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Handbook of Uncertainty Calculations for Ultrasonic, Turbine, and Coriolis Oil Flow Metering Stations

Documentation of Uncertainty Models and Internet Tool

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Summary

This Handbook documents uncertainty models for fiscal oil metering stations using ultrasonic, turbine or Coriolis flow meters. Proving device is either a displacement prover, an ultrasonic flow master meter, a turbine flow master meter or a Coriolis meter (in case of Coriolis duty meter). The uncertainty models cover volumetric flow rate at standard conditions, volumetric flow rate at line conditions and mass flow rate. Volumetric water fractions of up to 5 % are covered and are either measured online or obtained through sampling and laboratory analysis. The density is either measured by an online densitometer or obtained through sampling and laboratory analysis. The uncertainty models are implemented on the web application (OilMetApp) using HTML and WebAssembly. The implemented models can be accessed free of charge from www.nfogm.no

The present work is related to similar work on fiscal oil metering stations, see (Frøysa, et al., 2020), (Frøysa, et al., 2018) and (Frøysa, et al., 2015). It is also based on (Dahl, et al., 2003), (Lunde, et al., 2002) and (Lunde, et al., 2010).

Revision history

Year	Revision number	Reason for revision
2015	00	First version of Handbook published with title *Handbook of uncertainty calculations for ultrasonic oil flow metering stations*. This handbook covered uncertainty models for fiscal oil metering stations using ultrasonic flow meter. Proving device could be either a displacement prover, an ultrasonic flow master meter or a turbine flow master meter. The uncertainty models covered volumetric flow rate at standard conditions, volumetric flow rate at line conditions and mass flow rate. The density was either measured by an online densitometer or obtained through sampling and laboratory analysis. The uncertainty models were implemented on a web-based Microsoft Silverlight technology which could be accessed free of charge from www.nfogm.no.
2018	01	Updated version of the Handbook published titled *Handbook of uncertainty calculations for Ultrasonic, Turbine and Coriolis oil flow metering stations.* This handbook covered uncertainty models for fiscal oil metering stations using turbine and Coriolis as duty meters in addition to ultrasonic flow meter as duty meter (as before). Proving device could be either a displacement prover, an ultrasonic flow master meter, a turbine flow master meter or a Coriolis meter (in case of Coriolis duty meter).
2020	02	Upgraded software platform. The uncertainty models were originally implemented on a Microsoft Silverlight technology. The Silverlight technology would not be supported after October 2021. The uncertainty models were thus upgraded to a new web application using HTML and WebAssembly.
2024	03	Updated version of Handbook published with same title as in 2020. Handbook and uncertainty models were updated to include corrections for cases with water in oil, and the models are designed for volumetric water contents of up to 5 %.

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1. Introduction

Documentation of uncertainty of flow rates measured by fiscal flow metering stations is essential as part of the evaluation of the condition of such metering stations. Authorities have requirements with respect to maximum uncertainty to secure the national interests. The partners selling the oil have interests in the uncertainty to secure their incomes. Finally, buyers of oil have interest in ensuring that they are not getting a lower amount of oil than what they pay for.

For all parties to accept an uncertainty analysis, it is important to obtain standardized ways of carrying out such analyses. The ISO Guide to the expression of uncertainty in measurement, (ISO_GUM, 2008) provides general methodology for carrying out uncertainty analyses. This methodology can also be applied in uncertainty analysis of fiscal oil metering stations. However, the ISO GUM does not give detailed methods for the specific uncertainty analyses for such metering stations (or other applications). Therefore, models must be developed based on the ISO GUM methodology. Similarly, the ISO 5168 (ISO5168, 2005) provides general procedures for evaluation of uncertainty for the measurement of fluid flow. Also, the procedures in this standard must be developed further to approach the uncertainty evaluation of a specific metering station.

The Norwegian Society for Oil and Gas Measurement (NFOGM) in cooperation with the Norwegian Offshore Directorate (NOD) and The Norwegian Society of Graduate Technical and Scientific Professionals (Tekna) have earlier issue a Handbook for uncertainty calculations for oil metering stations, which has been updated and extended several times (Frøysa, et al., 2020), (Frøysa, et al., 2018), (Frøysa, et al., 2015) and (Frøysa, et al., 2014). The handbook agrees with the ISO GUM methodology. The calculation of the uncertainty according to that Handbook can be done free of charge using a calculation program available at www.nfogm.no.

Prior to that handbook, some more technology-specific uncertainty handbooks for a fiscal ultrasonic gas metering station (Lunde, et al., 2002) and for a fiscal orifice gas metering station and a turbine oil metering station (Dahl, et al., 2003) have been issued. These works are also in agreement with the ISO GUM methodology and were based on a previous version of the ISO GUM from 1995. The calculations of the uncertainty have in these works been based on an Excel spread sheet that can be downloaded for free from www.nfogm.no. In addition, uncertainty models for fiscal turbine oil metering stations (Dahl, et al., 2003) and fiscal ultrasonic oil metering stations (Lunde, et al., 2010) have been established.

The present work is a similar Handbook as (Frøysa, et al., 2020) and (Frøysa, et al., 2018) covering fiscal oil metering stations with ultrasonic, turbine or Coriolis flow meters used as duty meters, but also encompasses uncertainty calculations for water in oil measurements of up to 5 % volumetric water contents. The intention of this work is to establish uncertainty analysis models covering common fiscal oil metering station configurations in use on the Norwegian Sector. The intention is also to make a tool in which a complete uncertainty analysis for an oil metering station can be performed within one tool in a minimum of time. This is achieved as the tool calculates all necessary parameters from a minimum of inputs, based upon reasonable default values and default input values for uncertainty in accordance with requirements in the Norwegian measurement regulations and NORSOK. Furthermore, a main focus is to make it easy to define the most common metering station configurations in the tool.

The uncertainty model has been made flexible, allowing (i) ultrasonic master meter prover (ii) turbine mater meter prover or (iii) volume displacement prover or (iv) Coriolis master meter prover. The uncertainty models are implemented on a web application (OilMetApp) using HTML and WebAssembly.

This Handbook is a documentation of the uncertainty models developed and the web-based calculation tool. It should be noted that the example input values in that calculation tool are just examples and should not be regarded as recommended values by NFOGM, NORCE, NOD or any other party.

Chapter 2 describes on an overview level the metering stations covered in the Handbook. In Chapter 3, uncertainties related to secondary instrumentation temperature, pressure and density are covered. Chapter 4 presents the functional relationships defining the metering stations, Chapter 5 documents the uncertainty models for the metering stations and Chapter 6 documents the web-based uncertainty calculation program. Chapter 7 includes a brief summary of the Handbook.

Appendix A contains some details with respect to the uncertainty model related to adjustments of a flow meter after flow calibration. Appendix B contains a list of symbols.

The uncertainty models presented here are based on the ISO GUM uncertainty methodology. The measurement regulations by the Norwegian Offshore Directorate and the NORSOK standard I-106 on fiscal measurement systems for hydrocarbon gas (NORSOK I-106, 2014) have been important references with respect to layout of the metering stations and requirements to the uncertainty of individual instruments and the operation of the metering station as a whole. A series of ISO, API MPMS and other international standards and reports have also been essential in this work. The details are covered in the relevant sections of the Handbook. It is also referred to the reference list in Chapter 8. Water-in-oil corrections are based on the work on a previous sensitivity study (Hallanger, et al., 2007).

The present work has been carried out for the Norwegian Society for Oil and Gas Measurement (NFOGM) with financial support also from Norwegian Offshore Directorate and Tekna.

2. Description of metering stations

In the present Handbook, the following meter station configurations are covered:

Primary flow meter can be one of the following:

- Ultrasonic flow meter
- Turbine flow meter
- Coriolis flow meter

The different primary flow meters can be proved according to the following setup:

- The ultrasonic flow meter is proved by a displacement prover or a master meter:
  1. Configuration 1: Displacement prover (API MPMS 4.2, 2003).
  2. Configuration 2: Ultrasonic master meter prover (API MPMS 4.5, 2016) and (API MPMS 5.8, 2011).
  3. Configuration 3: Turbine master meter prover (API MPMS 4.5, 2016) and (API MPMS 5.3, 2005).

- The turbine flow meter is proved by a displacement prover or a master meter:
  4. Configuration 4: Displacement prover (API MPMS 4.2, 2003).
  5. Configuration 5: Ultrasonic master meter prover (API MPMS 4.5, 2016) and (API MPMS 5.8, 2011).
  6. Configuration 6: Turbine master meter prover (API MPMS 4.5, 2016) and (API MPMS 5.3, 2005).

- The Coriolis flow meter is proved by a Coriolis master meter:
  7. Configuration 7: Coriolis (mass flow) prover (API MPMS 4.5, 2016) and (API MPMS 5.6, 2021).

Note that the USM and turbine flow meters are proved with volume flow meters, while the Coriolis flow meter is proved with mass flow meters (Coriolis).

The seven different configurations are illustrated in Figure 2.1.

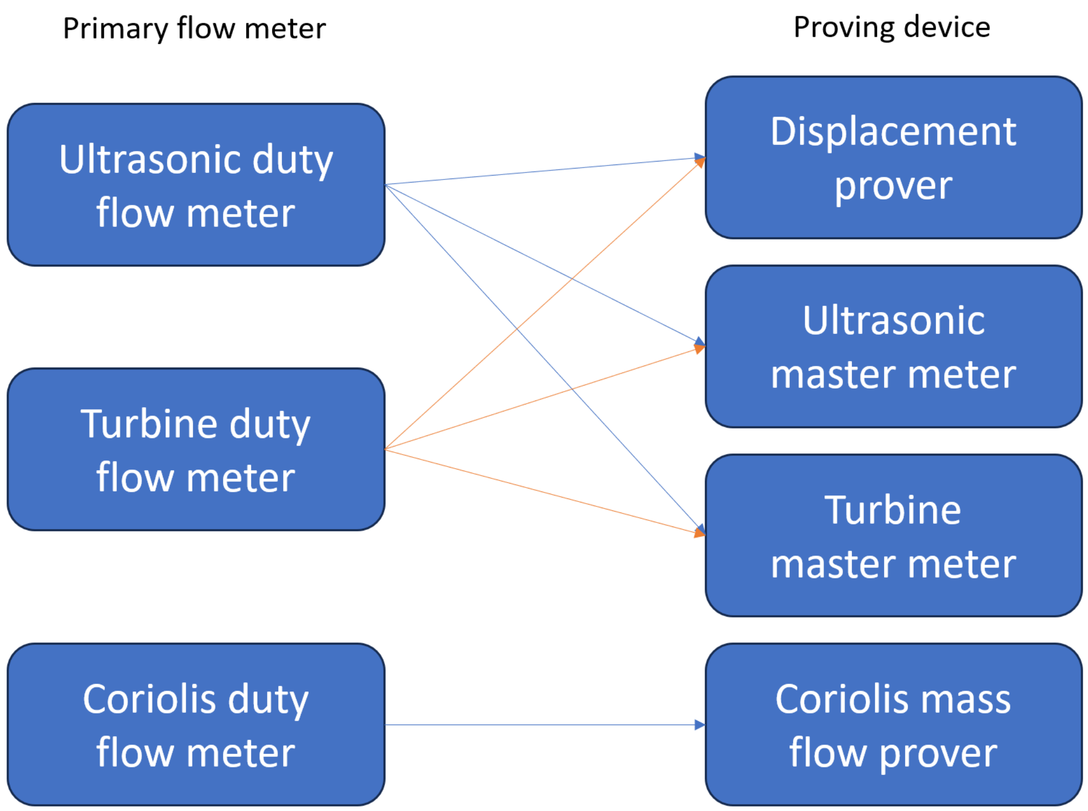


Figure 2.1: Overview of the seven different configurations of primary flow meter versus proving device.

It is assumed that the proving of the primary flow meter is carried out at a single flow rate. The calibration of the proving device will in case of master meter provers be carried out at a series of flow rates. In case of a displacement prover, the prover is calibrated at a single flow rate only.

The metering station is also equipped either with densitometer giving the density at the densitometer pressure and temperature conditions, or provided e.g., with sampling and laboratory analysis, which gives the standard density with a given uncertainty. If the density is from a Coriolis meter, the uncertainty tool described in this handbook, will consider this as a densitometer. The uncertainty input should in that case be given at an overall level. Note that the Coriolis meter providing the density measurement, is not the same Coriolis meter providing the mass flow rate.

The densitometer, the flow meter and the proving device are all equipped with pressure and temperature measurements.

3. Oil measurement uncertainties

This chapter will address the uncertainty models for the measurements of temperature in Section3.1, pressure in Section 3.2 and density in Section 3.3.

3.1. Temperature measurement

The uncertainty model for the temperature measurement follows the similar model in (Lunde, et al., 2002) and (Dahl, et al., 2003).

The uncertainty in the measured temperature can be specified in two ways:

- Overall level.
- Detailed level.

In case of the overall level, the absolute uncertainty in the measured temperature is specified directly by the user of the uncertainty calculation program.

In case of the detailed level, the following uncertainty model is used,

$$u(T)^2 = u\left(T_{elem,transm}\right)^2 + u\left(T_{stab,transm}\right)^2 + u(T_{RFI})^2 + u\left(T_{temp}\right)^2 + u\left(T_{stab,elem}\right)^2 + u(T_{misc})^2 \quad (3\text{-}1)$$

where the terms of the equation are defined as follows:

$u\left(T_{elem,transm}\right)$: standard uncertainty of the temperature element and temperature transmitter, calibrated as a unit. Typically found either in product specifications or in calibration certificates.

$u\left(T_{stab,transm}\right)$: standard uncertainty related to the stability of the temperature transmitter, with respect to drift in readings over time. Typically found in product specifications.

$u(T_{RFI})$: standard uncertainty due to radio-frequency interference (RFI) effects on the temperature transmitter.

$u\left(T_{temp}\right)$: standard uncertainty of the effect of temperature on the temperature transmitter, for change of temperature relative to the temperature at calibration. Typically found in product specifications.

$u\left(T_{stab,elem}\right)$: standard uncertainty related to the stability of the temperature element. Instability may relate e.g., to drift during operation, as well as instability and hysteresis effects due to oxidation and moisture inside the encapsulation, and mechanical stress during operation. Typically found in product specifications.

$u(T_{misc})$: standard uncertainty of other (miscellaneous) effects on the temperature transmitter.

This uncertainty model is quite generic and can be used on a series of industrial products. In cases where this model does not fit with the product specifications, the miscellaneous uncertainty contributions can be used for specification of other uncertainty contributions. Alternatively, the uncertainty of the temperature measurements can be calculated manually, and the result can be given to the program using the overall input level.

When the average of two temperature measurements is used, it is assumed that the two temperature measurements are uncorrelated. The reason for this assumption is that often the two probes are not calibrated at the same time. This means that even if they are calibrated using the same procedure, the time difference generates an uncorrelated drifting term, both in the reference and in the temperature measurement itself. This means that the uncertainty in the average of two temperature measurements is assumed to be equal to the uncertainty for one measurement, divided by the square root of two.

3.2. Pressure measurement

The uncertainty model for the pressure measurement follows the similar model in (Lunde, et al., 2002) and (Dahl, et al., 2003).

The uncertainty in the measured pressure can be specified in two ways:

- Overall level.
- Detailed level.

In case of the overall level, the relative uncertainty in the measured pressure is specified directly by the user of the uncertainty calculation program.

In case of the detailed level, the following uncertainty model is used,

$$u(P)^2 = u(P_{transmitter})^2 + u\left(P_{stability}\right)^2 + u(P_{RFI})^2 + u\left(P_{temp}\right)^2 + u(P_{atm})^2 + u(P_{misc})^2 \qquad (3\text{-}2)$$

where the terms of the equation are defined as follows:

$u(P_{transmitter})$: standard uncertainty of the pressure transmitter, including hysteresis, terminal-based linearity, repeatability, and the standard uncertainty of the pressure calibration laboratory.

$u\left(P_{stability}\right)$: standard uncertainty of the stability of the pressure transmitter, with respect to drift in readings over time.

$u(P_{RFI})$: standard uncertainty due to radio-frequency interference (RFI) effects on the pressure transmitter.

$u(P_{temp})$: standard uncertainty of the effect of ambient air temperature on the pressure transmitter, for change of ambient temperature relative to the temperature at calibration.

$u(P_{atm})$: standard uncertainty of the atmospheric pressure, relative to 1 atm. = 1.01325 bar (or another nominal value that is used), due to local meteorological effects. This effect is only of relevance for units measuring gauge pressure. It can be reduced by using the actually measured barometric pressure instead of a nominal atmospheric pressure.

$u(P_{misc})$: standard uncertainty due to other (miscellaneous) effects on the pressure transmitter, such as mounting effects, etc.

This uncertainty model is quite generic and can be used on a series of industrial products. In cases where this model does not fit with the product specifications, the miscellaneous uncertainty contributions can be used for specification of other uncertainty contributions. Alternatively, the uncertainty of the pressure measurements can be calculated manually, and the result can be given to the program using the overall input level.

When the average of two pressure measurements is used, it is assumed that the two pressure measurements are uncorrelated. The reason for this assumption is that often the two probes are not calibrated at the same time. This means that even if they are calibrated using the same procedure, the time difference generates an uncorrelated drifting term, both in the reference and in the pressure measurement itself. This means that the uncertainty in the average of two pressure measurements is assumed to be equal to the uncertainty for one measurement, divided by the square root of two.

3.3. Density measurement

The uncertainty model for the density measurement follows the similar model as in (Dahl, et al., 2003).

The uncertainty in the measured density at densitometer conditions can be specified in two ways:

- Overall level.
- Detailed level.

In case of the overall level, the relative uncertainty in the measured density is specified directly by the user of the uncertainty calculation program.

In case of the detailed level, the uncertainty model is more complicated than for the temperature and pressure measurements above. The density measurement consists of several steps:

- Measurement of an uncorrected density from the period measurement of a vibrating string.
- Corrections based on temperature difference between calibration and measurement.
- Corrections based on pressure difference between calibration and measurement.

This will in total form the functional relationship for the density measurement as follows:

$$\begin{aligned}\rho_{dens} = \{&\rho_u[1 + K_{18}(T_{dens} - T_{cal})] + K_{19}(T_{dens} - T_{cal})\} \\ &\cdot \left(1 + [K_{20A} + K_{20B} \cdot (P_{dens} - P_{cal})] \right. \\ &\left. \cdot (P_{dens} - P_{cal})\right) \\ &+ [K_{21A} + K_{21B} \cdot (P_{dens} - P_{cal})] \\ &\cdot (P_{dens} - P_{cal})\end{aligned} \tag{3-3}$$

In this equation subscript “*dens”* means densitometer conditions and subscript “*cal”* means calibration conditions. The following variables are used in this equation:

ρ_u: indicated (uncorrected) density, in density transducer [kg/m^3].

K_{18} , K_{19},

K_{20} , K_{20},

K_{21} , K_{21}: constants from the calibration certificate.

T_d: oil temperature in density transducer [°C].

T_{cal}: calibration temperature [°C].

P_{dens}: oil pressure in density transducer [bar].

P_{cal}: calibration pressure [bar].

By using the general uncertainty model approach in ISO GUM (ISO_GUM, 2008) the uncertainty model will be,

$$\begin{aligned}u_c^2(\rho_{dens}) = s_{\rho_u}^2 u^2(\rho_u) + u^2(\rho_{stab}) + u^2(\rho_{rept}) + s_{\rho,T_{dens}}^2 u^2(T_{dens}) \\ + s_{\rho,P_{dens}}^2 u^2(P_{dens}) + u^2(\rho_{temp}) + u^2(\rho_{pres}) \\ + u^2(\rho_{misc})\end{aligned} \tag{3-4}$$

where the terms of the equation are defined as follows:

$u(\rho_u)$: standard uncertainty of the indicated (uncorrected) density, ρ_u, including the calibration laboratory uncertainty, the reading error during calibration, and hysteresis.

$u(\rho_{stab})$: standard uncertainty of the stability of the indicated (uncorrected) density, ρ_u.

$u(\rho_{rept})$: standard uncertainty of the repeatability of the indicated (uncorrected) density, ρ_u.

$u(T_{dens})$: standard uncertainty of the oil temperature in the densitometer, T_{dens}.

$u(P_{dens})$: standard uncertainty of the oil pressure in the densitometer, T_{dens}.

$u(\rho_{temp})$: standard uncertainty of the temperature correction factor for the density, ρ, representing the *model uncertainty* of the temperature correction model used, $\{\rho_u[1 + K_{18}(T_{dens} - T_{cal})] + K_{19}(T_{dens} - T_{cal})\}$ and the pressure correction model used,
$\rho \cdot \big(1 + [K_{20A} + K_{20B} \cdot (P_{dens} - P_{cal})] \cdot (P_{dens} - P_{cal})\big) + [K_{21A} + K_{21B} \cdot (P_{dens} - P_{cal})] \cdot (P_{dens} - P_{cal})$. This also includes the uncertainty of the various *K*-coefficients, and the measurement of the pressure and temperature during calibration.

$u(\rho_{misc})$: standard uncertainty of the density, accounting for miscellaneous uncertainty contributions, such as due to:

- reading error during measurement (for digital display instruments),
- possible deposits on the vibrating element,
- possible corrosion of the vibrating element,
- mechanical (structural) vibrations on the oil line,
- variations in power supply,
- self-induced heat,
- flow in the bypass density line,
- possible liquid viscosity effects,
- effect of a by-pass installation of the densitometer,
- other possible effects.

The sensitivity coefficients in Eq. (3-4) can be calculated from the functional relationship Eq. (3-3) by use of the ISO GUM methodology:

$$s_{\rho_u} = \{1 + K_{18}(T_{dens} - T_{cal})\}\big(1 + [K_{20A} + K_{20B} \cdot (P_{dens} - P_{cal})] \cdot (P_{dens} - P_{cal})\big), \tag{3-5}$$

$$s_{\rho,T_{dens}} = \{\rho_u K_{18} + K_{19}\}\big(1 + [K_{20A} + K_{20B} \cdot (P_{dens} - P_{cal})] \cdot (P_{dens} - P_{cal})\big), \tag{3-6}$$

$$s_{\rho,P_{dens}} = \{\rho_u[1 + K_{18}(T_{dens} - T_{cal})]\} \cdot \big([K_{20A} + K_{20B} \cdot (P_{dens} - P_{cal})] \cdot (P_{dens} - P_{cal})\big) + K_{21A} + K_{21B} \cdot (P_{dens} - P_{cal}) \tag{3-7}$$

These expressions are in practice obtained by partially derivation of the density (Eq. (3-3)) with respect to ρ_u, T_{dens} and P_{dens}, respectively.

4. Functional relationships

In this Chapter, overall functional relationships for the volumetric flow rates at standard and line conditions, and for mass flow rate are presented in Section 4.1. The functional relationships for the oil expansion coefficients for the expansion of oil due to pressure and temperature are covered in Section 4.2. The functional relationships for the steel expansion coefficients for the expansion of steel due to pressure and temperature are covered in Section 4.3. The functional relationships for water in oil corrections are covered in Section 4.4.

4.1. Overall functional relationships

In this section, the functional relationship for the volumetric flow rate at standard conditions is covered in Section 4.1.1. In Section 4.1.2 the functional relationship for the volumetric flow rate at line conditions is covered, and in Section 4.1.3 the functional relationship for the mass flow rate is covered.

4.1.1. Volumetric flow rates at standard conditions

4.1.1.1 Functional relationship when volume flow rate is the primary measurement

The standard volume of oil measured by the primary flow meter is traceable through the following chain:

- The standard volume of oil measured by the primary flow meter is compared to the standard volume of oil measured by a proving device using a single flow rate. This is denoted "proving".
- The standard volume of oil measured by the proving device is compared to a reference standard volume of oil using a single flow rate if the proving device is a displacement prover, and by multiple flow rates if the proving device is a master meter. This is denoted "calibration".
- The reference standard volume is provided by an external party. The traceability of this device is outside the scope of this Handbook.

This can formally be written in the following manner:

$$V_{0,meas} = \left(\frac{V_{0,ref}^{calibration}}{V_{0,prover}^{calibration}}\right)\left(\frac{V_{0,prover}^{proving}}{V_{0,flowmeter}^{proving}}\right)V_{0,flowmeter}^{metering} \qquad (4\text{-}1)$$

Here,

$V_{0,meas}$: the standard volume of oil measured by the primary flow meter volume, after corrections from the proving and calibration.

$V_{0,ref}^{calibration}$: the standard volume of oil measured by the reference instrumentation during calibration of the proving device.

$V_{0,prover}^{calibration}$: the standard volume of oil measured by the proving device during calibration of the proving device.

$V_{0,prover}^{proving}$: the standard volume of oil measured by the proving device during proving of the primary flow meter.

$V_{0,flowmeter}^{proving}$: the standard volume of oil measured by the primary flow meter during proving of the primary flow meter.

$V_{0,flowmeter}^{metering}$: the standard volume of oil measured by the primary flow meter volume at metering, without the corrections form the proving and calibration.

The second parenthesis is the correction from the proving (using a single flow rate) and the first parenthesis is the correction from the calibration, for the flow rate used at proving.

The standard volume of oil through the primary flow meter and the proving device is typically found from the actual volume of oil (at a measured pressure and temperature), through volume correction factors. However, at calibration, standard volumes are compared. After a calibration, the calibration certificate including uncertainty is usually given for this comparison of standard volumes. Therefore, the above equation is modified as follows:

$$V_{0,meas} = \left(\frac{V_{0,ref}^{calibration}}{V_{0,prover}^{calibration}}\right)\left(\frac{C_{tlp}^{prov} C_{plp}^{prov} V_{prover}^{proving}}{C_{tlm}^{prov} C_{plm}^{prov} V_{flowmeter}^{proving}}\right) C_{tlm}^{met} C_{plm}^{met} V_{flowmeter}^{metering} \tag{4-2}$$

Volumes without subscript "0" are here actual volumes at the relevant pressure and temperature. Furthermore C_{tlx} is the temperature volume expansion coefficient for oil and C_{plx} is the pressure volume expansion coefficient for oil, from actual temperature and pressure to standard temperature and pressure. "*x*" is replaced by "*m*" when the actual oil temperature and pressure are the ones at the primary flow meter. "*x*" is replaced by "*p*" when the actual oil temperature and pressure are the ones at the proving device. The superscript "*prov*" or "*met*" indicates whether the actual temperature and pressure during proving or during normal measurement shall be used.

The volumes defined by the flow meter and the proving device have also to be corrected for steel expansion due to pressure and temperature, relative to a reference temperature and pressure, at which a nominal volume is given:

$$V_{0,meas} = \left(\frac{V_{0,ref}^{calibration}}{C_{tsp}^{cal} C_{psp}^{cal} V_{0,nom,prover}^{calibration}}\right) \times$$
$$\left(\frac{C_{tlp}^{prov} C_{plp}^{prov} C_{tsp}^{prov} C_{psp}^{prov} V_{nom,prover}^{proving}}{C_{tlm}^{prov} C_{plm}^{prov} C_{tsm}^{prov} C_{psm}^{prov} V_{nom,flowmeter}^{proving}}\right) \times \tag{4-3}$$
$$C_{tlm}^{met} C_{plm}^{met} C_{tsm}^{met} C_{psm}^{met} V_{nom,flowmeter}^{metering}$$

Here C_{tsx} is the temperature volume expansion coefficient for steel and C_{psx} is the pressure volume expansion coefficient for steel, from a base temperature and pressure to actual temperature and pressure. "*x*" is replaced by "*m*" when the actual steel temperature and pressure are the ones at the primary flow meter. "*x*" is replaced by "*p*" when the actual steel temperature and pressure are the ones at the proving device. The superscript "*cal*", "*prov*" or "*met*" indicates whether the actual temperature and pressure during calibration, during proving or during normal measurement shall be used.

Furthermore,

$V_{0,nom,prover}^{calibration}$: the standard volume of oil that would have been measured by the proving device during calibration of the proving device if temperature and pressure expansions in steel had not been taken into account.

$V_{nom,prover}^{proving}$: the actual volume of oil (line conditions) that would have been measured by the proving device during proving of the primary flow meter if temperature and pressure expansions in steel had not been taken into account.

$V_{nom,flowmeter}^{proving}$: the actual volume of oil (line conditions) that would have been measured by the primary flow meter during proving of the primary flow meter if temperature and pressure expansions in steel had not been taken into account.

$V_{nom,flowmeter}^{metering}$: the actual volume of oil (line conditions) that would have been measured by the primary flow meter volume at metering, without the corrections from the proving and calibration, if temperature and pressure expansions in steel had not been taken into account.

Eq. (4-3) can be re-formulated as

$$V_{0,meas} = \left(\frac{C_{tlp}^{prov} C_{plp}^{prov} C_{tlm}^{met} C_{plm}^{met}}{C_{tlm}^{prov} C_{plm}^{prov}} \right) \cdot \left(\frac{C_{tsp}^{prov} C_{psp}^{prov} C_{tsm}^{met} C_{psm}^{met}}{C_{tsp}^{cal} C_{psp}^{cal} C_{tsm}^{prov} C_{psm}^{prov}} \right) V_{nom,flowmeter}^{metering} \frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \tag{4-4}$$

The two parentheses in this expression will for simplicity be denoted

$$A_{liq}^{m,\Delta p} = \left(\frac{C_{tlp}^{prov} C_{plp}^{prov} C_{tlm}^{met} C_{plm}^{met}}{C_{tlm}^{prov} C_{plm}^{prov}} \right) \tag{4-5}$$

and

$$A_{steel}^{m,\Delta p,c} = \left(\frac{C_{tsp}^{prov} C_{psp}^{prov} C_{tsm}^{met} C_{psm}^{met}}{C_{tsp}^{cal} C_{psp}^{cal} C_{tsm}^{prov} C_{psm}^{prov}} \right) \tag{4-6}$$

The three superscripts *m*, *p* and *c* in these expressions refer to metering, proving and calibration, respectively. The Δ in the superscripts is given when there are two temperature and pressure corrections of relevance for the given process (metering, proving or calibration).

In this way, Eq. (4-4) can be simplified as follows:

$$V_{0,meas} = A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} V_{nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \tag{4-7}$$

The volumetric flow rate at standard conditions, $q_{v_0,meas}$ can now be written as

$$q_{v_0,meas} = A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \tag{4-8}$$

where $q_{v,nom,flowmeter}^{metering}$ is the volumetric flow rate of oil at line conditions that would have been measured by the primary flow meter during metering, without the corrections from the proving and calibration, and if temperature and pressure expansions in steel had not been taken into account.

4.1.1.2 Functional relationship when standard volume flow rate is estimated from measured mass flow

Analogously to standard volume (refer to Eq. (4-1)), the standard mass of oil measured by the primary Coriolis flow meter is traceable through the following chain:

- The mass of oil measured by the primary flow meter is compared to the standard mass of oil measured by a proving device using a single flow rate. This is denoted “proving”.
- The mass of oil measured by the proving device is compared to a reference mass of oil using a single flow rate if the proving device is a displacement prover, and by multiple flow rates if the proving device is a master meter. This is denoted “calibration”.
- The reference mass is provided by an external party. The traceability of this device is outside the scope of this Handbook.

This can formally be written in the following manner:

$$M_{meas} = \left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}} \right) \left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}} \right) M_{flowmeter}^{metering} \tag{4-9}$$

Here,

M_{meas}: the mass of oil measured by the primary flow meter, after corrections from the proving and calibration.

$M_{ref}^{calibration}$: the mass of oil measured by the reference instrumentation during calibration of the proving device.

$M_{prover}^{calibration}$: the mass of oil measured by the proving device during calibration of the proving device.

$M_{prover}^{proving}$: the mass of oil measured by the proving device during proving of the primary flow meter.

$M_{flowmeter}^{proving}$: the mass of oil measured by the primary flow meter during proving of the primary flow meter.

$M_{flowmeter}^{metering}$: the mass of oil measured by the primary flow meter at metering, without the corrections from the proving and calibration

The second parenthesis is the correction from the proving (using a single flow rate) and the first parenthesis is the correction from the calibration, for the flow rate used at proving.

The mass flow rate for a Coriolis meter can now be written as:

$$q_{m,meas} = q_{m,flowmeter}^{metering}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right) \tag{4-10}$$

The standard volume flow can be found by dividing the mass flow by density at standard conditions, which is expressed as:

$$\rho_0 = \frac{\rho_{dens}}{C_{tld}^{met} C_{pld}^{met}} \tag{4-11}$$

Here,

ρ_0: the density at standard conditions

ρ_{dens}: the density at densitometer conditions

C_{tld}^{met}: the temperature correction factor for the liquid from densitometer to standard conditions.

C_{pld}^{met}: the pressure correction factor for the liquid from densitometer to standard conditions.

DENSITY FROM ONLINE DENSITOMETER

If the density is measured with an online densitometer, the standard volume flow for a Coriolis meter can now be written as:

$$q_{v0,meas} = \frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}} q_{m,flowmeter}^{metering} \left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}} \right) \left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}} \right) \tag{4-12}$$

DENSITY FROM LABORATORY ANALYSIS

If the density is found by using standard density obtained from laboratory samples, the standard volume flow for a Coriolis meter can be written as:

$$q_{v0,meas} = \frac{q_{m,flowmeter}^{metering}}{\rho_0} \left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}} \right) \left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}} \right) \tag{4-13}$$

4.1.2. Volumetric flow rate at line conditions

4.1.2.1 Functional relationship when volume flow rate is the primary measurement

The volumetric flow rate at line conditions, $q_{v,meas}$, can be found from the volumetric flow rate at standard conditions as follows:

$$q_{v,meas} = \frac{q_{v_0,meas}}{C_{tlm}^{met} C_{plm}^{met}} \tag{4-14}$$

By use of Eqs. (4-5), (4-6) and (4-8) above, the volumetric flow rate at line conditions, Eq. (4-14), can be written as

$$q_{v,meas} = A_{liq}^{\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \tag{4-15}$$

where

$$A_{liq}^{\Delta p} = \frac{C_{tlp}^{prov} C_{plp}^{prov}}{C_{tlm}^{prov} C_{plm}^{prov}} \tag{4-16}$$

4.1.2.2 Functional relationship when standard volume flow is estimated from measured mass flow

The volume flow can be found by dividing the mass flow by density at line conditions. The density at line conditions is expressed as:

$$\rho_{line} = A_{liq}^{\Delta m} \rho_{dens} \tag{4-17}$$

Here,

ρ_{line}: density at line conditions

ρ_{dens}: density at densitometer conditions

and

$$A_{liq}^{\Delta m} = \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} \tag{4-18}$$

DENSITY FROM ONLINE DENSITOMETER

If the density is measured with an online densitometer, the volume flow rate at line conditions for a Coriolis meter can now be written as:

$$q_{v,meas} = \frac{q_{m,flowmeter}^{metering}}{A_{liq}^{\Delta m} \rho_{dens}} \left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}} \right) \left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}} \right) \tag{4-19}$$

DENSITY FROM LABORATORY ANALYSIS

If the density is found by using standard density obtained from laboratory samples, the standard volume flow for a Coriolis meter can be written as:

$$q_{v,meas} = \frac{q_{m,flowmeter}^{metering}}{C_{tlm}^{met} C_{plt}^{met} \rho_0} \left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}} \right) \left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}} \right) \tag{4-20}$$

4.1.3. Mass flow rate

4.1.3.1 Functional relationship when mass flow rate is estimated from measured volume flow

DENSITY FROM LABORATORY ANALYSIS

The mass flow rate, $q_{m,meas}$, can be found by multiplying the volumetric flow rate at standard conditions, Eq. (4-8), with the standard density (density at standard temperature and pressure), ρ_0:

$$q_{m,meas} = \rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \quad (4\text{-}21)$$

When the density is found from laboratory analysis, this is the relevant functional relationship.

DENSITY FROM ONLINE DENSITOMETER

When the density is measured by a densitometer, oil volume correction factors must be applied to get the standard density. In that case, Eq. (4-21) must be elaborated on in the following manner:

$$\begin{aligned} q_{m,meas} &= \rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \\ &= \frac{\rho_{dens}}{C_{tld}^{met} C_{pld}^{met}} A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \\ &= \rho_{dens} \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} \frac{C_{tlm}^{prov} C_{plm}^{prov}}{C_{tlm}^{prov} C_{plm}^{prov}} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \end{aligned} \quad (4\text{-}22)$$

The condition "*d*" in the oil volume correction factors means that the actual temperature and pressure are the ones at the densitometer (i.e., densitometer conditions). ρ_{dens} is the density at the densitometer conditions. This means that the functional relationship for the mass flow rate in the case where the density is measured by a densitometer can be written as

$$q_{m,meas} = \rho_{dens} A_{liq}^{\Delta m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \quad (4\text{-}23)$$

where

$$A_{liq}^{\Delta m \Delta p} = \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} \frac{C_{tlp}^{prov} C_{plp}^{prov}}{C_{tlm}^{prov} C_{plm}^{prov}} \quad (4\text{-}24)$$

4.1.3.2 Functional relationship when mass flow rate is the primary measurement

The mass flow rate for a Coriolis meter was explained in section 4.1.1.2, where it was needed to explain the calculations of volume flow when mass flow is the primary measurement. The mass flow rate for a Coriolis meter is here repeated for convenience:

$$q_{m,meas} = q_{m,flowmeter}^{metering}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right) \tag{4-25}$$

4.2. Oil volume expansion coefficients

In this Section, the oil volume expansion formulas used in this Handbook will be presented. In Section 4.2.1, the overall correction model is given. In Section 4.2.2 the specific formula for the temperature expansion coefficient is given and in Section 4.2.3 the specific formula for the pressure expansion coefficient is given.

4.2.1. Overall expression

The volume of a given quantity of oil depends on the temperature and pressure. Temperature and pressure expansion factors are provided to calculate how much such a volume is changed when pressure and temperature are changed. More specific, if the volume V of oil at a temperature T_x and pressure P_x is known, the volume of the same amount of oil, V_0, at standard temperature T_0 and pressure P_0can be found from the following expression,

$$V_0 = C_{tlx}C_{plx}V \tag{4-26}$$

where C_{tlx} is the temperature expansion coefficient and C_{plx} is the pressure expansion coefficient. Here "*x*" denotes the condition (either "*p*" for prover, "*m*" for flow meter or "*d*" for densitometer).

4.2.2. Temperature volume expansion coefficient

The temperature volume expansion coefficients for oil can be found in (API MPMS 11.1, 2004). Based on that standard, the following formula is used in this work,

$$C_{tlx} = e^{-\alpha\Delta T_x - 0.8\alpha^2\Delta T_x^2} \tag{4-27}$$

where

$$\alpha = \frac{K_0}{\rho_0^2} + \frac{K_1}{\rho_0} + K_2 \tag{4-28}$$

and

$$\Delta T_x = T_x - T_0 \tag{4-29}$$

This formula corrects the volume from the temperature T_x to the standard reference temperature T_0. ρ_0 is the density at standard reference pressure and temperature. K_0, K_1, and K_2 are coefficients that depend on the type of oil.

The formula above includes two minor approximations compared to the (API MPMS 11.1, 2004) standard.

The first approximation is that no calculation between the old ITS-68 and the new ITS-90 temperature scales have been included. For the purpose of this uncertainty model, this is acceptable as the difference between the two scales is minimal, much less than the typical uncertainty of a temperature measurement.

The second approximation is that these formulas here are used with another standard reference temperature than 60 °F. To be strict, a calculation of the volume change from the temperature T_x to the standard reference temperature of 15 °C shall according to (API MPMS 11.1, 2004) be carried out by first calculating the volume change in the temperature change from T_x to 60 °F, with 60 °F as standard reference temperature, and standard reference density at 60 °F. Thereafter the volume change in a temperature change from 60 °F to 15 °C is calculated, also with a standard reference temperature of 60 °F. See API MPMS 11.1.3.6 and 11.1.3.7 for reference. Instead, the above formulas have been used with 15 °C as standard reference temperature and the standard density is referred to 15 °C. The difference in volume correction factor due to this approximation is minimal, typical in the order of 0.001 % and thus several orders of magnitude below the uncertainty requirements for fiscal oil metering of about 0.25 - 0.30 % (depending on country). For the uncertainty model in focus here, this model is therefore sufficient.

(API MPMS 11.1, 2004) gives several sets of values for the coefficients K_0 and K_1, depending on the type of oil. Among these are (i) crude oil, (ii) fuel oil, (iii) jet group and (iv) gasoline. Crude oil is there classified with API gravity at 60 °F between 100 and -10, corresponding to reference density at 15 °C and 1 atm between 611.16 and 1163.79 kg/m³.

In API MPMS 11.1.6.1, the coefficients K_0 and K_1 are given for the four liquid hydrocarbon types mentioned above. These coefficients are multiplied by 1.8 to convert from Fahrenheit to Celsius temperature scale. The data set then obtained is shown in Table 4.1.

Table 4.1 Temperature expansion coefficient for selected types of oil, for use with temperature on Celsius scale.

	Crude Oil	Fuel Oil	Jet Group	Gasoline
K_0	613.97226	186.9696	594.5418	346.42277
K_1	0	0.48618	0	0.43883
K_2	0	0	0	0

4.2.3. Pressure volume expansion coefficient

The pressure volume expansion coefficients for oil can be found in (API MPMS 11.1, 2004). Based on that standard, the following formula is used in this work,

$$C_{plx} = \frac{1}{1 - 100\,(P_x - P_e)F} \tag{4-30}$$

where

$$F = 10^{-6} e^{A + BT_x + 10^6 \rho_0^{-2}(C + DT_x)} \tag{4-31}$$

The coefficients A, B, C, and D have the following values:

- A = -1.6208,
- B = 0.00021592,
- C = 0.87096,
- D = 0.0042092.

In Eqs. (4-30) and (4-31) it is important to use the unit bar for the pressure and °C for the temperature.

4.3. Steel volume expansion coefficients

The steel volume expansion coefficients refer to expansions from a base temperature and pressure, typically but not necessarily 15 °C and 1 atm = 1.01325 bar. Below, the base temperature and base pressure have the subscript *b*.

4.3.1. Temperature volume expansion coefficients

The temperature volume expansion coefficients are given differently for provers, ultrasonic flow meters and turbine meters.

For displacement provers, (API MPMS 12.2, 2021) gives the following expression for single-walled provers,

$$C_{tsx} = 1 + ((T_x - T_b)\; G_c) \tag{4-32}$$

where G_c is the mean coefficient of cubic expansion per degree temperature of the material of which the container is made, between the temperatures T_b and T_x.

For ultrasonic flow meters, (ISO12242, 2012) gives the following expression in Annex A:

$$C_{tsx} = 1 + \left(\alpha(T_x - T_b)\right)^3 \approx 1 + 3\alpha(T_x - T_b) \tag{4-33}$$

where α is the linear thermal expansion coefficient.

For turbine flow meters, (NORSOK I-105, 1998) rev. 2 gives the following expression,

$$C_{tsx} = \left(1 - Eh(T_x - T_0)\right)^2 \left(1 - Er(T_x - T_0)\right) \quad (4\text{-}34)$$

where Eh and Er are the linear temperature expansion coefficients for the meter housing and the meter rotor, respectively. The notation from the NORSOK-standard is used here. In newer versions of NORSOK I-105 and the successor (NORSOK I-106, 2014), this formula is not present. Furthermore, it has not been possible for the authors to identify another general formula for the pressure expansion for a turbine flow meter in international standards. Therefore, this formula will not be used here.

In this work, the following expression will be used for all three types of equipment:

$$C_{tsx} = 1 + 3\alpha\,(T_x - T_b) \quad (4\text{-}35)$$

This is a valid approximation if the correction factor is not far from 1. That means that extreme temperature differences are not taken into account. It also assumes that in the case of a turbine meter, the rotor is of the same material (metal type) as the meter housing.

4.3.2. Pressure volume expansion coefficients

The pressure volume expansion coefficients are given differently for provers, ultrasonic flow meters and turbine meters.

For displacement provers, (API MPMS 12.2, 2021) gives the following expression for single-walled provers,

$$C_{psx} = 1 + \frac{(P_x - P_b)ID}{E \cdot WT} \quad (4\text{-}36)$$

where

ID:	Inner pipe diameter.
E:	Young's modulus of the pipe metal.
WT:	Pipe wall thickness.

The notation from (API MPMS 12.2, 2021) is used here.

For ultrasonic flow meters (Per Lunde, 2007) demonstrated that the expansion depends on a series of issues. These include:

- Type of material (steel) in the meter spool.
- Pipe wall thickness.
- Upstream and downstream piping.
- Geometry of ultrasonic transducers.
- Number of and location of the acoustic paths, including distance between the acoustic path and the flanges.
- Etc.

In (ISO12242, 2012), Appendix A, and in (ISO17089-1, 2010), Appendix E, this is addressed. A worst-case expression is given as follows:

$$C_{psx} = 1 + 4\left(\frac{R^2 + r^2}{R^2 - r^2}\right)\frac{P_x - P_b}{E} \quad (4\text{-}37)$$

where

r: Inner diameter of pipe.
R: Outer diameter of pipe.
E: Young's modulus of the pipe metal.
μ: Poisson's ratio of the pipe metal.

The notation from the ISO-standards is used here. This expression will be used here because the topic is an uncertainty model. Indicative numbers of the size of the coefficient and on dependencies on the pressure are therefore sufficient for the purpose in focus here.

For turbine flow meters, the (NORSOK I-105, 1998) rev. 2 contains the following expression:

$$C_{psx} = \left(1 + (P - P_b)\frac{(2 - e)2R}{E\left(1 - \frac{AT}{\pi R^2}\right)2t}\right) \quad (4\text{-}38)$$

where

R: Inner diameter of pipe.
t: Pipe wall thickness of pipe.
E: Young's modulus of the pipe metal.
e: Poisson's ratio of the pipe metal.
AT: The area in the pipe cross section that is occupied by the rotor blades. (In the program to be given in percent of the total cross-sectional area.)

The notation from the NORSOK-standard is used here. In newer versions of NORSOK I-105 and the successor (NORSOK I-106, 2014), this formula is not present. This means that generally, the pressure expansion coefficient can by written as

$$C_{psx} = 1 + \beta(P_x - P_b) \quad (4\text{-}39)$$

The expression for β depends on the equipment, and is given as follows:
Displacement prover:

$$\beta = \frac{R}{E \cdot d_w} \quad (4\text{-}40)$$

Ultrasonic flow meter:

$$\beta = \frac{4}{E}\left(\frac{(R + d_w)^2 + R^2}{(R + d_w)^2 - R^2} + \mu\right) \tag{4-41}$$

Turbine flow meter:

$$\beta = \frac{(2 - \mu)2R}{E\left(1 - \frac{A_T}{\pi R^2}\right)2d_w} \tag{4-42}$$

Here the notation of the steel expansion factors is uniformed between the different technologies, as follows:

- *R:* Inner diameter of pipe.
- *d_w:* Pipe wall thickness of pipe.
- *E:* Young's modulus of the pipe metal.
- *μ:* Poisson's ratio of the pipe metal.
- *A_T:* The area in the pipe cross section that is occupied by the rotor blades. (In the program to be given in percent of the total cross-sectional area.)

4.4. Water in oil corrections

In this section, the functional relationships for flow rates are corrected to encompass volumetric water-in-oil contents for up to 5 %. This section is based on the work by the work on a previous sensitivity study (Hallanger, et al., 2007).

The volumetric flow rate at standard conditions is covered in Section 4.4.1. The volumetric flow rate at line conditions is covered in Section 4.4.2, and the mass flow rate is covered in Section 4.4.3.

An assumption for this section is that for turbine and ultrasonic meters, with no slip, the mixed volume of oil and water is the sum of the oil and water volumes at line conditions, such that the volume flow rate at line conditions becomes:

$$q_v^{oil} = q_v^{mix}\left(1 - \phi_{v,line}^{water}\right) \tag{4-43}$$

Here,

q_v^{oil}: The oil volume flow at line conditions

q_v^{mix}: The mixed oil and water volume flow at line conditions and is identical to $q_{v,meas}$ as defined in Eq. (4-15).

$\phi_{v,line}^{water}$: The volumetric water fraction at line conditions

For Coriolis meters the assumption is that the mixed mass of oil and water is the sum of the oil and water masses, such that the mass flow rate becomes:

$$q_m^{oil} = q_m^{mix}\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}}\phi_{v,line}^{water}\right) \tag{4-44}$$

Here,

q_m^{oil}: The oil mass flow

q_m^{mix}: The mixed oil and water mass flow and is identical to $q_{m,meas}$ as defined in Eq. (4-19).

ρ_{line}^{water}: The water density at line conditions

ρ_{line}^{mix}: The density of the oil and water mix at line conditions

The main assumption in this section is that the liquid expansion coefficients are valid for the oil water mix given a volumetric water fraction of less than 5 %.

4.4.1. Water in oil – volumetric flow rate at standard conditions

4.4.1.1 Correction when volume flow rate is the primary measurement

For turbine and ultrasonic meters, the net oil volume flow at standard conditions is found by applying expansion coefficients for temperature and pressure as defined in Eqs. (4-27) and (4-30) to the net oil flow. This can be written as:

$$q_{v_0}^{oil} = q_v^{mix}\left(1 - \phi_{v,line}^{water}\right)C_{tlm}^{met}C_{plm}^{met} \tag{4-45}$$

Here,

$q_{v_0}^{oil}$: The net oil volume flow at standard conditions

C_{tlm}^{met}: The temperature volume expansion coefficient for oil to meter conditions

C_{plm}^{met}: The pressure volume expansion coefficient for oil to meter conditions

Equation (4-45) can be rewritten as:

$$q_{v_0}^{oil} = A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering}\left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}}\right)\left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}}\right)\left(1 - \phi_{v,line}^{water}\right) \tag{4-46}$$

4.4.1.2 Correction when standard volume flow rate is estimated from measured mass flow

DENSITY FROM LABORATORY ANALYSIS

For Coriolis meters, the net oil volume flow at standard conditions is found by dividing the mass flow by the density ρ_0^{oil} at standard conditions. The net oil volume flow at standard conditions can now be written as:

$$q_{v_0}^{oil} = \frac{q_m^{mix}}{\rho_0^{oil}}\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}}\phi_{v,line}^{water}\right) \tag{4-47}$$

Equation (4-47) can be rewritten as:

$$q_{v_0}^{oil} = \frac{q_{m,flowmeter}^{metering}}{\rho_0^{oil}}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right)\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}}\phi_{v,line}^{water}\right) \tag{4-48}$$

The density of the water-oil mix at line conditions, ρ_{line}^{mix}, can be calculated in the following manner:

$$\rho_{line}^{mix} = \rho_0^{oil}\left(1 - \phi_{v,line}^{water}\right)C_{tlm}^{met}C_{plm}^{met} + \phi_{v,line}^{water}\rho_{line}^{water} \tag{4-49}$$

Where:

- ρ_{line}^{mix}: density of the oil-water mix, at line conditions.
- ρ_{line}^{oil}: oil density at line conditions.
- ρ_{line}^{water}: water density at line conditions.
- $\phi_{v,line}^{water}$: volumetric water fraction at line conditions.

DENSITY FROM ONLINE DENSITOMETER

In cases where there is a densitometer installed, the density at reference conditions is found by substitution with Eq. (4-11). The expression for standard volume flow then becomes:

$$\begin{aligned} q_{v_0}^{oil} &= \frac{q_{m,flowmeter}^{metering}}{\rho_{dens}^{mix}A_{liq}^{\Delta m}}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right)\left(1 - \phi_{v,line}^{water}\right)C_{tlm}^{met}C_{plm}^{met} \\ &= \frac{q_{m,flowmeter}^{metering}C_{tld}^{met}C_{pld}^{met}}{\rho_{dens}^{mix}}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right)\left(1 - \phi_{v,line}^{water}\right) \end{aligned} \tag{4-50}$$

The density of the mix is here multiplied by $A_{liq}^{\Delta m}$ to get to the density at line conditions. With low water fractions, this is considered a negligible simplification.

4.4.2. Water in oil – volumetric flow rate at line conditions

4.4.2.1 Correction when volume flow rate is the primary measurement

For turbine and ultrasonic meters, the net oil volume flow at line conditions can be written as shown in (4-43), here repeated for convenience:

$$q_v^{oil} = q_v^{mix}\left(1 - \phi_{v,line}^{water}\right) \tag{4-51}$$

Equation (4-51) can be rewritten as:

$$q_v^{oil} = A_{liq}^{\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}}\right)\left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}}\right)\left(1 - \phi_{v,line}^{water}\right) \tag{4-52}$$

4.4.2.2 Correction when standard volume flow is estimated from measured mass flow

DENSITY FROM LABORATORY ANALYSIS

For Coriolis meters, the net oil volume flow at line conditions is found by dividing the mass flow by the density line conditions. The density at line conditions is:

$$\rho_{line}^{oil} = \rho_0^{oil} C_{tlm}^{met} C_{plm}^{met} \tag{4-53}$$

Here,

ρ_{line}^{oil}: The oil density at line conditions

ρ_0^{oil}: The oil density at standard conditions

The net oil volume flow at line conditions can now be written as:

$$q_v^{oil} = \frac{q_m^{mix}}{\rho_0^{oil} C_{tlm}^{met} C_{plm}^{met}}\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}} \phi_{v,line}^{water}\right) \tag{4-54}$$

Equation (4-54) can be rewritten as:

$$q_v^{oil} = \frac{q_{m,flowmeter}^{metering}}{C_{tlm}^{met} C_{plm}^{met} \rho_0^{oil}}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right)\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}} \phi_{v,line}^{water}\right) \tag{4-55}$$

DENSITY FROM ONLINE DENSITOMETER

In cases where there is a densitometer installed, the density at reference conditions is found by substitution with Eq. (4-11). The expression for volume flow at line conditions then becomes:

$$q_v^{oil} = \frac{q_{m,flowmeter}^{metering}}{\rho_{dens}^{mix} A_{liq}^{\Delta m}} \left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}} \right) \left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}} \right) \left(1 - \phi_{v,line}^{water}\right) \tag{4-56}$$

The density of the mix at densitometer conditions is here multiplied by $A_{liq}^{\Delta m}$ to get to the density at line conditions. With low water fractions, this is considered a negligible simplification.

4.4.3. Water in oil – mass flow rate

4.4.3.1 Correction when mass flow rate is estimated from measured volume flow

For turbine and ultrasonic meters, the net mass flow can be found by multiplying the net volume flow rate at standard conditions by the oil density at standard conditions. The net oil mass flow can now be written as:

$$q_m^{oil} = q_v^{mix}\left(1 - \phi_{v,line}^{water}\right) C_{tlm}^{met} C_{plm}^{met} \rho_0^{oil} \tag{4-57}$$

DENSITY FROM LABORATORY ANALYSIS

If the oil density is found from laboratory analysis, Eq. (4-57) the net oil mass flow can be rewritten as:

$$q_m^{oil} = \rho_0^{oil} A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \left(1 - \phi_{v,line}^{water}\right) \tag{4-58}$$

DENSITY FROM ONLINE DENSITOMETER

If the oil density is based on densitometer measurements, Eq. (4-57) is rewritten as:

$$q_m^{oil} = \rho_{dens}^{mix} A_{liq}^{\Delta m} A_{liq}^{\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}} \right) \left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}} \right) \left(1 - \frac{\rho_{line}^{water}}{\rho_{dens}^{mix} A_{liq}^{\Delta m}} \phi_{v,line}^{water}\right) \tag{4-59}$$

The density of the mix at densitometer conditions is here multiplied by $A_{liq}^{\Delta m}$ to get to the density at line conditions. With low water fractions, this is considered a negligible simplification. Correlations with $A_{liq}^{\Delta p}$ are also considered negligible, as the effects of temperature and pressure on the term cancel out almost completely.

4.4.3.2 Correction when mass flow rate is the primary measurement

For Coriolis meters, the net mass flow can be written as:

$$q_m^{oil} = q_m^{mix}\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}}\phi_{v,line}^{water}\right) \tag{4-60}$$

Equation (4-60) can be rewritten as:

$$q_m^{oil} = q_{m,flowmeter}^{metering}\left(\frac{M_{prover}^{proving}}{M_{flowmeter}^{proving}}\right)\left(\frac{M_{ref}^{calibration}}{M_{prover}^{calibration}}\right)\left(1 - \frac{\rho_{line}^{water}}{\rho_{line}^{mix}}\phi_{v,line}^{water}\right) \tag{4-61}$$

DENSITY FROM LABORATORY ANALYSIS

The density of the water-oil mix at line conditions, ρ_{line}^{mix}, can be calculated using Eq. (4-49).

DENSITY FROM ONLINE DENSITOMETER

In cases where there is a densitometer installed, the density of the water-oil mix at line conditions, ρ_{line}^{mix}, is found by multiplying the density of the water-oil mix at densitometer conditions, ρ_{dens}^{mix}, by $A_{liq}^{\Delta m}$. With low water fraction, this is considered a negligible simplification.

4.5. Water fraction at line conditions

The volumetric water fraction at line conditions can be found by two means; online water cut measurement, or laboratory sampling. In both cases the water at line conditions is found by a correction calculation. Some additional correction factors that have not previously been defined, will be needed. These are defined as follows (ISO8222, 2020):

$$C_{twx} = \frac{\rho^w(T_2)}{\rho^w(T_1)} \tag{4-62}$$

$$\rho^w = 999.84382 \cdot \left(\frac{1 + 1.4639386\left(\frac{T}{100}\right) - 0.015505\left(\frac{T}{100}\right)^2 - 0.0309777\left(\frac{T}{100}\right)^3}{1 + 1.4572099\left(\frac{T}{100}\right) + 0.0648931\left(\frac{T}{100}\right)^2}\right) \tag{4-63}$$

$$C_{pwx} = \frac{1}{1 - P \cdot F_w} \tag{4-64}$$

$$F_w = \frac{1}{19.69 + 0.1418 \cdot T - 1.934 \cdot 10^{-3} \cdot T^2 + 5.866 \cdot 10^{-6} \cdot T^3} \cdot 10^{-3} \tag{4-65}$$

Here,

C_{twx}: Temperature volume expansion coefficient for water.

C_{pwx}: Pressure volume expansion coefficient for water.

T_1: The temperature to which the correction is made (reference conditions, °C).

T_2: The temperature from which the correction is made (°C).

P: The pressure from which the correction is made.

4.5.1. Online water cut metering

If an online water cut meter is installed, the water at line conditions can be found as follows:

$$\phi_{v,line}^{water} = \frac{A_{online}\phi_{WIO}}{A_{online}\phi_{WIO} + B_{online}(1-\phi_{WIO})} = \frac{C_{tww}C_{pww}C_{tlm}^{met}C_{plm}^{met}\phi_{WIO}}{C_{tww}C_{pww}C_{tlm}^{met}C_{plm}^{met}\phi_{WIO} + C_{twm}C_{pwm}C_{tlw}C_{plw}(1-\phi_{WIO})} \quad (4\text{-}66)$$

Here,

ϕ_{WIO}: The volumetric water fraction as measured by the water cut meter

A_{online}: $C_{tww}C_{pww}C_{tlm}^{met}C_{plm}^{met}$

B_{online}: $C_{twm}C_{pwm}C_{tlw}C_{plw}$

C_{tww}: Temperature volume expansion coefficient for water at water cut meter conditions

C_{pww}: Pressure volume expansion coefficient for water at water cut meter conditions

C_{twm}: Temperature volume expansion coefficient for water at flow meter conditions

C_{pwm}: Pressure volume expansion coefficient for water at flow meter conditions

C_{tlw}: Temperature volume expansion coefficient for oil at water cut meter conditions

C_{plw}: Pressure volume expansion coefficient for oil at water cut meter conditions

4.5.2. Laboratory water sampling

If an online water cut meter is installed, the water at line conditions can be found as follows:

$$\phi_{v,line}^{water} = \frac{A_{lab}\phi_{WIO}}{A_{lab}\phi_{WIO} + B_{lab}(1-\phi_{WIO})} = \frac{C_{twl}C_{pwl}C_{tlm}^{met}C_{plm}^{met}\phi_{LAB}}{C_{twl}C_{pwl}C_{tlm}^{met}C_{plm}^{met}\phi_{LAB} + C_{twm}C_{pwm}C_{tll}C_{pll}(1-\phi_{LAB})} \quad (4\text{-}67)$$

Here,

A_{lab}: $C_{twl}C_{pwl}C_{tlm}^{met}C_{plm}^{met}$

B_{lab}: $C_{twm}C_{pwm}C_{tll}C_{pll}$

ϕ_{LAB}: The volumetric water fraction as measured at laboratory conditions

C_{twl}: Temperature volume expansion coefficient for water at laboratory conditions

C_{pwl}: Pressure volume expansion coefficient for water at laboratory conditions

C_{twm}: Temperature volume expansion coefficient for water at flow meter conditions

C_{pwm}: Pressure volume expansion coefficient for water at flow meter conditions

C_{tll}: Temperature volume expansion coefficient for oil at laboratory conditions

C_{pll}: Pressure volume expansion coefficient for oil at laboratory conditions

5. Uncertainty models

In this Chapter, the uncertainty models for volumetric flow rate at standard conditions, volumetric flow rate at line conditions and mass flow rate, including water-in-oil corrections, are given in Sections 5.1, 5.2, and 5.3, respectively. These uncertainty models are quite general, and the various components of them are detailed in the next sections. In Sections 5.4, 5.5, and 5.6, respectively, the uncertainty contributions related to the calibration process, the proving process and the flow metering are addressed. In Section 5.7, the uncertainty contribution related to the oil and steel expansion coefficient is addressed. In Section 5.8 the model uncertainties of these oil and steel expansion coefficients are addressed more in detail. Uncertainties related to water in oil are discussed further in Section 5.9, note also that these contributions are set to zero if there is no water in the oil.

5.1. Volumetric flow rate at standard conditions

5.1.1. Uncertainty model when volume flow rate is the primary measurement

For turbine and ultrasonic meters, the relative standard uncertainty of the volumetric flow rate at standard conditions can be deduced from Eq. (4-8) and (4-45), and is here written as follows:

$$\left(\frac{u\left(q_{v_0}^{oil}\right)}{q_{v_0}^{oil}}\right)^2 = \left(\frac{u\left(A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}\right)}{A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}}\right)^2 + \left(\frac{u\left(q_{v_0}^{cal}\right)}{q_{v_0}^{cal}}\right)^2 + \left(\frac{u\left(q_{v_0}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 + \left(\frac{u\left(q_{v_0}^{met}\right)}{q_{v_0}}\right)^2 + \left(\frac{\phi_{v,line}^{water}}{1-\phi_{v,line}^{water}}\right)^2 \left(\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 \tag{5-1}$$

The interpretation of this equation is that the uncertainty consists of contributions related to expansion of oil and steel (first term), contributions related to the calibration process (second term), contributions related to the proving process (third term) and contributions related to the flow metering (fourth term), as well as the uncertainty related to the volumetric water fraction (last term). The last term is only relevant in cases where water-in-oil considerations are necessary.

The first term on the right-hand side of Eq. (5-1), related to the expansion of oil and steel, is discussed further in Section 5.85.7.

The second term on the right-hand side of Eq. (5-1), related to the calibration process, is discussed further in Section 5.4.

The third term on the right-hand side of Eq. (5-1), related to the proving process, is discussed further in Section 5.5.

The fourth term on the right-hand side of Eq. (5-1), related to the flow metering, is discussed further in Section 5.6.

The fifth term on the right-hand side of Eq. (5-1), related to the volumetric water fraction, is discussed further in Section 5.9.

5.1.2. Uncertainty model when standard volume flow rate is estimated from measured mass flow

DENSITY FROM ONLINE DENSITOMETER

For Coriolis meters, the relative standard uncertainty of the volumetric flow rate at standard conditions can be deduced from Eq. (4-12) and Eq. (4-50) and is here written as follows for the case where a densitometer is used:

$$\begin{aligned}\left(\frac{u\left(q_{v_0}^{oil}\right)}{q_{v_0}^{oil}}\right)^2 = &\left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 \\ &+ \left(\frac{u\left(C_{tld}^{met} C_{pld}^{met} / \rho_{dens}^{mix}\right)}{C_{tld}^{met} C_{pld}^{met} / \rho_{dens}^{mix}}\right)^2 \\ &+ \left(\frac{\phi_{v,line}^{water}}{1-\phi_{v,line}^{water}} \frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2\end{aligned} \qquad (5\text{-}2)$$

The interpretation of this equation is that the uncertainty consists of contributions related to the calibration process (first term), contributions related to the proving process (second term) and contributions related to the flow metering (third term), the uncertainty related to the liquid expansion coefficients and measured density (fourth term) as well as the uncertainty related to the volumetric water fraction (last term). The last term is only relevant in cases where water-in-oil considerations are necessary.

The first term on the right-hand side of Eq. (5-2), related to the calibration process, is discussed further in Section 5.4.

The second term on the right-hand side of Eq. (5-2), related to the proving process, is discussed further in Section 5.5.

The third term on the right-hand side of Eq. (5-2), related to the flow metering, is discussed further in Section 5.6.

The fourth term on the right-hand side of Eq. (5-2) is related to the density measurement and the liquid volume expansion coefficients.

The fifth term on the right-hand side of Eq. (5-2), related to the volumetric water fraction, is discussed further in Section 5.9.

DENSITY FROM LABORATORY ANALYSIS

For the case where an online densitometer is not used, and the reference density is found from laboratory samples, the uncertainty model, using also Eq. (4-48) becomes:

$$\left(\frac{u\left(q_{v_0}^{oil}\right)}{q_{v_0}^{oil}}\right)^2 = \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 + \left(\frac{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right)}{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right)+\phi_{v,line}^{water}\rho^{water}}\frac{u\left(\rho_0^{oil}\right)}{\rho_0^{oil}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right)+\phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(\rho_{line}^{water}\right)}{\rho_{line}^{water}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{\left(1-\phi_{v,line}^{water}\right)\left(C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right)+\phi_{v,line}^{water}\rho_{line}^{water}\right)}\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right)+\phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(C_{tlm}C_{plm}\right)}{C_{tlm}C_{plm}}\right)^2 + X_{Corr} \tag{5-3}$$

The interpretation of this equation is that the uncertainty consists of contributions related to liquid density, as well as the calibration process, the proving process, the flow metering, as well as the uncertainty in volumetric water fraction and water density.

The first term on the right-hand side of Eq. (5-3) and related to the calibration process, is discussed further in Section 5.4.

The second term on the right-hand side of Eq. (5-3), related to the proving process, discussed further in Section 5.5.

The third term on the right-hand side of Eq. (5-3), related to the flow metering, is discussed further in Section 5.6.

The fourth term and last term on the right-hand side of Eq. (5-3), related to the expansion of liquid, are discussed further in Section 5.8.

The last terms on the right-hand side of Eq. (5-3), account for correlations between densities and volume correction factors. It is found by numerical derivation of the functional relationship, refer to Eq. (4-48), thus finding the total variance, and subtracting the sum of variances of the individual contributions (arising from previous terms on the right-hand side of Eq. (5-3)). Prior to the numerical derivation, Eq. (4-48) is re-written in terms of its independent variables, e.g., all calculated densities are written as functions of pressures, temperatures, and measured densities. Numerical derivation is covered in Section 5.10.

The fourth term on the right-hand side of Eq. (5-3), is related to the standard density.

The remaining terms are related to the water density and volumetric water fraction, which is discussed in Section 5.9.

In cases where the density is calculated from laboratory samples, when water-in-oil considerations are not necessary, the uncertainty model is given by Eq. (5-4).

$$\left(\frac{u\left(q_{v_0,meas}\right)}{q_{v_0,meas}}\right)^2 = \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 + \left(\frac{u(\rho_0)}{\rho_0}\right)^2 \tag{5-4}$$

5.2. Volumetric flow rate at line conditions

5.2.1. Uncertainty model when volume flow rate is the primary measurement

For turbine and ultrasonic meters, the relative standard uncertainty of the volumetric flow rate at standard conditions can be deduced from Eq. (4-52), and is here written as follows:

$$\begin{aligned}\left(\frac{u\left(q_v^{oil}\right)}{q_v^{oil}}\right)^2 = &\left(\frac{u\left(A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}\right)}{A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2 + \left(\frac{u\left(q_{v_0}^{cal}\right)}{q_{v_0}^{cal}}\right)^2 + \left(\frac{u\left(q_{v_0}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 \\ &+ \left(\frac{u\left(q_{v_0}^{met}\right)}{q_{v_0}^{met}}\right)^2 + \left(\frac{\phi_{v,line}^{water}}{1-\phi_{v,line}^{water}}\right)^2 \left(\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2\end{aligned} \tag{5-5}$$

The interpretation of this equation is that the uncertainty consists of contributions related to expansion of oil and steel (first term), contributions related to the calibration process (second term), contributions related to the proving process (third term), contributions related to the flow metering (fourth term), and contributions related to the water fraction measurement (last term).

The first term on the right-hand side of Eq. (5-5), related to the expansion of oil and steel, is discussed further in Section 5.8.

The second term on the right-hand side of Eq. (5-5), related to the calibration process, is discussed further in Section 5.4.

The third term on the right hand-side of Eq. (5-5), related to the proving process, is discussed further in Section 5.5.

The fourth term on the right hand-side of Eq. (5-5), related to the flow metering, is discussed further in Section 5.6.

The fifth term on the right-hand side of Eq. (5-5), related to the volumetric water fraction, is discussed further in Section 5.9

The last term is only relevant in cases where water-in-oil considerations are necessary.

5.2.2. Uncertainty model when standard volume flow rate is estimated from measured mass flow

DENSITY FROM ONLINE DENSITOMETER

For Coriolis meters, the relative standard uncertainty of the volumetric flow rate at standard conditions can be deduced from Eq. (4-56) and is here written as follows for the case where a densitometer is used:

$$\left(\frac{u\left(q_v^{oil}\right)}{q_v^{oil}}\right)^2 = \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 + \left(\frac{u\left(A_{liq}^{\Delta m}\rho_{dens}^{mix}\right)}{A_{liq}^{\Delta m}\rho_{dens}^{mix}}\right)^2 + \left(\frac{\phi_{v,line}^{water}}{1-\phi_{v,line}^{water}}\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 \qquad (5\text{-}6)$$

The interpretation of this equation is that the uncertainty consists of contributions related to the calibration process (first term), contributions related to the proving process (second term) and contributions related to the flow metering (third term), the uncertainty related to the liquid expansion coefficients and measured density (fourth term) as well as the uncertainty related to the volumetric water fraction (last term). The last term is only relevant in cases where water-in-oil considerations are necessary.

The first term on the right-hand side of Eq. (5-6), related to the calibration process, is discussed further in Section 5.4.

The second term on the right-hand side of Eq. (5-6), related to the proving process, is discussed further in Section 5.5.

The third term on the right-hand side of Eq. (5-6), related to the flow metering, is discussed further in Section 5.6.

The fourth term on the right-hand side of Eq. (5-6), is related to the density measurement and the liquid volume expansion coefficients.

The fifth term on the right-hand side of Eq. (5-6), related to the volumetric water fraction, is discussed further in Section 5.9.

DENSITY FROM LABORATORY ANALYSIS

For the case where an online densitometer is not used, and the reference density is found from laboratory samples, the uncertainty model, using also Eq. (4-55) becomes:

$$
\begin{aligned}
&\left(\frac{u\left(q_v^{oil}\right)}{q_v^{oil}}\right)^2 \\
&= \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 \\
&+ \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{C_{tlm}C_{plm}\rho_0^{oil}(1-\phi_{line}) + \phi_{v,line}^{water}\rho_{line}^{water}} \frac{u\left(\rho_{line}^{water}\right)}{\rho_{line}^{water}}\right)^2 \\
&+ \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{(1-\phi_{v,line}^{water})(C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right) + \phi_{v,line}^{water}\rho_{line}^{water})} \frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 \\
&+ \left(-\frac{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right)}{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right) + \phi_{v,line}^{water}\rho_{line}^{water}} \frac{u\left(C_{tlm}C_{plm}\rho_0^{oil}\right)}{C_{tlm}C_{plm}\rho_0^{oil}}\right)^2
\end{aligned}
\tag{5-7}
$$

The interpretation of this equation is that the uncertainty consists of contributions related to liquid density, as well as the calibration process, the proving process, the flow metering, as well as the uncertainty in volumetric water fraction and water density.

The first on the right-hand side of Eq. (5-7), related to the calibration process, is discussed further in Section 5.4.

The second term on the right-hand side of Eq. (5-7), related to the proving process, discussed further in Section 5.5.

The third term on the right-hand side of Eq. (5-7), related to the flow metering, is discussed further in Section 5.6.

The fourth and fifth terms on the right-hand side of Eq. (5-7), are related to the water density and volumetric water fraction, respectively. The uncertainty of in volumetric water fraction is discussed in Section 5.9.

The last term on the right-hand side of Eq. (5-7) are related to the expansion of liquid, and densities. These uncertainties are discussed further in sections 5.7 and 5.8.

In cases where water-in-oil considerations are not necessary, the uncertainty model for cases with laboratory density measurements are given with Eq. (5-8).

$$
\begin{aligned}
\left(\frac{u\left(q_{v,meas}\right)}{q_{v,meas}}\right)^2 &= \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 \\
&+ \left(\frac{u\left(C_{tlm}^{met}C_{plm}^{met}\rho_0^{oil}\right)}{C_{tlm}^{met}C_{plm}^{met}\rho_0^{oil}}\right)^2
\end{aligned}
\tag{5-8}
$$

The last term in (5-8) is related to the reference density and the liquid volume expansion coefficients.

5.3. Mass flow rate

5.3.1. Uncertainty model when mass flow rate is estimated from measured volume flow

DENSITY FROM LABORATORY ANALYSIS

For turbine and ultrasonic meters, the relative standard uncertainty of the mass flow rate can in the case when the density is determined from laboratory analysis be deduced from Eq. (4-58), and is here written as follows:

$$\left(\frac{u(q_m^{oil})}{q_m^{oil}}\right)^2 = \left(\frac{u\left(\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}\right)}{\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}}\right)^2 + \left(\frac{u\left(q_{v_0}^{cal}\right)}{q_{v_0}^{cal}}\right)^2 + \left(\frac{u\left(q_{v_0}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 + \left(\frac{u(q_v^{met})}{q_v^{met}}\right)^2 + \left(\frac{\phi_{v,line}^{water}}{1-\phi_{v,line}^{water}}\right)^2 \left(\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 \qquad (5\text{-}9)$$

The interpretation of this equations is that the uncertainty consists of contributions related to density measurement and expansion of oil and steel (first term), contributions related to the calibration process (second term), contributions related to the proving process (third term), contributions related to the flow metering (fourth term), and contributions related to the volumetric water fraction (last term).

The first term on the right-hand side of Eq. (5-9), related to the density measurement and expansion of oil and steel, is discussed further in Section 5.8.

The second term on the right-hand side of Eq. (5-9), related to the calibration process, is discussed further in Section 5.4.

The third term on the right-hand side of Eq. (5-9), related to the proving process, is discussed further in Section 5.5.

The fourth term on the right-hand side of Eq. (5-9), related to the flow metering, is discussed further in Section 5.6.

The fifth term on the right-hand side of Eq. (5-9), related to the volumetric water fraction, is discussed further in Section 5.9.

The last term is only relevant in cases where water-in-oil considerations are necessary.

DENSITY FROM ONLINE DENSITOMETER

When the density is measured by a densitometer, the relative standard uncertainty of the mass flow rate can be deduced from Eq. (4-59), and is here written as follows:

(5-10)

$$\left(\frac{u\left(q_m^{oil}\right)}{q_m^{oil}}\right)^2 = \left(\frac{u\left(A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}\right)}{A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2 + \left(\frac{u\left(q_{v_0}^{cal}\right)}{q_{v_0}^{cal}}\right)^2 + \left(\frac{u\left(q_{v_0}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 + \left(\frac{u\left(q_{v_0}^{met}\right)}{q_{v_0}^{met}}\right)^2 + \left(\frac{\rho_{dens}^{mix}A_{liq}^{\Delta m}}{\rho_{dens}^{mix}A_{liq}^{\Delta m} - \rho_{line}^{water}\phi_{v,line}^{water}}\frac{u\left(\rho_{dens}^{mix}A_{liq}^{\Delta m}\right)}{\rho_{dens}^{mix}A_{liq}^{\Delta m}}\right)^2 + \left(\frac{\rho_{line}^{water}\phi_{v,line}^{water}}{\rho_{dens}^{mix}A_{liq}^{\Delta m} - \rho_{line}^{water}\phi_{v,line}^{water}}\frac{u\left(\rho_{line}^{water}\right)}{\rho_{line}^{water}}\right)^2 + \left(\frac{\rho_{line}^{water}\phi_{v,line}^{water}}{\rho_{dens}^{mix}A_{liq}^{\Delta m} - \rho_{line}^{water}\phi_{v,line}^{water}}\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2$$

The interpretation of this equations is that the uncertainty consists of contributions related to density measurement and expansion of oil and steel (first and second terms), contributions related to the calibration process (third term), contributions related to the proving process (fourth term), contributions related to the flow metering (fifth term), contributions related to the water density (sixth term) and contributions related to the volumetric water fraction (last term).

The second term on the right-hand side of Eq. (5-10), related to the density measurement and expansion of oil and steel, is discussed further in Section 5.8.

The third term on the right-hand side of Eq. (5-10), related to the calibration process, is discussed further in Section 5.4.

The fourth term on the right-hand side of Eq. (5-10), related to the proving process, is discussed further in Section 5.5.

The fifth and sixth terms on the right-hand side of Eq. (5-10) are related to the water density and volumetric water fraction, respectively. The uncertainty of in volumetric water fraction is discussed in Section 5.9.

5.3.2. Uncertainty model when mass flow rate is the primary measurement

DENSITY FROM ONLINE DENSITOMETER

For Coriolis meters, the relative standard uncertainty of the mass flow rate at standard conditions can be deduced from Eq. (4-61), and is here written as follows for the case where a densitometer is used:

$$\left(\frac{u\left(q_m^{oil}\right)}{q_m^{oil}}\right)^2 = \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{\rho_{dens}^{mix}A_{liq}^{\Delta m} + \phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(\rho_{line}^{water}\right)}{\rho_{line}^{water}}\right)^2 + \left(\frac{\phi_{line}^{water}\rho_{line}^{water}}{\rho_{dens}^{mix}A_{liq}^{\Delta m} + \phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{\rho_{dens}^{mix}A_{liq}^{\Delta m} + \phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(A_{liq}^{\Delta m}\rho_{dens}\right)}{A_{liq}^{\Delta m}\rho_{dens}}\right)^2 \quad (5\text{-}11)$$

DENSITY FROM LABORATORY ANALYSIS

For the case where an online densitometer is not used, and the reference density is found from laboratory samples, using Eqs. (4-61) and (4-49) the uncertainty model becomes:

$$\left(\frac{u\left(q_m^{oil}\right)}{q_m^{oil}}\right)^2 = \left(\frac{u\left(q_m^{cal}\right)}{q_m^{cal}}\right)^2 + \left(\frac{u\left(q_m^{prov}\right)}{q_m^{prov}}\right)^2 + \left(\frac{u\left(q_m^{met}\right)}{q_m^{met}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right) + \phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(\rho_{line}^{water}\right)}{\rho_{line}^{water}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{(1-\phi_{v,line}^{water})(C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right) + \phi_{v,line}^{water}\rho_{line}^{water})}\frac{u\left(\phi_{v,line}^{water}\right)}{\phi_{v,line}^{water}}\right)^2 + \left(\frac{\phi_{v,line}^{water}\rho_{line}^{water}}{C_{tlm}C_{plm}\rho_0^{oil}\left(1-\phi_{v,line}^{water}\right) + \phi_{v,line}^{water}\rho_{line}^{water}}\frac{u\left(C_{tlm}C_{plm}\rho_0^{oil}\right)}{C_{tlm}C_{plm}\rho_0^{oil}}\right)^2 \quad (5\text{-}12)$$

The interpretation of Eqs. (5-11) and (5-12) is that the uncertainty consists of contributions related to liquid density, as well as the calibration process, the proving process, the flow metering, as well as the uncertainty in volumetric water fraction and water density.

The first terms on the right-hand side of Eqs. (5-11) and (5-12), related to the calibration process, are discussed further in Section 5.4.

The second terms on the right-hand side of Eqs. (5-11) and (5-12), related to the proving process, are discussed further in Section 5.5.

The third terms on the right-hand side of Eqs. (5-11) and (5-12), related to the flow metering, are discussed further in Section 5.6.

The fourth and fifth terms on the right-hand side of Eqs. (5-11) and (5-12), are related to the water density and volumetric water fraction, respectively. The uncertainty of in volumetric water fraction is discussed in Section 5.9.

The last term on the right-hand side of Eqs. (5-11) and (5-12) are related to the expansion of liquid, and densities. These uncertainties are discussed further in sections 5.7 and 5.8.

In cases where water-in-oil considerations are not necessary, the uncertainty model is given by Eq. (5-13).

$$\left(\frac{u(q_{m,meas})}{q_{m,meas}}\right)^2 = \left(\frac{u(q_m^{cal})}{q_m^{cal}}\right)^2 + \left(\frac{u(q_m^{prov})}{q_m^{prov}}\right)^2 + \left(\frac{u(q_m^{met})}{q_m^{met}}\right)^2 \qquad (5\text{-}13)$$

5.4. Calibration uncertainties

The calibration uncertainty can be understood as the uncertainty in a newly calibrated device that operates under the same flow rates, oil quality, pressure, and temperature, and in case of a master meter, installed in the same place in the same flow loop as under calibration. This means that it is the uncertainty in the meter output related to an as-left flow test carried out for the same flow rates as where the flow calibration was carried out.

The calibration uncertainty for turbine and ultrasonic meters refers to the second term on the right hand of Eqs. (5-1), (5-5), (5-9), and (5-10) and can be written as follows:

$$\left(\frac{u(q_{v_0}^{cal})}{q_{v_0}^{cal}}\right)^2 = \left(\frac{u\left(q_{v_0,ref}^{cal}\right)}{q_{v_0}^{cal}}\right)^2 + \left(\frac{u(q_{v_0,rept-prover}^{cal})}{q_{v_0}^{cal}}\right)^2 \qquad (5\text{-}14)$$

The calibration uncertainty related to a Coriolis meter can be written analogously to that of the turbine and ultrasonic meters, ref. Eq. (5-14), replacing the standard volume flow q_{v_0} by mass flow q_m, as given in the following equation:

$$\left(\frac{u(q_m^{cal})}{q_m^{cal}}\right)^2 = \left(\frac{u(q_{m,ref}^{cal})}{q_m^{cal}}\right)^2 + \left(\frac{u(q_{m,rept-prover}^{cal})}{q_m^{cal}}\right)^2 \qquad (5\text{-}15)$$

Here the first term on the right-hand side is the relative standard uncertainty of the calibration reference:

- If the proving device is a displacement prover, this uncertainty term refers to the reference system used at the on-site calibration of the prover.
- If the proving device is a master meter (flow meter), there are two options.
    - If the master meter is calibrated off-site, at a flow laboratory, the uncertainty term refers to the uncertainty of the flow reference at the flow laboratory.
    - If the master meter is calibrated on-site, typically by use of a compact portable prover with transfer meter, the uncertainty term refers to the uncertainty in that prover/transfer meter system.

The second term on the right-hand side is the relative standard uncertainty due to the repeatability obtained during calibration of the proving device. This is found from the repeatability checks carried out under such calibrations. Note that the term relates to the uncertainty and not the repeatability itself.

5.5. Proving uncertainties

The proving uncertainty can be understood as the extra uncertainty contributions related to a proving device when used in the proving process, compared to the calibration uncertainty. It is assumed that the proving is carried out at a single flow rate only.

The proving uncertainty for turbine and ultrasonic meters refers to the third term on the right hand of Eqs. (5-1), (5-5), (5-9), and (5-10) and can be written as follows:

$$\left(\frac{u\left(q_{v_0}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 = \left(\frac{u\left(q_{v_0,rept-prover}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 + \left(\frac{u\left(q_{v_0,rept-flowmeter}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 + \left(\frac{u\left(q_{v_0,linearity}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 + \left(\frac{u\left(q_{v_0,profile}^{prov}\right)}{q_{v_0}^{prov}}\right)^2 \tag{5-16}$$

Similarly for the mass flow rate measurement, referring to the third terms in Eqs. (5-2), (5-3), (5-6), (5-7), (5-11), and (5-12), and the second terms in Eqs. (5-4), (5-8), and (5-13), the proving uncertainty is written as follows:

$$\left(\frac{u\left(q_{m}^{prov}\right)}{q_{m}^{prov}}\right)^2 = \left(\frac{u\left(q_{m,rept-prover}^{prov}\right)}{q_{m}^{prov}}\right)^2 + \left(\frac{u\left(q_{m,rept-flowmeter}^{prov}\right)}{q_{m}^{prov}}\right)^2 + \left(\frac{u\left(q_{m,linearity}^{prov}\right)}{q_{m}^{prov}}\right)^2 + \left(\frac{u\left(q_{m,profile}^{prov}\right)}{q_{m}^{prov}}\right)^2 + \left(\frac{u\left(q_{m,pressure}^{prov}\right)}{q_{m}^{prov}}\right)^2 + \left(\frac{u\left(q_{m,temperature}^{prov}\right)}{q_{m}^{prov}}\right)^2 \tag{5-17}$$

The first two terms on the right-hand side of Eqs. (5-16) and (5-17) correspond to the repeatability of the prover and the flow meter. Normally they can be merged to a single term, which is found from the repeatability check carried out under proving. Note that the term relates to the uncertainty and not the repeatability itself.

The third term on the right-hand side of Eqs. (5-16) and (5-17) accounts for the effect that the proving is not carried out at the same flow rate as used in the flow calibration. It is only relevant when the proving device is a master meter. This is dealt with in the same way as in (Frøysa, et al., 2014) (Uncertainty of the correction factor estimate). The uncertainty contribution is described in Appendix A. It is calculated from the deviation between the master meter flow rate and the flow rate measured by the reference meter at flow calibration, at a series of flow rates. The adjustment of the master meter is assumed to be carried out by linear interpolation. The actual expression for any uncorrected percentage deviation, δp, and the related uncertainty of the master meter after adjustment of the master meter is given in Appendix A.

As described in Appendix A, the relative standard uncertainty of the correction factor estimate can be written as

$$\left(\frac{u\left(q_{v_0,linearity}^{prov}\right)}{q_{v_0}^{prov}}\right) = \left(\frac{u(K)}{K}\right) = \frac{1}{K}\frac{\partial p}{\partial K}u(p) = \frac{\delta p/\sqrt{3}}{100+p} \tag{5-18}$$

The relative standard uncertainty of the correction factor estimate related to a Coriolis meter can be written analogously to that of the turbine and ultrasonic meters, ref. Eq. (5-18), replacing the standard volume flow q_{v_0} by mass flow q_m, as given in the following equation:

$$\left(\frac{u\left(q_{m,linearity}^{prov}\right)}{q_{m}^{prov}}\right) = \left(\frac{u(K)}{K}\right) = \frac{1}{K}\frac{\partial p}{\partial K}u(p) = \frac{\delta p/\sqrt{3}}{100+p} \tag{5-19}$$

The last term on the right-hand side of Eq. (5-16) and the fourth term in Eq. (5-17) accounts for the effect on the master meter by changes in flow profile from the flow calibration to the proving. Typically, this effect is larger when the master meter has been calibrated at an off-site calibration facility than when the master meter has been calibrated on-site. The size of this term depends on the care taken for having upstream pipe work at proving as close as possible to the pipe work at flow calibration. Flow meter specifications and type tests can give indications of the size of this term in case of an off-site calibration. It is, however, difficult to give specific and general numbers for this term. In the case of on-site calibration, the term usually is expected to be smaller, because the master meter is not physically moved between flow calibration and proving.

The two last terms in Eq. (5-17) are related to the influence of pressure and medium temperature, respectively, if the calibration pressure differs from the process pressure during proving, or if the zero-point adjustment temperature at calibration differs from the process temperature during proving.

5.6. Metering uncertainties

The metering uncertainty can be understood as the extra uncertainty contributions related to a duty flow meter when used in normal operation, compared to the uncertainty of the same meter just after proving, and with the same flow rate as during proving.

The metering uncertainty for turbine and ultrasonic meters refers to the second term on the right hand of Eqs. (5-1), (5-5), (5-9), and (5-10) and can be written as follows:

$$\left(\frac{u(q_{v_0}^{met})}{q_{v_0}^{met}}\right)^2 = \left(\frac{u\left(q_{v_0,rept-flowmeter}^{met}\right)}{q_{v_0}^{met}}\right)^2 + \left(\frac{u\left(q_{v_0,linearity}^{met}\right)}{q_{v_0}^{met}}\right)^2 + \left(\frac{u\left(q_{v_0,profile}^{met}\right)}{q_{v_0}^{met}}\right)^2 \tag{5-20}$$

Similarly for the mass flow rate measurement, referring to the fourth terms in Eqs. (5-2), (5-3), (5-6), (5-7), (5-11), and (5-12), and the third terms in (5-4), (5-8), and (5-13), the metering uncertainty is written as follows:

$$\left(\frac{u(q_{m}^{met})}{q_{m}^{met}}\right)^2 = \left(\frac{u\left(q_{m,rept-flowmeter}^{met}\right)}{q_{m}^{met}}\right)^2 + \left(\frac{u\left(q_{m,linearity}^{met}\right)}{q_{m}^{met}}\right)^2 + \left(\frac{u\left(q_{m,profile}^{met}\right)}{q_{m}^{met}}\right)^2 + \left(\frac{u\left(q_{m,pressure}^{met}\right)}{q_{m}^{met}}\right)^2 + \left(\frac{u\left(q_{m,temperature}^{met}\right)}{q_{m}^{met}}\right)^2 \tag{5-21}$$

The first term on the right-hand side in Eqs. (5-20) and (5-21) corresponds to the repeatability of the flow meter. This is usually not measured directly (as in the flow calibration and proving operations) but can be based on vendor specifications, proving repeatability tests, or found in other ways. Note that the term relates to the uncertainty and not the repeatability itself.

For a Coriolis flowmeter, the repeatability is assumed to be specified as follows, where zero-point stability in units of kg/h is input for the uncertainty estimation:

$$\frac{u\left(q_{m,rept-flowmeter}^{met}\right)}{q_{m}^{met}} = x\% + \left(\frac{1}{2}\frac{zero\ point\ stability \cdot 100\%}{q_{m}^{met}}\right)\%o.r. \tag{5-22}$$

The second term on the right-hand side in Eqs. (5-20) and (5-21) accounts for the effect that the flow meter is not measuring on the same flow rate as the flow rate where proving was carried out. It is calculated based on specification of the total linearity of the meter over the calibrated range of volumetric flow rate at standard conditions. This linearity, given in percents, is denoted *L*. It is assumed that as a maximum there is a linear drift of *L* % over the calibrated flow rate range of the flow meter. This means that when the flow meter is proved at a volumetric flow rate at standard conditions $q_{v_0}^{prov}$, and used at a volumetric flow rate at standard conditions $q_{v_0}^{met}$, the linearity uncertainty contribution can be written as

$$\left(\frac{u\left(q_{v_0,linearity}^{met}\right)}{q_{v_0}^{met}}\right) = \frac{L}{\sqrt{3}}\frac{|q_{v_0}^{met} - q_{v_0}^{prov}|}{q_{v_{0,max}}^{cal} - q_{v_{0,min}}^{cal}} \tag{5-23}$$

It is here assumed a rectangular probability function for the uncertainty, which means that the relative standard uncertainty is found by dividing the maximum drift by the square root of 3. The metering uncertainty related to linearity for a Coriolis meter can be written analogously to that of the turbine and ultrasonic meters, replacing the standard volume flow q_{v_0} by mass flow q_m,as follows:

$$\left(\frac{u\left(q_{m,linearity}^{met}\right)}{q_m^{met}}\right) = \frac{L}{\sqrt{3}} \frac{|q_m^{met} - q_m^{prov}|}{q_{m_{max}}^{cal} - q_{m_{min}}^{cal}} \tag{5-24}$$

The last term on the right-hand side in Eq. (5-20) and the third term in Eq. (5-21) accounts for the effect on the duty meter by changes in flow profile from the proving to normal operation. Typically, this term is expected to be small, because the duty meter is not physically moved between flow calibration and proving. In addition, a new proving is carried out at each ship loading. In case of continuous operation, a new proving is carried out with some days' time interval.

The two last terms of Eq. (5-21) are related to the influence of pressure and medium temperature, respectively, if the proving pressure differs from the process pressure during metering, or if the zero-point adjustment temperature during proving differs from the process temperature during metering.

5.7. Oil and steel expansion factor uncertainties

In this Section, the uncertainty models for the combined oil and steel expansion factors the first term on the right hand of Eqs. (5-1), (5-5), and (5-9) are addressed. Note that this uncertainty term is different for the each of the four situations covered in Eqs. (5-1), (5-5), and (5-9).

In Section 5.7.1 the model for the uncertainty of the expansion factor found in Eq. (5-1), and related to the volumetric flow rate at standard conditions, is presented.

In Section 5.7.2 the model for the uncertainty of the expansion factor found in Eq. (5-5), and related to the volumetric flow rate at line conditions, is presented.

In Section 5.7.3 the models for the uncertainty of the expansion factors found in Eqs. (5-9) and (5-10) and related to the mass flow rate, are presented.

Section 5.7.4 presents the models for the uncertainty found in Eq. (5-4) related to the volumetric flow rate at standard conditions from measured mass flow rate.

Section 5.7.5 presents the models for the uncertainty found in Eq. (5-8) related to the volumetric flow rate at line conditions.

5.7.1. Volumetric flow rate at standard conditions – from measured volume flow

The uncertainty model for the volumetric flow rate at standard conditions is given in Eq. (5-1). The relative standard uncertainty of $A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$ is part of that equation. This relative standard uncertainty will now be discussed. $A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$ can from Eqs. (4-5) and (4-6) be found as

$$A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} = \left(\frac{C_{tlp}^{prov} C_{plp}^{prov} C_{tlm}^{met} C_{plm}^{met} C_{tsp}^{prov} C_{psp}^{prov} C_{tsm}^{met} C_{psm}^{met}}{C_{tlm}^{prov} C_{plm}^{prov} C_{tsp}^{cal} C_{psp}^{cal} C_{tsm}^{prov} C_{psm}^{prov}}\right) \tag{5-25}$$

When the expressions for all expansion coefficients (see Sections 4.2 and 4.3) are inserted, Eq. (5-25) can formally be written as

$$A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} = f\left(T_p^{cal}, P_p^{cal}, T_p^{prov}, P_p^{prov}, T_m^{prov}, P_m^{prov}, T_m^{met}, P_m^{met}, \rho_0\right) \tag{5-26}$$

Each of these input parameters have uncertainty. In addition, there are material constants for the oil and the steel. These also have uncertainty. In this work they are considered as part of the model uncertainty for each type of expansion factors that are included in $A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$. That means that uncertainty in temperature expansion coefficient and pressure expansion coefficient for oil, and the same for steel are assumed to be part of the model uncertainty.

It should be noted here that the temperature T_p^{cal} (the temperature at the proving device during calibration) and the temperature T_p^{prov} (the temperature at the proving device during proving) are measured by the same temperature measurement device. Similarly, pressure P_p^{cal} (the pressure at the proving device during calibration) and the pressure P_p^{prov} (the pressure at the proving device during proving) are measured by the same pressure measurement device. However, there can be several months or more between the measurements during calibration and the measurements during proving. The temperature measurement device and the pressure measurement device can drift in-between these measurements, and there may also be re-calibrations of the equipment. Therefore, in the uncertainty model, T_p^{cal} and P_p^{cal} will be assumed to be uncorrelated with T_p^{prov} and P_p^{prov}.

In the same way, the temperature T_m^{prov} (the temperature at the flow meter during proving) and the temperature T_m^{met} (the temperature at the flow meter during ordinary flow metering) are measured by the same temperature measurement device. Similarly, pressure P_m^{prov} (the pressure at the flow meter during proving) and the pressure P_m^{met} (the pressure at the flow meter during ordinary flow metering) are measured by the same pressure measurement device. Opposite to the case for the temperature and pressure measurements at the proving the device, the temperature and pressure measurements at the flow meter (during proving and at ordinary flow metering) are carried out within few days or less. Therefore, in the uncertainty model, T_m^{prov} is assumed to be totally correlated with T_m^{met}, and P_m^{prov} is assumed to be totally correlated with P_m^{met}.

The uncertainty model for $A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$ can then be written as

$$
\begin{aligned}
&\left(\frac{u(A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c})}{A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}}\right)^2 \\
&\quad = \left(\frac{1}{A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}}\right)^2 \left\{\left(\frac{\partial f}{\partial T_p^{cal}} u\left(T_p^{cal}\right)\right)^2\right. \\
&\quad + \left(\frac{\partial f}{\partial P_p^{cal}} u\left(P_p^{cal}\right)\right)^2 + \left(\frac{\partial f}{\partial T_p^{prov}} u\left(T_p^{prov}\right)\right)^2 \\
&\quad + \left(\frac{\partial f}{\partial P_p^{prov}} u\left(P_p^{prov}\right)\right)^2 \\
&\quad + \left(\frac{\partial f}{\partial T_m^{prov}} u\left(T_m^{prov}\right) + \frac{\partial f}{\partial T_m^{met}} u(T_m^{met})\right)^2 \\
&\quad + \left(\frac{\partial f}{\partial P_m^{prov}} u\left(P_m^{prov}\right) + \frac{\partial f}{\partial P_m^{met}} u(P_m^{met})\right)^2 \\
&\quad \left. + \left(\frac{\partial f}{\partial \rho_0} u(\rho_0)\right)^2\right\} + \left(\frac{u(A_{liq,\mathrm{mod}}^{m,\Delta p})}{A_{liq}^{m,\Delta p}}\right)^2 + \left(\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel,\mathrm{mod}}^{m,\Delta p,c}}\right)^2
\end{aligned} \tag{5-27}
$$

where the two last terms represent model uncertainties. It will further be assumed that when a temperature or pressure device is used two times (in both calibration and proving operation or both in proving and normal flow metering operation), the absolute standard uncertainty of this temperature or pressure device is the same in the two cases. This is consistent with the specifications of most temperature and pressure devices and in agreement with the uncertainty models for temperature and pressure given in Sections 3.1 and 3.2. This means that the notation will be simplified as follows:

$$
\begin{aligned}
u\left(T_p\right) &= u\left(T_p^{cal}\right) = u\left(T_p^{prov}\right), \\
u\left(P_p\right) &= u\left(P_p^{cal}\right) = u\left(P_p^{prov}\right), \\
u(T_m) &= u\left(T_m^{prov}\right) = u(T_m^{met}), \\
u(P_m) &= u\left(P_m^{prov}\right) = u(P_m^{met})
\end{aligned} \tag{5-28}
$$

Eq. (5-27) now simplifies to

$$\left(\frac{u(A_{liq}^{m,\Delta p}A_{steel}^{m,\Delta p,c})}{A_{liq}^{m,\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2 = \left(\frac{1}{A_{liq}^{m,\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2 \left\{\left(\left(\frac{\partial f}{\partial T_p^{cal}}\right)^2 + \left(\frac{\partial f}{\partial T_p^{prov}}\right)^2\right)u(T_p)^2 + \left(\left(\frac{\partial f}{\partial P_p^{cal}}\right)^2 + \left(\frac{\partial f}{\partial P_p^{prov}}\right)^2\right)u(P_p)^2 + \left(\frac{\partial f}{\partial T_m^{prov}} + \frac{\partial f}{\partial T_m^{met}}\right)^2 u(T_m)^2 + \left(\frac{\partial f}{\partial P_m^{prov}} + \frac{\partial f}{\partial P_m^{met}}\right)^2 u(P_m)^2 + \left(\frac{\partial f}{\partial \rho_0}u(\rho_0)\right)^2\right\} + \left(\frac{u(A_{liq,\mathrm{mod}}^{m,\Delta p})}{A_{liq}^{m,\Delta p}}\right)^2 + \left(\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}\right)^2 \tag{5-29}$$

The model uncertainty term for oil expansion factors, $\frac{u(A_{liq,\mathrm{mod}}^{m,\Delta p})}{A_{liq}^{m,\Delta p}}$, is addressed in Section 5.8.3. The model uncertainty term for steel expansion factors, $\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}$, is addressed in Section 5.8.7.

5.7.2. Measured volumetric flow rate at line conditions

The uncertainty model for the volumetric flow rate at line conditions is given in Eq. (5-5). The relative standard uncertainty of $A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}$ is part of that equation. This relative standard uncertainty will now be discussed. $A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}$ can from Eqs. (4-6) and (4-16) be found as

$$A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c} = \left(\frac{C_{tlp}^{prov}C_{plp}^{prov}C_{tsp}^{prov}C_{psp}^{prov}C_{tsm}^{met}C_{psm}^{met}}{C_{tlm}^{prov}C_{plm}^{prov}C_{tsp}^{cal}C_{psp}^{cal}C_{tsm}^{prov}C_{psm}^{prov}}\right) \tag{5-30}$$

When the expressions for all expansion coefficients (see Sections 4.2 and 4.3) are inserted, Eq. (5-30) can formally be written as

$$A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c} = f_2\left(T_p^{cal}, P_p^{cal}, T_p^{prov}, P_p^{prov}, T_m^{prov}, P_m^{prov}, T_m^{met}, P_m^{met}, \rho_0\right) \tag{5-31}$$

The methodology and assumptions for finding the uncertainty $A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}$ is similar to the methodology and assumptions for finding the uncertainty $A_{liq}^{m\Delta p}A_{steel}^{m,\Delta p,c}$ in Section 5.7.1. The relative standard uncertainty of $A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}$ then becomes

$$\left(\frac{u(A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c})}{A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2$$

$$= \left(\frac{1}{A_{liq}^{\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2 \left\{\left(\left(\frac{\partial f_2}{\partial T_p^{cal}}\right)^2 + \left(\frac{\partial f_2}{\partial T_p^{prov}}\right)^2\right)u(T_p)^2 + \left(\left(\frac{\partial f_2}{\partial P_p^{cal}}\right)^2 + \left(\frac{\partial f_2}{\partial P_p^{prov}}\right)^2\right)u(P_p)^2 + \left(\frac{\partial f_2}{\partial T_m^{prov}} + \frac{\partial f_2}{\partial T_m^{met}}\right)^2 u(T_m)^2 + \left(\frac{\partial f_2}{\partial P_m^{prov}} + \frac{\partial f_2}{\partial P_m^{met}}\right)^2 u(P_m)^2 + \left(\frac{\partial f_2}{\partial \rho_0}u(\rho_0)\right)^2\right\} + \left(\frac{u(A_{liq,\mathrm{mod}}^{\Delta p})}{A_{liq}^{m,\Delta p}}\right)^2 + \left(\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}\right)^2 \quad (5\text{-}32)$$

The model uncertainty term for oil expansion factors, $\frac{u(A_{liq,\mathrm{mod}}^{\Delta p})}{A_{liq}^{m,\Delta p}}$, is addressed in Section 5.8.4. The model uncertainty term for steel expansion factors, $\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}$, is addressed in Section 5.8.7.

5.7.3. Mass flow rate – from measured volume flow rate

The functional relationship and the uncertainty model for the mass flow rate depends on whether the density is determined from laboratory analysis or measured by an online densitometer. These two cases must therefore be addressed individually.

DENSITY FROM LABORATORY ANALYSIS

The uncertainty model for the mass flow rate when the density is determined by laboratory analysis is given in Eq. (5-9). The relative standard uncertainty of $\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$ is part of that equation. This relative standard uncertainty will now be discussed. $\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$ can from Eqs. (4-5) and (4-6) be found as

$$\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c} = \left(\frac{C_{tlp}^{prov} C_{plp}^{prov} C_{tlm}^{met} C_{plm}^{met} C_{tsp}^{prov} C_{psp}^{prov} C_{tsm}^{met} C_{psm}^{met}}{C_{tlm}^{prov} C_{plm}^{prov} C_{tsp}^{cal} C_{psp}^{cal} C_{tsm}^{prov} C_{psm}^{prov}}\right) \quad (5\text{-}33)$$

When the expressions for all expansion coefficients (see Sections 4.2 and 4.3) are inserted, Eq. (5-33) can formally be written as

$$\rho_0 A_{liq}^{m\Delta p} A_{steel}^{m,\Delta p,c} = f_3\left(T_p^{cal}, P_p^{cal}, T_p^{prov}, P_p^{prov}, T_m^{prov}, P_m^{prov}, T_m^{met}, P_m^{met}, \rho_0\right) \tag{5-34}$$

Each of these input parameters have uncertainty. In addition, there are material constants for the oil and the steel. These also have uncertainty. In this work they are considered as part of the model uncertainty for each type of expansion factors that are included in $A_{liq}^{m\Delta p} A_{steel}^{m,\Delta p,c}$. That means temperature expansion coefficient and pressure expansion coefficient for oil, and the same for steel.

The methodology and assumptions for finding the uncertainty $\rho_0 A_{liq}^{m\Delta p} A_{steel}^{m,\Delta p,c}$ is similar to the methodology and assumptions for finding the uncertainty $A_{liq}^{m\Delta p} A_{steel}^{m,\Delta p,c}$ in Section 5.7.1. The relative standard uncertainty of $\rho_0 A_{liq}^{m\Delta p} A_{steel}^{m,\Delta p,c}$ then becomes

$$\begin{aligned}
\left(\frac{u(\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c})}{\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}}\right)^2
&= \left(\frac{1}{\rho_0 A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}}\right)^2 \Bigg\{\left(\left(\frac{\partial f_3}{\partial T_p^{cal}}\right)^2 + \left(\frac{\partial f_3}{\partial T_p^{prov}}\right)^2\right) u\left(T_p\right)^2 \\
&+ \left(\left(\frac{\partial f_3}{\partial P_p^{cal}}\right)^2 + \left(\frac{\partial f_3}{\partial P_p^{prov}}\right)^2\right) u\left(P_p\right)^2 \\
&+ \left(\frac{\partial f_3}{\partial T_m^{prov}} + \frac{\partial f_3}{\partial T_m^{met}}\right)^2 u(T_m)^2 \\
&+ \left(\frac{\partial f_3}{\partial P_m^{prov}} + \frac{\partial f_3}{\partial P_m^{met}}\right)^2 u(P_m)^2 + \left(\frac{\partial f_3}{\partial \rho_0} u(\rho_0)\right)^2 \Bigg\} \\
&+ \left(\frac{u(A_{liq,\mathrm{mod}}^{m,\Delta p})}{A_{liq}^{m,\Delta p}}\right)^2 + \left(\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}\right)^2
\end{aligned} \tag{5-35}$$

The model uncertainty term for oil expansion factors, $\frac{u(A_{liq,\mathrm{mod}}^{m,\Delta p})}{A_{liq}^{m,\Delta p}}$, is addressed in Section 5.8.3. The model uncertainty term for steel expansion factors, $\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}$, is addressed in Section 5.8.7.

DENSITY FROM ONLINE DENSITOMETER

The uncertainty model for the mass flow rate when the density is determined by densitometer is given. The relative standard uncertainty of $\rho_{dens} A_{liq}^{\Delta m,\Delta p} A_{steel}^{m,\Delta p,c}$ is part of that equation. This relative standard uncertainty will now be discussed. $\rho_{dens} A_{liq}^{\Delta m,\Delta p} A_{steel}^{m,\Delta p,c}$ from Eqs. (4-6) and (4-24) can be found as

$$\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c} = \left(\rho_{dens}\frac{C_{tlp}^{prov}C_{plp}^{prov}C_{tlm}^{met}C_{plm}^{met}C_{tsp,prover}^{prov}C_{psp,prover}^{prov}C_{tsm,flowmeter}^{met}C_{psm,flowmeter}^{met}}{C_{tlm}^{prov}C_{plm}^{prov}C_{tld}^{met}C_{pld}^{met}C_{tsp,prover}^{cal}C_{psp,prover}^{cal}C_{tsm,flowmeter}^{prov}C_{psm,flowmeter}^{prov}}\right) \tag{5-36}$$

When the expressions for all expansion coefficients (see Sections 4.2 and 4.3) are inserted, Eq. (5-36) can formally be written as

$$\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c} = f_4\left(T_p^{cal},P_p^{cal},T_p^{prov},P_p^{prov},T_m^{prov},P_m^{prov},T_m^{met},P_m^{met},T_d^{met},P_d^{met},\rho_0,\rho_{dens}\right) \tag{5-37}$$

It is here assumed that the standard density, ρ_0, is calculated from the measured density at the densitometer, and used in the volume correction coefficients. Each of these input parameters have uncertainty. In addition, there are material constants for the oil and the steel. These also have uncertainty. In this work they are considered as part of the model uncertainty for each type of expansion factors that are included in $A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c}$. That means temperature expansion coefficient and pressure expansion coefficient for oil, and the same for steel.

The methodology and assumptions for finding the uncertainty $\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c}$ is similar to the methodology and assumptions for finding the uncertainty $A_{liq}^{m,\Delta p}A_{steel}^{m,\Delta p,c}$ in Section 5.7.1. The relative standard uncertainty of $\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c}$ then becomes

$$\begin{aligned}
\left(\frac{u(\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c})}{\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2
&= \left(\frac{1}{\rho_{dens}A_{liq}^{\Delta m,\Delta p}A_{steel}^{m,\Delta p,c}}\right)^2 \Bigg\{\left(\left(\frac{\partial f_4}{\partial T_p^{cal}}\right)^2 + \left(\frac{\partial f_4}{\partial T_p^{prov}}\right)^2\right)u(T_p)^2 \\
&+ \left(\left(\frac{\partial f_4}{\partial P_p^{cal}}\right)^2 + \left(\frac{\partial f_4}{\partial P_p^{prov}}\right)^2\right)u(P_p)^2 \\
&+ \left(\frac{\partial f_4}{\partial T_m^{prov}} + \frac{\partial f_4}{\partial T_m^{met}}\right)^2 u(T_m)^2 \\
&+ \left(\frac{\partial f_4}{\partial P_m^{prov}} + \frac{\partial f_4}{\partial P_m^{met}}\right)^2 u(P_m)^2 + \left(\frac{\partial f_4}{\partial T_d^{met}}\right)^2 u(T_d)^2 \\
&+ \left(\frac{\partial f_4}{\partial P_d^{met}}\right)^2 u(P_d)^2 + \left(\frac{\partial f_4}{\partial \rho_{dens}}u(\rho_{dens})\right)^2\Bigg\} \\
&+ \left(\frac{u(A_{liq,\mathrm{mod}}^{\Delta m,\Delta p})}{A_{liq}^{\Delta m,\Delta p}}\right)^2 + \left(\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}\right)^2
\end{aligned} \tag{5-38}$$

It should be noted here that a small approximation has been carried out. It is assumed that the uncertainty of the relative density used in the volume correction coefficients is uncorrelated with the density ρ_{dens} that is explicitly written in Eq. (5-36). As the sensitivity for the relative density in the volume correction coefficients is quite small, this is a reasonable approximation.

The model uncertainty term for oil expansion factors, $\frac{u(A_{liq,\mathrm{mod}}^{\Delta m,\Delta p})}{A_{liq}^{\Delta m,\Delta p}}$, is addressed in Section 5.8.2. The model uncertainty term for steel expansion factors, $\frac{u(A_{steel,\mathrm{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}$, is addressed in Section 5.8.7.

5.7.4. Volume flow rate at standard conditions – from measured mass flow rate

The functional relationship and the uncertainty model for the volume flow rate depend on whether the density is determined from laboratory analysis or measured by an online densitometer. These two cases must therefore be addressed individually.

DENSITY FROM LABORATORY ANALYSIS

The uncertainty model for the volume flow rate at standard conditions from measured mass flow rate, when the density is determined by laboratory analysis, is given in Eq. (5-4). The relative standard uncertainty of $\rho_0$is part of that equation. When the primary measurement is mass flow, and there is no water in the oil, there will be no influence of expansion coefficients on the uncertainty model. In the case of water in oil, the effect of the expansion coefficients is described in Eq. (5-3) in Section 5.8.6.

DENSITY FROM ONLINE DENSITOMETER

The uncertainty model for the volume flow rate at standard conditions from measured mass flow rate, when the density is determined by densitometer is given in Eq. (5-2). The relative standard uncertainty of $\frac{C_{tld}^{met}C_{pld}^{met}}{\rho_{dens}}$ is part of that equation. When the expressions for all expansion coefficients (see Sections 4.2.2 and 4.2.3) are inserted, this ratio can formally be written as

$$\frac{C_{tld}^{met}C_{pld}^{met}}{\rho_{dens}} = f_5(T_d^{met}, P_d^{met}, \rho_0, \rho_{dens}) \qquad (5\text{-}39)$$

It is here assumed that the standard density, ρ_0, is calculated from the measured density at the densitometer, and used in the volume correction coefficients. Each of these input parameters have uncertainty. In addition, there are material constants for the oil. These also have uncertainty. In this work they are considered as part of the model uncertainty for each type of expansion factors that are included in $C_{tld}^{met} C_{pld}^{met}$. That means temperature expansion coefficient and pressure expansion coefficient for oil.

The methodology and assumptions for finding the uncertainty $\frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}}$, is similar to the methodology and assumptions for finding the uncertainty of $A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$, in Section 5.7.1. The relative standard uncertainty of $\frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}}$, then becomes:

$$\left(\frac{u\left(\frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}} \right)}{\frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}}} \right)^2 = \left(\frac{1}{\frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}}} \right)^2 \left\{ \left(\frac{\partial f_5}{\partial T_d^{met}} \right)^2 u(T_d)^2 + \left(\frac{\partial f_5}{\partial P_d^{met}} \right)^2 u(P_d)^2 + \left(\frac{\partial f_5}{\partial \rho_0} u(\rho_0) \right)^2 + \left(\frac{\partial f_5}{\partial \rho_{dens}} u(\rho_{dens}) \right)^2 \right\} + \left(\frac{u(C_{tld,\mathrm{mod}}^{met} C_{pld,\mathrm{mod}}^{met})}{C_{tld,\mathrm{mod}}^{met} C_{pld,\mathrm{mod}}^{met}} \right)^2 \quad (5\text{-}40)$$

It should be noted here that a small approximation has been carried out. It is assumed that the uncertainty of the relative density used in the volume correction coefficients is uncorrelated with the density ρ_{dens}, of the ratio $\frac{C_{tld}^{met} C_{pld}^{met}}{\rho_{dens}}$. As the sensitivity for the relative density in the volume correction coefficients is quite small, this is a reasonable approximation.

The model uncertainty term for oil expansion factors, $\frac{u(C_{tld,\mathrm{mod}}^{met} C_{pld,\mathrm{mod}}^{met})}{C_{tld,\mathrm{mod}}^{met} C_{pld,\mathrm{mod}}^{met}}$, is addressed in Section 5.8.6.

5.7.5. Volume flow rate at flow meter conditions – from measured mass flow rate

The functional relationship and the uncertainty model for the volume flow rate depend on whether the density is determined from laboratory analysis or measured by an online densitometer. These two cases must therefore be addressed individually.

DENSITY FROM LABORATORY ANALYSIS

The uncertainty model for the volume flow rate at flow meter conditions from measured mass flow rate, when the density is determined by laboratory analysis, is given in Eq. (5-8). The relative standard uncertainty of $C_{tlm}^{met} C_{plm}^{met} \rho_0$ is part of that equation. When the expressions for the expansion coefficients (see Sections 4.2.2 and 4.2.3) are inserted, this ratio can formally be written as

$$\frac{1}{C_{tlm}^{met} C_{plm}^{met} \rho_0} = f_6(T_m^{met}, P_m^{met}, \rho_0) \tag{5-41}$$

The standard density, ρ_0, is derived from the laboratory analysis, and are assumed used in the volume correction coefficients. Each of these input parameters have uncertainty. In addition, there are material constants for the oil. These also have uncertainty. In this work, they are considered as part of the model uncertainty for each of the expansion factors. The methodology and assumptions for finding the uncertainty of $\frac{1}{C_{tlm}^{met} C_{plm}^{met} \rho_0}$ is similar to the methodology and assumptions for finding the uncertainty of $A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p,c}$, in Section 5.7.1. The relative standard uncertainty of $\frac{1}{C_{tlm}^{met} C_{plm}^{met} \rho_0}$ then becomes:

$$\begin{aligned}\left(\frac{u\left(\frac{1}{C_{tlm}^{met} C_{plm}^{met} \rho_0}\right)}{\frac{1}{C_{tlm}^{met} C_{plm}^{met} \rho_0}}\right)^2 &= \left(\frac{1}{\frac{1}{C_{tlm}^{met} C_{plm}^{met} \rho_0}}\right)^2 \left\{\left(\frac{\partial f_6}{\partial T_m^{met}}\right)^2 u(T_m)^2 + \left(\frac{\partial f_6}{\partial P_m^{met}}\right)^2 u(P_m)^2 \right. \\ &+ \left(\frac{\partial f_6}{\partial T_d^{met}}\right)^2 u(T_d)^2 + \left(\frac{\partial f_6}{\partial P_d^{met}}\right)^2 u(P_d)^2 \\ &\left. + \left(\frac{\partial f_6}{\partial \rho_0}\right)^2 \left(u(\rho_0)\right)^2 + \left(\frac{\partial f_6}{\partial \rho_{dens}}\right)^2 \left(u(\rho_{dens})\right)^2 \right\} \\ &+ \left(\frac{u(C_{tlm,\mathrm{mod}}^{met} C_{plm,\mathrm{mod}}^{met})}{C_{tlm,\mathrm{mod}}^{met} C_{plm,\mathrm{mod}}^{met}}\right)^2\end{aligned} \tag{5-42}$$

The model uncertainty term for oil expansion factors $\frac{u(C_{tlm,\mathrm{mod}}^{met} C_{plm,\mathrm{mod}}^{met})}{C_{tlm,\mathrm{mod}}^{met} C_{plm,\mathrm{mod}}^{met}}$ is addressed in Section 5.8.6.

DENSITY FROM ONLINE DENSITOMETER

The uncertainty model for the volume flow rate at line conditions from measured mass flow rate, when the density is determined by densitometer, is given in Eqs. (5-6) and (5-11). The relative standard uncertainty of $A_{liq}^{\Delta m} \rho_{dens}$ is part of that equation. When the expressions for all expansion coefficients (see Sections 4.2.2 and 4.2.3) are inserted, this ratio can formally be written as

$$A_{liq}^{\Delta m} \rho_{dens} = \rho_{dens} \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} = f_7(T_m^{met}, P_m^{met}, T_d^{met}, P_d^{met}, \rho_0, \rho_{dens}) \tag{5-43}$$

It is here assumed that the standard density, ρ_0 , is calculated from the measured density at the densitometer, and used in the volume correction coefficients. Each of these input parameters have uncertainty. In addition, there are material constants for the oil. These also have uncertainty. In this work they are considered as part of the model uncertainty for each type of expansion factors that are included in $A_{liq}^{\Delta m}$. That means temperature expansion coefficient and pressure expansion coefficient for oil.

The methodology and assumptions for finding the uncertainty of $A_{liq}^{\Delta m}\rho_{dens}$ is similar to the methodology and assumptions for finding the uncertainty of $A_{liq}^{m,\Delta p}A_{steel}^{m,\Delta p,c}$, in Section 5.7.1. The relative standard uncertainty of $A_{liq}^{\Delta m}\rho_{dens}$ then becomes:

$$\left(\frac{u\left(A_{liq}^{\Delta m}\rho_{dens}\right)}{A_{liq}^{\Delta m}\rho_{dens}}\right)^2 = \left(\frac{1}{A_{liq}^{\Delta m}\rho_{dens}}\right)^2 \left\{\left(\frac{\partial f_7}{\partial T_m^{met}}\right)^2 u(T_m)^2 + \left(\frac{\partial f_7}{\partial P_m^{met}}\right)^2 u(P_m)^2 + \left(\frac{\partial f_7}{\partial T_d^{met}}\right)^2 u(T_d)^2 + \left(\frac{\partial f_7}{\partial P_d^{met}}\right)^2 u(P_d)^2 + \left(\frac{\partial f_7}{\partial \rho_0}\right)^2 \left(u(\rho_0)\right)^2 + \left(\frac{\partial f_7}{\partial \rho_{dens}}\right)^2 \left(u(\rho_{dens})\right)^2\right\} + \left(\frac{u(A_{liq,\mathrm{mod}}^{\Delta m})}{A_{liq,\mathrm{mod}}^{\Delta m}}\right)^2 \quad (5\text{-}44)$$

It should be noted here that a small approximation has been carried out. It is assumed that the uncertainty of the relative density used in the volume correction coefficients is uncorrelated with the density ρ_{dens} that is explicitly written in Eq. (5-43). As the sensitivity for the relative density in the volume correction coefficients is quite small, this is a reasonable approximation.

The model uncertainty term for oil expansion factors, $\frac{u(A_{liq,\mathrm{mod}}^{\Delta m})}{A_{liq,\mathrm{mod}}^{\Delta m}}$, is addressed in Section 0.

5.8. Expansion factor model uncertainties

In this Section the model uncertainty of the different combined expansion factors is addressed. First in Section 5.8.1, some general assumptions and approach for the oil expansion factors is presented. Then in Sections 5.8.2, 5.8.3, 5.8.4, 5.8.5, and 5.8.6 the model uncertainties for respectively the oil expansion factors $A_{liq}^{\Delta m,\Delta p}$, $A_{liq}^{m,\Delta p}$, $A_{liq}^{\Delta p}$, $A_{liq}^{\Delta m}$, and $C_{tld}^{met}C_{pld}^{met}$ and $C_{tlm}^{met}C_{plm}^{met}$ are derived. In Section 5.8.7, the model uncertainty for the steel expansion factor $A_{steel}^{m,\Delta p,c}$ is derived.

5.8.1. Oil expansion factor – introduction

The model uncertainty term for oil expansion factors, $\frac{u\left(A_{liq}^{\Delta m,\Delta p}\right)}{A_{liq}^{\Delta m,\Delta p}}$, $\frac{u(A_{liq}^{m,\Delta p})}{A_{liq}^{m,\Delta p}}$, $\frac{u\left(A_{liq}^{\Delta p}\right)}{A_{liq}^{\Delta p}}$, $\frac{u(A_{liq}^{\Delta m})}{A_{liq}^{\Delta m}}$, and $\frac{u(C_{tld}^{met}C_{pld}^{met})}{C_{tld}^{met}C_{pld}^{met}}$ and $\frac{u(C_{tlm}^{met}C_{plm}^{met})}{C_{tlm}^{met}C_{plm}^{met}}$ need some more focus. They are related to the model uncertainties of the temperature and pressure expansion factors C_{tlx} and C_{plx}. These factors calculate the volume change from a given temperature and pressure to standard temperature and pressure. The model uncertainties are given in (API MPMS 11.1, 2004). When calculations between other temperatures and pressures are carried out by combining several expansion factors, the model uncertainty for the total calculation will be different. For example, if a volume correction is carried out from 50 °C to 51 °C, it is expected that the model uncertainty is much less than for a volume correction from 50 °C to 15 °C. In Sections 5.8.2, 5.8.3, 5.8.4, 5.8.5, and 5.8.6 models for model uncertainty that account for this will be derived, for $A_{liq}^{\Delta m,\Delta p}$, $A_{liq}^{m,\Delta p}$, $A_{liq}^{\Delta p}$, $A_{liq}^{\Delta m}$, and $C_{tld}^{met}C_{pld}^{met}$ and $C_{tlm}^{met}C_{plm}^{met}$. For the derivations, it is assumed that the temperature correction factor depends on a parameter B_T which represents the model uncertainty. This is an artificial constant but can be thought of as a combination of the oil quality input parameters (K_0, K_1, and K_2). The model uncertainty of a temperature expansion coefficient can then be written as

$$u(C_{tlx})_{mod} = \frac{\partial C_{tlx}}{\partial B_T} u(B_T) \tag{5-45}$$

Similarly, it is assumed that the pressure correction factor depends on a parameter. The model uncertainty of a pressure expansion coefficient can then be written as

$$u\left(C_{plx}\right)_{mod} = \frac{\partial C_{plx}}{\partial B_P} u(B_P) \tag{5-46}$$

This will be the basis for the derivations in the next three Sections.

It should also be mentioned that in (API MPMS 11.1, 2004), the model uncertainty of the volume correction coefficients (C_{tlx} and C_{plx}) are specified as a fixed percentage (fixed relative uncertainty) over wide ranges of pressure and temperature. This is implicitly used below in the derivation of the uncertainty models.

An alternative, and maybe better approach would have been a model for the model uncertainty similar to the approach for steel expansion factors, see Section 5.8.7. However, in such a case, the model uncertainty of the oil volume correction factors would have to be defined as a linear function of the difference between line and standard temperature and between line and standard pressure. Because this is not the case in (API MPMS 11.1, 2004), such an approach has not been selected here.

5.8.2. Oil expansion factor $A_{liq}^{\Delta m \Delta p}$

The oil expansion factor $A_{liq}^{\Delta m,\Delta p}$ is given in Eq. (4-24) and repeated here for convenience:

$$A_{liq}^{\Delta m \Delta p} = \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} \frac{C_{tlp}^{prov} C_{plp}^{prov}}{C_{tlm}^{prov} C_{plm}^{prov}} \tag{5-47}$$

The model uncertainty $u\left(A_{liq}^{\Delta m,\Delta p}\right)$ of $A_{liq}^{\Delta m,\Delta p}$ is of relevance for the mass flow rate, when the density is measured by a densitometer, see Eq. (5-38). It can be found as

$$\left(u\left(A_{liq}^{\Delta m,\Delta p}\right)_{mod}\right)^2 = \left(\frac{\partial A_{liq}^{\Delta m\Delta p}}{\partial B_T}u(B_T)\right)^2 + \left(\frac{\partial A_{liq}^{\Delta m\Delta p}}{\partial B_P}u(B_P)\right)^2 + \left(\frac{C_{plp}^{prov}C_{plm}^{met}}{C_{plm}^{prov}C_{pld}^{met}} \cdot \frac{\frac{\partial}{\partial B_T}(C_{tlp}^{prov})C_{tlm}^{met} + C_{tlp}^{prov}\frac{\partial}{\partial B_T}(C_{tlm}^{met})C_{tlm}^{prov}C_{tld}^{met} - C_{tlp}^{prov}C_{tlm}^{met}\frac{\partial}{\partial B_T}(C_{tlm}^{prov})C_{tld}^{met} + C_{tlm}^{prov}\frac{\partial}{\partial B_T}(C_{tld}^{met})}{\left(C_{tlm}^{prov}C_{tld}^{met}\right)^2}u(B_T)\right)^2 + \left(\frac{C_{tlp}^{prov}C_{tlm}^{met}}{C_{tlm}^{prov}C_{tld}^{met}} \cdot \frac{\frac{\partial}{\partial B_P}(C_{plp}^{prov})C_{plm}^{met} + C_{plp}^{prov}\frac{\partial}{\partial B_P}(C_{plm}^{met})C_{plm}^{prov}C_{pld}^{met} - C_{plp}^{prov}C_{plm}^{met}\frac{\partial}{\partial B_P}(C_{plm}^{prov})C_{pld}^{met} + C_{plm}^{prov}\frac{\partial}{\partial B_P}(C_{pld}^{met})}{\left(C_{plm}^{prov}C_{pld}^{met}\right)^2}u(B_P)\right)^2 \tag{5-48}$$

It is now assumed that all the derivatives related to temperature are equal and that all derivatives related to pressure are equal. This is similar to assuming that the temperatures ($T_p^{prov}, T_m^{prov}, T_m^{met}, T_d^{met}$) in question here are not too far away from each other, and similarly that the pressures ($P_p^{prov}, P_m^{prov}, P_m^{met}, P_d^{met}$) in question here are not too far away from each other. This means that the notation will be simplified as follows:

$$\frac{\partial}{\partial B_T}C_{tl} = \frac{\partial}{\partial B_T}C_{tlp}^{prov} = \frac{\partial}{\partial B_T}C_{tlm}^{prov} = \frac{\partial}{\partial B_T}C_{tlm}^{met} = \frac{\partial}{\partial B_T}C_{tld}^{met},$$
$$\frac{\partial}{\partial B_P}C_{pl} = \frac{\partial}{\partial B_P}C_{plp}^{prov} = \frac{\partial}{\partial B_P}C_{plm}^{prov} = \frac{\partial}{\partial B_P}C_{plm}^{met} = \frac{\partial}{\partial B_P}C_{pld}^{met} \tag{5-49}$$

Eq. (5-48) then simplifies to

$$\begin{aligned}\left(u\left(A_{liq}^{\Delta m,\Delta p}\right)_{mod}\right)^2 &= \left(\frac{C_{plp}^{prov}C_{plm}^{met}}{C_{plm}^{prov}C_{pld}^{met}} \cdot \frac{\left(C_{tlm}^{met} + C_{tlp}^{prov}\right)C_{tlm}^{prov}C_{tld}^{met} - C_{tlp}^{prov}C_{tlm}^{met}\left(C_{tlm}^{prov} + C_{tld}^{met}\right)}{\left(C_{tlm}^{prov}C_{tld}^{met}\right)^2}\frac{\partial}{\partial B_T}(C_{tl})u(B_T)\right)^2 \\ &+ \left(\frac{C_{tlp}^{prov}C_{tlm}^{met}}{C_{tlm}^{prov}C_{tld}^{met}} \cdot \frac{\left(C_{plm}^{met} + C_{plp}^{prov}\right)C_{plm}^{prov}C_{pld}^{met} - C_{plp}^{prov}C_{plm}^{met}\left(C_{plm}^{prov} + C_{pld}^{met}\right)}{\left(C_{plm}^{prov}C_{pld}^{met}\right)^2}\frac{\partial}{\partial B_P}(C_{pl})u(B_P)\right)^2 \\ &+ \left(\frac{C_{plp}^{prov}C_{plm}^{met}}{C_{plm}^{prov}} \cdot \frac{\left(C_{tlm}^{met} + C_{tlp}^{prov}\right)C_{tlm}^{prov}C_{tld}^{met} - C_{tlp}^{prov}C_{tlm}^{met}\left(C_{tlm}^{prov} + C_{tld}^{met}\right)}{\left(C_{tlm}^{prov}C_{tld}^{met}\right)^2}u(C_{tl})_{mod}\right)^2 \\ &+ \left(\frac{C_{tlp}^{prov}C_{tlm}^{met}}{C_{tlm}^{prov}} \cdot \frac{\left(C_{plm}^{met} + C_{plp}^{prov}\right)C_{plm}^{prov}C_{pld}^{met} - C_{plp}^{prov}C_{plm}^{met}\left(C_{plm}^{prov} + C_{pld}^{met}\right)}{\left(C_{plm}^{prov}C_{pld}^{met}\right)^2}u\left(C_{pl}\right)_{mod}\right)^2\end{aligned} \tag{5-50}$$

Expressed with relative standard uncertainties, Eq. (5-50) becomes

$$\left(\frac{u(A_{liq}^{\Delta m,\Delta p})_{mod}}{A_{liq}^{\Delta m,\Delta p}}\right)^2 = \left(1+\frac{C_{tlm}^{met}}{C_{tlp}^{prov}}-\frac{C_{tlm}^{met}}{C_{tlm}^{prov}}-\frac{C_{tlm}^{met}}{C_{tld}^{met}}\right)^2\left(\frac{u(C_{tl})_{mod}}{C_{tlm}^{met}}\right)^2 + \left(1+\frac{C_{plm}^{met}}{C_{plp}^{prov}}-\frac{C_{plm}^{met}}{C_{plm}^{prov}}-\frac{C_{plm}^{met}}{C_{pld}^{met}}\right)^2\left(\frac{u(C_{pl})_{mod}}{C_{plm}^{met}}\right)^2 \quad (5\text{-}51)$$

5.8.3. Oil expansion factor $A_{liq}^{m\Delta p}$

The oil expansion factor $A_{liq}^{m,\Delta p}$ is given in Eq. (4-5) and repeated here for convenience:

$$A_{liq}^{m,\Delta p} = \left(\frac{C_{tlp}^{prov}C_{plp}^{prov}C_{tlm}^{met}C_{plm}^{met}}{C_{tlm}^{prov}C_{plm}^{prov}}\right) \quad (5\text{-}52)$$

The model uncertainty $u(A_{liq}^{m,\Delta p})$ of $A_{liq}^{m,\Delta p}$ is of relevance for volumetric flow rate at standard conditions, see Eq. (5-29), and for mass flow rate, when the reference density is measured in a laboratory, see Eq. (5-35). It can now be found:

$$\begin{aligned}\left(u(A_{liq}^{m,\Delta p})_{mod}\right)^2 &= \left(\frac{\partial A_{liq}^{m\Delta p}}{\partial B_T}u(B_T)\right)^2 + \left(\frac{\partial A_{liq}^{m\Delta p}}{\partial B_P}u(B_P)\right)^2 \\ &+ \left(\frac{C_{plp}^{prov}C_{plm}^{met}}{C_{plm}^{prov}} \cdot \frac{\frac{\partial}{\partial B_T}(C_{tlp}^{prov})C_{tlm}^{met} + C_{tlp}^{prov}\frac{\partial}{\partial B_T}(C_{tlm}^{met})C_{tlm}^{prov} - C_{tlp}^{prov}C_{tlm}^{met}\frac{\partial}{\partial B_T}(C_{tlm}^{prov})}{(C_{tlm}^{prov})^2}u(B_T)\right)^2 \\ &+ \left(\frac{C_{tlp}^{prov}C_{tlm}^{met}}{C_{tlm}^{prov}} \cdot \frac{\frac{\partial}{\partial B_P}(C_{plp}^{prov})C_{plm}^{met} + C_{plp}^{prov}\frac{\partial}{\partial B_P}(C_{plm}^{met})C_{plm}^{prov} - C_{plp}^{prov}C_{plm}^{met}\frac{\partial}{\partial B_P}(C_{plm}^{prov})}{(C_{plm}^{prov})^2}u(B_P)\right)^2\end{aligned} \quad (5\text{-}53)$$

As in Section 5.8.2, it is assumed that all the derivatives related to temperature are equal and that all derivatives related to pressure are equal. Eq. (5-53) then simplifies to

$$
\begin{aligned}
\left(u\left(A_{liq}^{m,\Delta p}\right)_{mod}\right)^2 = & \left(\frac{C_{plp}^{prov} C_{plm}^{met}}{C_{plm}^{prov}} \cdot \frac{\left(C_{tlm}^{met} + C_{tlp}^{prov}\right) C_{tlm}^{prov} - C_{tlp}^{prov} C_{tlm}^{met}}{\left(C_{tlm}^{prov}\right)^2} \frac{\partial}{\partial B_T}(C_{tl}) u(B_T)\right)^2 \\
& + \left(\frac{C_{tlp}^{prov} C_{tlm}^{met}}{C_{tlm}^{prov}} \cdot \frac{\left(C_{plm}^{met} + C_{plp}^{prov}\right) C_{plm}^{prov} - C_{plp}^{prov} C_{plm}^{met}}{\left(C_{plm}^{prov}\right)^2} \frac{\partial}{\partial B_P}(C_{pl}) u(B_P)\right)^2 \\
& + \left(\frac{C_{plp}^{prov} C_{plm}^{met}}{C_{plm}^{prov}} \cdot \frac{\left(C_{tlm}^{met} + C_{tlp}^{prov}\right) C_{tlm}^{prov} - C_{tlp}^{prov} C_{tlm}^{met}}{\left(C_{tlm}^{prov}\right)^2} u(C_{tl})_{mod}\right)^2 \\
& + \left(\frac{C_{tlp}^{prov} C_{tlm}^{met}}{C_{tlm}^{prov}} \cdot \frac{\left(C_{plm}^{met} + C_{plp}^{prov}\right) C_{plm}^{prov} - C_{plp}^{prov} C_{plm}^{met}}{\left(C_{plm}^{prov}\right)^2} u\left(C_{pl}\right)_{mod}\right)^2
\end{aligned}
\tag{5-54}
$$

Expressed with relative standard uncertainties, Eq. (5-54) becomes

$$
\begin{aligned}
\left(\frac{u\left(A_{liq}^{m,\Delta p}\right)_{mod}}{A_{liq}^{m,\Delta p}}\right)^2 = & \left(1 + \frac{C_{tlm}^{met}}{C_{tlp}^{prov}} - \frac{C_{tlm}^{met}}{C_{tlm}^{prov}}\right)^2 \left(\frac{u(C_{tl})_{mod}}{C_{tlm}^{met}}\right)^2 \\
& + \left(1 + \frac{C_{plm}^{met}}{C_{plp}^{prov}} - \frac{C_{plm}^{met}}{C_{plm}^{prov}}\right)^2 \left(\frac{u\left(C_{pl}\right)_{mod}}{C_{plm}^{met}}\right)^2
\end{aligned}
\tag{5-55}
$$

5.8.4. Oil expansion factor $A_{liq}^{\Delta p}$

The oil expansion factor $A_{liq}^{\Delta p}$ is given in Eq. (4-16) and repeated here for convenience:

$$
A_{liq}^{\Delta p} = \frac{C_{tlp}^{prov} C_{plp}^{prov}}{C_{tlm}^{prov} C_{plm}^{prov}}
\tag{5-56}
$$

The model uncertainty $u(A_{liq}^{\Delta p})$ of $A_{liq}^{\Delta p}$ is of relevance for volumetric flow rate at line conditions, see Eq. (5-32). It can now be found as

$$
\begin{aligned}
\left(u\left(A_{liq}^{\Delta p}\right)_{mod}\right)^2 = & \left(\frac{\partial A_{liq}^{\Delta p}}{\partial B_T} u(B_T)\right)^2 + \left(\frac{\partial A_{liq}^{\Delta p}}{\partial B_P} u(B_P)\right)^2 \\
& + \left(\frac{C_{plp}^{prov}}{C_{plm}^{prov}} \cdot \frac{\frac{\partial}{\partial B_T}\left(C_{tlp}^{prov}\right) C_{tlm}^{prov} - C_{tlp}^{prov} \frac{\partial}{\partial B_T}\left(C_{tlm}^{prov}\right)}{\left(C_{tlm}^{prov}\right)^2} u(B_T)\right)^2 \\
& + \left(\frac{C_{tlp}^{prov}}{C_{tlm}^{prov}} \cdot \frac{\frac{\partial}{\partial B_P}\left(C_{plp}^{prov}\right) C_{plm}^{prov} - C_{plp}^{prov} \frac{\partial}{\partial B_P}\left(C_{plm}^{prov}\right)}{\left(C_{plm}^{prov}\right)^2} u(B_P)\right)^2
\end{aligned}
\tag{5-57}
$$

As in Sections 5.8.2 and 5.8.3, it is assumed that all the derivatives related to temperature are equal and that all derivatives related to pressure are equal. Eq. (5-57) then simplifies to

$$\left(u\left(A_{liq}^{\Delta p}\right)_{mod}\right)^2 = \left(\frac{C_{plp}^{prov}}{C_{plm}^{prov}} \cdot \frac{C_{tlm}^{prov} - C_{tlp}^{prov}}{\left(C_{tlm}^{prov}\right)^2} \frac{\partial}{\partial B_T}(C_{tl}) u(B_T)\right)^2 + \left(\frac{C_{tlp}^{prov}}{C_{tlm}^{prov}} \cdot \frac{C_{plm}^{prov} - C_{plp}^{prov}}{\left(C_{plm}^{prov}\right)^2} \frac{\partial}{\partial B_P}(C_{pl}) u(B_P)\right)^2 + \left(\frac{C_{plp}^{prov}}{C_{plm}^{prov}} \cdot \frac{C_{tlm}^{prov} - C_{tlp}^{prov}}{\left(C_{tlm}^{prov}\right)^2} u(C_{tl})_{mod}\right)^2 + \left(\frac{C_{tlp}^{prov}}{C_{tlm}^{prov}} \cdot \frac{C_{plm}^{prov} - C_{plp}^{prov}}{\left(C_{plm}^{prov}\right)^2} u\left(C_{pl}\right)_{mod}\right)^2 \quad (5\text{-}58)$$

Expressed with relative standard uncertainties, Eq. (5-58) becomes:

$$\left(\frac{u\left(A_{liq}^{\Delta p}\right)_{mod}}{A_{liq}^{m,\Delta p}}\right)^2 = \left(1 - \frac{C_{tlm}^{met}}{C_{tlm}^{prov}}\right)^2 \left(\frac{u(C_{tl})_{mod}}{C_{tlm}^{met}}\right)^2 + \left(1 - \frac{C_{plm}^{met}}{C_{plp}^{prov}}\right)^2 \left(\frac{u\left(C_{pl}\right)_{mod}}{C_{plm}^{met}}\right)^2 \quad (5\text{-}59)$$

5.8.5. Oil expansion factor $A_{liq}^{\Delta m}$

The oil expansion factor $A_{liq}^{\Delta m}$ is given in Eq. (4-18) and repeated here for convenience:

$$A_{liq}^{\Delta m} = \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} \quad (5\text{-}60)$$

The model uncertainty $u(A_{liq}^{\Delta m})$ of $A_{liq}^{\Delta m}$ can now be found as

$$\begin{aligned} \left(u\left(A_{liq}^{\Delta m}\right)_{mod}\right)^2 &= \left(\frac{\partial A_{liq}^{\Delta m}}{\partial B_T} u(B_T)\right)^2 + \left(\frac{\partial A_{liq}^{\Delta m}}{\partial B_P} u(B_P)\right)^2 \\ &= \left(\frac{\frac{C_{plm}^{met}}{C_{pld}^{met}}\left(\frac{\partial}{\partial B_T}\left(C_{tlm}^{met}\right) C_{tld}^{met} - \frac{\partial}{\partial B_T}\left(C_{tld}^{met}\right) C_{tlm}^{met}\right)}{\left(C_{tld}^{met}\right)^2}\right)^2 \\ &+ \left(\frac{\frac{C_{plm}^{met}}{C_{pld}^{met}}\left(\frac{\partial}{\partial B_P}\left(C_{tlm}^{met}\right) C_{tld}^{met} - \frac{\partial}{\partial B_P}\left(C_{tld}^{met}\right) C_{tlm}^{met}\right)}{\left(C_{tld}^{met}\right)^2}\right)^2 \end{aligned} \quad (5\text{-}61)$$

As in Sections 5.8.2, 5.8.3, and 5.8.4 it is assumed that all the derivatives related to temperature are equal and that all derivatives related to pressure are equal. Eq. (5-61) then simplifies to

$$\left(u\left(A_{liq}^{\Delta m}\right)_{mod}\right)^2 = \left(\frac{C_{plm}^{met}}{C_{tld}^{met}}\frac{C_{tld}^{met}-C_{tlm}^{met}}{(C_{tld})^2}\frac{\partial}{\partial B_T}(C_{tl})u(B_T)\right)^2 + \left(\frac{C_{tlm}^{met}}{C_{tld}^{met}}\frac{C_{pld}^{met}-C_{plm}^{met}}{\left(C_{pld}\right)^2}\frac{\partial}{\partial B_P}\left(C_{pl}\right)u(B_P)\right)^2 = \left(\frac{C_{plm}^{met}}{C_{tld}^{met}}\frac{C_{tld}^{met}-C_{tlm}^{met}}{(C_{tld})^2}u(C_{tl})_{mod}\right)^2 + \left(\frac{C_{tlm}^{met}}{C_{tld}^{met}}\frac{C_{pld}^{met}-C_{plm}^{met}}{\left(C_{pld}\right)^2}u\left(C_{pl}\right)_{mod}\right)^2 \quad (5\text{-}62)$$

Expressed with relative standard uncertainties, Eq. (5-62) becomes

$$\left(\frac{u\left(A_{liq}^{\Delta m}\right)_{mod}}{A_{liq}^{\Delta m}}\right)^2 = \left(1-\frac{C_{tlm}^{met}}{C_{tld}^{met}}\right)^2\left(\frac{u(C_{tl})_{mod}}{C_{tlm}^{met}}\right)^2 + \left(1-\frac{C_{plm}^{met}}{C_{pld}^{met}}\right)^2\left(\frac{u\left(C_{pl}\right)_{mod}}{C_{plm}^{met}}\right)^2 \quad (5\text{-}63)$$

5.8.6. Oil expansion factors $C_{tld}^{met}C_{pld}^{met}$ and $C_{tlm}^{met}C_{plm}^{met}$

The model uncertainty $u(C_{tlx,\mathrm{mod}}^{met}C_{plx,\mathrm{mod}}^{met})$ of the oil expansion factor $C_{tlx,\mathrm{mod}}^{met}C_{plx,\mathrm{mod}}^{met}$ is of relevance for volumetric flow rate from measured mass flow rate, at standard conditions, see Eq. (5-39) (x = d) and at line conditions see Eq. (5-41) (x = m). It can be found as:

$$\left(u(C_{tlx,\mathrm{mod}}^{met}C_{plx,\mathrm{mod}}^{met})\right)^2 = \left(\frac{\partial(C_{tlx}^{met}C_{plx}^{met})}{\partial B_T}u(B_T)\right)^2 + \left(\frac{\partial(C_{tlx}^{met}C_{plx}^{met})}{\partial B_P}u(B_P)\right)^2 + \left(C_{plx}^{met}\frac{\partial}{\partial B_T}(C_{tlx}^{met})u(B_T)\right)^2 + \left(C_{tlx}^{met}\frac{\partial}{\partial B_P}\left(C_{plx}^{met}\right)u(B_P)\right)^2 \quad (5\text{-}64)$$

As in Sections 5.8.2, 5.8.3, and 5.8.4 it is assumed that all the derivatives related to temperature are equal and that all derivatives related to pressure are equal. Eq. (5-64) then simplifies to

$$\left(u(C_{tlx,\mathrm{mod}}^{met}C_{plx,\mathrm{mod}}^{met})\right)^2 = \left(C_{plx}^{met}\frac{\partial}{\partial B_T}(C_{tl})u(B_T)\right)^2 + \left(C_{tlx}^{met}\frac{\partial}{\partial B_P}\left(C_{pl}\right)u(B_P)\right)^2 + \left(C_{plx}^{met}u(C_{tl})_{mod}\right)^2 + \left(C_{tlx}^{met}u\left(C_{pl}\right)_{mod}\right)^2 \quad (5\text{-}65)$$

Expressed with relative standard uncertainties Eq. (5-65) becomes:

$$\left(\frac{u(C_{tlx,\mathrm{mod}}^{met}C_{plx,\mathrm{mod}}^{met})}{C_{tlx,\mathrm{mod}}^{met}C_{plx,\mathrm{mod}}^{met}}\right)^2 = \left(\frac{C_{plx}C_{tlx}}{C_{tlx}C_{plx}}\right)^2\left(\frac{u(C_{tl})_{mod}}{C_{tlx}^{met}}\right)^2 + \left(\frac{C_{tlx}^{met}C_{plx}^{met}}{C_{tlx}C_{plx}}\right)^2\left(\frac{u\left(C_{pl}\right)_{mod}}{C_{plx}^{met}}\right)^2 = \left(\frac{u(C_{tl})_{mod}}{C_{tlx}^{met}}\right)^2 + \left(\frac{u\left(C_{pl}\right)_{mod}}{C_{plx}^{met}}\right)^2 \tag{5-66}$$

5.8.7. Steel expansion factor $A_{steel}^{m\Delta p,c}$

The steel expansion factor $A_{steel}^{m\Delta p,c}$ is given in Eq. (4-6) and repeated here for convenience:

$$A_{steel}^{m,\Delta p,c} = \left(\frac{C_{tsp}^{prov}C_{psp}^{prov}C_{tsm}^{met}C_{psm}^{met}}{C_{tsp}^{cal}C_{psp}^{cal}C_{tsm}^{prov}C_{psm}^{prov}}\right) \tag{5-67}$$

The model uncertainty $u(A_{steel}^{m\Delta p,c})$ of $A_{steel}^{m\Delta p,c}$ is of relevance for all flow rates covered in this Handbook, See Eqs. (5-29), (5-32), (5-35), and (5-38).

As discussed in Sections 4.3.1 and 4.3.2, the temperature and pressure volume correction factors for steel can be written as

$$C_{tsx} = 1 + 3\alpha(T_x - T_b) \tag{5-68}$$

and

$$C_{psx} = 1 + \beta(P_x - P_b) \tag{5-69}$$

see Eqs. (4-35) and (4-39). The parameters α and β depends on the type of flow meter / prover and on the type of steel quality. The model uncertainty of $A_{steel}^{m\Delta p,c}$ will be calculated from the uncertainty in the α and β parameters.

The derivation will be divided into two different cases:

- Ultrasonic flow meter as master meter
- Turbine flow meter as master meter, or a displacement prover

First, the case where an ultrasonic flow meter is used as master meter, is addressed. In that case it is assumed that it is the same type of flow meter that is in use both for the duty meter and the master meter. Therefore, the α and β coefficients will be the same for the two meters. In this case $A_{steel}^{m\Delta p,c}$ can be written as

$$A_{steel}^{m\Delta p,c} = \frac{\left(1+3\alpha_m\left(T_p^{prov}-T_b\right)\right)\left(1+\beta_m\left(P_p^{prov}-P_b\right)\right)\left(1+3\alpha_m(T_m^{met}-T_b)\right)\left(1+\beta_m(P_m^{met}-P_b)\right)}{\left(1+3\alpha_m\left(T_p^{cal}-T_b\right)\right)\left(1+\beta_m\left(P_p^{cal}-P_b\right)\right)\left(1+3\alpha_m\left(T_m^{prov}-T_b\right)\right)\left(1+\beta_m\left(P_m^{prov}-P_b\right)\right)} \quad (5\text{-}70)$$

The model uncertainty of $A_{steel}^{m\Delta p,c}$ can be written as

$$\left(u(A_{steel,\text{mod}}^{m,\Delta p,c})\right)^2 = \left(\frac{\partial A_{steel}^{m,\Delta p,c}}{\partial \alpha_m} u(\alpha_m)\right)^2 + \left(\frac{\partial A_{steel}^{m,\Delta p,c}}{\partial \beta_m} u(\beta_m)\right)^2 \quad (5\text{-}71)$$

By inserting Eq. (5-70) into Eq. (5-71) and carrying out the differentiation, the relative standard model uncertainty can be written as

$$\begin{aligned}\left(\frac{u(A_{steel,\text{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}\right)^2 = &\left(\frac{3\alpha_m(T_p^{prov}-T_b)}{1+3\alpha_m(T_p^{prov}-T_b)} + \frac{3\alpha_m(T_m^{met}-T_b)}{1+3\alpha_m(T_p^{prov}-T_b)} - \frac{3\alpha_m\left(T_p^{cal}-T_b\right)}{1+3\alpha_m\left(T_p^{cal}-T_b\right)}\right. \\ &\left. - \frac{3\alpha_m(T_m^{prov}-T_b)}{1+3\alpha_m(T_m^{prov}-T_b)}\right)^2 \left(\frac{u(\alpha_m)}{\alpha_m}\right)^2 \\ &+ \left(\frac{\beta_m(P_p^{prov}-P_b)}{1+\beta_m(P_p^{prov}-P_b)} + \frac{\beta_m(P_m^{met}-P_b)}{1+\beta_m(P_p^{prov}-P_b)} - \frac{\beta_m\left(P_p^{cal}-P_b\right)}{1+\beta_m\left(P_p^{cal}-P_b\right)}\right. \\ &\left. - \frac{\beta_m(P_m^{prov}-P_b)}{1+\beta_m(P_m^{prov}-P_b)}\right)^2 \left(\frac{u(\beta_m)}{\beta_m}\right)^2\end{aligned} \quad (5\text{-}72)$$

Next, the case of where a turbine flow meter is used as master meter, or a displacement prover is used, is addressed. In that case there will be different α and β coefficients for the flow meter and the proving device, and they must be treated as four uncorrelated variables $\left(\alpha_m, \alpha_p, \beta_m, \beta_p\right)$ with respect to the uncertainty. In this case $A_{steel}^{m\Delta p,c}$ can be written as

$$A_{steel}^{m\Delta p,c} = \frac{\left(1+3\alpha_p\left(T_p^{prov}-T_b\right)\right)\left(1+\beta_p\left(P_p^{prov}-P_b\right)\right)\left(1+3\alpha_m(T_m^{met}-T_b)\right)\left(1+\beta_m(P_m^{met}-P_b)\right)}{\left(1+3\alpha_p\left(T_p^{cal}-T_b\right)\right)\left(1+\beta_p\left(P_p^{cal}-P_b\right)\right)\left(1+3\alpha_m\left(T_m^{prov}-T_b\right)\right)\left(1+\beta_m\left(P_m^{prov}-P_b\right)\right)} \quad (5\text{-}73)$$

The model uncertainty of $A_{steel}^{m\Delta p,c}$ can be written as

$$\left(u(A_{steel,\text{mod}}^{m,\Delta p,c})\right)^2 = \left(\frac{\partial A_{steel}^{m,\Delta p,c}}{\partial \alpha_p} u(\alpha_p)\right)^2 + \left(\frac{\partial A_{steel}^{m,\Delta p,c}}{\partial \beta_p} u\left(\beta_p\right)\right)^2 + \left(\frac{\partial A_{steel}^{m,\Delta p,c}}{\partial \alpha_m} u(\alpha_m)\right)^2 + \left(\frac{\partial A_{steel}^{m,\Delta p,c}}{\partial \beta_m} u(\beta_m)\right)^2 \quad (5\text{-}74)$$

By inserting Eq. (5-73) into Eq. (5-74) and carrying out the differentiation, the relative standard model uncertainty can be written as

$$\left(\frac{u(A_{steel,\text{mod}}^{m,\Delta p,c})}{A_{steel}^{m,\Delta p,c}}\right)^2 = \left(\frac{3\alpha_p\left(T_p^{prov} - T_p^{cal}\right)}{(1+3\alpha_p(T_p^{prov} - T_b))(1+3\alpha_p(T_p^{cal} - T_b))}\right)^2 \left(\frac{u(\alpha_p)}{\alpha_p}\right)^2 + \left(\frac{3\beta_p\left(P_p^{prov} - P_p^{cal}\right)}{(1+3\beta_p(P_p^{prov} - P_b))(1+3\beta_p(P_p^{cal} - P_b))}\right)^2 \left(\frac{u(\beta_p)}{\beta_p}\right)^2 + \left(\frac{3\alpha_m(T_m^{met} - T_m^{prov})}{(1+3\alpha_m(T_m^{met} - T_b))(1+3\alpha_m(T_m^{prov} - T_b))}\right)^2 \left(\frac{u(\alpha_m)}{\alpha_m}\right)^2 + \left(\frac{3\beta_m(T_m^{met} - T_m^{prov})}{(1+3\beta_m(T_m^{met} - T_b))(1+3\beta_m(T_m^{prov} - T_b))}\right)^2 \left(\frac{u(\beta_m)}{\beta_m}\right)^2 \quad (5\text{-}75)$$

5.9. Uncertainty in volumetric water fraction

The uncertainty model for the volumetric volume fraction can generally be written as follows:

$$\left(\frac{u(\phi_{v,line}^{water})}{\phi_{v,line}^{water}}\right)^2 = \left(\frac{u(C_\phi)}{C_\phi}\right)^2 + \left(\frac{B}{A\phi + B(1-\phi)}\right)^2 \left(\frac{u(\phi)}{\phi}\right)^2 \quad (5\text{-}76)$$

Where:

C_ϕ: The collective correction of the volumetric water fraction, consisting of the expansion coefficients presented in Eqs. (4-27), (4-30), (4-64), and (4-67). This uncertainty contribution is found numerically and is further discussed below.

If the volumetric water fraction is found by online water measurement:

ϕ: ϕ_{WIO}

A: $C_{tww} C_{pww} C_{tlm}^{met} C_{plm}^{met}$

B: $C_{twm} C_{pwm} C_{tlw} C_{plw}$

If the volumetric water fraction is found by sampling and laboratory analysis:

ϕ: ϕ_{LAB}

A: $C_{twl} C_{pwl} C_{tlm}^{met} C_{plm}^{met}$

B: $C_{twm} C_{pwm} C_{tll} C_{pll}$

Model uncertainties are negligible.

$$\left(\frac{u(C_\phi)}{C_\phi}\right)^2 = \left(\frac{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)} - \frac{A(T-T_\delta,P)\phi}{A(T-T_\delta,P)\phi + B(1-\phi)}}{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)}\sqrt{3}}\right)^2 + \left(\frac{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)} - \frac{A\phi}{A\phi + B(T-T_\delta,P)(1-\phi)}}{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)}\sqrt{3}}\right)^2 + \left(\frac{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)} - \frac{A(T,P-P_\delta)\phi}{A(T,P-P_\delta)\phi + B(1-\phi)}}{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)}\sqrt{3}}\right)^2 + \left(\frac{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)} - \frac{A\phi}{A\phi + B(T,P-P_\delta)(1-\phi)}}{\frac{A(T,P)\phi}{A(T,P)\phi + B(T,P)(1-\phi)}\sqrt{3}}\right)^2 \quad (5\text{-}77)$$

Where:

T_δ: the The difference in temperature T_δ at the flow meter and where the water content is measured (at online water cut meter or laboratory).

P_δ: The difference in pressure P_δ at the flow meter and where the water content is measured (at the online water cut meter or laboratory).

5.10. Numerical derivation

It is difficult to calculate the derivative of the functional relationships for standard volume flow with water in oil corrections, due to correlations between variables that are not factored out collectively (refer to Eq. (4-47)). Therefore, analytical derivatives are calculated for the case that the variables are not correlated, and a correction term to account for the correlations is added. This correlation term is found by numerical derivation of the functional relationship, thus finding the total variance, and subtracting the sum of variances of the individual contributions (arising from previous terms on the right-hand side of Eqs. (5-2) and (5-3)). Prior to the numerical derivation, Eq. (4-47) is re-written in terms of its independent variables, e.g., all calculated densities are written as functions of pressures, temperatures, and measured densities. The numerical derivation is covered in below.

In general, the functional relationship for a quantity, Q, can be written

$$Q = f(x_1, x_2, x_3, \dots, x_n) \quad (5\text{-}78)$$

where $x_1, x_2, x_3, \dots, x_n$ are the independent variables of the function f. The uncertainty model for Eq. (5-78) is written

$$u(Q)^2 = \left(\frac{\partial f}{\partial x_1}u(x_1)\right)^2 + \left(\frac{\partial f}{\partial x_2}u(x_2)\right)^2 + \left(\frac{\partial f}{\partial x_3}u(x_3)\right)^2 + \cdots + \left(\frac{\partial f}{\partial x_n}u(x_n)\right)^2 \quad (5\text{-}79)$$

The partial derivatives are numerically calculated as follows:

$$\frac{\partial f}{\partial x_i} \approx \frac{f\left(x_1, x_2, x_3, \dots, x_{i-1,} x_i + \delta x_i, x_{i+1,} \dots, x_n\right) - f(x_1, x_2, x_3, \dots, x_{i-1,} x_i - \delta x_i, x_{i+1,} \dots, x_n)}{2\delta x_i} \tag{5-80}$$

where $2\delta x_i$ is a small perturbation of the independent variable.

6. OilMetApp Web Application

This chapter documents the OilMetApp web application for carrying out uncertainty analyses based on the uncertainty models described in this Handbook. It should here be emphasized that the example input values in that calculation tool are just examples and should not be regarded as recommended values by NFOGM, NORCE, NOD or any other party.

6.1. Software platform

The new web application was developed in 2020 and is based on the open-source Blazor platform, developed by Microsoft (https://blazor.net). This platform only uses open web standards recommended by W3C (https://www.w3.org), including HTML5 and WebAssembly (https://webassembly.org).

When the user visits a web page all the files that constitutes the application will be downloaded and the application will run in the browser without need for any further communication with the web server. The application files are stored in the web browser cache and will only be downloaded again if there is a new version available.

The web application is supported on all web browsers that supports WebAssembly 1.0 or later, and this includes all major browsers on both Windows, Linux, Mac, and also mobile and other operating systems (https://caniuse.com/#feat=wasm). Note that Internet Explorer 11 does not support WebAssembly and cannot be used to run the new web application.

6.2. Installation and use

The web address for the application will be published on nfogm.no. By visiting the published address, all the files that constitutes the application will be downloaded and the application will run in the browser without need for any further communication with the web server. The application files are stored in the web browser cache and will only be downloaded again if there is a new version available.

6.3. Program overview

The “OilMetering” application uses input consisting of

- Metering station template (the general type of instruments and layout of these).
- Oil properties.
- Properties for the different equipment included in the template.
- Process conditions and measurement results from the calibration, proving and metering phases.

From this input the application then can

1. Compute and visualize the resulting uncertainty in flow measurement values.
2. Compute additional relevant properties of the oil and process conditions.
3. Generate a report in different formats and print the report.
4. Save work in a file for future use and reference.

The following sections describe the functionality in more detail and uses screenshots from the application to illustrate.

6.3.1. Specify metering station template

The start page of the application (Figure 6.1) is also the page where the user specifies the metering station template, meaning the general type of instruments and the layout of these. There are several aspects of the metering station that is modeled:

- **Flow Meter:** What type of meter is used and what configuration of sensors is used to measure the line temperature and line pressure. Ultrasonic, Turbine or Coriolis flow meters are available.
- **Stationary prover / master meter:** What type of stationary prover / master meter is used (Ultrasonic, Turbine, Displacement Prover or Coriolis) and what configuration of sensors is used to measure the temperature and pressure of the prover / master meter (single, dual or average).
- **Density measurement:** What type of density measurement is used, laboratory measurement or installed densitometer. If an installed densitometer is used, what configuration of sensors is used to measure the densitometer temperature and pressure (single, dual or average).
- **Water in oil measurement: W**hat type of water in oil measurement is used, laboratory, online or no metering.

By specifying choices for each of these aspects, the user is in effect selecting a metering station template. When the user then presses the “Accept and Continue”-button a copy of the selected template is created and the application moves to the first of several input pages, “Oil Properties” (Figure 6.2). A page navigation menu below the application header is also displayed, where the user now can move freely between different pages (Figure 6.3), some related to input and others related to computed results and visualizations. The pages typically organize content in several sections, and the user can select a section with some form of navigation control.

The selected metering station template is set up with some example values, so the user can explore the application functionality without first finishing all the data input.

The following pages are available after the metering station template has been selected:

- **Metering Station:** start page where the selected template is displayed. The user can also create a new or open an existing from a file.
- **Liquid:** input regarding oil product type, specifying water density and volumetric fraction, as well as other properties like base pressure and temperature.
- **Equipment:** input regarding properties and uncertainty in the metering station equipment, for example flow meter, master meter, and the temperature and pressure sensors used.

- **Calibration:** input regarding calibration conditions and uncertainty in the calibration procedure. This input page is not applicable when the selected stationary prover is of type "Displacement Prover".
- **Proving:** input regarding proving conditions and uncertainty in the proving procedure.
- **Metering:** input regarding metering conditions and uncertainty in the metering procedure.
- **Results**: computed uncertainty of the main flow measurement variables (absolute volume flow, standard volume flow and mass flow), volume correction factors and density measurement, all displayed as uncertainty budgets tables.
- **Charts:** computed uncertainty of the main flow measurement variables (absolute volume flow, standard volume flow and mass flow), volume correction factors and density measurement, all displayed as uncertainty budgets tables.
- **Report**: summary of the uncertainty analysis formatted as an on-screen report. This can be printed (and using typical system functionality exported to PDF format), and it is possible to save the analysis in a file for later use, sharing and reference.

The user can move between the input pages in any order, but due to computational dependencies the following work flow is recommended when input data: "Oil"->"Equipment"->"Calibration"->"Proving"->"Metering". In addition, the logical flow between sections in each page is typically from left to right.

The following discusses each of the pages.

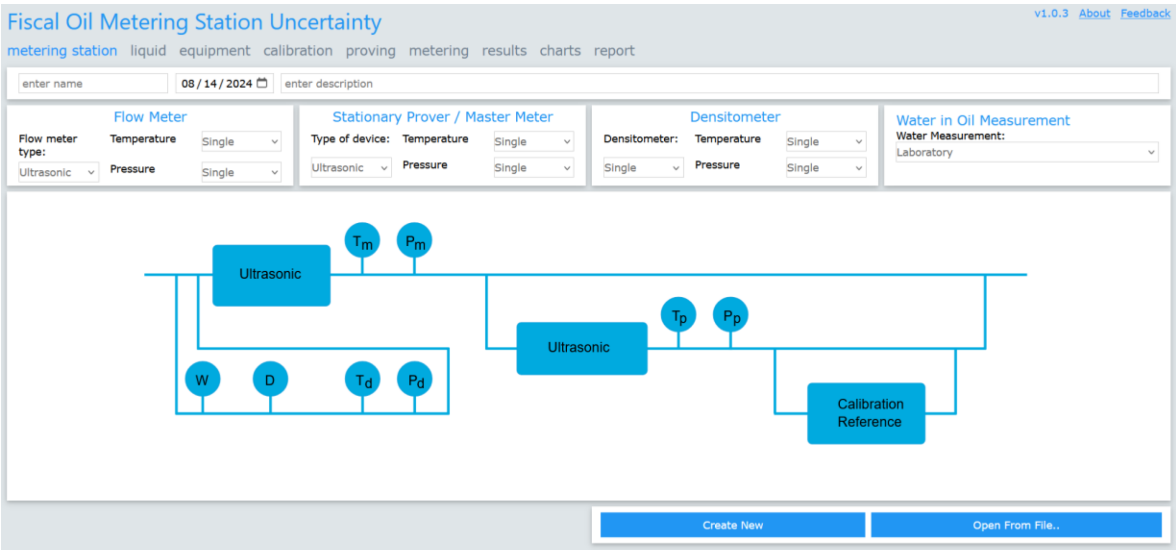


Figure 6.1. Oil metering application start page, where the user specifies the metering station template. It is also possible to open a previously saved file.

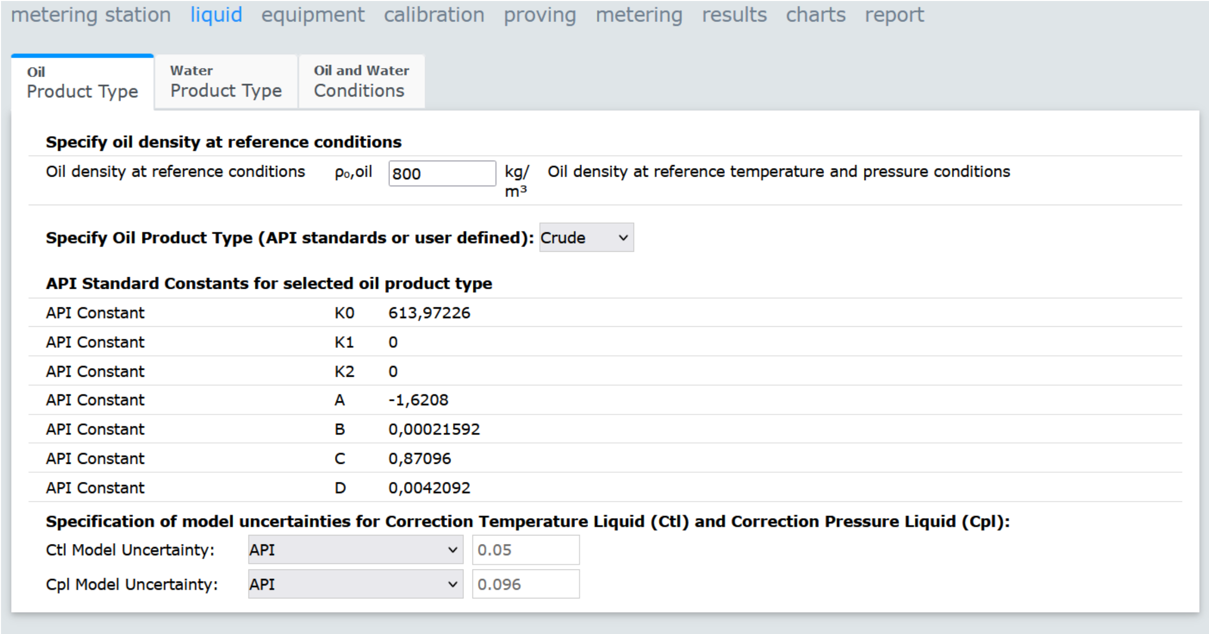


Figure 6.2. "Liquid"-page with "Oil Product Type"-section selected, where the user specifies the oil product type from a set of API standards, or model another product type by entering values for a set of API constants.

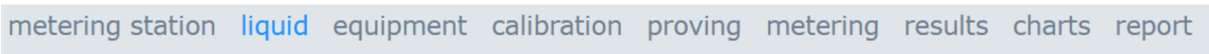


Figure 6.3. Page navigation menu where the user can move freely between different pages, some related to input and others related to computed results and visualizations.

6.3.2. Liquid page

There are three sections on the liquid page, one concerning the reference density and API model of the oil, one concerning the water density and volumetric fraction, and the last concerning the operating conditions:

- Oil Product Type: specification of oil density at reference conditions and API standard oil product type, defined by a set of API constants (Figure 6.2).
- Water Product Type: specification of water density at reference temperature and pressure conditions, as well as the liquid volumetric water fraction at reference conditions. (Figure 6.4).
- Oil and Water Conditions: specification of base pressure and temperature and equilibrium vapor pressure (Figure 6.5).

The available oil product types are as defined in API. An oil product not in the API standard can be specified by choosing the "Other"-checkbox. The table listing the API standard constants (Figure 6.2) will then be enabled (it is read-only otherwise) and relevant model parameters can be input and will be used in the calculation. The model uncertainties for "C_{tl}" and "C_{pl}" is also given as either API standards, or user defined. When specified as "API" the actual values (temperature and pressure dependent) are displayed for reference in corresponding read-only text fields. When specified as "User Defined" these text fields become enabled and the user can enter values directly.

Water density at reference conditions, as well as volumetric water fraction are specified in the "Water Product Type" section.

In the "Oil and Water Conditions" section, the "normal process conditions" is defined. These are the typical system pressure and temperature values, meaning that these values will be used as default values for different pressure and temperature values until specified otherwise. For example, when later specifying "Proving Conditions" (Figure 6.11), the "flow meter conditions" and "master meter conditions" will both be equal to the "normal process conditions". The user can then change this as necessary. Water sampling and measurement, however, is usually performed were typical system pressure and temperature values do not apply. Therefore, an offset to the typical system pressure and temperature values are provided for the water measurement point.

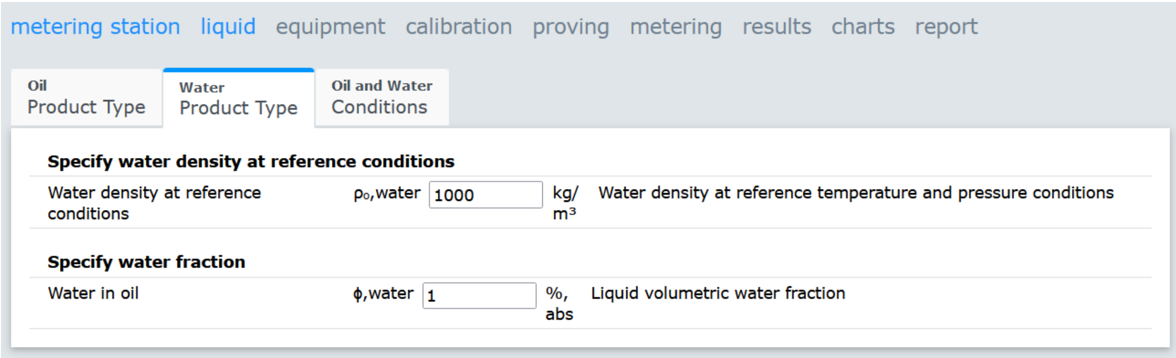


Figure 6.4. "Liquid"-page with "Water Product Type"-section selected, where the user specifies the water density at reference conditions, as well as the volumetric water fraction.

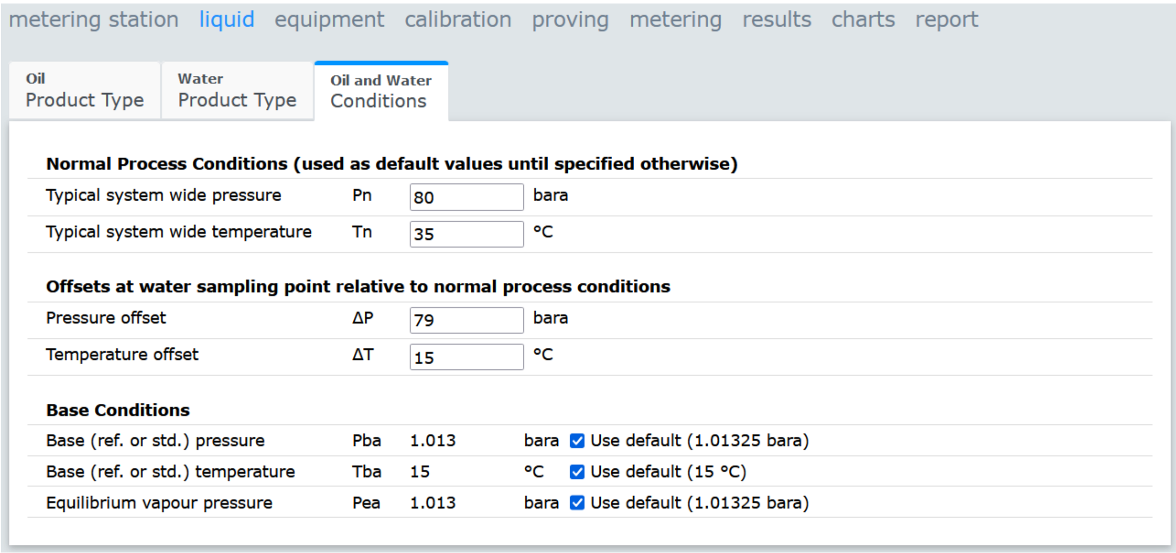


Figure 6.5. "Liquid"-page with "Oil and Water Condtions"-section selected, where the user specifies the oil operating conditions.

6.3.3. Equipment page

The “Equipment”-page contains input regarding properties and uncertainty in the metering station equipment (Figure 6.6). The content of this page depends on the selected metering station template, but it can include

- Flow meter with temperature and pressure sensors.
- Stationary prover/master meter with temperature and pressure sensors.
- Densitometer with temperature and pressure sensors.
- Online water cut meter.

Some of the equipment uncertainty models can be specified either as an overall measurement uncertainty or as a detailed model of a typical sensor. This choice is controlled by checkbox as shown in (Figure 6.7) and (Figure 6.8) for a temperature sensor. The online water cut meter (Figure 6.9) is included only if online water measurement was selected while specifying the metering station template.

Note that in some views there is functionality for storing frequently used specifications in files for later retrieval (as shown in Figure 6.8 where the detailed input for a temperature transmitter). The “Save” -button can be used to save the complete specification to a file, and the “Load”-button can then later be used for quickly loading the saved specification for a temperature transmitter.

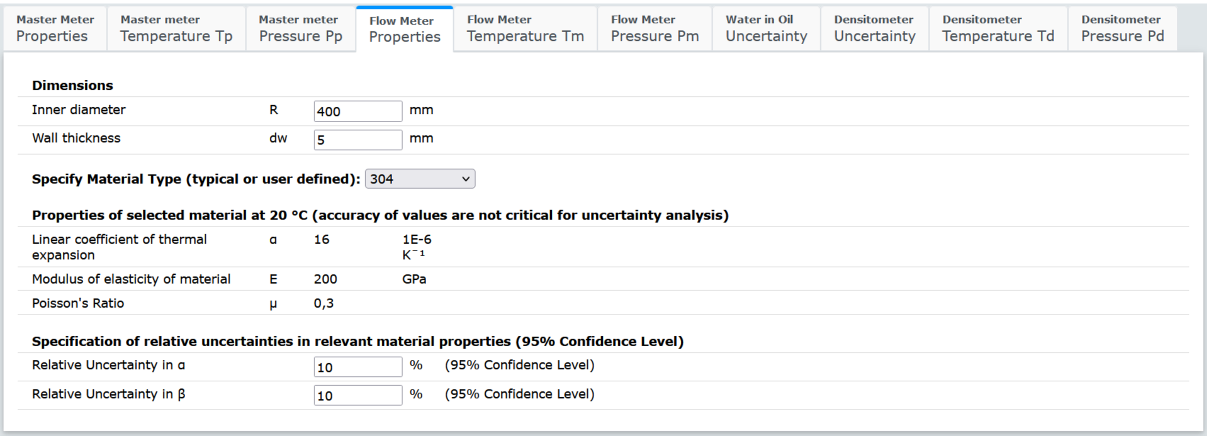


Figure 6.6. “Equipment”-page with “Flow Meter”-section selected. This page concerns uncertainty in the metering station equipment.

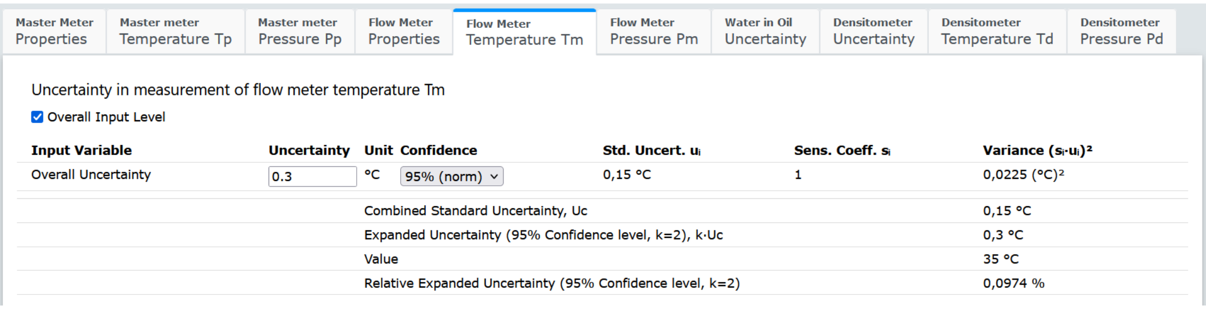
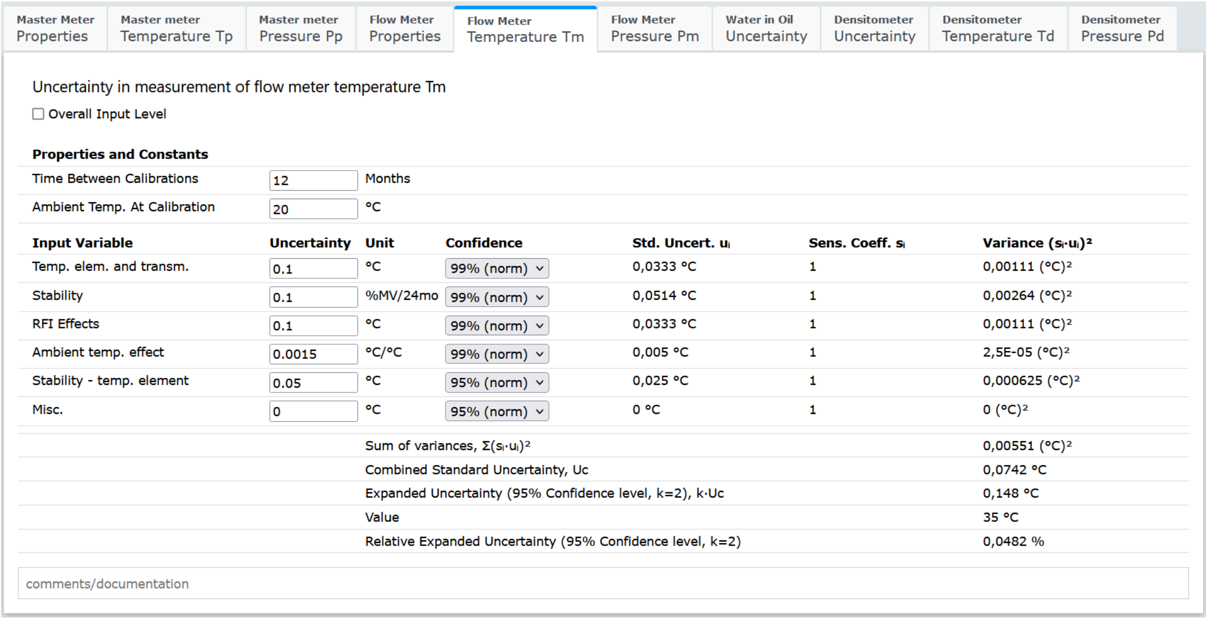


Figure 6.7. Overall input for a temperature transmitter.



Master Meter Properties | Master meter Temperature Tp | Master meter Pressure Pp | Flow Meter Properties | Flow Meter Temperature Tm | Flow Meter Pressure Pm | Water in Oil Uncertainty | Densitometer Uncertainty | Densitometer Temperature Td | Densitometer Pressure Pd

Uncertainty in measurement of flow meter temperature Tm

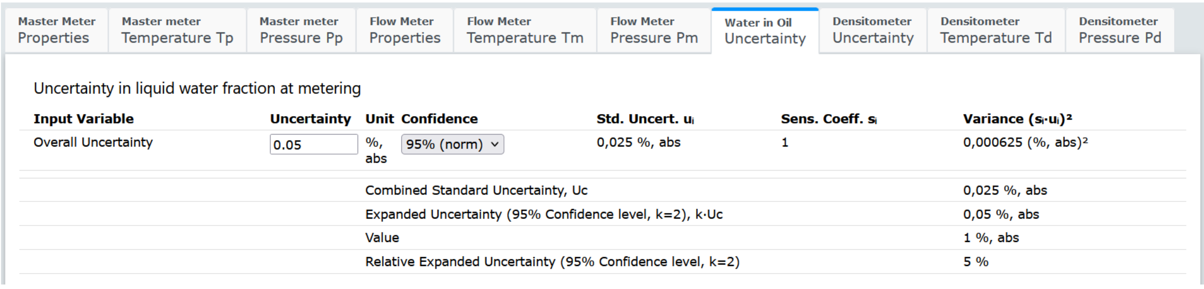
☐ Overall Input Level

Properties and Constants		
Time Between Calibrations	12	Months
Ambient Temp. At Calibration	20	°C

Input Variable	Uncertainty	Unit	Confidence	Std. Uncert. u_i	Sens. Coeff. s_i	Variance $(s_i \cdot u_i)^2$
Temp. elem. and transm.	0.1	°C	99% (norm)	0,0333 °C	1	0,00111 (°C)²
Stability	0.1	%MV/24mo	99% (norm)	0,0514 °C	1	0,00264 (°C)²
RFI Effects	0.1	°C	99% (norm)	0,0333 °C	1	0,00111 (°C)²
Ambient temp. effect	0.0015	°C/°C	99% (norm)	0,005 °C	1	2,5E-05 (°C)²
Stability - temp. element	0.05	°C	95% (norm)	0,025 °C	1	0,000625 (°C)²
Misc.	0	°C	95% (norm)	0 °C	1	0 (°C)²
		Sum of variances, $\Sigma(s_i \cdot u_i)^2$				0,00551 (°C)²
		Combined Standard Uncertainty, Uc				0,0742 °C
		Expanded Uncertainty (95% Confidence level, k=2), k·Uc				0,148 °C
		Value				35 °C
		Relative Expanded Uncertainty (95% Confidence level, k=2)				0,0482 %

comments/documentation

Figure 6.8. Detailed input for a temperature transmitter. The "Save"-button can be used to save the complete specification to a file, and the "Load"-button can the later be used for quickly loading the saved specification.



Master Meter Properties | Master meter Temperature Tp | Master meter Pressure Pp | Flow Meter Properties | Flow Meter Temperature Tm | Flow Meter Pressure Pm | Water in Oil Uncertainty | Densitometer Uncertainty | Densitometer Temperature Td | Densitometer Pressure Pd

Uncertainty in liquid water fraction at metering

Input Variable	Uncertainty	Unit	Confidence	Std. Uncert. u_i	Sens. Coeff. s_i	Variance $(s_i \cdot u_i)^2$
Overall Uncertainty	0.05	%, abs	95% (norm)	0,025 %, abs	1	0,000625 (%, abs)²
		Combined Standard Uncertainty, Uc				0,025 %, abs
		Expanded Uncertainty (95% Confidence level, k=2), k·Uc				0,05 %, abs
		Value				1 %, abs
		Relative Expanded Uncertainty (95% Confidence level, k=2)				5 %

Figure 6.9. Input for an online water cut meter.

6.3.4. Calibration page

The "Calibration"- page (Figure 6.10) contains input regarding calibration conditions and uncertainty in the calibration procedure of the master meter (Figure 6.8). Note that this input page is not applicable when the selected stationary prover is of type "Displacement Prover". It contains the following sections:

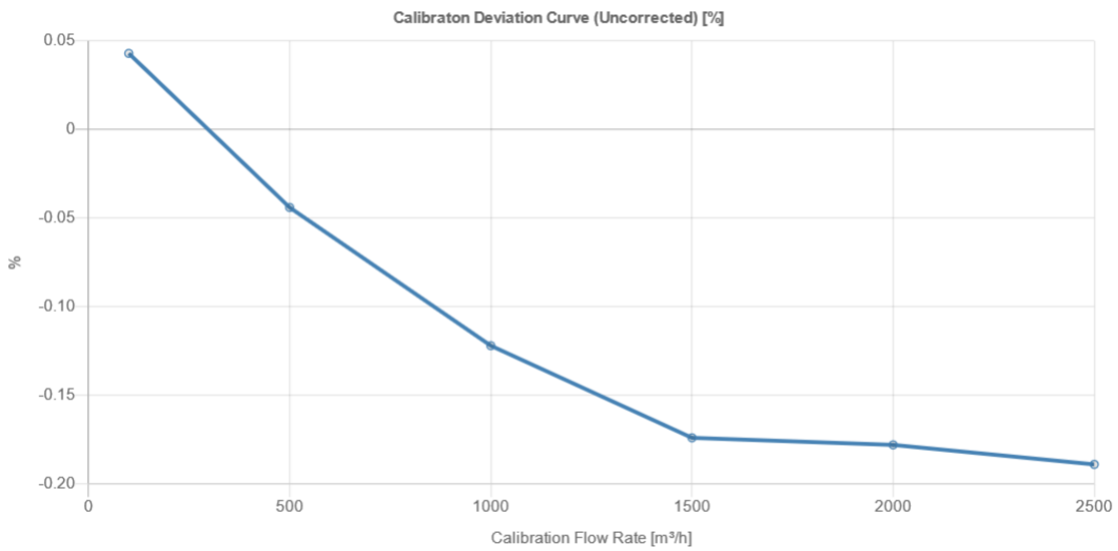
- **Calibration Conditions:** pressure and temperature conditions for master meter at calibration. These are used to calculate corresponding liquid and steel volume correction factors.
- **Master Meter Calibration:** specifies the result from the master meter calibration procedure. The "deviation"-curve together with calibration reference uncertainty and repeatability for the master meter at the different calibration flow rates is specified. The calibration curve is also displayed for convenience.

Calibration Conditions | Master Meter Calibration

Master Meter, Calibration

Add Calibration Point | Remove Last Point

#	Rate m³/h	Calib. Ref. Uncertainty %, 95% Conf.	Deviation Curve (Uncorrected) %	Master Meter Repeat. %, 95% Conf.	Total %, 95% Conf.
1	100	0.031	0.043	0.027	0,0411
2	500	0.031	-0.044	0.027	0,0411
3	1000	0.031	-0.122	0.027	0,0411
4	1500	0.031	-0.174	0.027	0,0411
5	2000	0.031	-0.178	0.027	0,0411
6	2500	0.031	-0.189	0.027	0,0411



comments/documentation

Figure 6.10. "Calibration"-page with "Master Meter Calibration"-section selected. This page concerns calibration conditions and uncertainty in the calibration procedure.

6.3.5. Proving page

The "Proving"-page contains input regarding proving conditions and uncertainty in the proving procedure. There are significant differences between the scenario where proving is performed with an ultrasonic or turbine master meter, and the scenario where a displacement prover is used. Therefore, these are now described separately.

Proving with Ultrasonic or Turbine Master Meter

The input page contains the following sections:

- **Proving Conditions:** pressure and temperature conditions for duty meter and master meter, at proving (Figure 6.11). These are used to calculate corresponding liquid and steel volume correction factors.
- **Proving Uncertainty:** specifies the proving flow rate and the uncertainty in proving of duty meter against master meter at this flow rate (Figure 6.12). Repeatability for both duty meter and master meter at the proving flow rate can be specified, and in addition an uncertainty due to flow profile and fluid effects on master meter. There is also an uncertainty of the proving result for the flow meter due to difference between proving flow rate and nearest calibration flow rate for the master meter, and this is automatically computed and included in the model. This contribution is also visualized as shown in Figure 6.12.

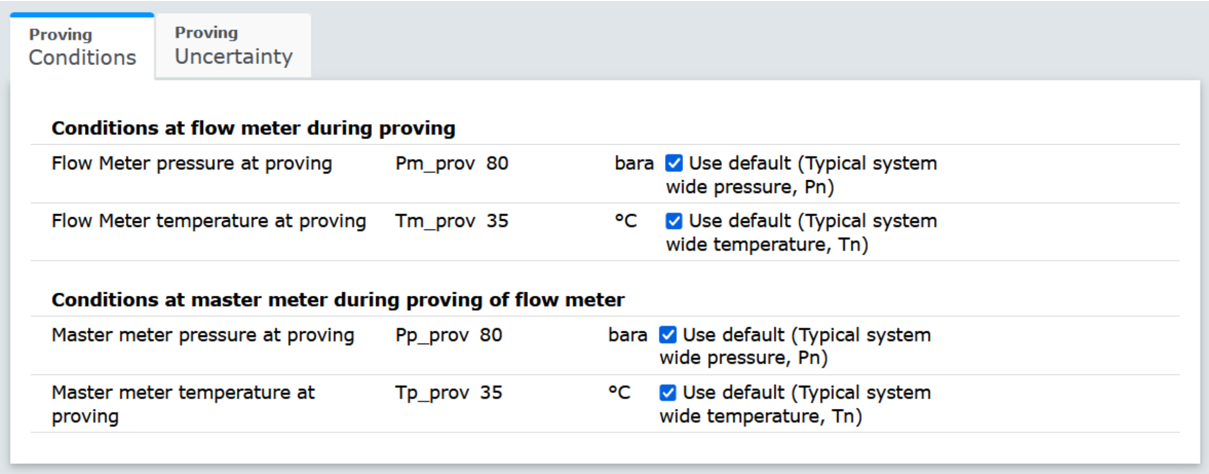


Figure 6.11. "Proving"-page with "conditions"-section for scenario with duty meter proving by ultrasonic or turbine master meter.

Proving Conditions | Proving Uncertainty

Flow Rate: 800 m³/h

Uncertainty in proving of flow meter against master meter

Input Variable	Uncertainty	Unit	Confidence	Std. Uncert. u_i	Sens. Coeff. s_i	Variance $(s_i \cdot u_i)^2$
Flow meter repeatability at proving	0.027	%	95% (norm)	0.0135 %	1	0.000182 (%)²
Master meter repeatability at proving	0.027	%	95% (norm)	0.0135 %	1	0.000182 (%)²
Flow profile and fluid effects on master meter	0.03	%	95% (norm)	0.015 %	1	0.000225 (%)²
Uncertainty contribution from difference in proving flow rate and calibration flow rates	0.0361	%	95% (norm)	0.018 %	1	0.000325 (%)²
		Sum of variances, $\Sigma(s_i \cdot u_i)^2$				0.000915 (%)²
		Relative Combined Standard Uncertainty				0.0302 %
		Relative Expanded Uncertainty (95% Confidence level, k=2)				0.0605 %

comments/documentation

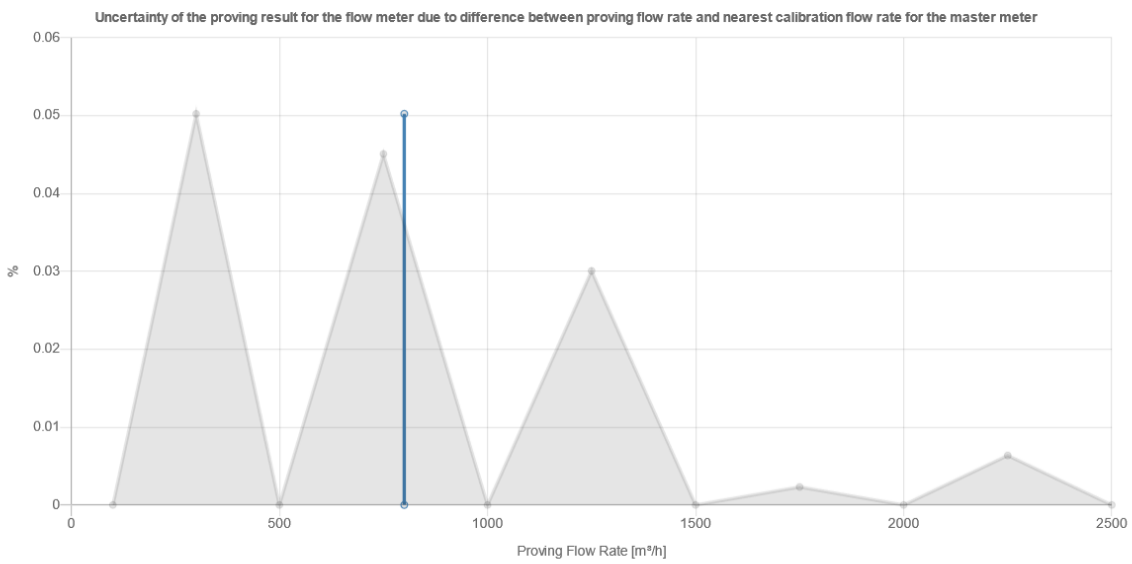


Figure 6.12. Uncertainty in proving of duty meter against master meter at a given proving rate. Note the visualization of the uncertainty contribution due to difference between proving flow rate and nearest calibration flow rate for the master meter.

Proving with Ultrasonic or Turbine Master Meter

The input page contains the following sections:

- **Proving Conditions:** pressure and temperature conditions for duty meter and displacement prover, at proving (Figure 6.13). These are used to calculate corresponding liquid and steel volume correction factors.
- **Proving Uncertainty:** specifies the proving flow rate and the uncertainty in proving of duty meter against displacement prover at this flow rate (Figure 6.14). Repeatability for flow meter at the proving flow rate and displacement prover uncertainty at proving flow rate can be specified.

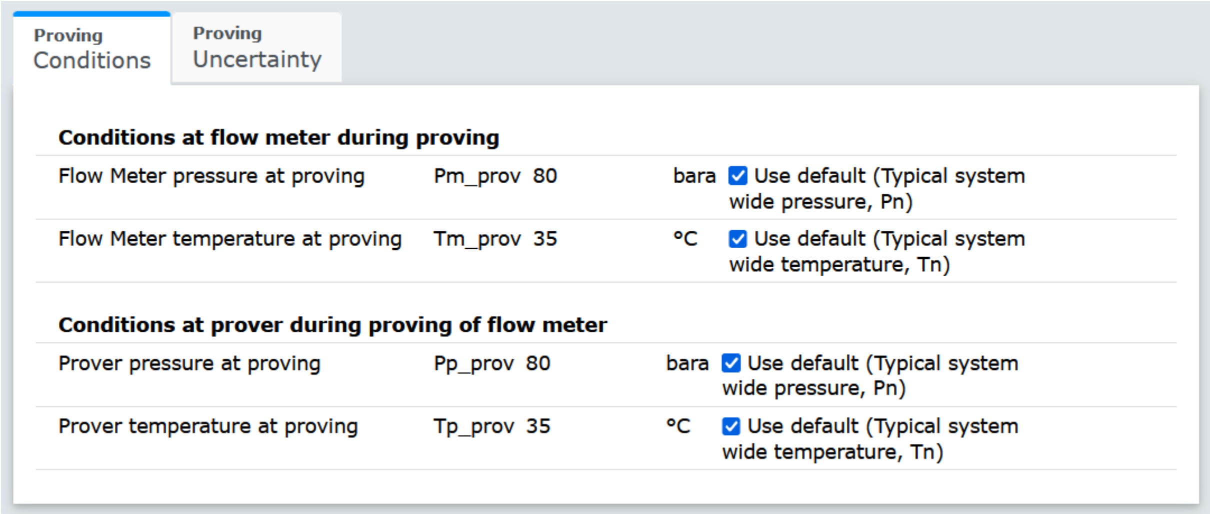


Figure 6.13. "Proving"-page with "conditions"-section for scenario with duty meter proving by displacement prover.

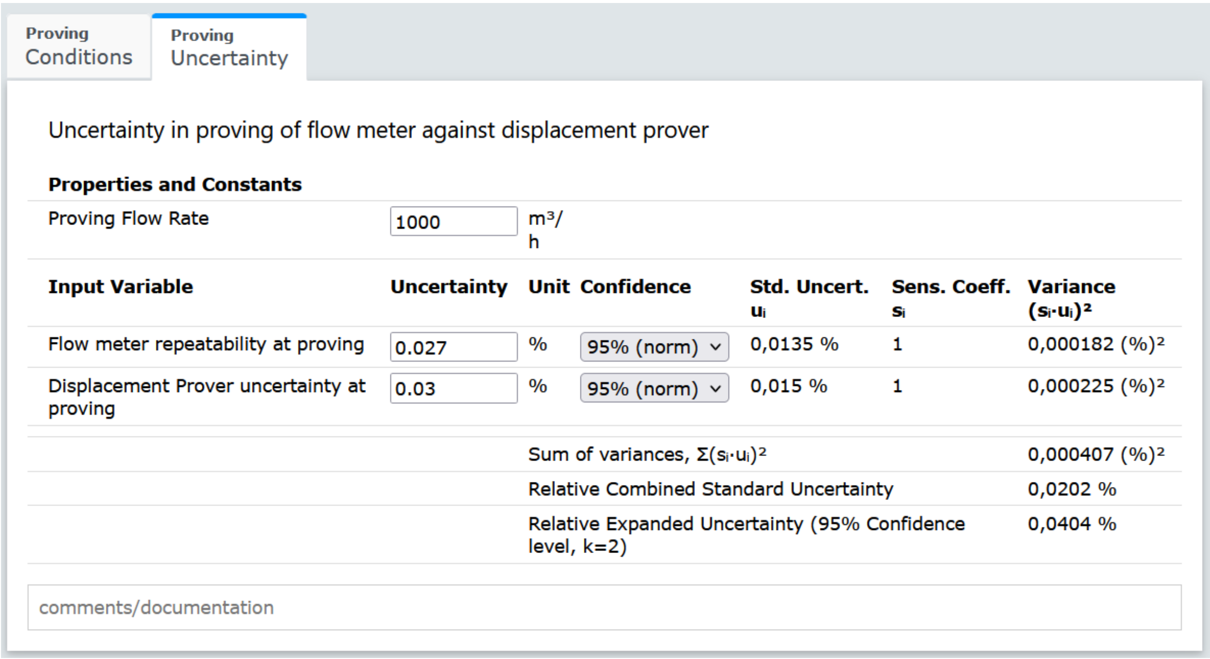


Figure 6.14. Uncertainty in proving of duty meter against displacement prover.

6.3.6. Metering page

The “Metering”-page contains input regarding metering conditions and uncertainty in the metering procedure. It contains the following sections:

- **Metering Conditions:** Pressure and temperature conditions for flow meter at metering (Figure 6.15). These are used to calculate corresponding liquid and steel volume correction factors.
- **Metering Uncertainty:** Specifies the metering flow rate, the operating range, and the linearity of the flow meter in the operating range, and the uncertainty in the flow measurement procedure at this flow rate (Figure 6.16). In the uncertainty model, repeatability for flow meter at the metering rate can be specified, and in addition an uncertainty due to flow profile and fluid effects on the flow meter. There is also an uncertainty contribution due to difference between metering flow rate and proving flow rate, and this is automatically computed using the linearity of the flow meter and included in the model. This contribution is also visualized as shown in Figure 6.16.
- **Reference Water in Oil Density:** If the laboratory water in oil measurement was selected in the metering station template, the uncertainty of the laboratory measurement of the water reference density is displayed as shown in Figure 6.17. If the laboratory water in oil measurement was not selected in the metering station template, this page section is not shown.

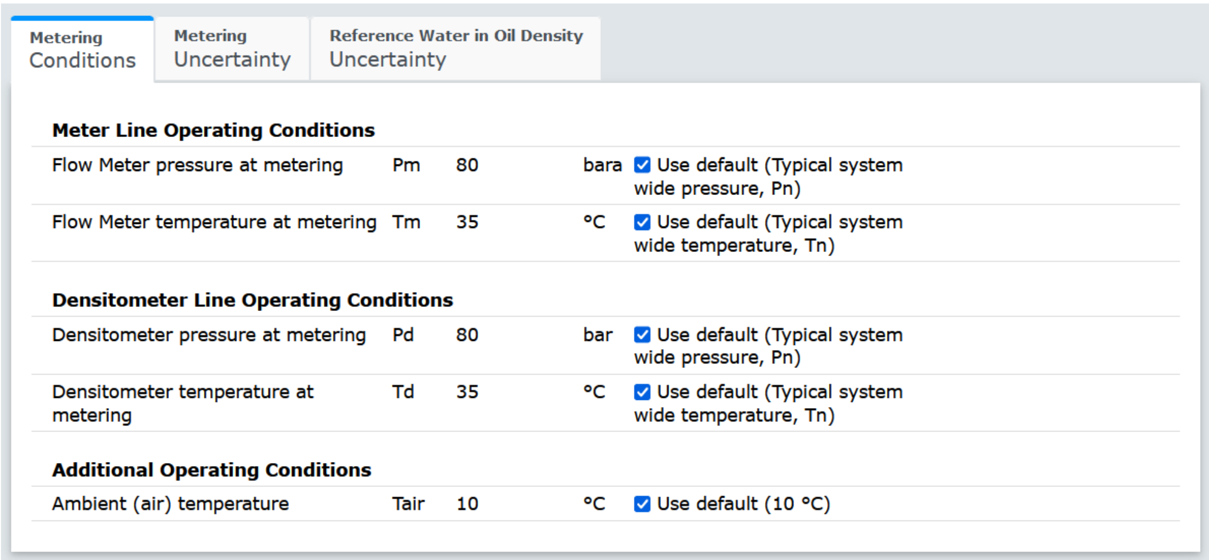


Figure 6.15. “Metering”-page with “Conditions”-section.

Metering Conditions | Metering Uncertainty | Reference Water in Oil Density Uncertainty

Flow Rate: 1000 Sm³/h

Uncertainty in flow meter at metering conditions

Properties and Constants		
Min. operating flow rate without reproving	100	Sm³/h
Max. operating flow rate without reproving	2500	Sm³/h
Linearity of flow meter in the operating range	0.2	%

Input Variable	Uncertainty	Unit	Confidence	Std. Uncert. u_i	Sens. Coeff. s_i	Variance $(s_i \cdot u_i)^2$
Flow meter repeatability at metering	0.027	%	95% (norm)	0.0135 %	1	0.000182 (%)²
Flow profile and fluid effects on flow meter	0.03	%	95% (norm)	0.015 %	1	0.000225 (%)²
Uncertainty contribution from difference in metering rate and proving rate	0.0159	%	100% (rect)	0.00917 %	1	8.41E-05 (%)²
		Sum of variances, $\Sigma(s_i \cdot u_i)^2$				0.000491 (%)²
		Relative Combined Standard Uncertainty				0.0222 %
		Relative Expanded Uncertainty (95% Confidence level, k=2)				0.0443 %

comments/documentation

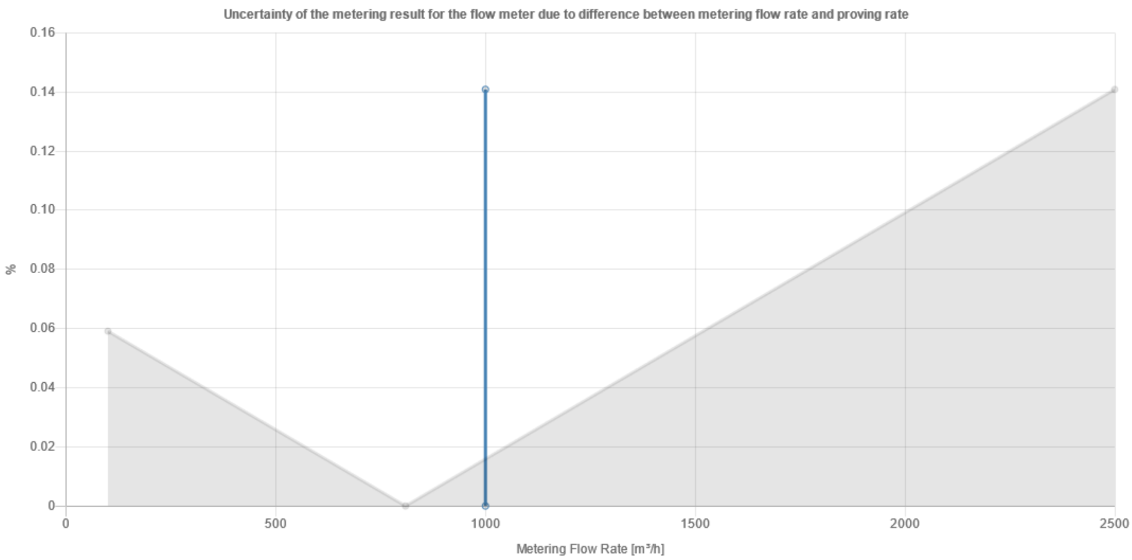


Figure 6.16. "Metering"-page with "Uncertainty"-section for uncertainty in flow metering at a given flow rate. Note the visualization of the uncertainty contribution due to difference between metering flow rate and proving flow rate.

Metering Conditions | Metering Uncertainty | Reference Water in Oil Density Uncertainty

Uncertainty in laboratory measurement of water reference density

Input Variable	Uncertainty	Unit	Confidence	Std. Uncert. u_i	Sens. Coeff. s_i	Variance $(s_i \cdot u_i)^2$
Overall Uncertainty	0.5	kg/m³	95% (norm)	0.25 kg/m³	1	0.0625 (kg/m³)²
		Combined Standard Uncertainty, Uc				0.25 kg/m³
		Expanded Uncertainty (95% Confidence level, k=2), k·Uc				0.5 kg/m³
		Value				1E+03 kg/m³
		Relative Expanded Uncertainty (95% Confidence level, k=2)				0.05 %

Figure 6.17. "Metering"-page with "Reference Water in Oil Density".

6.3.7. Results page

This page is the first of several pages that displays the result of the uncertainty calculation based on the input data (Figure 6.18). There is one section for each of the main flow measurement variables, standard volume flow, absolute volume flow and mass flow. Then there is one section for each of the relevant volume correction factors, where the number of factors depends on the chosen template. In addition, depending on whether oil densitometer is used, there will be a section for the uncertainty in the computation of reference density.

The uncertainty is displayed as uncertainty budgets tables, and the functional relationship is displayed for reference. Depending on the model displayed, there can also be a list of "computed values". These are values computed from the input data for use in the uncertainty calculation and listed here for convenience. An example of this is in Figure 6.19 displaying the uncertainty model for the volume correction factor AliqΔp and where the different computed values for the related factors are listed for reference (Ctlm, Cplm, Ctlp, Cplp, etc.).

Fiscal Oil Metering Station Uncertainty v1.0.3 About Feedback

metering station liquid equipment calibration proving metering results charts report

USM Meter, Actual Volume Flow

$$q_v^{oil} = A_{liq}^{\Delta p} A_{steel}^{m,\Delta p,c} q_{v,nom,flowmeter}^{metering} \left(\frac{V_{nom,prover}^{proving}}{V_{nom,flowmeter}^{proving}}\right)\left(\frac{V_{0,ref}^{calibration}}{V_{0,nom,prover}^{calibration}}\right)\left(1-\phi_{v,line}^{water}\right)$$

Input Variable	Uncertainty	Unit	Confidence	Std. Uncert. u_i	Sens. Coeff. s_i	Variance $(s_i \cdot u_i)^2$
Volume Correction factor AliqΔp	0.0315	%	95% (norm)	0.0158 %	1	0.000248 %²
Volume Correction factor Asteel	0.00579	%	95% (norm)	0.00289 %	1	8.37E-06 %²
Master meter repeatability at calibration	0.027	%	95% (norm)	0.0135 %	1	0.000182 %²
Calibration reference uncertainty	0.031	%	95% (norm)	0.0155 %	1	0.00024 %²
Flow meter repeatability at proving	0.027	%	95% (norm)	0.0135 %	1	0.000182 %²
Master meter repeatability at proving	0.027	%	95% (norm)	0.0135 %	1	0.000182 %²
Flow profile and fluid effects on master meter	0.03	%	95% (norm)	0.015 %	1	0.000225 %²
Uncertainty contribution from difference in proving flow rate and calibration flow rates	0.0361	%	95% (norm)	0.018 %	1	0.000325 %²
Flow meter repeatability at metering	0.027	%	95% (norm)	0.0135 %	1	0.000182 %²
Flow profile and fluid effects on flow meter	0.03	%	95% (norm)	0.015 %	1	0.000225 %²
Uncertainty contribution from difference in metering rate and proving rate	0.0183	%	95% (norm)	0.00917 %	1	8.41E-05 %²
Liquid volume correction factor C_(1-ϕLAB)	0.000194	%	95% (norm)	9.7E-05 %	1	9.42E-09 %²
Uncertainty in the water cut measurement	0.0505	%	95% (norm)	0.0253 %	1	0.000638 %²
		Sum of variances, $\Sigma(s_i \cdot u_i)^2$				0.00271 %²
		Relative Combined Standard Uncertainty				0.052 %
		Relative Expanded Uncertainty (95% Confidence level, k=2)				0.104 %

Figure 6.18. Computed uncertainty of the main flow measurement variables (standard volume flow, absolute volume flow and mass flow) and some other essential values, displayed as uncertainty budget tables. It is possible to select range with left mouse button pressed and use standard copy and paste operation into an Excel spreadsheet. There is one chart for each of the main flow measurement variables, standard volume flow, absolute volume flow and mass flow. Then there is one chart for each of the relevant volume correction factors, where the number of factors depends on the chosen template. In addition, depending on whether oil densitometer is used there will be a chart for the uncertainty in the computation of reference density.

It is also possible to copy all or parts of the different tables to Excel as a table. Select range with left mouse button pressed and use standard copy and paste operation into and Excel spreadsheet.

Volume Correction factor AliqΔmΔp

$$A_{liq}^{\Delta m,\Delta p} = \frac{C_{tlm}^{met} C_{plm}^{met}}{C_{tld}^{met} C_{pld}^{met}} \frac{C_{tlp}^{prov} C_{plp}^{prov}}{C_{tlm}^{prov} C_{plm}^{prov}}$$

Computed Values		
Ctld_met	0.9807	Liquid volume correction factor from densitometer temperature at metering to ref. cond.
Cpld_met	1.008	Liquid volume correction factor from densitometer pressure at metering to ref. cond.
Ctlm_met	0.9807	Liquid volume correction factor from flow meter temperature at metering to ref. cond.
Cplm_met	1.008	Liquid volume correction factor from flow meter pressure at metering to ref. cond.
Ctlm_prov	0.9807	Liquid volume correction factor from flow meter temperature at proving to ref. cond.
Cplm_prov	1.008	Liquid volume correction factor from flow meter pressure at proving to ref. cond.
Ctlp_prov	0.9807	Liquid volume correction factor from proving device temperature at proving to ref. cond.
Cplp_prov	1.008	Liquid volume correction factor from proving device pressure at proving to ref. cond.
AliqΔmΔp	1	Liquid volume correction factor from proving device to flow meter conditions at proving, and from flow meter to densitometer cond. at metering.

Input Variable		**Uncertainty**	**Unit**	**Confidence**	**Std. Uncert. u_i**	**Sens. Coeff. s_i**	**Variance $(s_i \cdot u_i)^2$**
Flow Meter Temperature	Tm	0.148	°C	95% (norm)	0.0742 °C	6.5E-06	2.33E-13 $\%^2$
Flow Meter Pressure	Pm	0.24	bara	95% (norm)	0.12 bar	1.17E-07	1.96E-16 $\%^2$
Master Meter Temperature	Tp	0.3	°C	95% (norm)	0.15 °C	-0.0936	0.000197 $\%^2$
Master Meter Pressure	Pp	0.24	bara	95% (norm)	0.12 bar	0.00986	1.4E-06 $\%^2$
Densitometer Temperature	Td	0.3	°C	95% (norm)	0.15 °C	0.0936	0.000197 $\%^2$
Densitometer Pressure	Pd	0.24	bar	95% (norm)	0.12 bar	-0.00986	1.4E-06 $\%^2$
Reference Density	ρ_0	0.98	kg/m^3	95% (norm)	0.49 kg/m^3	0	0 $\%^2$
Ctl model uncertainty		0.05	%	95% (norm)	0.025 %	0	0 $\%^2$
Cpl model uncertainty		0.096	%	95% (norm)	0.048 %	0	0 $\%^2$
			Sum of variances, $\Sigma(s_i \cdot u_i)^2$				0.000397 $\%^2$
			Relative Combined Standard Uncertainty				0.0199 %
			Relative Expanded Uncertainty (95% Confidence level, k=2)				0.0399 %

Figure 6.19. Uncertainty model for the volume correction factor AliqΔmΔp with the different computed values for the related factors listed for reference.

6.3.8. Uncertainty budgets charts page

This page displays the same data as the "Result"-page, but as uncertainty budget charts (Figure 6.20). The bar chart displays numerical values when the mouse pointer hover over a bar, and it is possible to copy the chart as an image by "right-clicking" on the chart and select either save or copy from the context menu.

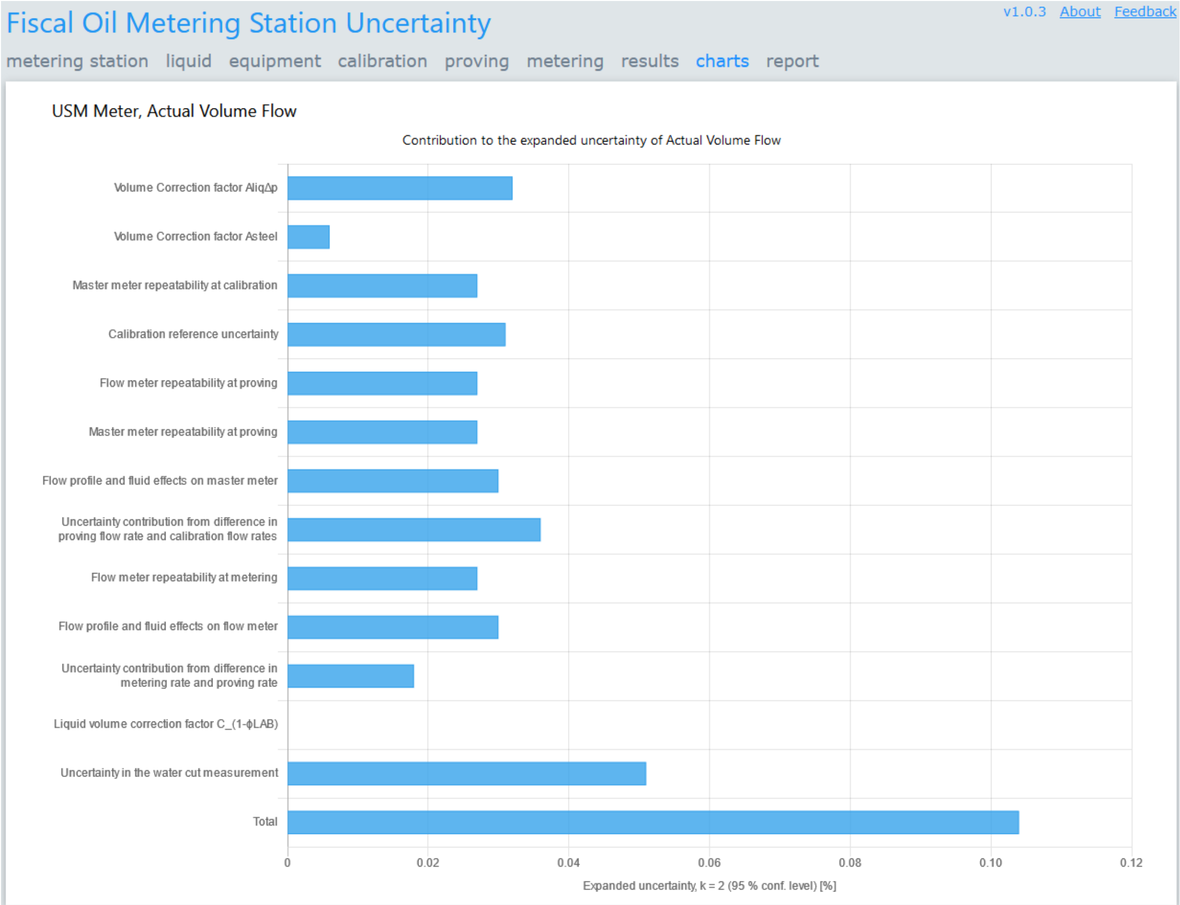


Figure 6.20. Computed uncertainty of the main flow measurement variables (standard volume flow, absolute volume flow, and mass flow) and some other essential values, displayed as uncertainty budget charts. Note that it is possible to copy the chart as an image by right-clicking on the chart and select either save or copy from the context menu.

6.3.9. Uncertainty report page

The “Report”-page contains a summary of the uncertainty analysis formatted as an on-screen report (Figure 6.21).

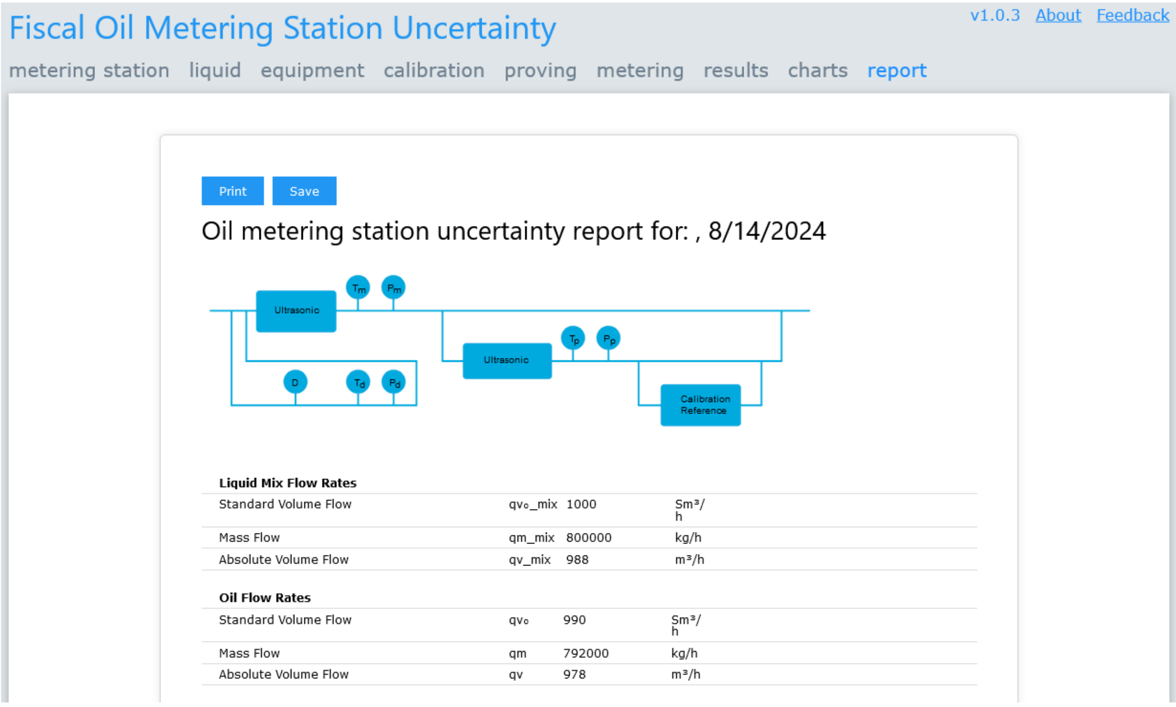


Figure 6.21

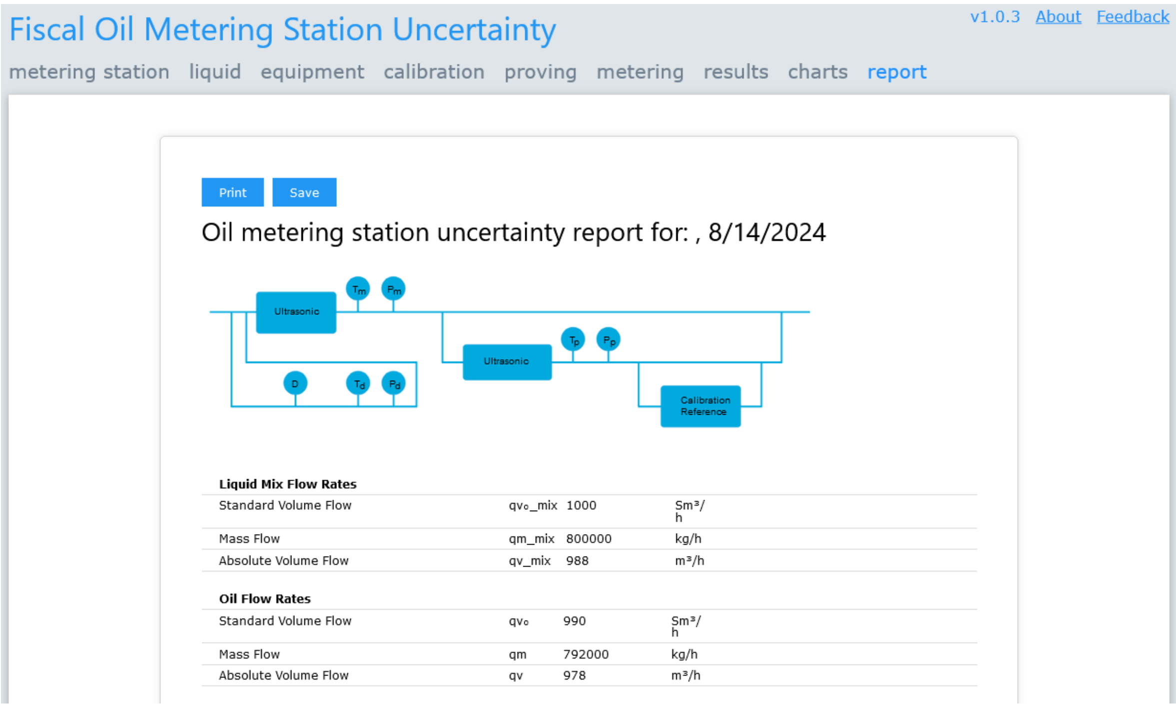


Figure 6.21. The “report”-page contains a summary of the uncertainty analysis formatted as an on-screen report. This can be printed, and it is possible to save the analysis in a file for later use and reference.

The on-screen report includes the following:

- **Print-button:** Shows the browser/system “print-preview” dialog where typical settings for the printout is selected, and a preview of the result is displayed.
- **Save-button:** Downloads a file with a name consisting of the “name” and “date” fields selected on the frontpage of the application.
- Header which integrates the <Name>, <Date> and <Description> input from start page.
- Graphic that displays the selected metering station template.
- Tables listing the line metering operating conditions, proving conditions and calibration conditions.

6.3.10. Note about use of “browser refresh”

The application holds all input and results in memory until the user explicitly saves an uncertainty analysis to file. If the user presses the browser refresh button at any point before saving, all input and results are lost. There is no reason for the user to do this, other than by accident. A “browser refresh” should be avoided, and if the user wants to start a new uncertainty analysis, the “Create New”-button on the application start page should be used. Note that the web-application is not influenced by network disconnects, since after the web-application is loaded there is no further communication with the webserver.

7. Summary

This Handbook documents uncertainty models for fiscal oil metering stations using ultrasonic, turbine or Coriolis flow meters. Proving device is either a displacement prover, an ultrasonic flow master meter or a turbine flow master meter in case of volume flow meter. If the flow meter is a Coriolis meter, the proving device will also be a Coriolis flow meter. The uncertainty models cover volumetric flow rate at standard conditions, volumetric flow rate at line conditions and mass flow rate. Volumetric water fractions of up to 5 % are covered and is either measured online or obtained through sampling and laboratory analysis. The density is either measured by an online densitometer or obtained through sampling and laboratory analysis.

The uncertainty models are implemented on the web application (OilMetApp) using HTML and WebAssembly. This can be accessed for free from nfogm.no.

The present work is related to similar work on fiscal oil metering stations, see (Frøysa, et al., 2020), (Frøysa, et al., 2018) and (Frøysa, et al., 2015) . It is also based on (Dahl, et al., 2003), (Lunde, et al., 2002) and (Lunde, et al., 2010).

The first version of this Handbook was published in 2015. The 2015-version focused on the uncertainty analysis for fiscal oil measurements performed with ultrasonic meters. In 2018, additional metering technologies were added. Cases with turbine and Coriolis meters as master meters were included, turbine meter as a possible proving meter, and in case of a Coriolis master meter; a Coriolis proving meter. In 2020, the software platform was moved from Silverlight to WebAssembly. In the present 2024-version, the uncertainty models are updated to include corrections for cases with water in oil, and the models are designed for volumetric water contents of up to 5 %. This accounts for added measurement uncertainty when small water fractions are measured.

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Appendix A

Detailed formulas for the linearity contribution to the proving uncertainty

This Appendix gives the details regarding the linearity contribution to the proving uncertainty in the case of a master meter proving device. The uncertainty contribution appears in the third term on the right-hand side of Eq. (5-16). This uncertainty contribution is caused by the fact that the flow calibration of the master meter is carried out at a limited number of flow rates. The correction of the flow (master) meter is based on an interpolation overflow rates of the deviations from reference at the flow rates used in the flow calibration. The results presented here are based on (Lunde, et al., 2002), (Lunde, et al., 2010), and (Frøysa, et al., 2014).

A1 Functional relationship

After flow calibration, an adjustment of the flow meter shall be performed. The flow calibration is carried out by comparing the output flow rate from the flow meter with the similar reading from a reference measurement. This is carried out at a set of N different flow rates where the reference meter measured the flow rate $q_{v,ref,i}$,, and the flow meter measured the flow rate $q_{v,Meter,i}$, $i = 1, \ldots, N$. A full correction of the flow meter at each of these flow rates can therefore be written as

$$q_{v,i} = K_i q_{v,Meter,i} \tag{A.1}$$

where

$$K_i = \frac{q_{v,ref,i}}{q_{v,Meter,i}} \tag{A.2}$$

The relative difference in per cent between the flow rate as measured by the flow meter and the reference meter can be written as

$$p_i = 100 \frac{q_{v,Meter,i} - q_{v,ref,i}}{q_{v,ref,i}} \tag{A.3}$$

The relation between these two quantities is

$$p_i = 100 \frac{1 - K_i}{K_i}; K_i = \frac{100}{100 + p_i} \tag{A.4}$$

From these correction factors a general correction factor valid for all flow rates (and not only at the specific flow rates where the flow calibration is carried out) is established. This can formally be written as

$$q_v = K q_{v,Meter} \tag{A.5}$$

where

$$K = f(K_1, K_2, \ldots, K_N, q_v). \tag{A.6}$$

This factor corresponds to correcting a percentage deviation of p %, where

$$p_i = 100\frac{1-K}{K}; K = \frac{100}{100+p} \tag{A.7}$$

In practice, such a correction could have been carried out by different methods, including

i. no correction,
ii. a constant percentage correction,
iii. linear interpolation, and
iv. other methods (splines and other curve fittings).

If one of the two first methods is used, there will be known systematic errors left which are not corrected for. This is not in accordance with the Regulations relating to fiscal measurement in the petroleum activities issued by the Norwegian Offshore Directorate (NOD, 2023), where one requirement in § 64 is that "Calibration shall take place in such a manner that systematic effects as a result of differences between calibration and operating conditions are avoided or compensated for." They will therefore not be covered here.

In the third method, the adjustment will be based on a linear interpolation between the adjustment factors established for the flow rates used in the flow calibration. Such an interpolation can be carried out either on K, or on the percentage deviation p. Here, a linear interpolation in p is described. Both for the correction and for the uncertainty analysis, the results will almost be the same whether the interpolation is carried out on p or on K. The linear interpolation can be written as

$$p = p_1 + \frac{p_{i+1} - p_i}{q_{v,Meter,i+1} - q_{v,Meter,i}}\left(q_{v,Meter} - q_{v,Meter,i}\right); \tag{A.8}$$

When $q_{v,Meter,i} < q_{v,Meter} < q_{v,Meter,i+1}$

K can then be found from Eq. (A.7). It should be commented that this third method provides a correction such that the flow meter's flow rate will be corrected to the reference meter flow rate, when the flow rate is equal to any of the flow rates used in the flow calibration. This case is therefore in agreement with the Regulations relating to fiscal measurement in the petroleum activities issued by the Norwegian Offshore Directorate.

The fourth method is a generalization of the third method, where the linear interpolation is replaced with a non-linear interpolation (e.g., based on splines) or a partially linear interpolation where more interpolation points than the ones used in the flow calibration (ref. method (iii)) are used. In such cases, it is recommended that for the uncertainty analysis, it is treated as method (iii).

A2 Uncertainty model

The uncertainty of the flow rate due to the above-mentioned adjustment of a flow meter after flow calibration will now be described. This is the linearity contribution to the proving uncertainty, as appearing in the third term on the right-hand side of Eq. (5-16). It can be written as

$$\left(\frac{u\left(q_{v0,linearity}^{prov}\right)}{q_{v0}^{prov}}\right)^2 = \left(\frac{u(K)}{K}\right)^2 \qquad \text{(A.9)}$$

with a reference to Eq. (A.5). The term is related to the percentage difference, p, between flow rate from the flow meter and the reference measurement, because of Eq. (A.4). The actual expression depends on the adjustment method for the flow meter, and of any uncorrected percentage deviations, δp, between the flow meter and the reference meter. As discussed above, only a linear interpolation correction method will be addressed here.

More specifically, the correction is carried out using a linear interpolation in the percentage deviation between the flow meter and the reference meter. The linear interpolation provides an approximate value for the deviation from reference for flow rates between the ones used in the flow calibration. This is illustrated in an example shown in Figure A.1, where a flow meter is flow calibrated at volume flow rates at standard conditions of 500 m³/h and 2000 m³/h. The deviation from reference at 500 m³/h is in this example 0.3 %. At 2000 m³/h it is 0.1 %. The blue curve represents the interpolated for volume flow rates at standard conditions between 500 m³/h and 2000 m³/h. The correction of the meter is based on this curve. However, such a linear interpolation is an approximation, and the exact shape of the deviation curve is not known. In this work it is assumed that the true curve is somewhere inside the red parallelogram. It is further assumed that the probability is the same for the curve to be anywhere inside the parallelogram. The maximum (and unknown) uncorrected percentage deviation after correction is therefore not larger than:

- when $q_{v0,Meter,i} < q_{v0,Meter} < (q_{v,Meter,i} + q_{v,Meter,i+1})/2$:

$$\delta p = \frac{q_{v0,Meter} - q_{v0,Meter,i}}{q_{v0,Meter,i+1} - q_{v0,Meter,i}} |p_{i+1} - p_i|; \qquad \text{(A.10)}$$

- when $\frac{q_{v,Meter,i} + q_{v,Meter,i+1}}{2} < q_{v0,Meter} < q_{v0,Meter,i}$:

$$\delta p = \frac{q_{v0,Meter,i+1} - q_{v0,Meter}}{q_{v0,Meter,i+1} - q_{v0,Meter,i}} |p_{i+1} - p_i|; \qquad \text{(A.11)}$$

This maximum percentage deviation is shown in Figure A.2.

For flow rates outside the calibrated range, extrapolation is carried out for getting an estimate for the uncorrected percentage deviation. In this case, the uncorrected percentage deviation increases as the flow rate leaves the calibrated range, and is calculated as

$$\delta p = \frac{q_{v0,Meter} - q_{v0,Meter,1}}{q_{v0,Meter,2} - q_{v0,Meter,1}} |p_2 - p_1|; \tag{A.12}$$

when $q_{v0,Meter} < q_{v0,Meter,1}$

and

$$\delta p = \frac{q_{v0,Meter} - q_{v0,Meter,n}}{q_{v0,Meter,n} - q_{v0,Meter,n-1}} |p_n - p_{n-1}|; \tag{A.13}$$

when $q_{v0,Meter} > q_{v0,Meter,n}$

The expression for δp is considered to be expanded uncertainty of p with 100 % confidence level and rectangular distribution function. The standard uncertainty of p is then found by dividing δp with the square root of 3. The relative standard uncertainty of the correction factor estimate can now be written as

$$\left(\frac{u\left(q_{v0,linearity}^{prov}\right)}{q_{v0,linearity}^{prov}}\right) = \left(\frac{u(K)}{K}\right) = \frac{1}{K}\frac{\partial p}{\partial K}u(p) = \frac{\delta p/\sqrt{3}}{100 + p} \tag{A.14}$$

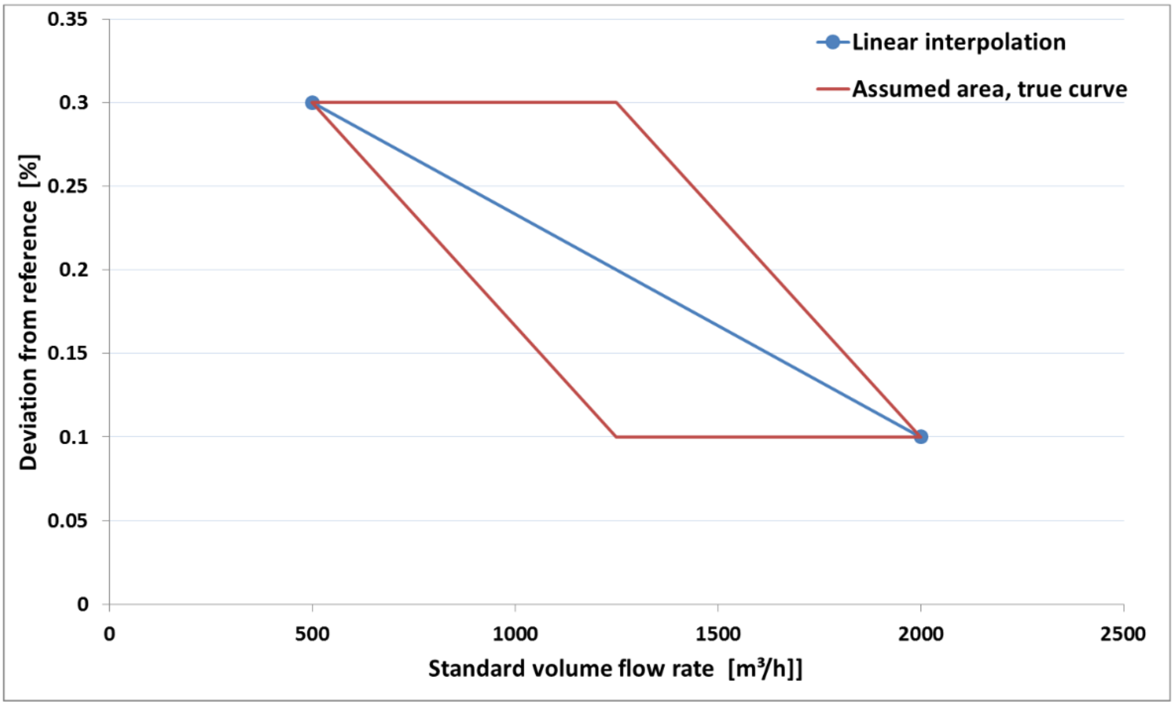

Figure A.1. Example of deviation from reference at flow calibration at a standard volume flow rate of 500 m³/h (here deviation of 0.3 %) and 2000 m³/h (here deviation of 0.1%). For the correction of the flow meter, the deviation at flow rates between 500 m³/h and 2000 m³/h are found by linear interpolation (blue curve). It is assumed that the "true" deviation curve is somewhere within the red parallelogram.

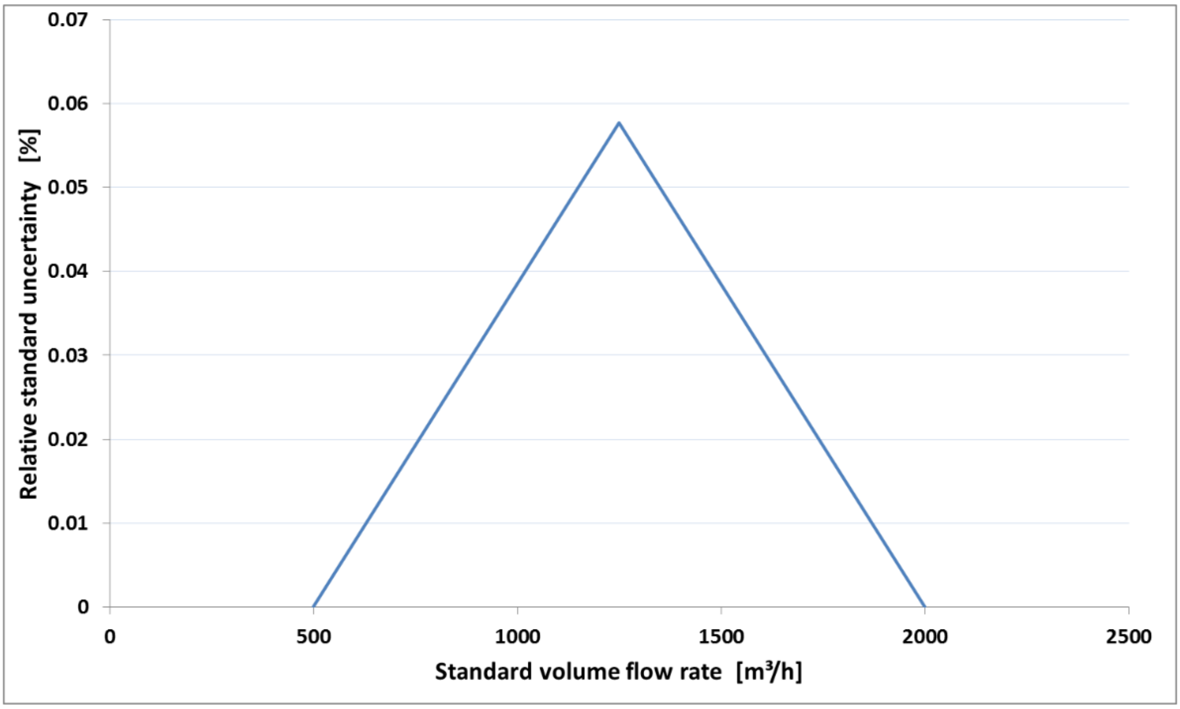

Figure A.2. Relative standard uncertainty related to the correction factor for the example shown in Figure A.1.

Appendix B

List of symbols

This appendix contains a list of the most relevant parameters used throughout this Handbook.

A_{liq}: Combinations of pressure and temperature volume coefficients, of relevance when calculating the volumetric flow rate at standard conditions, the volumetric flow rate at line conditions and the mass flow rate. A_{liq} is combined with superscripts “p” (for proving) and/or “m” (for metering). A delta (Δ) in front of “p” or “m” means that volume correction between two conditions during proving or metering. No delta (Δ) means that the volume correction to standard pressure and temperature is calculated.

A_{steel}: Combinations of pressure and temperature volume coefficients, of relevance when calculating the volumetric flow rate at standard conditions, the volumetric flow rate at line conditions and the mass flow rate. A_{steel} is combined with superscripts “c” (for calibration), “p” (for proving) and/or “m” (for metering). A delta (Δ) in from of “c”, “p” or “m” means that volume correction between two conditions during proving or metering. No delta (Δ) means that the volume correction to standard pressure and temperature is calculated.

C_{plx}: Volume correction coefficient for oil (liquid), for pressure changes from the pressure at condition “x” to standard pressure.

C_{psx}: Volume correction coefficient for steel, for pressure changes from a base pressure to the pressure at condition “x”.

C_{pwx}: Volume correction coefficient for water, for pressure changes from a base pressure to the pressure at condition “x”.

C_{tlx}: Volume correction coefficient for oil (liquid), for temperature changes from the temperature at condition “x” to standard temperature.

C_{tsx}: Volume correction coefficient for steel, for temperature changes from a base temperature to the temperature at condition “x”.

C_{twx}: Volume correction coefficient for water for temperature changes from the temperature at condition “x” to standard temperature.

K: Correction factor to be applied after flow calibration, see Eq. (A.5).

p: Percentage deviation that is corrected after flow calibration, see Eq. (A.7).

P_x: Absolute pressure at condition “x”.

P_0: Absolute standard pressure (1 atm = 101325 Pa)

$q_{m,meas}$: Mass flow rate, as measured by the primary flow meter, after corrections from the proving and calibration.

$q_{v,meas}$: Volumetric flow rate at line (flow meter) conditions, as measured by the primary flow meter, after corrections from the proving and calibration.

$q_{v_0,meas}$: Volumetric flow rate at standard conditions (volumetric flow rate converted to standard temperature and pressure), as measured by the primary flow meter, after corrections from the proving and calibration.

$q_{nom,flowmeter}^{metering}$: Volumetric flow rate at line conditions that would have been measured by the primary flow meter during metering, without the corrections from the proving and calibration, and if temperature and pressure expansions in steel had not been taken into account.

T_x: Temperature (°C) at condition "x".

T_0: Standard temperature (15 °C).

$u(X)$: Standard uncertainty of quantity.

$u(X)/X$: Relative standard uncertainty of quantity.

$V_{0,meas}$: The standard volume of oil measured by the primary flow meter volume, after corrections from the proving and calibration.

$V_{0,flowmeter}^{metering}$: The standard volume of oil measured by the primary flow meter volume at metering (line conditions), without the corrections from the proving and calibration.

$V_{0,flowmeter}^{proving}$: The standard volume of oil measured by the primary flow meter during proving of the primary flow meter.

$V_{0,nom,prover}^{calibration}$: The standard volume of oil that would have been measured by the proving device during calibration of the proving device if temperature and pressure expansions in steel had not been taken into account.

$V_{0,ref}^{calibration}$: The standard volume of oil measured by the reference instrumentation during calibration of the proving device.

$V_{0,prover}^{calibration}$: The standard volume of oil measured by the proving device during calibration of the proving device.

$V_{0,prover}^{proving}$: The standard volume of oil measured by the proving device during proving of the primary flow meter.

$V_{nom,flowmeter}^{metering}$: The actual volume of oil (line conditions) that would have been measured by the primary flow meter volume at metering, without the corrections from the proving and calibration, if temperature and pressure expansions in steel had not been taken into account.

$V_{nom,flowmeter}^{proving}$: The actual volume of oil (line conditions) that would have been measured by the primary flow meter during proving of the primary flow meter if temperature and pressure expansions in steel had notbeen taken into account.

$V_{nom,prover}^{proving}$: The actual volume of oil (line conditions) that would have been measured by the proving device during proving of the primary flow meter if temperature and pressure expansions in steel not had been taken into account.

δp: Uncorrected percentage deviation after flow calibration, see Section A2.

ρ_{dens}: Oil density at densitometer conditions

ρ_0: Oil density at standard conditions

ϕ : The volumetric water fraction

Conditions substituting "x": Index "d" means densitometer conditions, index "m" means line (flow meter) conditions, index "p" means proving device condition and index "c" means flow calibration conditions.

Superscript "met" means during normal metering, "prov" means during proving and "cal" means during flow calibration.