Global Flow Measurement Workshop
24-26 October 2023
Technical Paper

Further Adventures in Integrated Risk Exposure for Cost Benefit Analysis – from Economics to Quarks

Phil Stockton, Accord Energy Solutions Ltd

1 INTRODUCTION

In 2009, the author presented a paper at the 27th North Sea Flow Measurement Workshop [1] in which a method was proposed that expressed the risk of loss associated with measurement or allocation uncertainty in terms of a monetary value. This was proposed as a method to be utilised in cost benefit analyses to allow more expensive meters, or methods of allocation, with lower uncertainties, to be compared against cheaper alternatives.

The approach has subsequently been cited in several Flow Measurement Workshop papers, for example [2], [3], [4] and appears in both the NORSOK Standard I-106 [5] and the UK Measurement Guidelines [6].

The original paper presented a relatively brief discussion and development of the approach. This paper expands on the underlying basis of the approach and discusses issues that have arisen subsequently:

- Is there any supporting evidence that it is the correct approach?
- What is the underlying basis of the aversion to risk of loss?
- Net present value (NPV) calculations; how do these fit in with the approach? Does a reduction in uncertainty change the discount rate?
- Timelines; how do the risks evolve with time?
- Do recent developments in economic analysis associated with ergodic theory inform the approach?

The paper is organised as follows:

- Section 2 presents the Integrated Risked Exposure to Loss (IRE Loss) equation, discusses its basis and development.
- Section 3 discusses the concept of utility in terms of attitudes to risk aversion.
- Section 4 describes the factors necessary to compute cash flows for NPV calculations and also the appropriate discount rate for evaluating measurement systems.
- Section 5 describes models for the evolution of measurement error over time.
- Section 6 highlights recent developments in economic theory associated with ergodicity and speculates whether the concepts are applicable to cost benefit analysis of measurement devices.

2 INTEGRATED RISKED EXPOSURE TO LOSS

2.1 Basic Equation

The risked exposure to loss of revenue in the original paper was expressed by:

$$L = \frac{U}{\sqrt{8\pi}} \qquad (1)$$

Where:

L	IRE Loss in revenue terms
U	Absolute uncertainty in the measured or allocated quantity expressed, at the 95% confidence level, converted to equivalent revenue.

This is a convenient form of the equation as uncertainties (usually at the 95% confidence level) are readily available for measurement devices and it is the most common metric of the expression of uncertainty in the energy industries.

The NORSOK Standard I-106 [5] abbreviates the equation even further:

$$L \sim 0.2\, U \qquad (2)$$

Hence, the exposure to loss is reduced to a factor, roughly 0.2 (i.e. $1/\sqrt{8\pi}$), which is multiplied by the uncertainty.

The absolute uncertainty of the measured quantity, e.g. flow of oil, can be readily converted to an equivalent revenue quantity using the oil price and then summed (integrated) over time to obtain a cumulative exposure to risk of loss of revenue or IRE Loss. This can be compared with the cost of the measurement equipment and allow a cost benefit analysis to be performed. In simple terms, the equation allows the benefit of the uncertainty associated with a measurement device to be compared with its cost.

2.2 Cost Benefit Analysis

The term ‘agent’ is used to represent decision makers associated with a project or company, tasked with comparing and selecting measurement equipment or allocation systems using cost benefit analysis.

Revenues and costs are expressed in dollars ($).

For example, an agent is faced with a decision about which of two flow measurement devices to install:

- Meter A, costs $1,000,000 and has a relative uncertainty of ±0.25%
- Meter B, costs $250,000 and has a relative uncertainty of ±1.0%

Assuming the selected meter is required to measure oil production at a constant flow of 1,000 t/d and the lifetime of the field is 5 years, which meter should the agent select?

Equation (1) allows a cost benefit analysis to be performed to answer this question and the calculation is illustrated in Table 1:

Table 1 – Simple Example of Cost Benefit Analysis

	Meter A	Meter B	Difference
Total oil produced over 5 years (t)	1,825,000	1,825,000	
Total oil revenue over 5 years ($)	1,368,750,000 (Note 1)	1,368,750,000 (Note 1)	
Meter relative uncertainty (±%)	±0.25%	±1.0%	

	Meter A	Meter B	Difference
Absolute produced oil uncertainty (t)	4,563	18,250	
Absolute revenue uncertainty, U ($)	3,421,875	13,687,500	
IRE loss of revenue, L ($)	696,495	2,785,981	-2,089,486
Cost ($)	1,000,000	250,000	750,000

Note 1: The conversion from produced oil uncertainty in tonnes to revenue terms is calculated based on an oil price of $750/t.

From the above analysis, the more expensive flow meter, Meter A, appears to be worth the additional cost of $750,000 as it reduces the IRE Loss of revenue by over $2,000,000. The reverse would be true if the production only lasted a year or would be marginal if it lasted two years.

In this simplified illustration no account has been taken of discounting the revenue cash flow over time; this is addressed in Section 4.

Also, a subtle assumption has been made in the above discussion in that the uncertainty in a flow rate, expressed in t/d, can be integrated over time by simple multiplication of the time interval to obtain the uncertainty in the total cumulative oil production in t. The validity of this assumption is discussed in Section 5 and extended in Section 6.

2.3 The Importance of the Standard Deviation

It should be emphasised that equation (1) is only valid if the uncertainty, U, is expressed at the 95% confidence level. There is no special significance about the 95% level uncertainty it is merely a useful convention with which to express uncertainties.

A more rigorous form of the equation is:

$$L = \frac{U_k}{k\sqrt{2\pi}} \qquad (3)$$

Where:

k	Coverage factor
U_k	Absolute expanded uncertainty with coverage factor k

At the 95% confidence level, k=1.96. In effect, U_k/k is the standard uncertainty or standard deviation. If the uncertainty was expressed at the 99.7% confidence level then k =3. However the uncertainty is expressed, it has to be converted to the standard deviation. In the 2009 paper the exposure to loss was expressed first in terms of the standard deviation:

$$L = \frac{\sigma}{\sqrt{2\pi}} \qquad (4)$$

Where:

σ	Standard deviation converted to equivalent revenue

The basis and development of the equation is described in the next sections.

2.4 Premise

The development of the equation is based on two postulates:

The uncertainty in the measured quantity is described by the normal distribution.

The agent has a linear aversion to loss, but is indifferent to gains; i.e. it is the exposure to loss alone that the agent is concerned about.

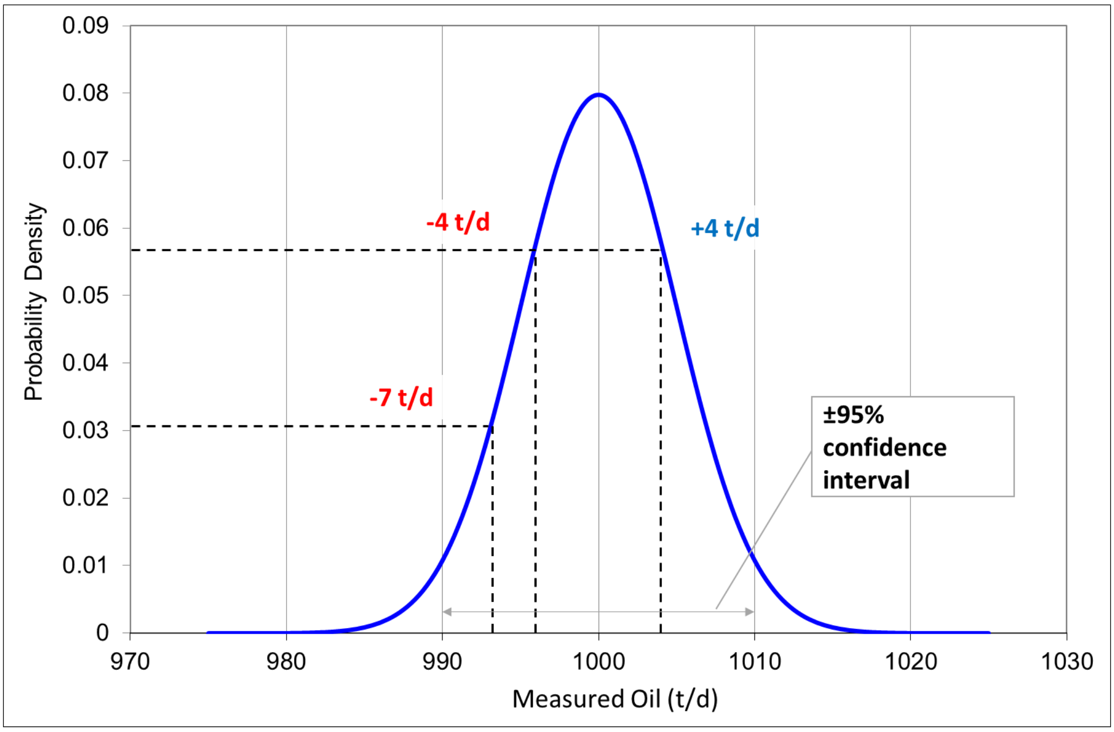
The justification for the first postulate and its implications are described in the next section. The second postulate is discussed further in Section 2.6 and Section 3.

2.5 Normal Distribution

The uncertainty in any output measurement or allocated quantity is almost certainly characterised by the normal distribution due to the Central Limit Theorem (discussed in Appendix G.2 of the Guide to the expression of uncertainty in measurement (GUM) [7]).

Figure 2-1 illustrates the normal probability distribution for the simple example of a flow of 1,000 t/d measured by a meter with an uncertainty of ±1% (expressed at the 95% confidence level) or ±10 t/d in absolute terms, (equivalent to a standard uncertainty or standard deviation of ±5.1 t/d):

Figure 2-1: Flow Measurement – Normal Distribution



The probability density of the vertical axis is not a probability itself as it relates to an individual flow rate. Probabilities can only be expressed as lying between two flow rates. For example, the familiar 95% uncertainty values express the range of values that 95% of measured flow rates will fall from the true value.

The graph is based on a hypothetical true oil flow of 1,000 t/d. The flow measurement device's reported flow rate is represented by the horizontal axis. The probability of the reported measurement is proportional to the height of the blue line (as represented by the vertical ordinate axis). Hence, the probability of reporting a value of 993 t/d, under-reading by 7 t/d, is roughly half that of under-reading by 4 t/d. There is an equal and opposite probability of over-reading by the same amounts, as indicated at +4 t/d.

As discussed in Annex D of the GUM [7], the true value and actual error resulting from an under- or over-reading are unknowable quantities. All that can be known is the probability distribution of the amount of deviation of the actual measurement from this true value. The term error used here does not mean a gross error due to a faulty measurement, it is used to refer to the difference between the reported (measurement reading) and (unknown) true value consistent with the legitimate uncertainty in the measurement device.

The probability density function (blue line) is described by the equation:

$$pdf = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (5)$$

Where:

pdf	Probability density function
x	Measured flow rate (t/d)
μ	True flow rate (t/d)

In the example, μ is 1,000 t/d and the standard deviation, σ is 5.1 t/d (i.e. 10/1.96).

2.6 Integrated Risked Exposure to Loss

It is more convenient to plot the distribution in terms of gains and losses about the true value:

$$z = x - \mu \qquad (6)$$

Where:

z	Gain/loss relative to true value (t/d)

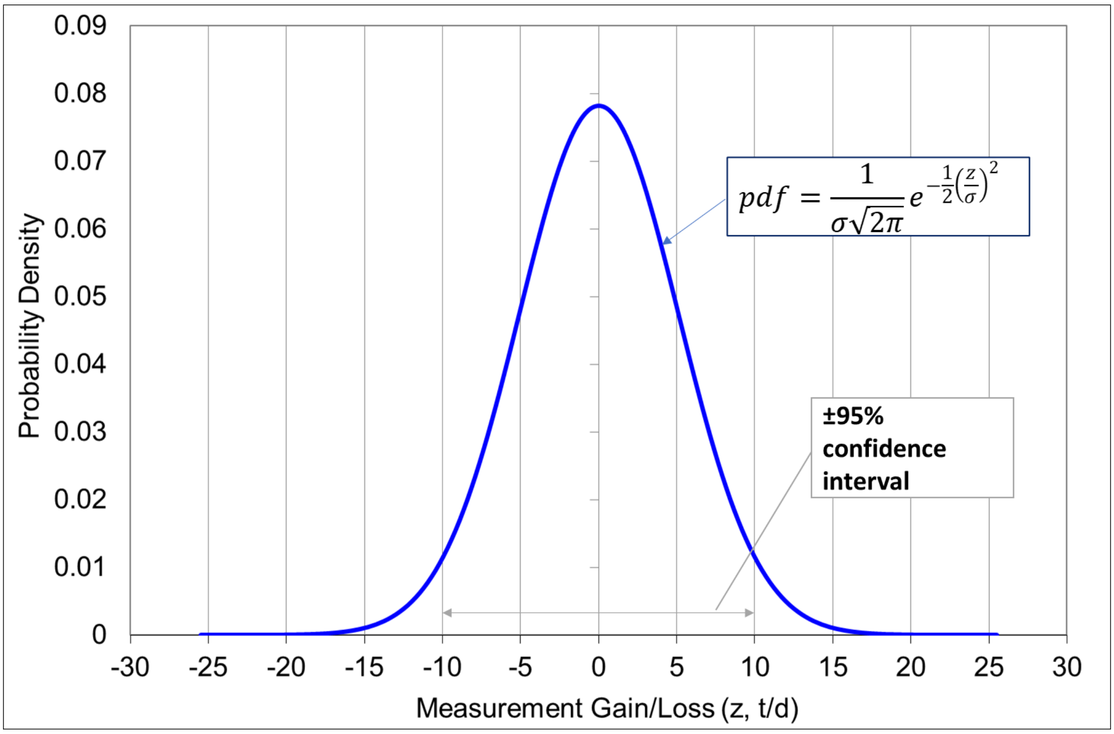
The gain/loss (z) is equivalent to the error mentioned above and the terms are used interchangeably.

Hence, (5) becomes:

$$pdf = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2} \qquad (7)$$

This is plotted in Figure 2-2:

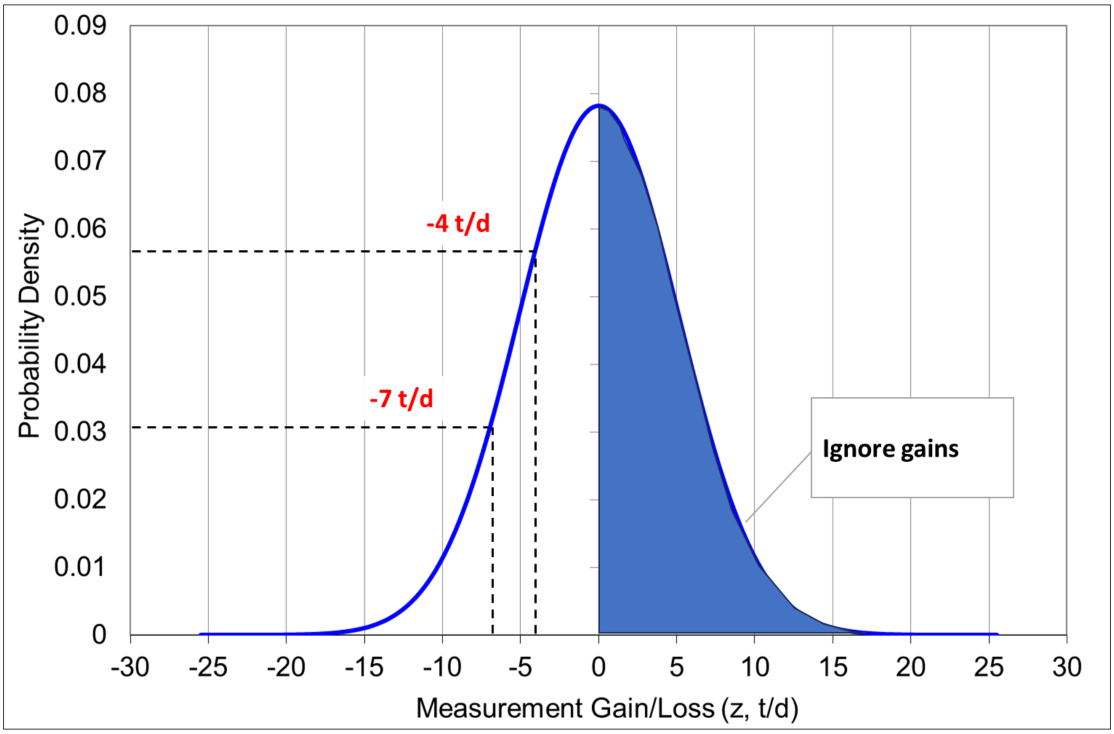
Figure 2-2: Gains and Losses – Normal Distribution



In the above plot, the probability of a 7 te/d loss is approximately half that of a 4 te/d loss. To capture the diminishing probability of larger losses the size of the loss can be multiplied by its probability of occurrence and summed, (or more strictly integrated), over the possible range of under-allocation.

Because the measured oil is normally distributed, there is an equal probability of an over-allocation, or gain, However, it is the exposure to under-measurement or loss alone that is being considered as illustrated in Figure 2-3:

Figure 2-3: Losses – Normal Distribution

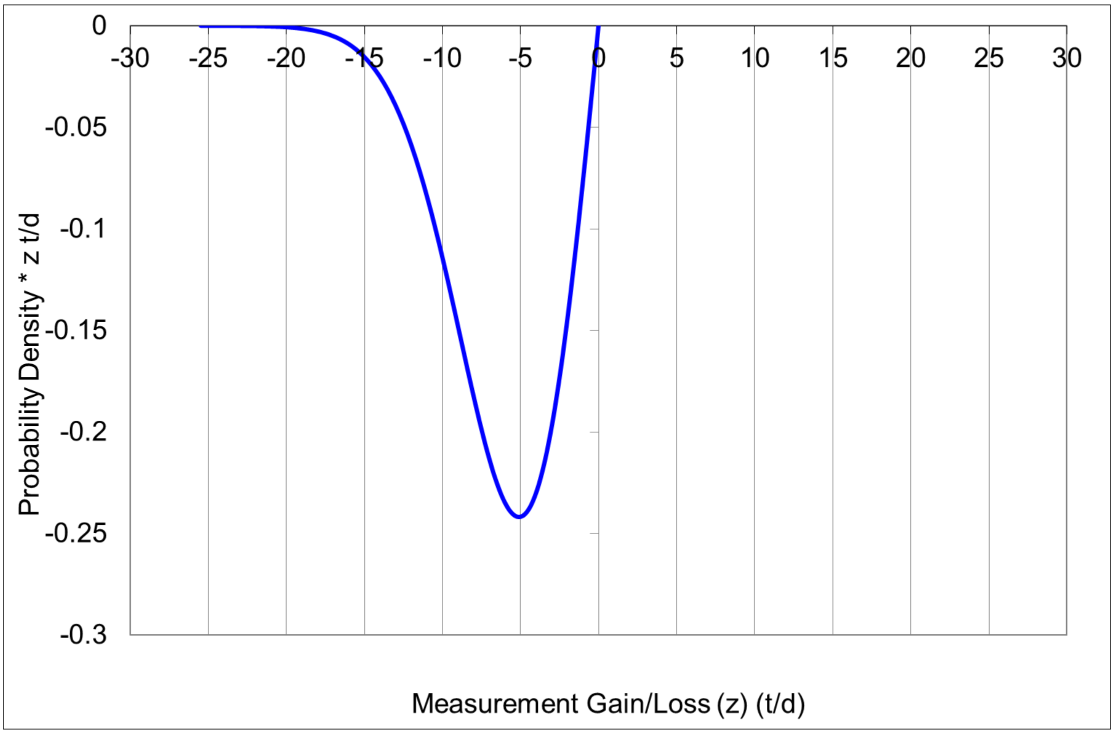


Multiplying each level of loss (negative values of z) by the probability of its occurrence:

$$pdf * z = \frac{z}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2} \tag{8}$$

Which is plotted in Figure 2-4:

Figure 2-4: Loss Multiplied by Probability Density



The product of z and the pdf is negative because z is a negative quantity for a loss. The integrated risked exposure to gain/loss (termed R) is obtained from the area under this curve, which is given by the integral:

$$R = \int_{-\infty}^{0} \frac{z}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2} dz \tag{9}$$

The gains, i.e. z>0, are ignored so the integral is over the limits minus infinity to zero. Integrating:

$$R = \left[\frac{-\sigma}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2}\right]_{-\infty}^{0} \tag{10}$$

Which, when evaluated, gives:

$$R = \frac{-\sigma}{\sqrt{2\pi}} \tag{11}$$

R is termed the integrated gain/loss, it is positive for a gain and negative for a loss. The IRE loss, L, is just the negative of R and hence equation (4) is obtained as the negative of (11).

2.7 Alternatives?

In the original 2009 paper an alternative method of evaluating the exposure to loss was considered. This was termed the uncertainty approach and the exposure to loss was given by:

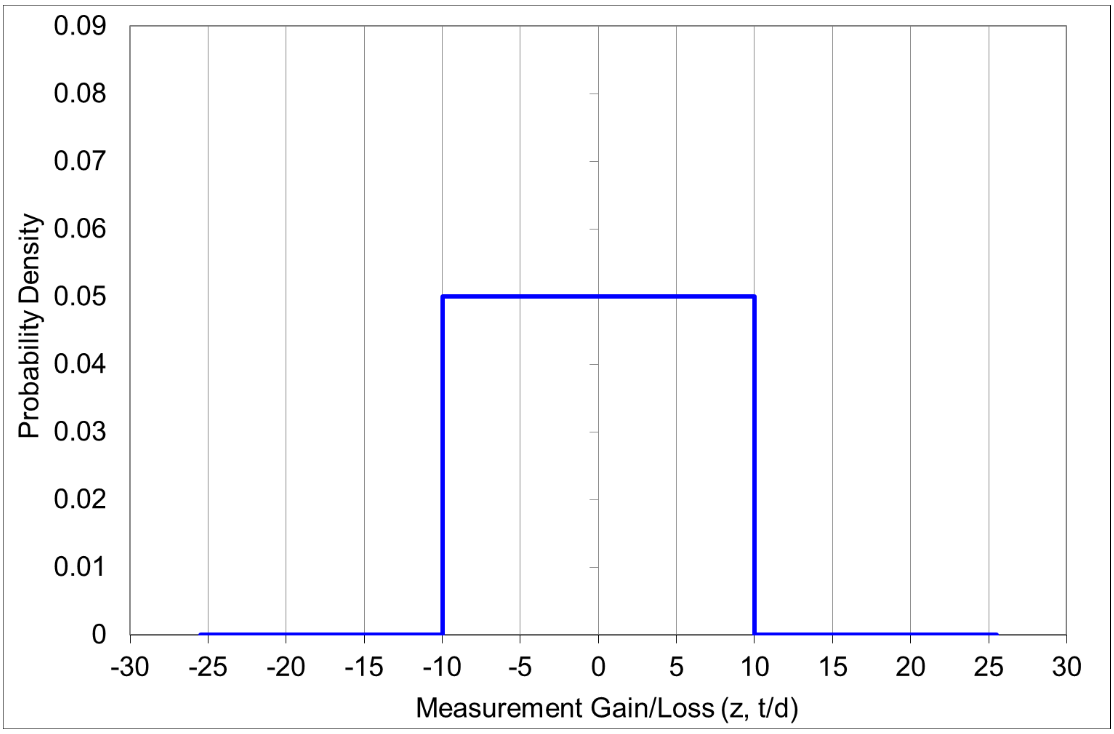
$$L_{alt} = U_{95} \tag{12}$$

Hence, the loss is equal to the uncertainty of the measurement device expressed at the 95% confidence level. This results in an exposure to loss roughly 5 times that of the IRE loss.

This equation was presented, as in the author's experience, it was, and is, a commonly used approach in performing cost benefit analyses of measurement devices. However, it does not appear to have an underlying developmental basis in contrast with the IRE loss.

The only case where it could conceivably be derived in the same way as equation (4), is for the case where the probability distribution of z is uniform as depicted in Figure 2-5:

Figure 2-5: Gains and Losses – Uniform Rectangular Distribution



However, due to the Central Limit Theorem, this is unlikely to be the case.

3 UTILITY

3.1 Loss Averse Agent

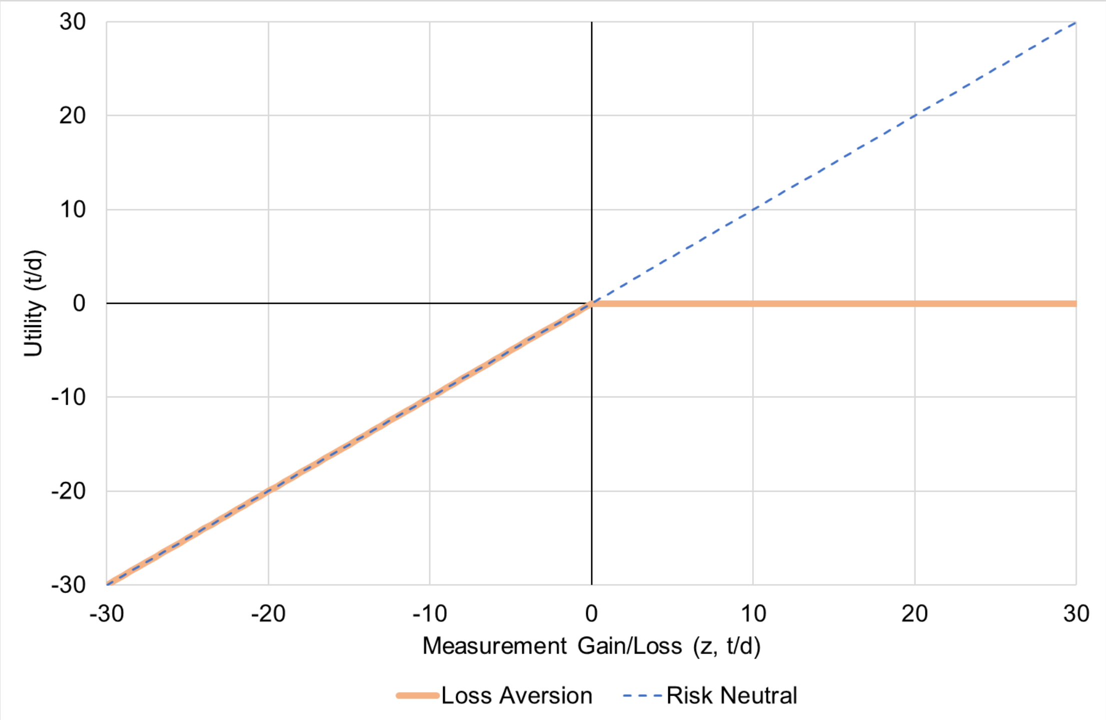
The second postulate only considers losses and ignores the equally likely gains. If the gains were included in the analysis the exposure to gain/loss would integrate to zero. Hence, the second postulate assumes that the agent is only concerned with the exposure to losses and ignores any upside from possible gains. Is there any evidence to support that this is appropriate?

The second postulate is, in effect, a utility function. The concept of utility is widely used in economics to model worth or value [8], for example in the quantification of risk versus return.

Also, the Introduction to Measurement Science and Engineering [9] presents utility analysis as a tool to compare and select instruments.

The utility described by the second postulate is presented by the orange line in Figure 3-1, in terms of gains and losses:

Figure 3-1: Utility Function of Second Postulate



To be consistent with the example above, again the graph is nominally based on a hypothetical true oil flow of 1,000 t/d. The horizontal axis plots a range of values in terms of gains or losses (z), relative to this true value, resulting from the measurement.

The vertical axis represents the utility of the agent. For measured values below the true flow, a loss is experienced, and the agent's utility is equal to the loss. For measured values above the true flow, i.e. a gain, the utility is zero. In effect, this is describing the loss aversion postulate in terms of utility. To the loss averse agent, losses loom as their full lost value and any gains are ignored. It is the exposure to loss alone that the agent is concerned about.

3.2 Neutral Agent

The dashed blue line represents the utility of a neutral agent who values gains and losses equal to their actual value. As pointed out in the 2009 paper an agent with a neutral utility would be indifferent to how accurate the measurement device is.

The logical consequence of this is that, since on average, the gains will even out with the losses, the neutral agent should always choose the cheapest measurement solution.

For example, if purchasing an ultrasonic meter, such a neutral agent should prefer the cheapest version of the meter, e.g. a clamp on device compared with a more expensive in-line meter.

This attitude, or approach, could be taken to absurd levels:

- why not purchase wildly inaccurate meters?

- Why maintain them?
- Why calibrate?

This is arguably the logical, though extreme, conclusion of acting as a neutral agent.

As stated above, the rationale is that gains losses due to measurement will average out and tend towards zero. This could be in two ways:

- over time for an individual meter
- across many systems of meters.

The evolution and averaging of measurement errors through time and across systems is discussed further in Section 5.

Importantly, the neutral agent described above does not account for other factors which affect the agent's decision-making process, for example:

- Reliability; cheaper, poorer quality measurement devices are more likely to fail.
- The reputation of the company as a reasonable and prudent operator.
- Poorer quality meters are more likely to develop gross errors and result in more frequent and larger mis-measurements. Mis-measurement corrections can be very significant resulting in multi-million dollar repayments. These can also be expensive in terms of adminstrative, accounting and legal costs which can easily exceed any savings made in meter costs.
- Regulatory requirements.
- Losses are not necessarily equal and opposite to gains.

The final bullet highlights a problem with the risk neutrality approach. The gains and losses have been considered in simple revenue terms. If an agent accumulates sufficient losses, this can result in liquidation of the company. It is unlikely that the consistent under-reading of a single meter would solely result in the liquidation of a company, but it would contribute to its combined losses. Too many gains cannot result in liquidation but too many losses can.

Finally, it appears inconceivable that a project constructed to produce a high value product such as oil, hydrocarbon gas, hydrogen, captured CO_2, etc. would not wish to measure this accurately to ensure the correct value is obtained.

3.3 Utility Functions

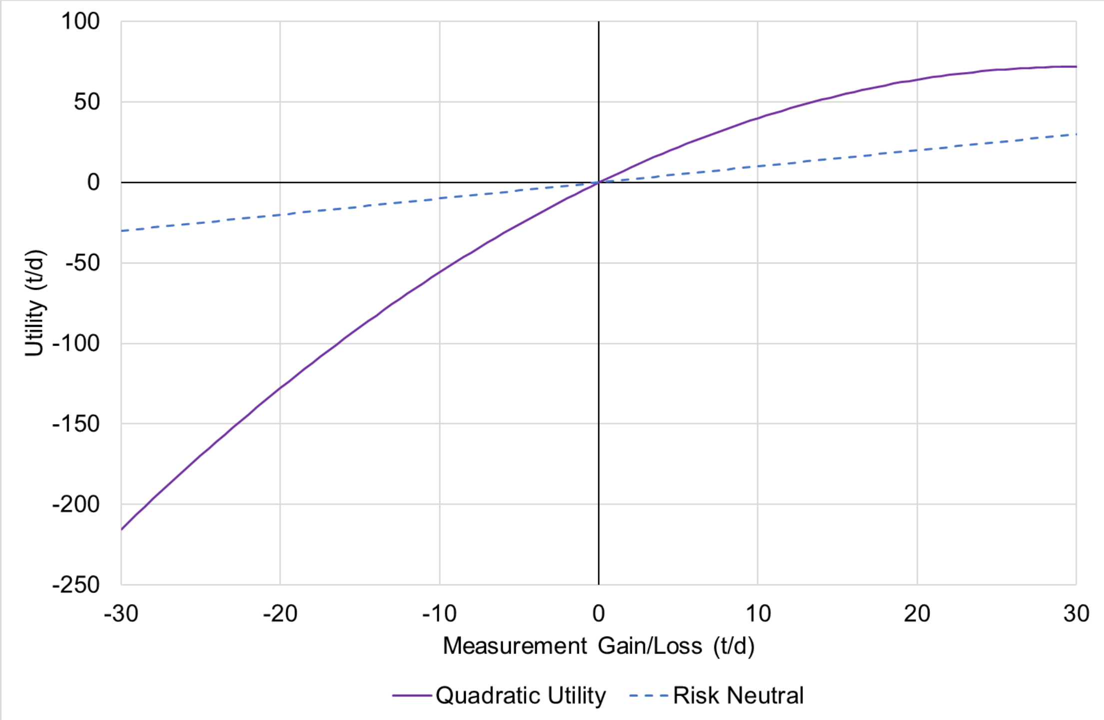
If losses are not simply equal and opposite to gains, how are they to be evaluated quantitatively? Or more generally, how are all the less tangibly quantifiable factors listed in the previous section to be incorporated into a cost benefit analysis?

As mentioned in Section 3.1, utility functions are an attempt to address these issues. Several utility functions have been proposed in the literature, for example:

- Quadratic
- Logarithmic
- Exponential

As an example, a quadratic utility function is presented in Figure 3-2:

Figure 3-2: Quadratic Utility Function



The quadratic utility function above does exhibit a non-zero positive utility for gains but the negative utility associated with losses are more significant. In fact, equation (4) can be derived using a quadratic utility.

Hence, the loss averse approach of the second postulate is one of a number of possible utility functions. Is there any supporting evidence that this loss averse function is a reasonable utility function and that it is widely applicable to agents in the real world?

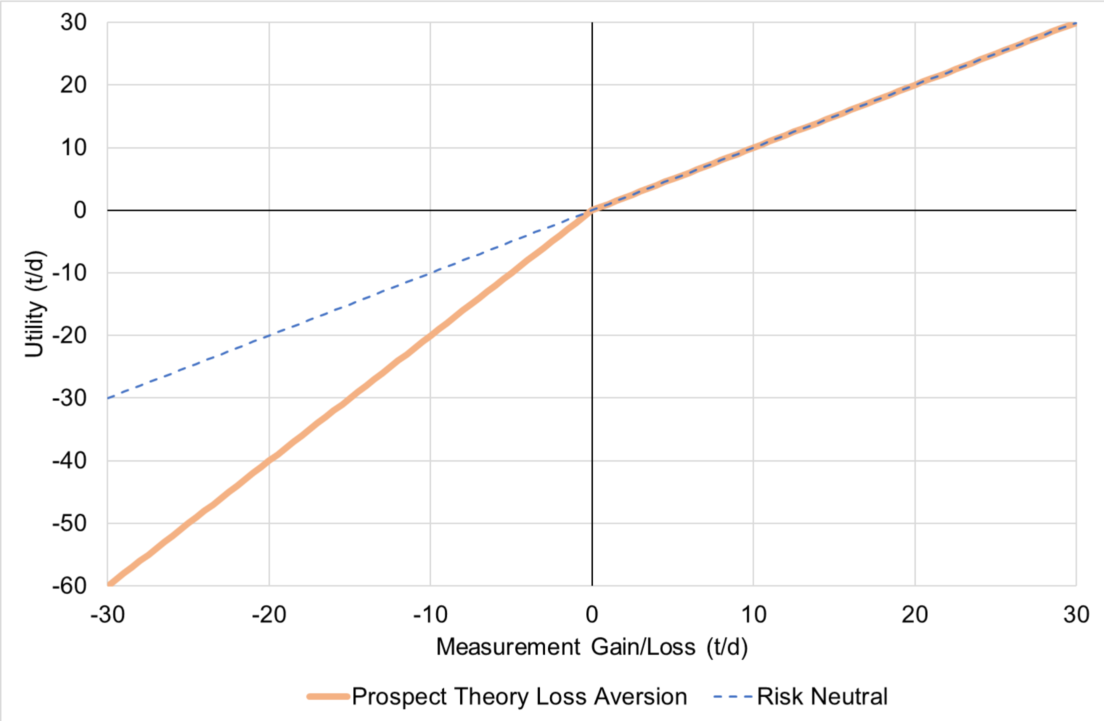
3.4 Prospect Theory

Prospect theory [10] is a development from expected utility theory and is based on the behavioural economics developed by Daniel Kahneman and Amos Tversky in 1979 (Daniel Kahneman won the Nobel Prize in Economics in 2002).

Loss aversion is one of three principles that underlies their development of Prospect Theory. They found experimentally that the loss aversion ratio of agents is in the range 1.5 to 2.5, an average of around 2. This means that agents value losses twice as much as gains.

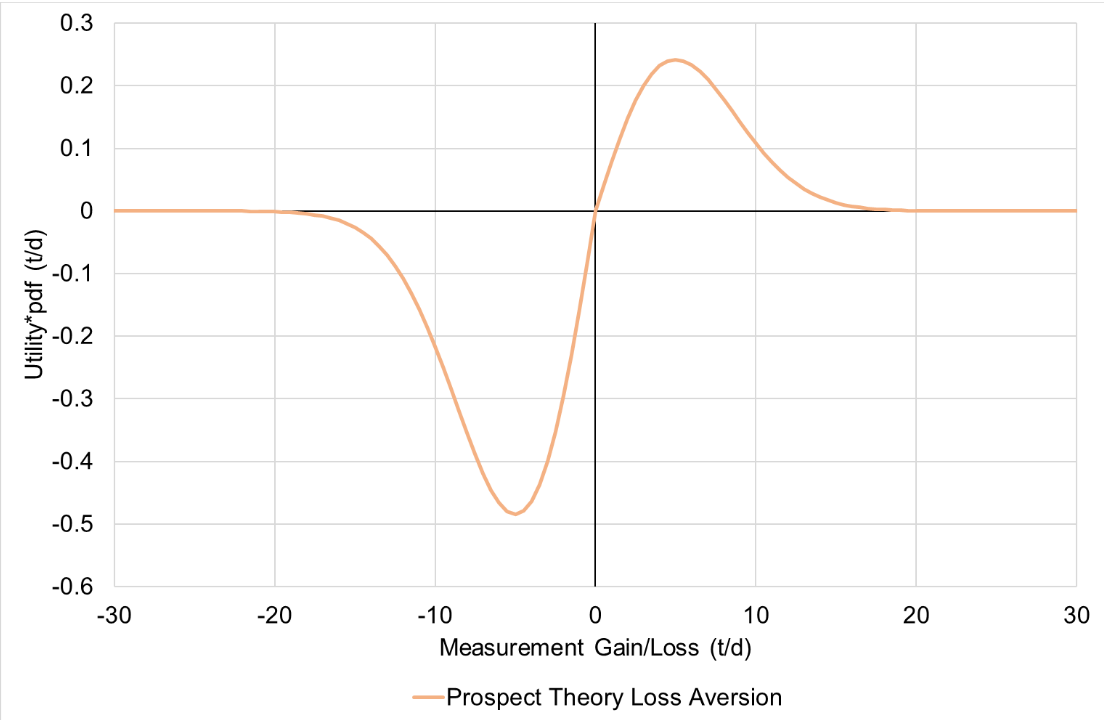
The utility function presented in Figure 3-1 can be modified to be consistent with this. Instead of the utility of a gain being zero, it can be assigned a value equal to the gain. However, the utility of losses now become twice the value of the loss which results in the following plot:

Figure 3-3: Prospect Theory Utility Function



If this utility function is inserted in equation (8) and a plot analogous to Figure 2-4 drawn:

Figure 3-4: Prospect Theory Utility Multiplied by Probability Density



and integrated from minus infinity to plus infinity, the result is identical with that of equation (11).

This provides valuable supporting evidence that the loss averse result is reflective of agents' behaviour in the real world.

4 NET PRESENT VALUE CALCULATIONS

4.1 Net Present Value

NORSOK 106 includes a term for the Net Present Value (NPV) in its presentation of the IRE Loss equation.

The NPV is calculated from:

$$NPV = \sum_{n=1}^{N} \frac{CF_n}{\left(1 + r_p\right)^n} - CM \tag{13}$$

Where:

NPV Net Present Value ($)

r_p Project discount rate

n Number of time period

N Total number of time periods

CF_n Cash flow in time period n

CM Cost of measurement device (CAPEX)

The NPV is based on discounted cash flow (DCF) and is the most commonly used tool for project evaluation [11].

4.2 Discounted Cash Flow

According to Brealey and Myers [12] in DCF calculations it is important to recognise that the cash flow is not the same as sales revenue. The cash flow is simply the difference between dollars received and dollars paid out.

Among other costs, tax needs to be deducted from the revenue. However, the depreciation in the measurement equipment costs can be offset against the tax liability.

Returning to the simplified example presented in Section 2.2, the undiscounted cash flow for one year is calculated:

Table 2 – Simple Example Yearly Cash Flows

		Meter A	Meter B
1	Sales revenue ($/y)	273,750,000	273,750,000
2	IRE loss of revenue ($/y)	139,299	557,196
3	Meter depreciation (over 5 years) ($/y)	200,000	50,000
4 (=1-2-3)	Pre-tax profit ($/y)	273,410,701	273,003,505
5	Tax at 40% ($/y)	109,364,280	109,257,122

6 (=4-5)	Profit after tax ($/y) = Cash Flow	164,246,421	163,935,682

The IRE loss has been subtracted from the revenue for each meter. To calculate the taxable profit the meter depreciation costs have been subtracted.

In the example, the tax rate has been assumed to be 40% for illustrative purposes and is not meant to be representative of any country. Similarly, the depreciation of the meter costs has been assumed to be straight line over the 5 year life of field.

The profit after tax is constant each year in this example but in the DCF calculation it is discounted each year in accordance with the project discount rate (r_p), assumed initially to be 10% (per annum):

Table 3 – Discounted Cash Flows, 10% Discount Rate

Year	Meter A	Meter B
1	149,314,928	149,032,438
2	135,740,843	135,484,035
3	123,400,767	123,167,305
4	112,182,515	111,970,277
5	101,984,105	101,791,161
Total DCF	622,623,158	621,445,215

The NPV of the two meters can now be calculated:

Table 4 – NPV Comparison, 10% Discount Rate

	Meter A	Meter B
CAPEX ($)	1,000,000	250,000
DCF ($)	622,623,158	621,445,215
NPV ($)	621,623,158	621,195,215

Meter A's NPV is greater than Meter B by $427,943 and therefore A should be selected.

The above calculations can be condensed into the equation:

$$NPV_A - NPV_B = \sum_{n=1}^{N} \frac{R_n(1-T)(\varepsilon_B - \varepsilon_A) + \frac{T(CM_A - CM_B)}{N}}{\sqrt{8\pi}\left(1+r_p\right)^n} + (CM_B - CM_A) \tag{14}$$

Where:

R_n	Sales revenue in year n
ε_A	Relative uncertainty, meter A
ε_B	Relative uncertainty, meter B
T	Tax rate
r_p	Project discount rate
CM_A	Capital cost, meter A
CM_B	Capital cost, meter B

The NPV calculations require a discount rate r_p. What is the correct discount rate and is it consistent with the development of the IRE Loss?

4.3 Discount Rate

The most widely used method to estimate the project discount rate is the Capital Asset Pricing Model (CAPM) [11] [12]. CAPM is itself based on Modern Portfolio Theory.

Investors in stocks and shares are assumed to be risk averse and require a higher return for more risky investments, relative to the market portfolio (i.e. a well-diversified portfolio).

CAPM maps projects to equivalent share returns. The project discount rate (r_p) is calculated from the following equation based on a similar security carrying the same risk as the project:

$$r_s = r_f + \beta_s(r_m - r_f) \tag{15}$$

Where:

r_s	Expected return of security s
r_f	Risk-free rate
r_m	Expected market return
β_s	beta of security s

The risk-free rate is the return from a safe asset such as government bonds. The market return is that expected from a well-diversified portfolio of stocks (market portfolio) and the difference between the two is the risk premium of the market. The returns from a market portfolio of stocks will be more volatile, especially over the short term, than those from government bonds. Hence, investors require a higher return from the market portfolio. For the case of an individual company, it may be more or less volatile than the market as a whole and this is what β accounts for.

Beta (β) is a statistical measure that compares the volatility of a stock against the volatility of the broader market, which is typically measured by a reference market index. In fact, it is the covariance of the stock with the market portfolio. Since the market is the benchmark, the market's beta is always 1. When a stock has a beta greater than 1, it means the stock is expected to increase by more than the market in up markets and decrease more than the market in down markets. Conversely, a stock with a beta lower than 1 is expected to rise less than the market when the market is moving up, but fall less than the market when the market is moving down.

Company betas or indeed wider sector betas, e.g. oil and gas sector, are quoted in financial reports and can be readily obtained. The oil and gas sector is generally well correlated with the wider market and hence its beta varies around 1.

Hence, equation (15) provides a level of return required by investors based on the volatility of the asset in relation to the market as whole. The volatility of stocks is measured by their variance or standard deviation.

The company carrying out the project is assumed to be a publicly traded one in which investors can buy stocks. The market will determine the price of the stocks in the company, which in turn is influenced by the expected return on those stocks. The stock price also reflects the company's cost of raising capital for the project.

If the project is typical of the company's usual activities, then the required project return r_p can be assumed to be equal to r_s calculated from equation (15) using the company's beta. If the project is more or less risky, then the required return is adjusted up or down by finding an analogue company who performs such projects and the beta changed accordingly. Hence, equation (15) provides a mechanism to calculate r_p for any project based on its riskiness. The idea is that more risky projects should provide a greater level of return.

However, investment in measurement equipment is motivated by different aims than the general motivation for the project as a whole, which is to generate profit from sales revenue of the product. Higher quality of the measurement equipment does not increase profits, but it does reduce uncertainty / risk / volatility in the revenues.

What is the correct beta for a measurement device? A common approach is to discount the revenue whose risk is specific to the project at the risk-free discount rate [11]. This is because the beta of the risk due to the measurement itself is zero as it is uncorrelated with the market volatility and indeed the project return volatility. The additional risks introduced by measurement equipment are unrelated to the riskiness of project itself.

The 10% discount rate assumed in the example in Section 4.2 is therefore not appropriate. The risk-free rate is more typically in the range 0% to 5%, hence assuming 3%, the NPV of the two meters is revised to:

Table 5 – NPV Comparison, 3% Discount Rate

	Meter A	Meter B
CAPEX ($)	1,000,000	250,000
DCF ($)	752,200,513	750,777,422
NPV ($)	751,200,513	750,527,422

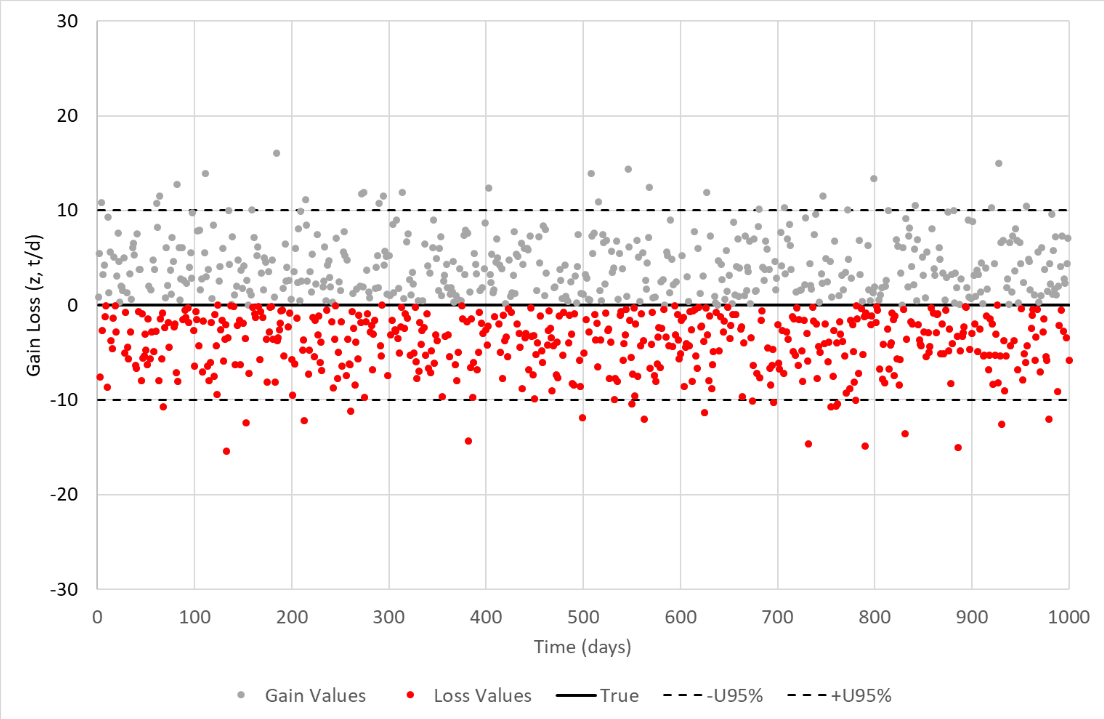
The case for Meter A is now more compelling as the difference in NPV has increased to $673,090.

5 MEASUREMENT ERROR OVER TIME AND SYSTEMS

5.1 Random Errors

If the measurement gains/losses (z) are randomly distributed about the true value in accordance with the normal distribution and the gains/losses at each time step are independent of each other (this is additive white Gaussian noise), a plot of the form in Figure 5-1 is obtained for the meter errors through time:

Figure 5-1: Random Gains/Losses Through Time



The above plot is based on the example presented in Section 2.2.

The randomly fluctuating gains and losses around the true value are colour coded: grey for gains, red for losses. If the red loss values are summed over the 1,000 days (in effect the gains are counted as zero in loss terms), the resultant summed loss is 2,165 t or 2.165 t/d on average. According to equation (3), the summed loss is predicted to be 2,035 t or 2.035 t/d on average. The plot above is only one instance of a randomly generated set of 1,000 data points. If the time is extended to 10,000 days or data points, the sum is 20,442 t or 2.044 t/d and if extended to a million data points, the sum is 2,037,145 t or 2.037 t/d. As the amount of data increases the accumulated loss tends to that predicted by equation (3).

This confirms numerically that the calculated loss exposure values can simply be summed over time, as was assumed in the simplified example in Table 1.

5.2 Components of Measurement Error

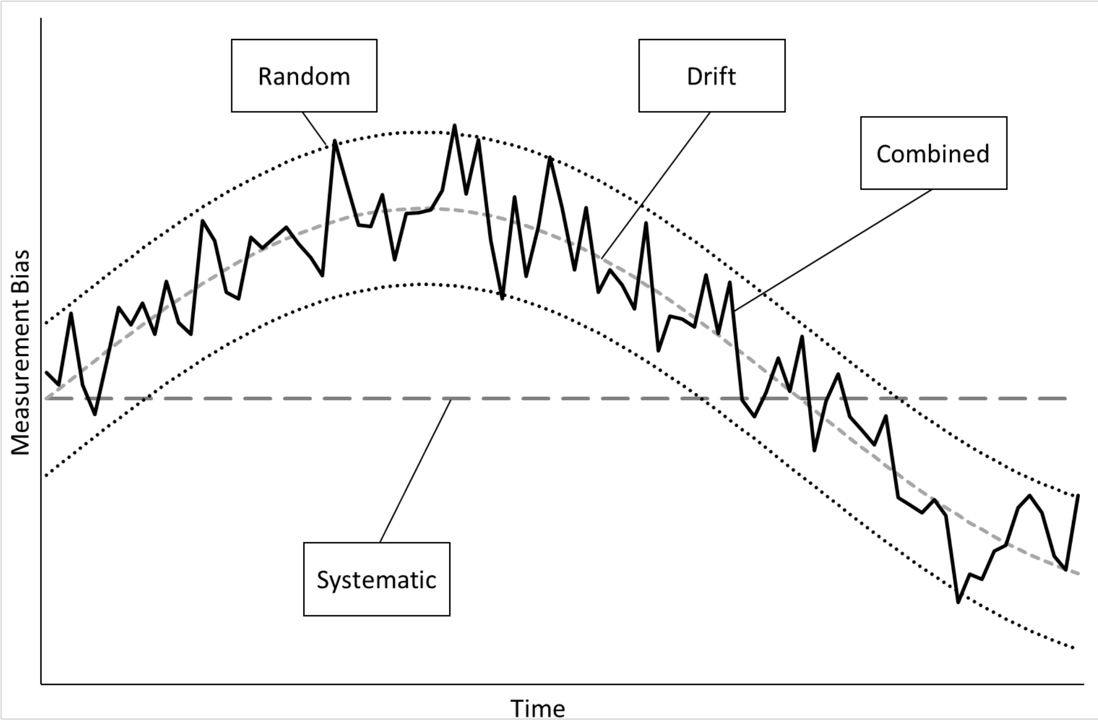
The above plot assumes random fluctuations from day to day. Do the same fluctuations happen from hour to hour or second to second and do real measurement devices behave this way?

Pashnina N, 2016 [4] discusses various components of measurement error:

- Random measurement error; due to process noise, thermal effects, etc.
- Systematic measurement error; remains relatively constant through time but is typically unknown, an example would be the orifice discharge coefficient calculated in accordance with ISO 5167-2 [13].
- Drift error; incremental change over time due to instrument ageing effects.

These three components are illustrated in Figure 5-2:

Figure 5-2: Components of Measurement Error Through Time

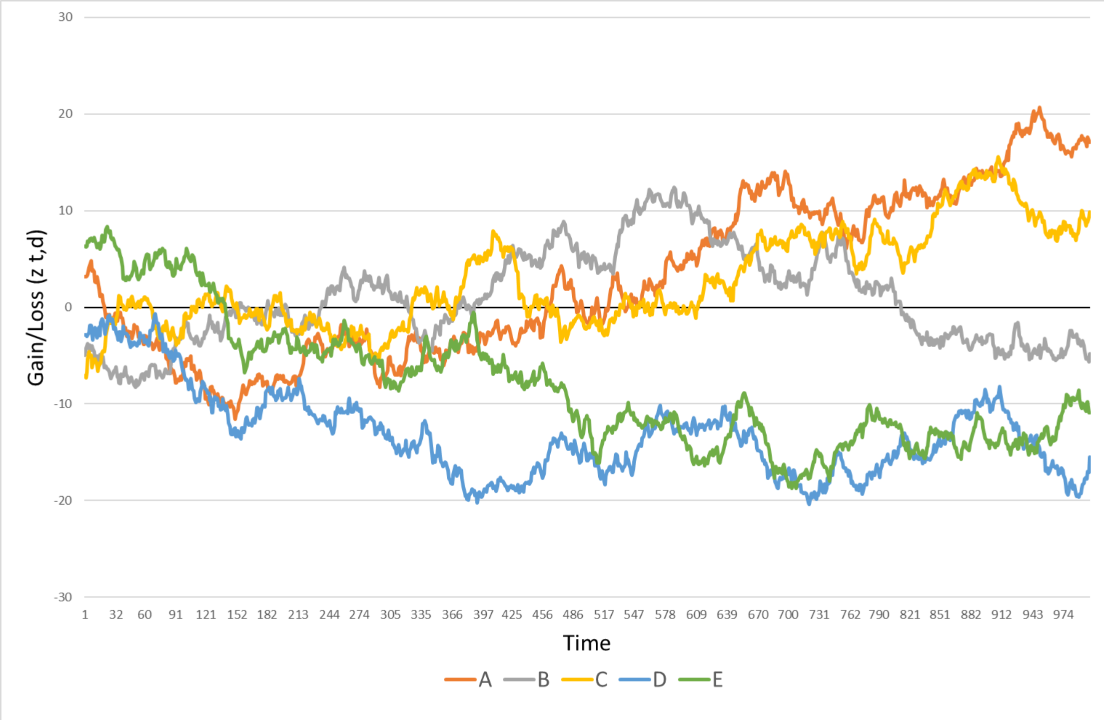


5.3 Error Evolution Through Time

The growth of measurement errors and hence uncertainty with time is poorly understood according to Sydenham [9]. NASA Reference Publication 1342 [14] describes eight models for uncertainty growth time series, including two random walk models. Gelb et al [15] in Applied Optimal Estimation also considered random walk, random ramp and exponentially correlated random variables as models for measurement uncertainty.

The random walk processes are specifically highlighted as they present an interesting and plausible mechanism for the evolution of random errors from one moment to the next. A simple discrete random walk process simulating measurement error is presented in Figure 5-3 for 5 systems labelled A to E:

Figure 5-3: Random Walk Error Through Time Multiple Systems



The size of the change in z (Δz), from one time step to the next is normally distributed.

What is apparent with this random walk model is that the variance is a function of time and hence the standard deviation increases with the square root of time. If uncalibrated the uncertainty in the measurement will continue to grow.

What is also apparent, is that the gains and losses over time across an individual trajectory do not even themselves out as was observed with the white noise model of Figure 5-1.

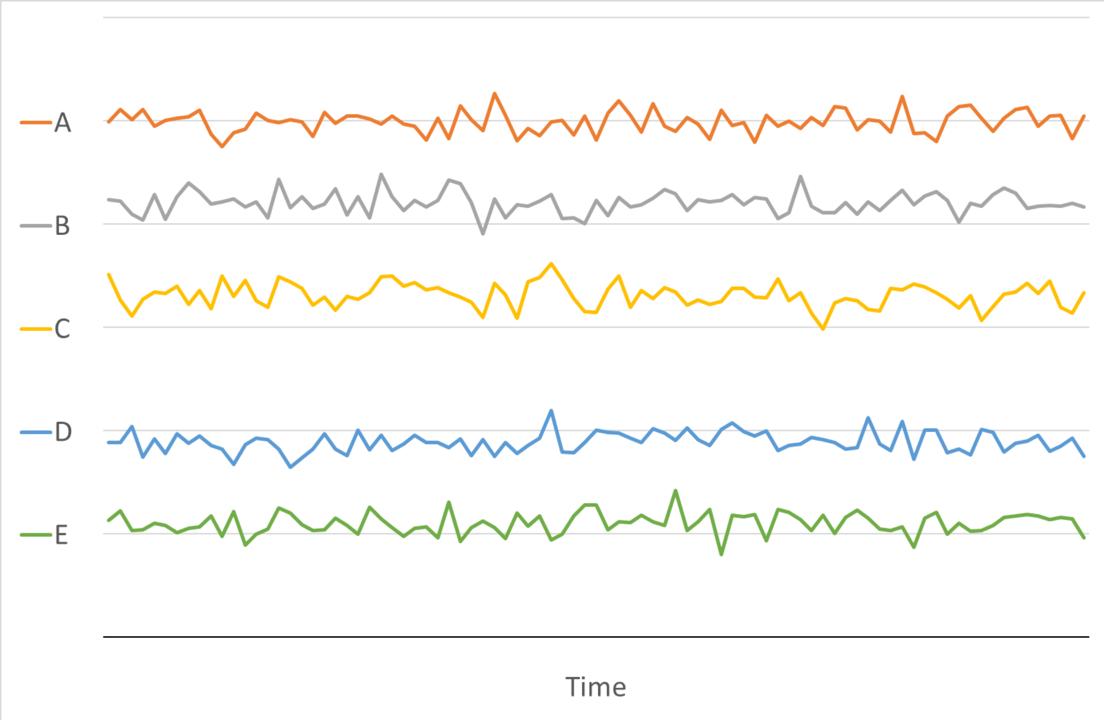
The averaging over time was presented as a possible justification for the adoption of the risk neutral approach described in Section 3.2 but this does not appear to be plausible if the random walk model of measurement error is at least in part true. Though the magnitude of the cumulative gain or loss over a period of operation will be random, it will not be zero. The distribution of the magnitude is Gaussian and increases with time. Hence, the assertion that gains and losses will average out over time is unlikely to be true and may be significant simply due to random effects.

The potential consequences of this are discussed further in Section 6.

5.4 Diversification Across Systems

The averaging of risk across multiple systems was also presented as a possible justification for the adoption of the risk neutral approach. It is theoretically plausible to mitigate the risks of persistent errors due to systematic and drift components owning multiple installations. This is illustrated in Figure 5-4:

Figure 5-4: Systematic Errors Across Multiple Systems



The true values are indicated by the horizontal grey lines and the measurement in parallel installations A to E are shown fluctuating randomly round some systematic offset. If the agent had a portfolio of such systems, the risks could be diversified away.

Assuming the agent has sufficient installations to reduce the exposure of such systematic and drift uncertainties, so the errors even themselves out across facilities is unlikely to be practical. Small companies may only have one or part ownership in a single or small number of facilities and hence not be able to diversify away the risks across many installations. Even large multinationals are unlikely to have so many installations that they can confidently diversify the risks of poor measurement away.

6 ERGODICITY

6.1 An Inconsistency

The original IRE Loss equation was derived based on fixed values of measurement uncertainties. The preceding section has just discussed how measurement errors and uncertainties of real systems grow with time.

It may be argued that this is mitigated by the regular calibration of measurement devices. However, not all measurement devices associated with allocation systems are subjected to the same rigorous calibration regime demanded of custody transfer meters.

Hence, there appears to be an inconsistency between plausible models of the time dynamics of measurement error growth and the notion of a fixed uncertainty.

6.2 Subjective Utility?

The discussion in the previous sections has encompassed uncertainty and risk from both a measurement error perspective and in economic terms with regard to calculating an appropriate discount rate.

The introduction of a utility function was used to quantify risk and loss aversion in the integrated risk exposure calculations. Utility is also the basis of the CAPM and hence discount rate for NPV calculations.

Though the loss averse utility function postulated in Section 2.4, was justified in Section 3 based on behavioural economics, and was required to develop the IRE loss, associated cost benefit and NPV calculations, at some level it appears subjective and incongruous with the rest of the analysis. In fact, there are multiple utility functions available of which the parameters can be adjusted to reflect different attitudes to risk.

6.3 Ergodicity Economics

Ole Peters of the London Mathematical Laboratory and Murray Gell-Mann[1] of the Sante Fe Institute published a paper in 2016 titled: "Evaluating gambles using dynamics" [16], which was the most downloaded article from the journal that year. This is one of series of papers generated from an ongoing area of active research [17].

Notably, they challenged the necessity for subjective utility functions.

The aim of ergodicity economics is to carry out a fundamental re-evaluation of the basis of more traditional economic theory.

In an ergodic scenario, the average outcome of a group is the same as the average outcome of the individual over time. This is not typically true for individual agents who are faced with a one-shot decision, which they invest in across a period of time.

By reframing in terms of the growth dynamics for an agent across time rather than across systems, some interesting results have been obtained. The dynamics of wealth growth can be modelled in terms of random walks. Depending on the precise dynamics they demonstrated that the equivalent of logarithmic utility is a rational approach in terms of optimising wealth growth, thus eliminating the subjectivity of the utility function.

6.4 Application to Measurement Systems

In the context of measurements, if ergodic, the uncertainty across a set of measurement devices would be identical to the uncertainty experienced by one measurement device through time.

This is evidently not true of measurement devices:

- Systematic biases vary randomly across devices but are fixed through time for an individual meter.
- Similarly the rate of drift across devices may be randomly distributed but the individual meter will experience a systematically increasing bias.

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[1] Murray Gell-Mann won the 1969 physics Nobel Prize for his work on elementary particles, which introduced the concept of the quark.

- If errors develop through time as a random walk the uncertainty grows as a function of the time, but the uncertainty does not increase across sytems.

The parallels of the factors affecting cost benefit analysis for measurement systems and those associated with ergodicity economics appear apparent. The purpose of this section is not to discredit the existing IRE Loss equation but to highlight the possibility of developing the cost benefit analysis of measurement devices using the methods of ergodicity economics into a more complete and robust methodology.

Ergodicity economics provides a potential way forward, eliminating the need for a subjective utility theory and reflecting actual error growth models.

7 CONCLUSIONS

The IRE Loss equation can be developed based on two postulates: that measurement errors are normally distributed, and agents have a loss averse utility function.

The approach was further justified on the basis of results from behavioural economics which showed experimentally that agents value losses twice as much as gains.

Expressing the IRE Loss equation in terms of measurement uncertainty at the 95% confidence level is merely a matter of convenience. More fundamentally it is expressed in terms of the standard uncertainty or standard deviation of the measurement errors.

A commonly adopted alternative to the IRE Loss equation in cost benefit analyses is to use the 95% confidence uncertainty as the exposure to loss. This has been demonstrated only to be applicable if the uncertainty follows a uniform rectangular distribution.

NPV calculations need to correctly account for tax and the depreciation of the measurement device. Since, the uncertainty introduced by measurement devices is uncorrelated with the wider risks associated with the revenue generation from the project, the correct discount rate in the NPV calculations should be the risk-free rate, typically that of government bonds.

The assumption that the gains and losses due to randomly distributed measurement errors will average out towards zero over time for a meter have been shown to be implausible.

Recent developments in ergodicity economics potentially offer the possibility of further developing the cost benefit analysis method into a more complete and robust methodology.

8 NOTATION

C_d	Discharge coefficient
CM	Cost of meter
CM_A	Capital cost, meter A
CM_B	Capital cost, meter B
DCF	Discounted cash flow
k	Coverage factor
L	Integrated risked exposure to loss
n	Time step number

N	Total number of time steps
NPV	Net present value
pdf	Probability density function
R	Integrated risked exposure to gain/loss (z)
R_f	Sales revenue, year n
r_f	Risk free interest rate
r_m	Expected market return
r_p	Project discount rate
r_s	Expected return on security s
T	Tax rate
U	Absolute uncertainty
x	Measured flow rate (t/d)
z	Gain/loss
β_s	Covariance of security s with market
Δz	Change in z between time steps
ε_A	Relative uncertainty meter A
ε_B	Relative uncertainty meter B
μ	True flow rate (t/d)
σ	Standard deviation

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