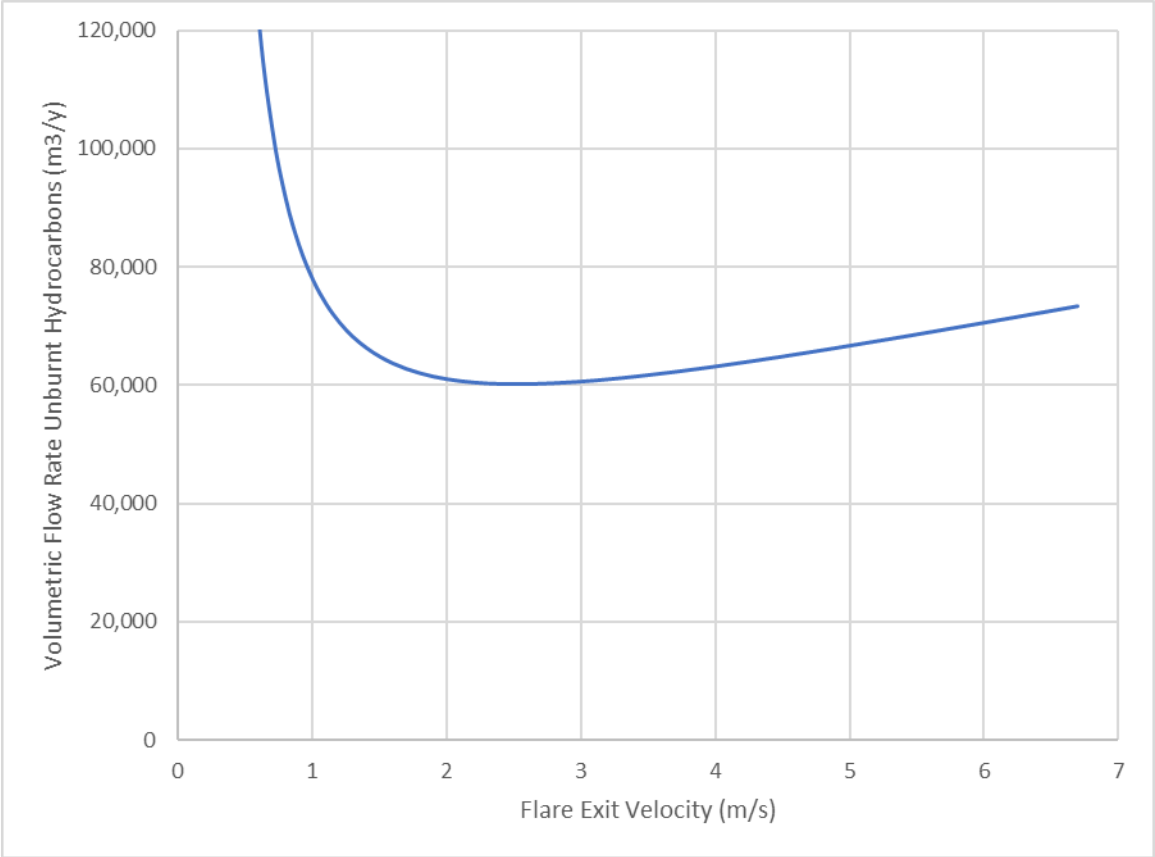

Figure 6 Unburnt Hydrocarbons vs Flare Jet Exit Velocity Gases

The minimum yearly flow of unburnt hydrocarbons in this case is 60,160 m^3, which occurs at an exit velocity of 2.5 m/s.

The authors have developed a proprietary equation with which to calculate the optimum fixed exit velocity. This is a function of flare diameter and flare gas LHV and provides a means to set the flare purge rate to minimise methane emissions, for the case when the purge flow is not controlled in real time.

It should be noted that there may be other factors, such as minimum purge rate to prevent air ingress, which will need to be considered also.

Standard volumes of unburned hydrocarbons (strictly it is unburned flare gas which can include inherent inerts also) have been presented here but these can be readily converted to tonnes or specifically methane quantities using the flare composition.

6.2 Minimisation of Unburnt Hydrocarbons with Adjusted Flare Purge Rate

If the wind speed is measured, then the purge flow can be adjusted in real time to minimise methane emissions.

Adopting a similar approach to that adopted in Section 6.1, but with Equation (3) instead of (15), we arrive at an expression of the instantaneous emissions of unburnt flare $Q_{UHC,inst}$, in actual m³/s:

$$Q_{UHC,inst} = \left(\frac{\pi d^2 U_f}{4}\right)\left(0.00166\left(\frac{LHV_{CH4}}{LHV_f}\right)^3 e^{\left(\frac{0.317U_w}{(gdU_f)^{1/3}}\right)}\right) \quad (20)$$

For the example case presented in Section 3.5, the instantaneous volumetric flow of unburnt hydrocarbons has been calculated and plotted as a function of flare exit velocity for various wind speeds in Figure 7 and Figure 8:

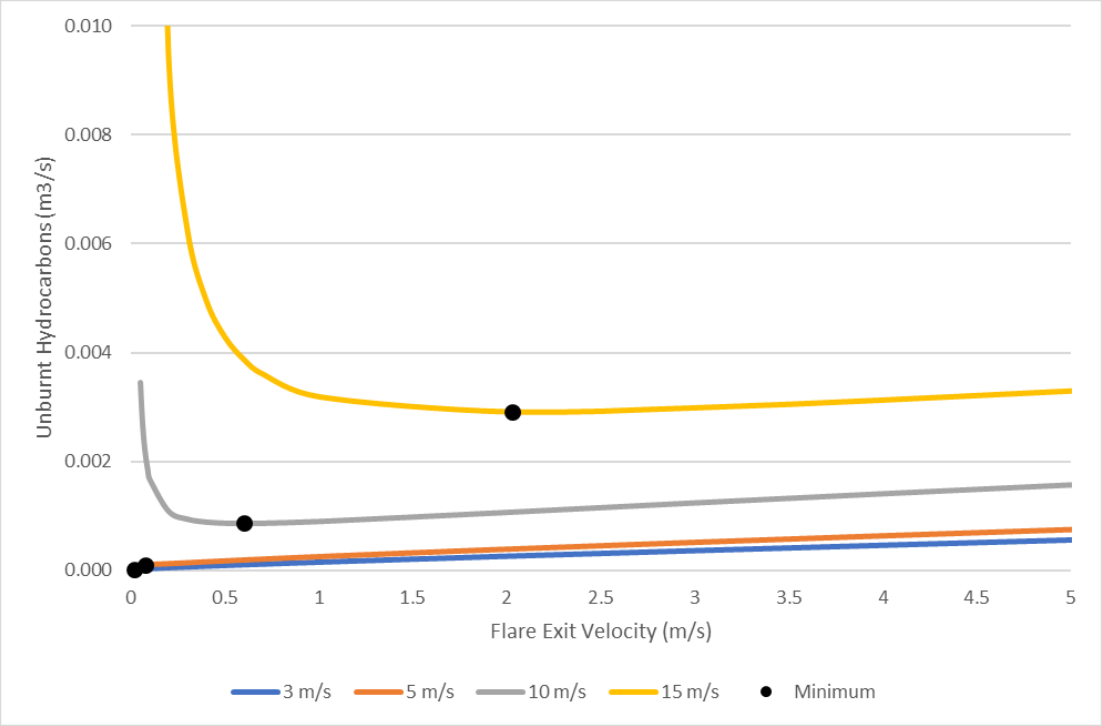

Figure 7 Unburnt Hydrocarbons vs Flare Jet Exit Velocity at Various Wind Speeds (3 to 15 m/s)

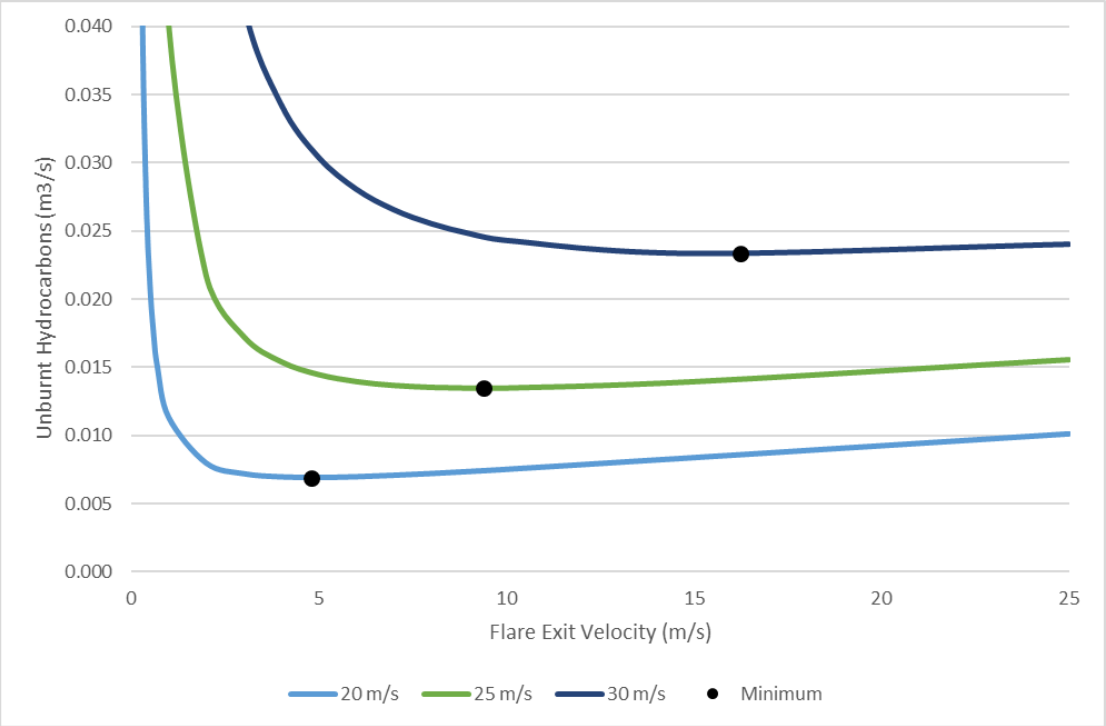

Figure 8 Unburnt Hydrocarbons vs Flare Jet Exit Velocity at Various Wind Speeds (20 to 30 m/s)

Again, an increase in flare exit velocity increases the total flow but improves combustion efficiency, thereby resulting in an optimum exit velocity corresponding to the minimum flow of unburnt hydrocarbons.

As can be observed the exit velocity for minimum unburnt hydrocarbon emissions changes significantly with wind speed (indicted by the black circular markers).

Again, the authors have developed a proprietary, relatively simple equation with which to calculate the optimum exit velocity. This is a function of wind speed, flare diameter and flare gas LHV and provides a set point target with which to minimise methane emissions, for the case when the flow is controlled in real time.

This can again be multiplied by the wind speed probability distribution and integrated over the full range of wind speeds to obtain an equation for the real time optimised minimum unburned hydrocarbon emissions over a year.

The minimum yearly flow of unburnt hydrocarbons for the example case presented in Section 3.5 is 48,240 m^3.

This is a 20% reduction in the emission of unburnt hydrocarbons compared with the fixed rate approach and illustrates the benefit of actively controlling the flare purge in response to wind speed.

Additionally, the above approach also provides a mechanism with which to forecast unburnt hydrocarbon emissions.

6.3 Flare Rate Optimisation to Minimise CO2e Emissions

The above optimisation calculations minimise the unburnt hydrocarbons and hence methane emissions associated with flaring. They do not take into account the quantity of CO_2 that is emitted due to the combustion of hydrocarbons by the flare. This requires a relationship between methane and CO_2 in terms of global warming impact. This is provided by the Global Warming Potential (GWP) value. It is a measure of how much energy the emission of 1 ton of a gas will absorb over a given period of time, relative to the emission of 1 ton of carbon dioxide (CO_2).

The associated carbon dioxide equivalent or CO_2e means the number of metric tons of CO_2 emissions with the same global warming potential as one metric ton of another greenhouse gas.

The analysis in Sections 6.1 and 6.2 was conducted in terms of standard volumes, to simplify the mathematics, but could equally have been presented in terms of mass. The inclusion of CO_2 from combusted hydrocarbons in this section is more naturally presented in mass terms.

The GWP of methane is multiple times that of CO_2. The GWP depends on the length of period it is specified over as methane has a half-life in the atmosphere of 12.4 years, whereas CO_2's half-life is considerably longer. The GWP of methane over 20 years is 84 [12], meaning that a methane emission is projected to have 84 times the impact on temperature of a carbon dioxide emission of the same mass over the following 20 years. The GWP falls to 28 if taken over a 100 year period [12].

Also required is the composition of the unburnt gas as not all of it is necessarily methane. The fuel stripping mechanism suggests that the unburnt gas has the same composition as the flare gas and the UoA research team found this to be the case at higher wind speeds. More recent work suggests however [11], that at low wind speeds other mechanisms are more dominant and that the unburnt gas is principally methane.

To simplify the calculations here, which serve as illustration rather than definitive, the unburnt gas has been assumed to be all methane. This appears conservative but the definition of the GWP over 20 vs 100 years has a much larger impact on the calculations. It is simple to modify the calculations to include the composition of the flare however.

The total emissions in terms of CO_2e are given by:

$$M_{CO2e} = \mu E_{CO2e} M_f + (1 - \mu) GWP_{CH4} M_f \qquad (21)$$

Where,

μ	combustion efficiency
M_{CO2e}	total mass flow of CO_2 equivalent emitted by the flare (kg/s)
M_f	total mass flow of unburnt flare jet (kg/s)
E_{CO2e}	mass of CO2 produced per mass of unburnt flare gas (kg/kg)
GWP_{CH4}	GWP of methane (kg/kg)

The CE from Equation (3) can be substituted in (21) to calculate the total CO_2e emissions.

Figure 9 illustrates the contribution of the combusted and uncombusted flare gases to the total CO2e emissions for the example case presented in Section 3.5 at a wind speed of 20 m/s.

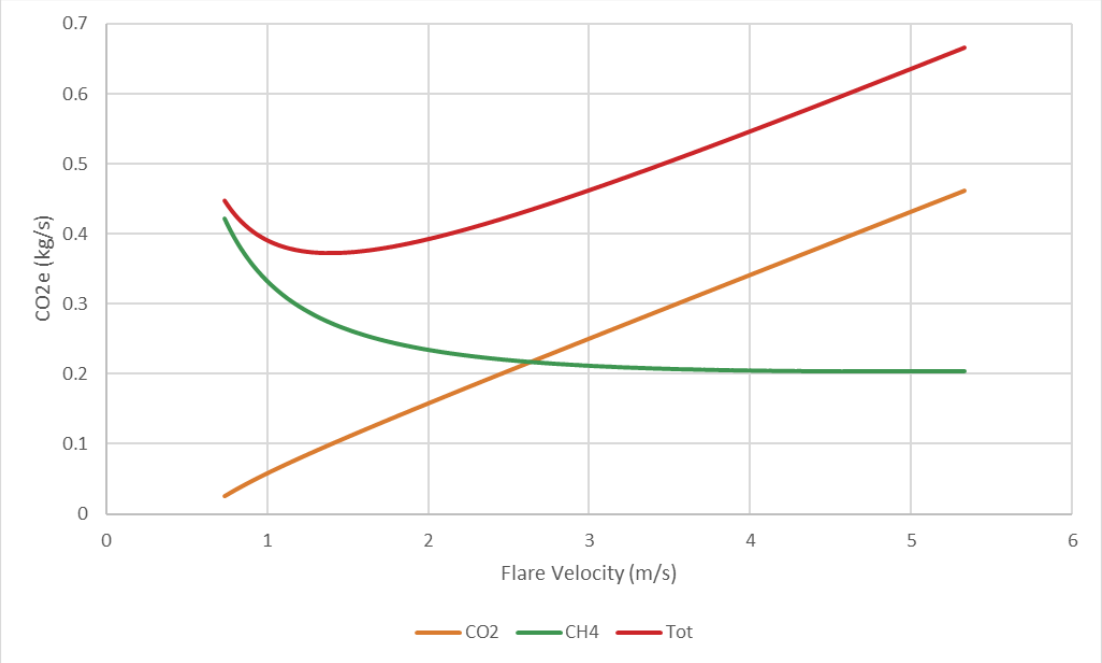

Figure 9 CO2e Emissions vs Flare Jet Exit Velocity at Wind Speed of 20 m/s and 100 Year GWP

As the flare velocity increases, the CO2 generated as a result of combustion rises nearly linearly as indicated by the orange line. The emission of unburnt methane decreases initially as the CE improves. The minimum CO2e due to methane occurs at an exit velocity of 4.8 m/s as indicated by green line. The combined effect of the two results in the red line which has a minimum value of total CO2e emissions of 0.372 kg/s and occurs at flare exit velocity of 1.4 m/s.

This assumes the 100 year GWP value for methane. If the 20 year figure is adopted the plot becomes:

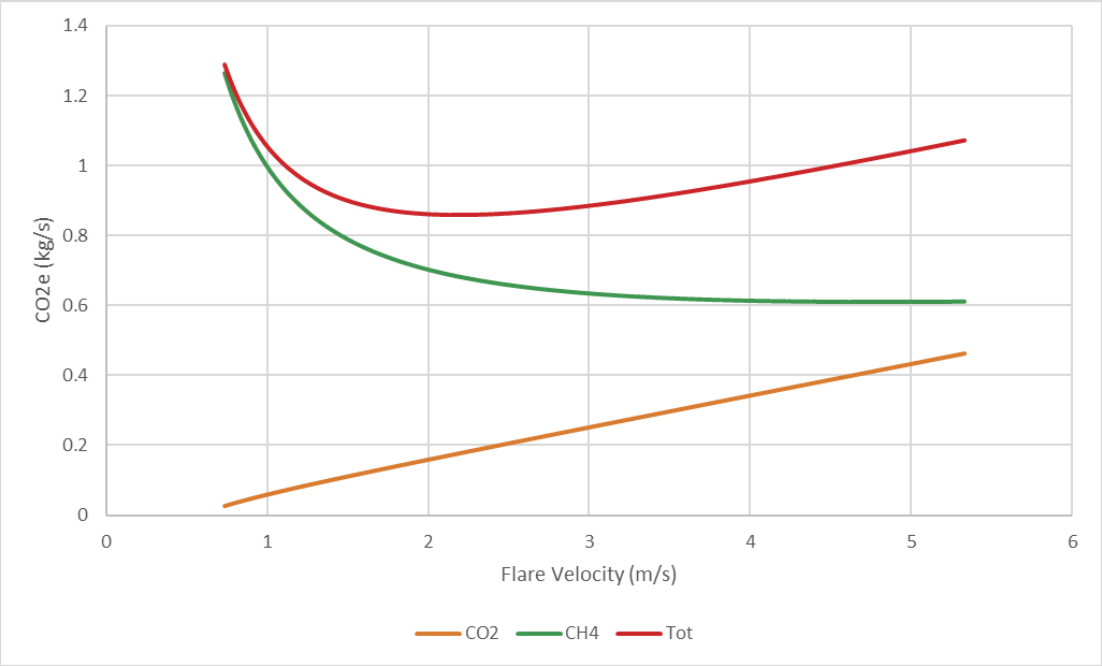

Figure 10 CO2e Emissions vs Flare Jet Exit Velocity at Wind Speed of 20 m/s and 20 Year GWP

The CO_2e due to methane increases significantly (the ordinate y-axis covers a wider range than in Figure 9) and the minimum total CO2e emissions more than double to 0.857 kg/s which occurs at flare exit velocity of 2.2 m/s.

Again, the authors have developed proprietary equations to calculate the optimal purge and flare rates to minimise total CO_2e emissions. This is in a suitable form to control the rate of purge to the flare in response to changes of wind speed.

7 CONCLUSIONS

The UoA equation is a recognised means to calculate the combustion efficiency of lit flares during normal operation. It is a semi-empirical equation based on two dimensionless groups: a modified Richardson number, which is the ratio of the flare buoyancy to crosswind momentum and an LHV term characterising the energy density of the flare stream.

As such, the application of the equation to flare diameters greater than those that could tested in the UoA wind-tunnel has been justified based on the recognised use of dimensionless groups in semi-empirical equations for scaling up experimental or pilot plant to full scale industrial processes.

The UoA research team reported that they could measure the CE to within ±0.6%.

There are four principal input parameters to the equation: flare diameter, flare jet exit velocity, flare gas LHV and wind speed. In the field, the uncertainties associated with the measurement of these inputs also contribute to the calculated CE uncertainty. A high-level uncertainty analysis illustrated that CE was most

sensitive to LHV and wind speed (of the four inputs), but the additional uncertainty introduced was typically small.

The determination of flare gas LHV has been demonstrated to be viable using a process simulation.

Wind is a major factor that affects CE, in that high wind speeds significantly reduce CE. Correct consideration and integration of the wind speed probability distribution with the UoA CE equation provides a method with which to forecast unburned hydrocarbon emissions. Using a single, average wind speed has been demonstrated to significantly under-estimate unburned hydrocarbon and hence, methane emissions.

A means to mitigate the deleterious impact of high wind speeds on CE is to adjust the flare exit velocity by altering the flare purge flow rate. Methods have been presented that optimise the flare purge rate for three scenarios:

- Minimise unburned hydrocarbon emissions using a fixed flare purge flow rate
- Minimise unburned hydrocarbon emissions using a dynamically controlled flare purge flow rate in response to wind speed
- Minimise CO2e emissions due to unburned hydrocarbons and combusted hydrocarbons using a dynamically controlled flare purge flow rate in response to wind speed.

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