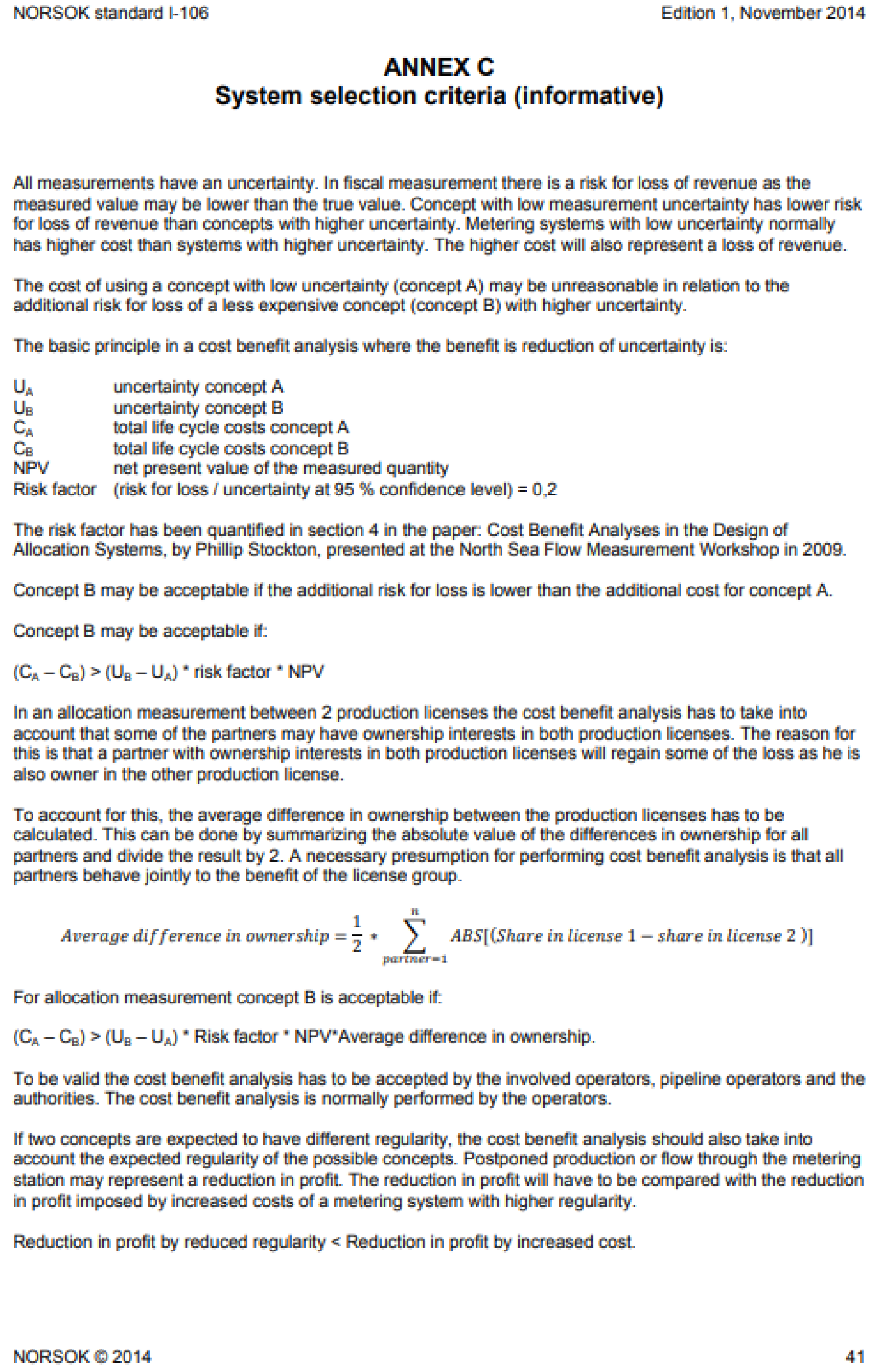
NORSOK I-106 Draft annex on:
Risk-cost-benefit analysis for allocation system

NFOGM HCM allocation Workshop 2025
2025-06-12
Dag Hendrik Flølo

NORSOK I-106

- ´Fiscal metering systems for hydrocarbon liquid and gas´
- 2014 version has an Annex C ´System selection criteria (informative)´
- A new revision will soon be issued for review
- Will be issued with a new Annex: ´Risk-Cost-Benefit analysis´

- NORSOK is concerned with design – not operation



ANNEX C
System selection criteria (informative)

All measurements have an uncertainty. In fiscal measurement there is a risk for loss of revenue as the measured value may be lower than the true value. Concept with low measurement uncertainty has lower risk for loss of revenue than concepts with higher uncertainty. Metering systems with low uncertainty normally has higher cost than systems with higher uncertainty. The higher cost will also represent a loss of revenue.

The cost of using a concept with low uncertainty (concept A) may be unreasonable in relation to the additional risk for loss of a less expensive concept (concept B) with higher uncertainty.

The basic principle in a cost benefit analysis where the benefit is reduction of uncertainty is:

U_A	uncertainty concept A
U_B	uncertainty concept B
C_A	total life cycle costs concept A
C_B	total life cycle costs concept B
NPV	net present value of the measured quantity
Risk factor	(risk for loss / uncertainty at 95 % confidence level) = 0,2

The risk factor has been quantified in section 4 in the paper: Cost Benefit Analyses in the Design of Allocation Systems, by Phillip Stockton, presented at the North Sea Flow Measurement Workshop in 2009.

Concept B may be acceptable if the additional risk for loss is lower than the additional cost for concept A.

Concept B may be acceptable if:

$(C_A - C_B) > (U_B - U_A)$ * risk factor * NPV

In an allocation measurement between 2 production licenses the cost benefit analysis has to take into account that some of the partners may have ownership interests in both production licenses. The reason for this is that a partner with ownership interests in both production licenses will regain some of the loss as he is also owner in the other production license.

To account for this, the average difference in ownership between the production licenses has to be calculated. This can be done by summarizing the absolute value of the differences in ownership for all partners and divide the result by 2. A necessary presumption for performing cost benefit analysis is that all partners behave jointly to the benefit of the license group.

$$Average\ difference\ in\ ownership = \frac{1}{2} * \sum_{partner=1}^{n} ABS[(Share\ in\ license\ 1 - share\ in\ license\ 2\ )]$$

For allocation measurement concept B is acceptable if:

$(C_A - C_B) > (U_B - U_A)$ * Risk factor * NPV*Average difference in ownership.

To be valid the cost benefit analysis has to be accepted by the involved operators, pipeline operators and the authorities. The cost benefit analysis is normally performed by the operators.

If two concepts are expected to have different regularity, the cost benefit analysis should also take into account the expected regularity of the possible concepts. Postponed production or flow through the metering station may represent a reduction in profit. The reduction in profit will have to be compared with the reduction in profit imposed by increased costs of a metering system with higher regularity.

Reduction in profit by reduced regularity < Reduction in profit by increased cost.

Why update NORSOK I-106 Annex on risk-cost-benefit analysis?

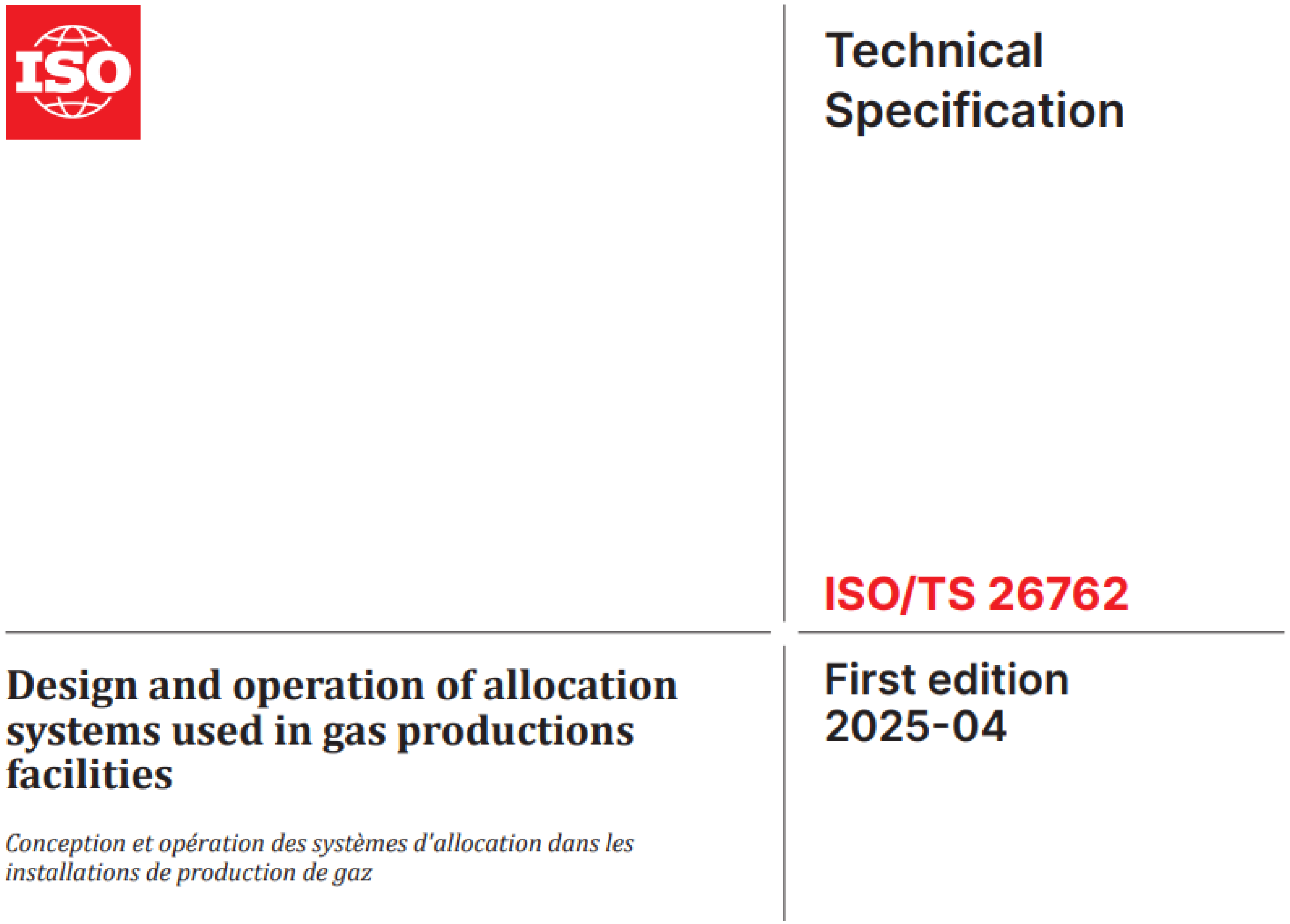
- Provide more complete description and reasoning for the method to facilitate:
  - Application of the method
  - Common understanding
  - Scrutiny and further improvement

- Need a universal method that covers:
  - The cases with more than two fields
  - The allocation method ('Pro rata' or 'By difference')
  - Calculation of present value
  - Correlated vs. uncorrelated uncertainties from year to year

- Facilitate implementation of principles in other regulations (e.g. EU CO2) and countries (e.g. Brazil)

Annex ?: Risk-Cost-Benefit analysis

Contents



Technical Specification

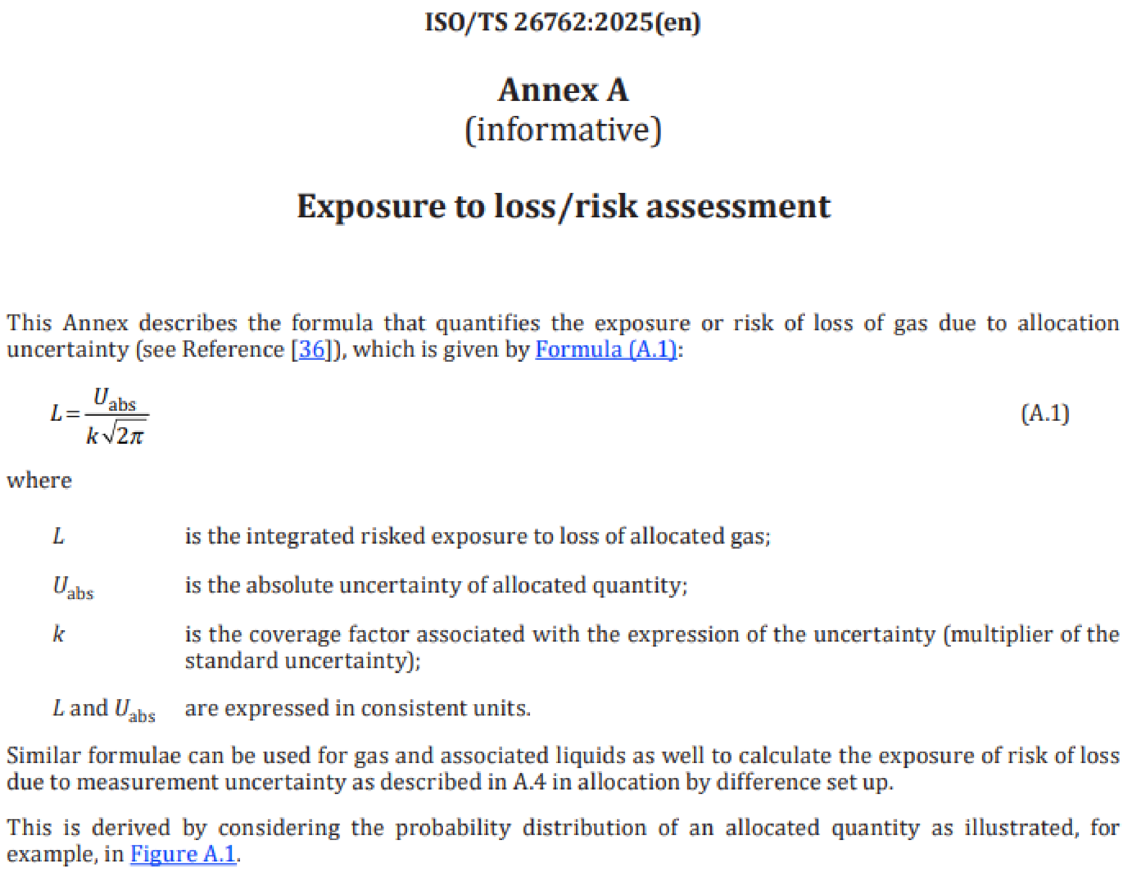
ISO/TS 26762

Design and operation of allocation systems used in gas productions facilities

Conception et opération des systèmes d'allocation dans les installations de production de gaz

First edition
2025-04

ISO/TS 26762:2025 Derives the relationship between uncertainty and risk



Annex A
(informative)

Exposure to loss/risk assessment

This Annex describes the formula that quantifies the exposure or risk of loss of gas due to allocation uncertainty (see Reference [36]), which is given by Formula (A.1):

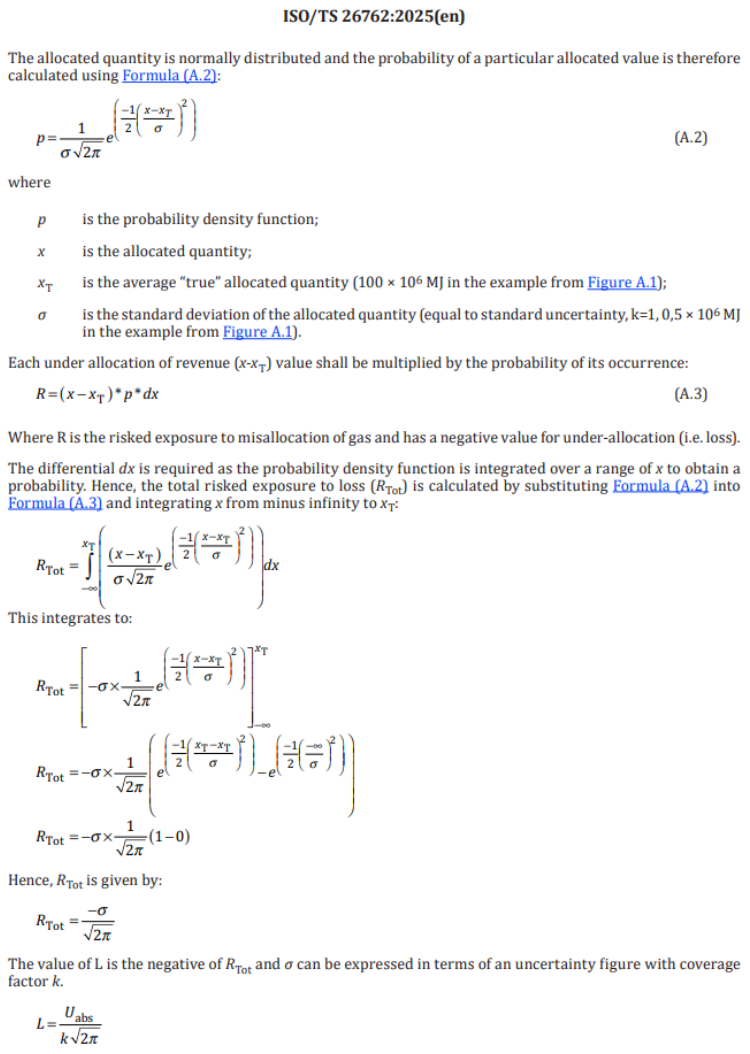
$$L = \frac{U_{\mathrm{abs}}}{k\sqrt{2\pi}} \tag{A.1}$$

where

L	is the integrated risked exposure to loss of allocated gas;
U_{abs}	is the absolute uncertainty of allocated quantity;
k	is the coverage factor associated with the expression of the uncertainty (multiplier of the standard uncertainty);
L and U_{abs}	are expressed in consistent units.

Similar formulae can be used for gas and associated liquids as well to calculate the exposure of risk of loss due to measurement uncertainty as described in A.4 in allocation by difference set up.

This is derived by considering the probability distribution of an allocated quantity as illustrated, for example, in Figure A.1.



The allocated quantity is normally distributed and the probability of a particular allocated value is therefore calculated using Formula (A.2):

$$p = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(\frac{-1}{2}\left(\frac{x - x_{\mathrm{T}}}{\sigma}\right)^2\right)} \tag{A.2}$$

where

p	is the probability density function;
x	is the allocated quantity;
x_{T}	is the average "true" allocated quantity (100 × 10^6 MJ in the example from Figure A.1);
σ	is the standard deviation of the allocated quantity (equal to standard uncertainty, k=1, 0,5 × 10^6 MJ in the example from Figure A.1).

Each under allocation of revenue (x-x_{T}) value shall be multiplied by the probability of its occurrence:

$$R = (x - x_{\mathrm{T}}) * p * dx \tag{A.3}$$

Where R is the risked exposure to misallocation of gas and has a negative value for under-allocation (i.e. loss).

The differential dx is required as the probability density function is integrated over a range of x to obtain a probability. Hence, the total risked exposure to loss (R_{Tot}) is calculated by substituting Formula (A.2) into Formula (A.3) and integrating x from minus infinity to x_{T}:

$$R_{\mathrm{Tot}} = \int_{-\infty}^{x_{\mathrm{T}}} \left(\frac{(x - x_{\mathrm{T}})}{\sigma\sqrt{2\pi}} e^{\left(\frac{-1}{2}\left(\frac{x - x_{\mathrm{T}}}{\sigma}\right)^2\right)} \right) dx$$

This integrates to:

$$R_{\mathrm{Tot}} = \left[-\sigma \times \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-1}{2}\left(\frac{x - x_{\mathrm{T}}}{\sigma}\right)^2\right)} \right]_{-\infty}^{x_{\mathrm{T}}}$$

$$R_{\mathrm{Tot}} = -\sigma \times \frac{1}{\sqrt{2\pi}} \left(e^{\left(\frac{-1}{2}\left(\frac{x_{\mathrm{T}} - x_{\mathrm{T}}}{\sigma}\right)^2\right)} - e^{\left(\frac{-1}{2}\left(\frac{-\infty}{\sigma}\right)^2\right)} \right)$$

$$R_{\mathrm{Tot}} = -\sigma \times \frac{1}{\sqrt{2\pi}} (1 - 0)$$

Hence, R_{Tot} is given by:

$$R_{\mathrm{Tot}} = \frac{-\sigma}{\sqrt{2\pi}}$$

The value of L is the negative of R_{Tot} and σ can be expressed in terms of an uncertainty figure with coverage factor k.

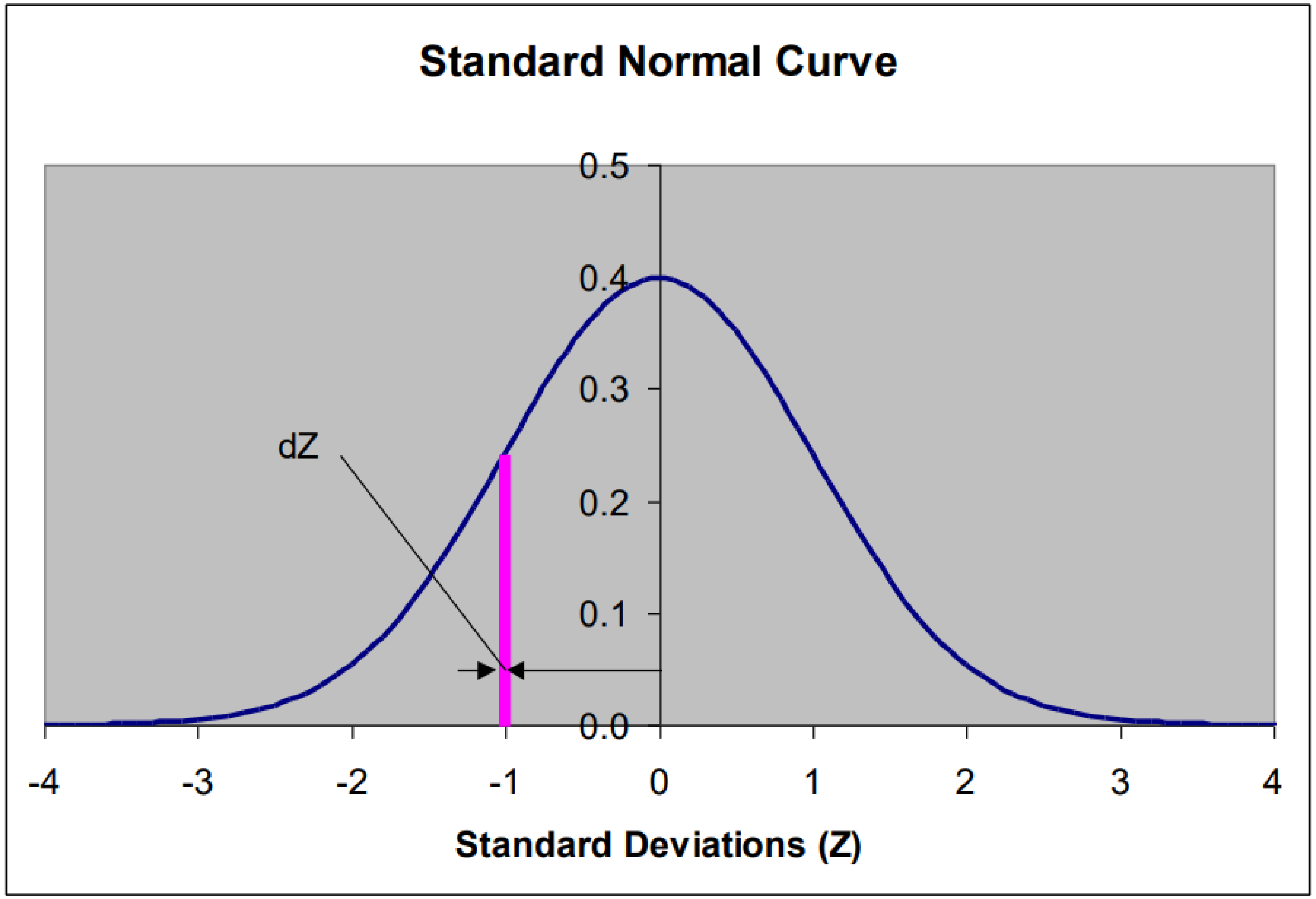
$$L = \frac{U_{\mathrm{abs}}}{k\sqrt{2\pi}}$$

risk = probability * consequence

Economic risk = probability * economic consequence

Economic risk: ´ Risk for loss of profit´

Stockton NSFMW 2009 – ´Cost Benefit Analyses in the Design of Allocation Systems



- **Probability** =

  Area of pink segment

- **Consequence** =

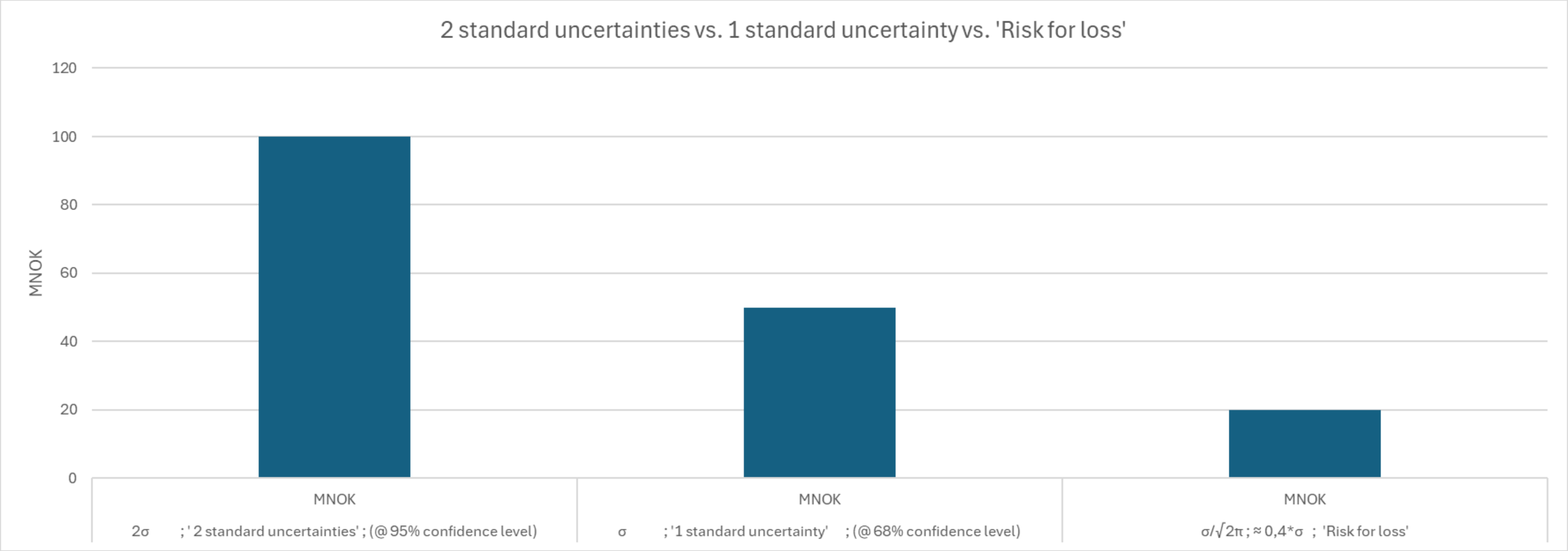
  Distance of that segment from zero error

- **'Risk for loss'** =

  Integral of probability * consequence
  over the loss side of the normal distribution

$$RLU = \frac{-\sigma}{\sqrt{2\pi}} \approx 0{,}4 * \sigma$$

RLU = Risk for Loss due to Uncertainty in measurement
σ = 1 standard deviation (equivalent to 1 standard uncertainty)


2 standard uncertainties vs. 1 standard uncertainty vs. 'Risk for loss'
MNOK
120
100
80
60
40
20
0
MNOK
2σ ; ' 2 standard uncertainties' ; (@ 95% confidence level)
MNOK
σ ; '1 standard uncertainty' ; (@ 68% confidence level)
MNOK
σ/√2π ; ≈ 0,4*σ ; 'Risk for loss'

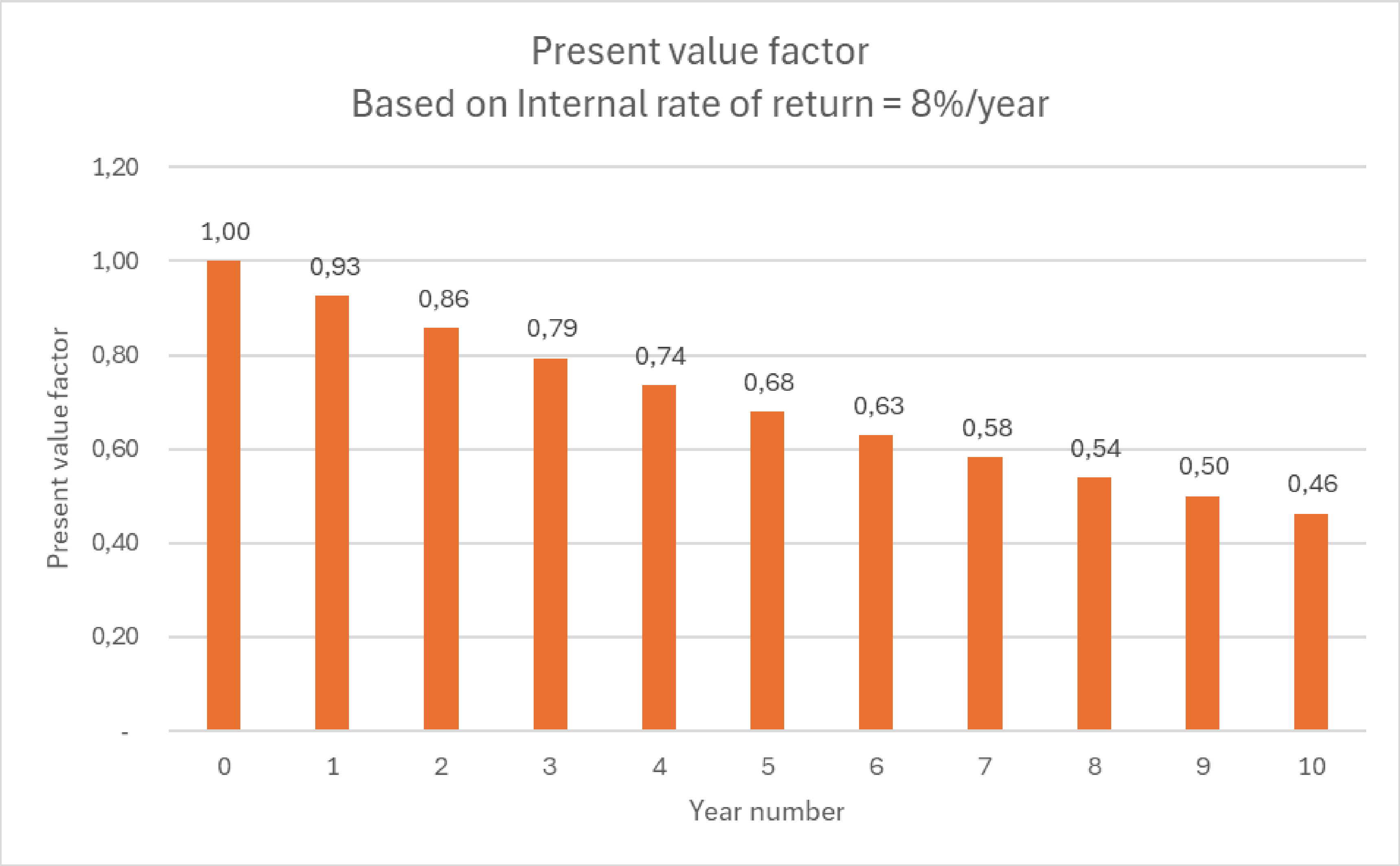
Cost is also a 'Risk for loss of profit'
Where:

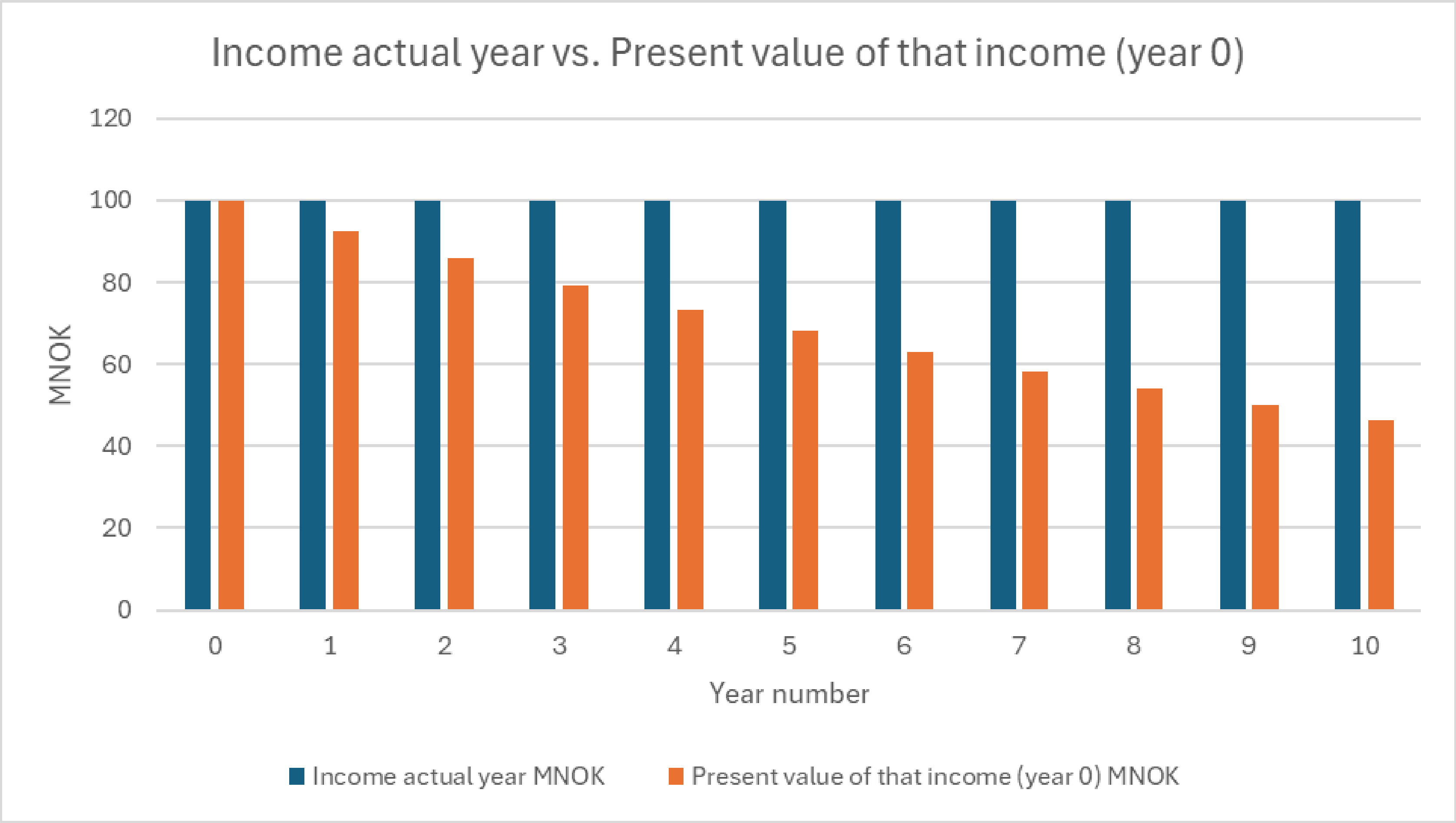
probability = 1
Economic consequence = Cost
Risk for loss due to cost = 1 * Cost

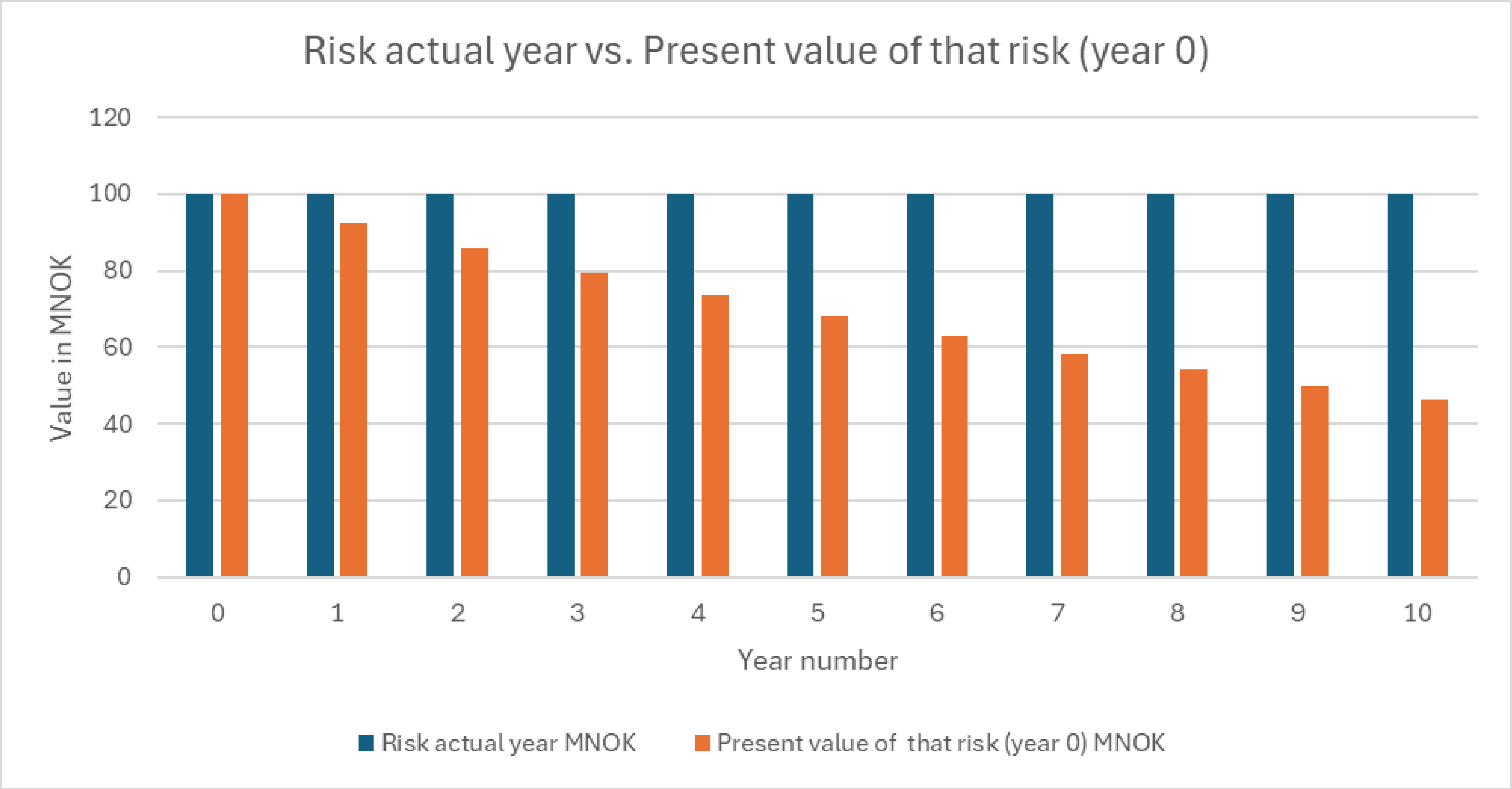
´Cost´ and ´Risk for Loss due to Uncertainty´

are both ´apples´

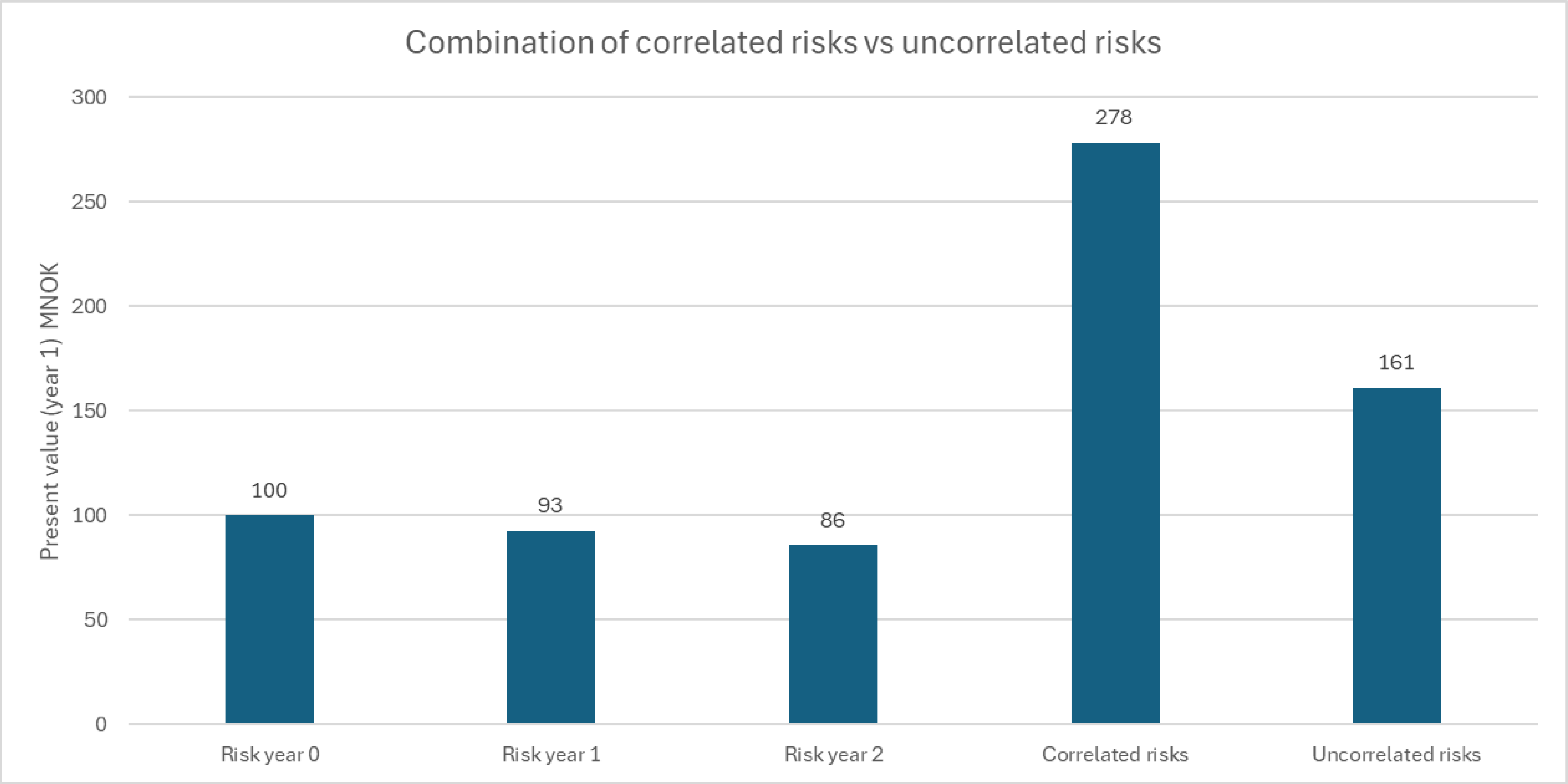
Same unit and same basical definition


Present value factor
Based on Internal rate of return = 8%/year
Present value factor
1,20
1,00
0,80
0,60
0,40
0,20
-
1,00
0,93
0,86
0,79
0,74
0,68
0,63
0,58
0,54
0,50
0,46
0
1
2
3
4
5
6
7
8
9
10
Year number

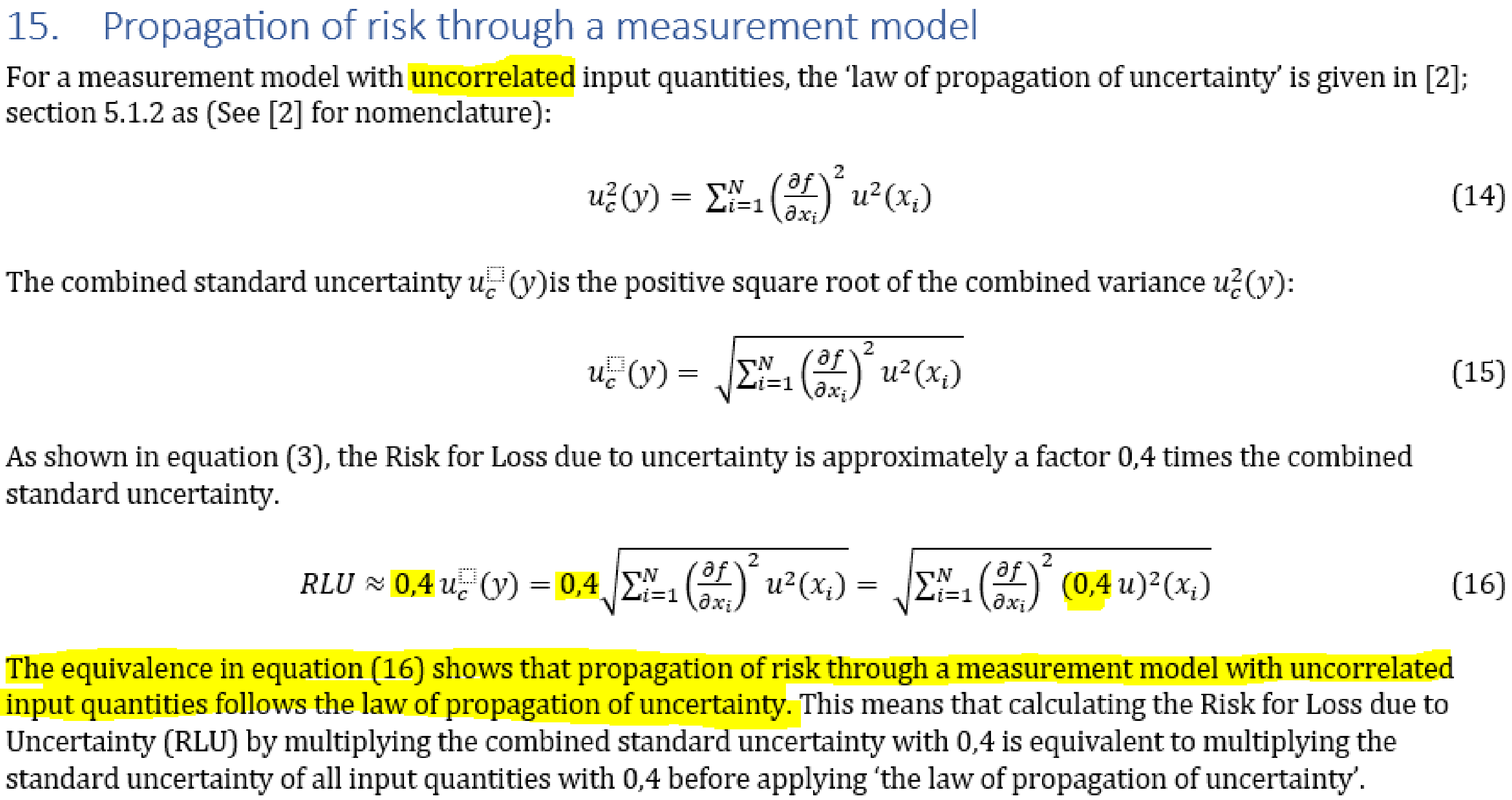

Income actual year vs. Present value of that income (year 0)
120
100
80
60
40
20
0
MNOK
0
1
2
3
4
5
6
7
8
9
10
Year number
Income actual year MNOK
Present value of that income (year 0) MNOK


Risk actual year vs. Present value of that risk (year 0)
120
100
80
60
40
20
0
Value in MNOK
0
1
2
3
4
5
6
7
8
9
10
Year number
Risk actual year MNOK
Present value of that risk (year 0) MNOK

Uncertainty, or risk, must be expressed in same unit: 'present value' - before combining uncertainties, or risk, of different years.


Combination of correlated risks vs uncorrelated risks
Present value (year 1) MNOK
300
250
200
150
100
50
0
100
93
86
278
161
Risk year 0
Risk year 1
Risk year 2
Correlated risks
Uncorrelated risks

Risks propagate through a measurement model by the "law of propagation of uncertainty'



15. Propagation of risk through a measurement model

For a measurement model with uncorrelated input quantities, the 'law of propagation of uncertainty' is given in [2]; section 5.1.2 as (See [2] for nomenclature):

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \tag{14}$$

The combined standard uncertainty $u_c(y)$is the positive square root of the combined variance $u_c^2(y)$:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} \tag{15}$$

As shown in equation (3), the Risk for Loss due to uncertainty is approximately a factor 0,4 times the combined standard uncertainty.

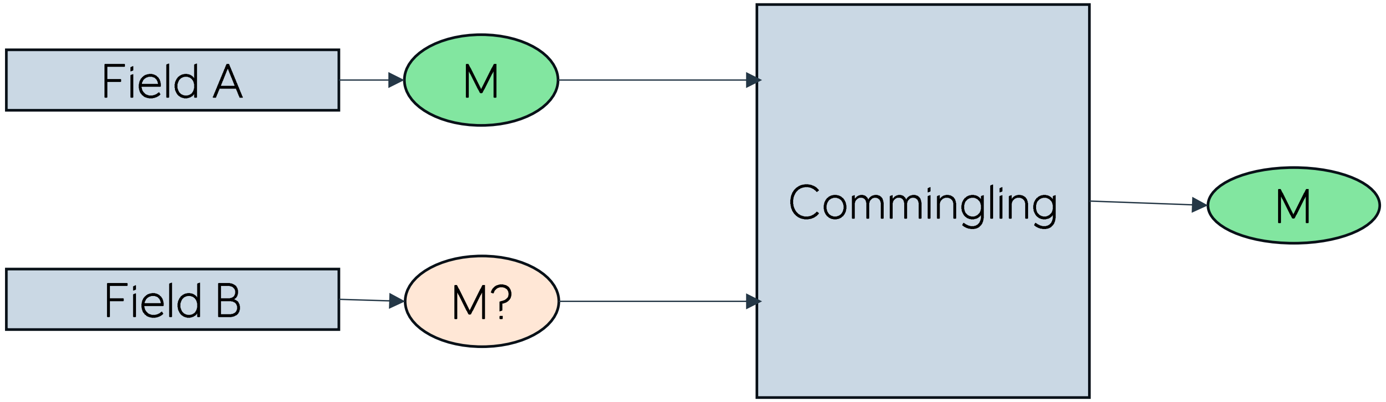
$$RLU \approx 0{,}4\, u_c(y) = 0{,}4 \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 (0{,}4\, u)^2(x_i)} \tag{16}$$

The equivalence in equation (16) shows that propagation of risk through a measurement model with uncorrelated input quantities follows the law of propagation of uncertainty. This means that calculating the Risk for Loss due to Uncertainty (RLU) by multiplying the combined standard uncertainty with 0,4 is equivalent to multiplying the standard uncertainty of all input quantities with 0,4 before applying 'the law of propagation of uncertainty'.

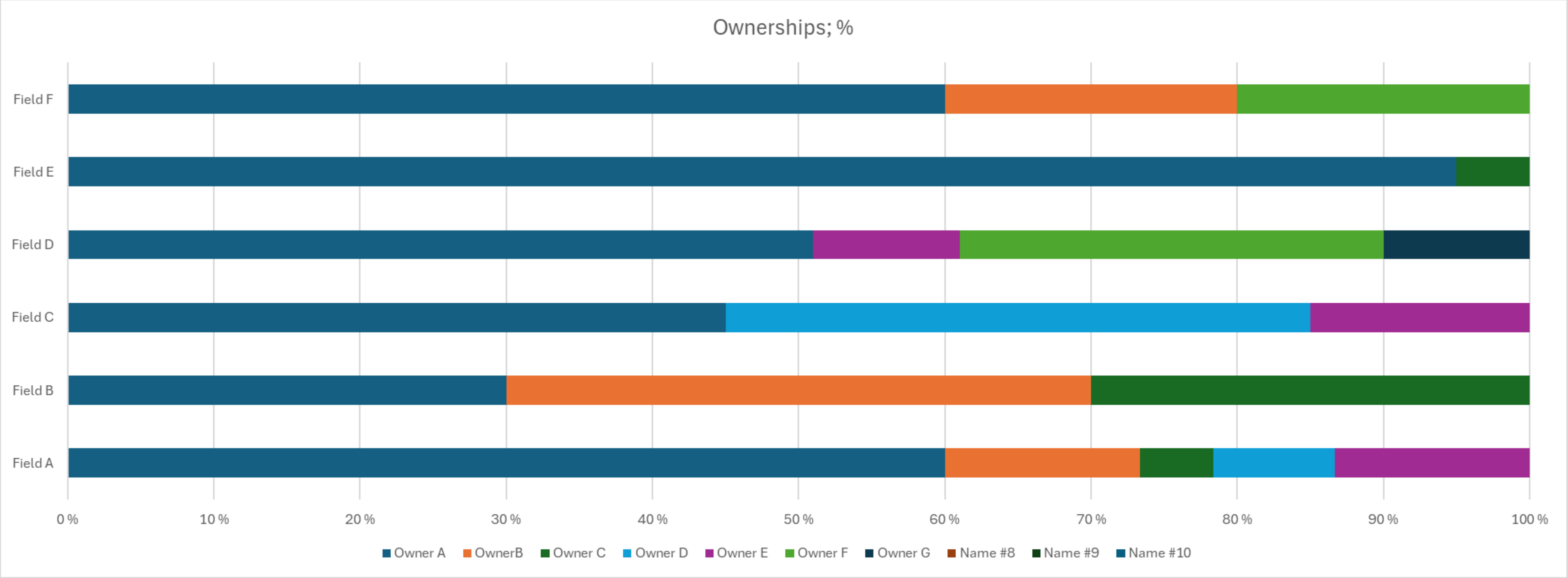
'laws of propagation of uncertainty' = ISO/IEC GUIDE 98-3:2008(E); 'Guide to the expression of uncertainty in measurement…' section 5.1.2 for uncorrelated input uncertainties and section 5.2.2 for correlated input uncertainties

The main allocation model must be included in risk calculations

- A risk-cost-benefit analysis is normally done to evalute the propagated risks of alternatives for one of the input measurements
- In a ´pro rata system´ there is an inherent risk reducing feature - which is **not** there in a ´by difference system´
- For evaluating a ´pro-rata system´ the ´pro-rata´ model must be applied when propagating risks through the model



Differences in ownership has an impact on risk analysis



´average difference in ownerships´:

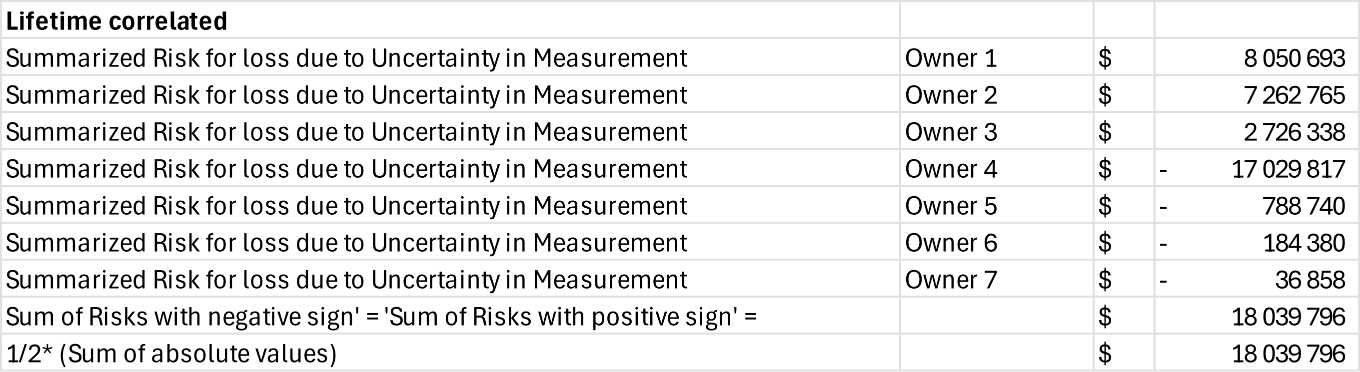
				Absolute
	Field A	Field B	Difference	Difference
Owner #1	80 %	50 %	30 %	30 %
Owner #2	20 %	5 %	15 %	15 %
Owner #3	0 %	45 %	-45 %	45 %
Sum	100 %	100 %		90 %
			Average difference in ownership:	45 %

- The ´average difference in ownership´ method from NORSOK I-106 (2014) collapses with more than two fields
- For multiple fields there is hardly a way around calculating risk for each individual owner in the whole system

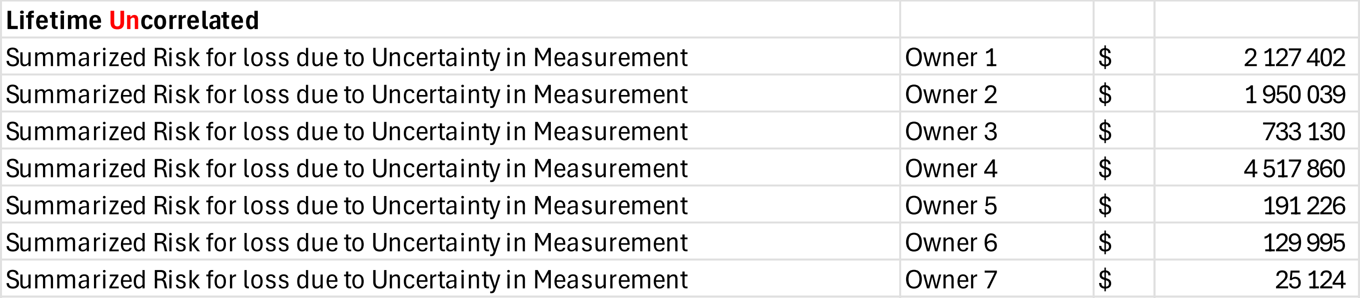
There may exists many alternative ways, reasonable simplifications or mathematically equivalent ways of performing the risk-cost-benefit evaluation. Anyhow, the following procedure is one example of how a risk-cost-benefit evaluation can be performed for the described context:

1. For each year, use the forecasted quantities and the allocation model to calculate the quantity allocated to each field.
2. Redo estimation in 1 with forecasted quantity for field A plus Risk for Loss from uncertainty (RLU) in equation (3) for metering system X).
3. Calculate the difference between 1. and 2. The difference is the Risk for Loss from uncertainty (RLU) in the quantity allocated to each field as propagated through the allocation model from RLU for metering system X for field A
4. For each field, calculate the RLU$ (equation 4), for each year, from the result of 3.
5. For each field, calculate the $PVRLU\$_{(year\ i)}$ (equation 6), for each year
6. Calculate $PVLRLU\$$ for each field by combining the risks for each year by a method including eventual correlation between individual years. For example, by equation (7) or (8).
7. Calculate the $PVLRLU\$$ for each individual owner in each individual field
   (By multiplying the $PVLRLU\$$ for each field with each owner's share in that field.)
8. Add $PVLRLU\$$ for each individual owner in all the fields to a total risk for loss for each owner
9. Calculate the total $PVLRLU\$$ by adding the risk for loss for each owner in the system having a calculated positive risk for loss from step 8.
10. Calculate $TRUMC\$$ by adding the cost of metering concept X of Field A ($RLC\$$) to total $PVLRLU\$$ from step 9.
11. Redo step 1 to 10 for metering concept Y.
12. The $TRUMC\$$'s for metering concept X and Y can then be compared.

One simplification can be to use the difference in RLU between metering concept X and Y in step 2. That difference must then be compared with the difference in cost ($RLC\$$) between metering concept X and Y.



Lifetime correlated			
Summarized Risk for loss due to Uncertainty in Measurement	Owner 1	$	8 050 693
Summarized Risk for loss due to Uncertainty in Measurement	Owner 2	$	7 262 765
Summarized Risk for loss due to Uncertainty in Measurement	Owner 3	$	2 726 338
Summarized Risk for loss due to Uncertainty in Measurement	Owner 4	$	- 17 029 817
Summarized Risk for loss due to Uncertainty in Measurement	Owner 5	$	- 788 740
Summarized Risk for loss due to Uncertainty in Measurement	Owner 6	$	- 184 380
Summarized Risk for loss due to Uncertainty in Measurement	Owner 7	$	- 36 858
Sum of Risks with negative sign' = 'Sum of Risks with positive sign' =		$	18 039 796
1/2* (Sum of absolute values)		$	18 039 796



Lifetime Uncorrelated			
Summarized Risk for loss due to Uncertainty in Measurement	Owner 1	$	2 127 402
Summarized Risk for loss due to Uncertainty in Measurement	Owner 2	$	1 950 039
Summarized Risk for loss due to Uncertainty in Measurement	Owner 3	$	733 130
Summarized Risk for loss due to Uncertainty in Measurement	Owner 4	$	4 517 860
Summarized Risk for loss due to Uncertainty in Measurement	Owner 5	$	191 226
Summarized Risk for loss due to Uncertainty in Measurement	Owner 6	$	129 995
Summarized Risk for loss due to Uncertainty in Measurement	Owner 7	$	25 124

The idea seemed to work for correlated uncertainties from year to year

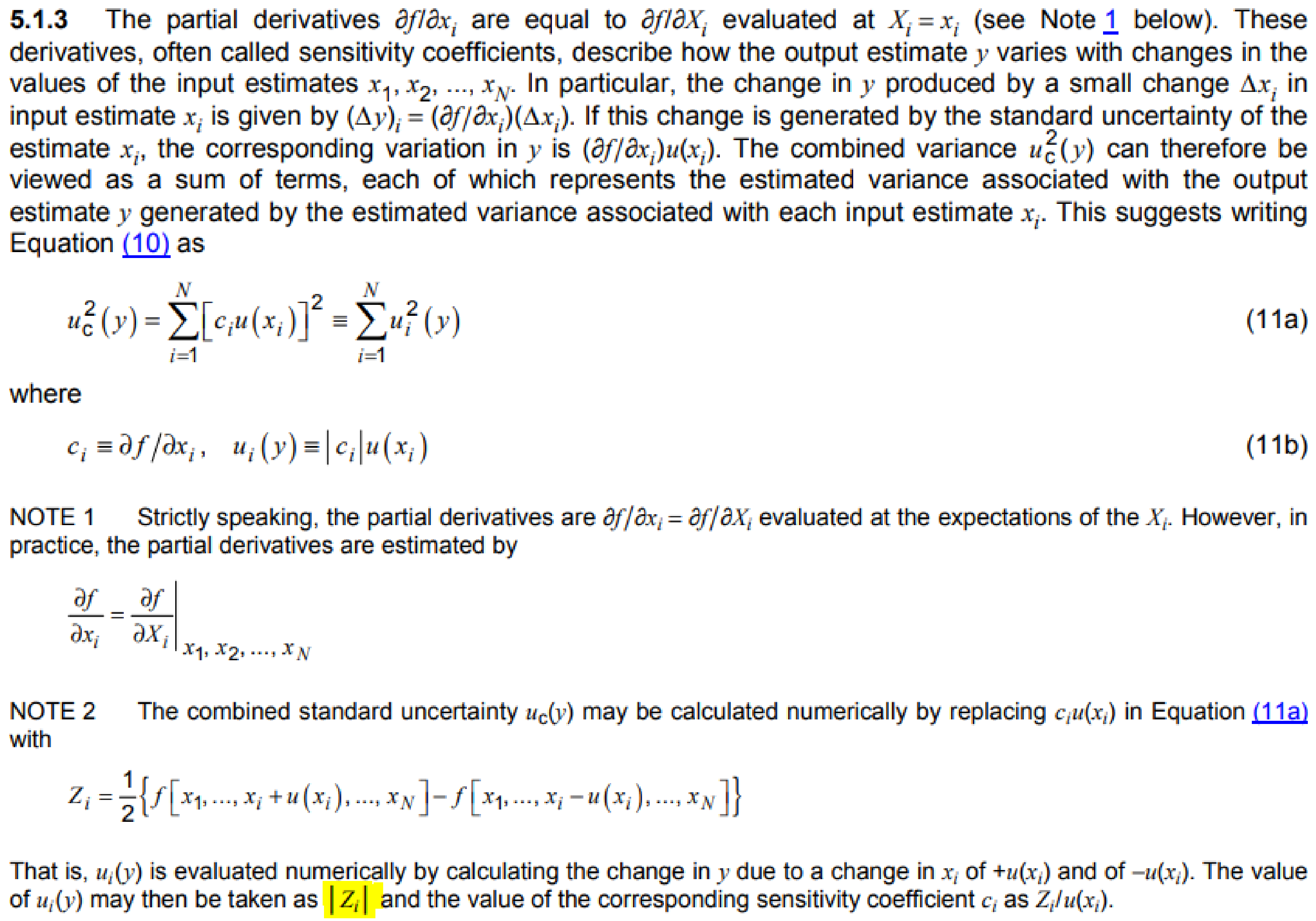
However

The information about sign was lost for uncorrelated uncertainties from year to year

This will also happen if we strictly follow the numerical method in ISO/IEC GUIDE 98-3:2008(E) (5.2.3 note 2)

(Presumably, there would be similar problems with a Monte Carlo analysis)

If the numerical method is done by the book – the sign will also be lost



5.1.3 The partial derivatives $\partial f/\partial x_i$ are equal to $\partial f/\partial X_i$ evaluated at $X_i = x_i$ (see Note 1 below). These derivatives, often called sensitivity coefficients, describe how the output estimate y varies with changes in the values of the input estimates $x_1, x_2, \ldots, x_N$. In particular, the change in y produced by a small change Δx_i in input estimate x_i is given by $(\Delta y)_i = (\partial f/\partial x_i)(\Delta x_i)$. If this change is generated by the standard uncertainty of the estimate x_i, the corresponding variation in y is $(\partial f/\partial x_i)u(x_i)$. The combined variance $u_c^2(y)$ can therefore be viewed as a sum of terms, each of which represents the estimated variance associated with the output estimate y generated by the estimated variance associated with each input estimate x_i. This suggests writing Equation (10) as

$$u_c^2(y) = \sum_{i=1}^{N} \left[c_i u(x_i)\right]^2 \equiv \sum_{i=1}^{N} u_i^2(y) \tag{11a}$$

where

$$c_i \equiv \partial f/\partial x_i, \quad u_i(y) \equiv |c_i| u(x_i) \tag{11b}$$

NOTE 1 Strictly speaking, the partial derivatives are $\partial f/\partial x_i = \partial f/\partial X_i$ evaluated at the expectations of the X_i. However, in practice, the partial derivatives are estimated by

$$\frac{\partial f}{\partial x_i} = \left.\frac{\partial f}{\partial X_i}\right|_{x_1, x_2, \ldots, x_N}$$

NOTE 2 The combined standard uncertainty $u_c(y)$ may be calculated numerically by replacing $c_i u(x_i)$ in Equation (11a) with

$$Z_i = \frac{1}{2}\left\{f\left[x_1, \ldots, x_i + u(x_i), \ldots, x_N\right] - f\left[x_1, \ldots, x_i - u(x_i), \ldots, x_N\right]\right\}$$

That is, $u_i(y)$ is evaluated numerically by calculating the change in y due to a change in x_i of $+u(x_i)$ and of $-u(x_i)$. The value of $u_i(y)$ may then be taken as $|Z_i|$ and the value of the corresponding sensitivity coefficient c_i as $Z_i/u(x_i)$.

The information about sign will be lost if risks are propagated by ´the laws of propagation of uncertainty´ (in accordance with ISO/IEC GUIDE 98-3:2008(E) ´The Guide to expression of uncertainty´)

There may exists many alternative ways, reasonable simplifications or mathematically equivalent ways of performing the risk-cost-benefit evaluation. Anyhow, the following procedure is one example of how a risk-cost-benefit evaluation can be performed for the described context:

1. For each year, use the forecasted quantities and the allocation model to calculate the quantity allocated to each field.
2. Redo estimation in 1 with forecasted quantity for field A plus Risk for Loss from uncertainty (RLU) in equation (3) for metering system X).
3. Calculate the difference betw [illegible] 2. The difference is th [illegible] oss from uncertainty (RLU) in the quantity allocated to each field as propagated thr [illegible] n model fr [illegible] ng system X for field A
4. For each field, calculate the [illegible] of 3.
5. For each field, calculate the PVR [illegible]
6. Calculate $PVLRLU\$$ for each field by [illegible] year by a method including eventual correlation between individual years. For example, by equ [illegible]
7. Calculate the $PVLRLU\$$ for each [illegible] ld (By multiplying the $PVLRL$ [illegible] field.)
8. Add $PVLRLU\$$ for each indi [illegible] the fields [illegible] ss for each owner
9. Calculate the total $PVLRLU\$$ [illegible] e risk for loss for ea [illegible] the system having a calculated positive risk for loss from step 8.
10. Calculate $TRUMC\$$ by adding the cost of metering concept X of Field A ($RLC\$$) to total $PVLRLU\$$ from step 9.
11. Redo step 1 to 10 for metering concept Y.
12. The $TRUMC\$$'s for metering concept X and Y can then be compared.

One simplification can be to use the difference in RLU between metering concept X and Y in step 2. That difference must then be compared with the difference in cost ($RLC\$$) between metering concept X and Y.

How do we combine the calculated benefit of risk reduction on each owner to a total benefit of risk reduction for a reduction of measurement uncertainty?

There is additional information:

1. A loss for one set of parties is gained by the other set of parties
2. There is a 50 % chance for loss

Lifetime Uncorrelated			
Summarized Risk for loss due to Uncertainty in Measurement	Owner 1	$	2 127 402
Summarized Risk for loss due to Uncertainty in Measurement	Owner 2	$	1 950 039
Summarized Risk for loss due to Uncertainty in Measurement	Owner 3	$	733 130
Summarized Risk for loss due to Uncertainty in Measurement	Owner 4	$	4 517 860
Summarized Risk for loss due to Uncertainty in Measurement	Owner 5	$	191 226
Summarized Risk for loss due to Uncertainty in Measurement	Owner 6	$	129 995
Summarized Risk for loss due to Uncertainty in Measurement	Owner 7	$	25 124
Sum	Owners	$	9 674 775
1/2* (Sum of absolute values)		$	4 837 388

To divide by 2 or to not divide by 2 – that is the question.

If so, the suggested chapter on : ´Risk for loss from uncertainty in allocation systems´, reduces to:

1. Follow Guide to the expression of Uncertainty in Measurement
2. Transform either all input uncertainties or all output uncertainties to risk domain
3. Calculate ´combined risk for loss´ as: $\frac{1}{2} * \sum_{i=1}^{i=n} ABS(Risk\ for\ owner\ i)^{*}$

*Taking the absolute value is superfluos as all values are positive – but it shows the link

Summary of hypotheses and questions

Combination of uncertainties by the ´laws of propagation of uncertainty´ also applies in ´risk domain´
Transformation to risk domain can either be done on all input uncertainties or on ouput uncertaintes
Uncertainty, or risk, should be expressed in the unit ´present value´ before combining years

Divide by 2 or not divide by 2 ?

Please review the draft to the NORSOK I-106 and help improve it

NORSOK I-106 Annex C:
Risk – cost - benefit evaluation for allocation systems

NFOGM HCM allocation workshop 2025
2025-06-12
Dag Hendrik Flølo
This presentation can be published by NFOGM